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Analytical model of current sheath path based upon shock wave trajectory of plasma layer in a plasma focus device

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1. Introduction

In plasma focus generators the magnetic energy is stored behind the moving current sheath (Mather,1971). A portion of this energy is converted into plasma energy during the rapid collapse of the current sheath towards the axis beyond the end of the central electrode. Electrical breakdown generates some initial plasma configuration through which the discharge current can flow and at very low pressure a discharge can develop within the whole inter-electrode volume. The current sheath formed at the end of the breakdown phase is accelerated by Lorentz force towards the open end of the inner electrode and then the current sheath sweeps around the end of the anode electrode and finally implodes due to the inward $J \times B$ force. When the current sheath reaches the end of the central electrode, it reverses over itself and collapses radially inward, heating the pinching plasma enclosed in it (Mather,1965). The radial compression of CS is open at one end. Hence a gas dynamic shock is propagated ahead of the CS into the undisturbed filling gas (Soto,2005). The snowplow model is used for axial acceleration of CS to obtain axial trajectory, CS speed and current profile. As the CS is assumed to be infinitesimally thin, no information of density is contained in the physics of the equation of motion, although an estimate of density may be obtained by invoking shock wave theory (Lee et al,1988). For a given density and temperature, plasma equilibrium models can be used to calculate the plasma states and emission spectra with the knowledge of the rates of transition and related parameters. The collisional radiative model was developed to fill the gap of several orders of magnitude in electron density where neither the LTE model nor the corona equilibrium is valid. This is a modification of the corona model which takes into account collisional transitions as well as radiative decay from higher bound levels, and three body as well as radiative recombination. The main difficulty with this model is its complication, need of computer time, etc. In the calculations the corona model has been applied as an approximation for simulating the argon plasma in the plasma focus. This could give sufficiently accurate simulation results (Bates et al,1962 & Yanagidaira,1999). A scheme of a plasma focus system and also the configuration of the 2D cylindrical geometry shock wave at the radial phase of plasma layer are shown in Figure 1. We have developed a new model

to predict accurately the CS trajectory before the dense plasma pinch formation driven by magnetic force. The path of CS and shock wave are correlated together and if we know the path of the shock front at the radial compression of CS, we can simulate the trajectory of imploding CS. This result can be used for the experimental design of plasma focus and also for stabilizing the hydrodynamic instabilities that affect the implosion of current sheath and the final pinch uniformity. As we will show, the results are in a good agreement with the simulation results of shock wave and CS trajectories which is obtained by the three phase theory of plasma focus performance (Mathuthu et al,1997).

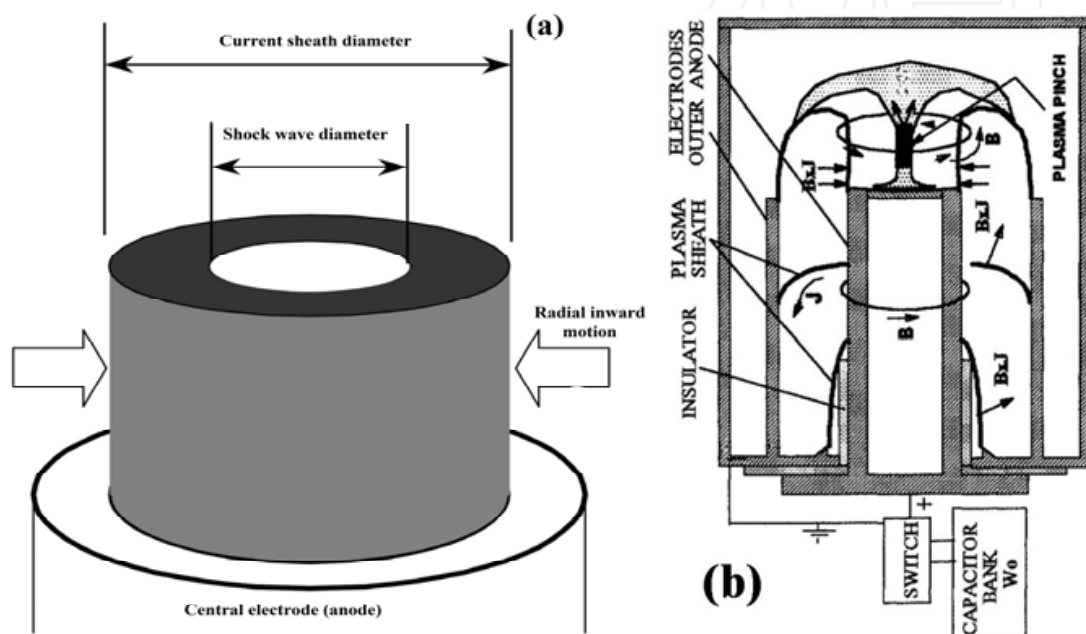


Fig. 1. (a) - conceptual drawing of plasma focus, (b)- configuration of the 2D cylindrical geometry shock wave

2. Analytical model

In the plasma focus model a radially implosive plasma slug is formed above the anode in the radial compression of CS (Lindemuth,1982). As it is simplified in Figure 2, plasma slug is formed and compresses radially inward until the shock front meet at the center axis. This slug is driven by the radial inward magnetic piston and plasma column continue compress to a narrow column region and results in a hot and dense plasma. Due to plasma focus geometry and plasma layer dynamic, the motion of the plasma slug can be described by the cylindrical geometry 2D shock wave equations. Conceptual design of the device performed precisely so that the plasma layer moves isentropic between two electrodes. Therefore we can consider an ideal cylindrical magnetic piston of dense plasma produced by electrical discharge between the electrodes of a plasma focus system. At the end of the axial rundown phase, the plasma will form a column at the axis and finally collapse and the period of the radial phase is approximate 50 ~ 200 ns, depending on the plasma focus machine characteristics. Because of such short living plasma we can suppose that the radius of this magnetic piston decreases so rapidly that a strong shock is driven in front of the wall toward the axis of the cylinder.

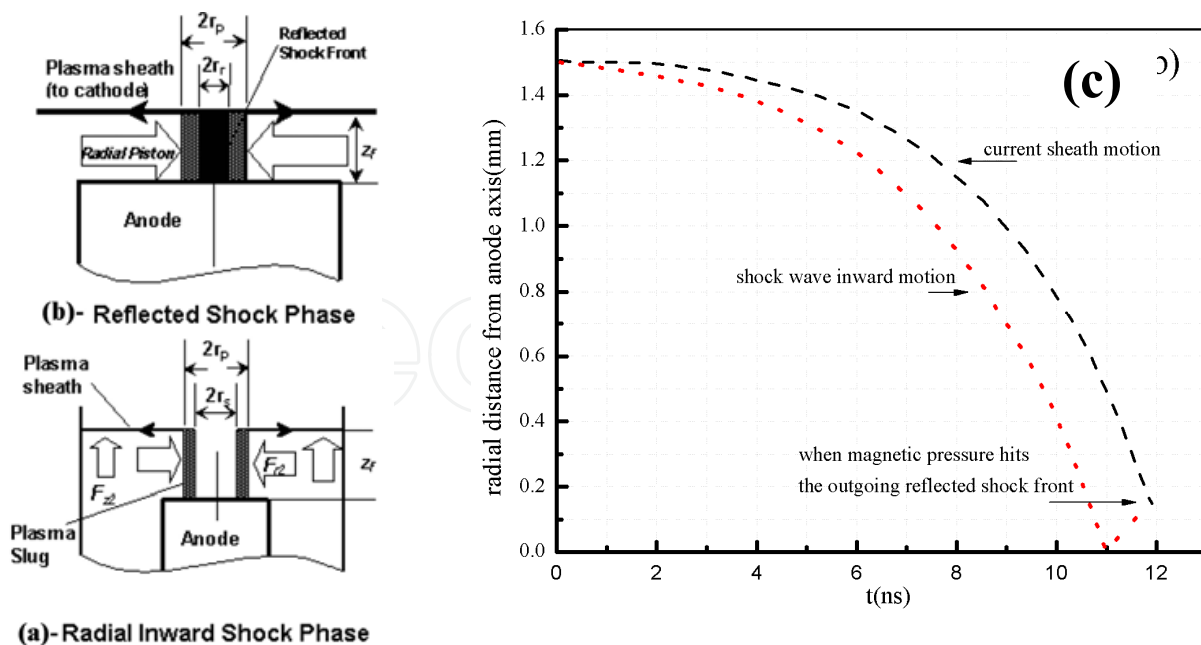


Fig. 2 .a) radial inward shock phase b) reflected shock phase of plasma focus, and c) formation of shock wave driven by CS inward motion

Due to high isentropic behavior of plasma slug we suppose that two particles that are located at different radii in the cylindrical CS, their respective radii will always be the same such that the particle initially closer to the axis will always be closer to the axis. This immediately leads to a law for conservation of mass and a method for labeling each particle. In Figure 3 the path of current sheath $CS(t)$, the path of shock wave $S(t)$, and the path of any particle $P(\xi, t)$ are plotted.

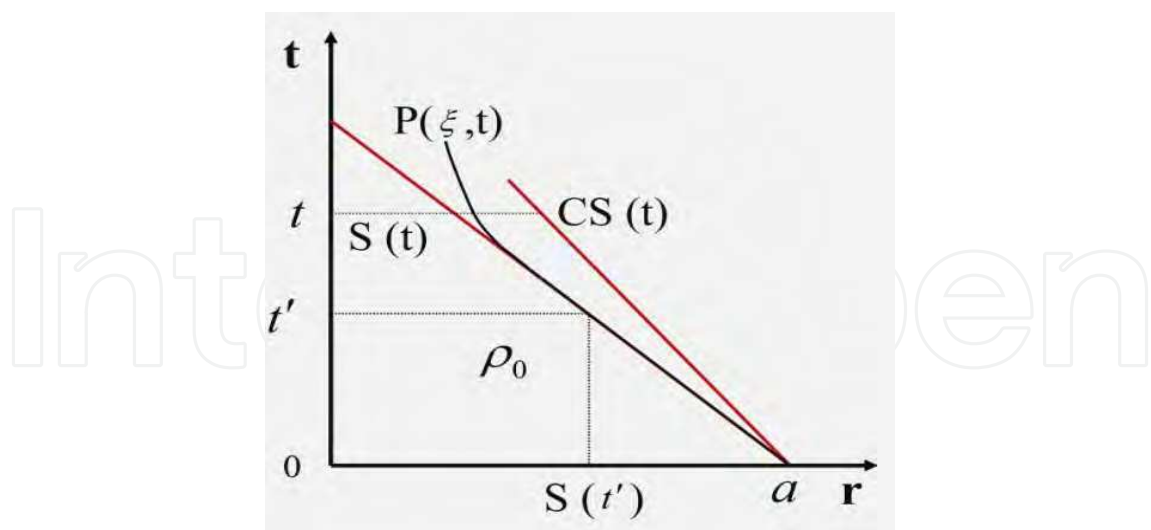


Fig. 3. the path of current sheath $CS(t)$, the path of shock wave $S(t)$, and the path of any particle $P(\xi, t)$

ξ corresponds to the mass between the CS and considered particle and t' refers to the time when the shock passes over the particle. Thus t' may be regarded as a function of ξ and

according to figures 1, and 2, we define $\xi = \pi\rho_0(a^2 - S(t')^2)$ in which ρ_0 and a are undisturbed gas density and radius of the central electrode respectively (Chernyi,1956 ,

Freeman,1956). The momentum equation is seen to be $\rho \frac{\partial^2 r}{\partial t^2} = -\frac{\partial P}{\partial r}$ where P is the

pressure acting on the particle to accelerate it. The process is assumed to be entirely isentropic except for a jump in entropy as the shock crosses the particle's pass. Therefore the ratio $\frac{P}{\rho^\gamma}$ is a constant for each particle as it travels from the shock toward the axis of cylinder. This constant is given by the conditions immediately after the shock. γ is the ratio of specific heats(for example $\gamma = 1.667$ for Ar as filling gas). The pressure and density of a

particle immediately after the shock (P_i and ρ_i) can be found by using the shock relations in conjunction with the perfect gas law as $P_i(t') = \rho_0(1 - \varepsilon)\dot{S}^2(t')$ and $\rho_i = \rho_0/\varepsilon$ in which $\varepsilon = \frac{\gamma - 1}{\gamma + 1}$ and $\dot{S}(t')$ is the velocity of the shock as it crosses the particle $\zeta(t')$. Thus the

isentropic condition is $\frac{P}{\rho^\gamma} = \rho_0^{1-\gamma} \varepsilon^\gamma (1 - \varepsilon) \dot{S}^2(t')$. The conservation relation will now be combined into one relationship. At any later time, ζ may be found by taking the integral

$\xi = 2\pi \int_{CS(t)}^{P(\zeta,t)} \rho(r,t) r dr$ $\xi = \pi\rho_0(a^2 - S(t')^2)$ in which $\rho(r,t)$ is the density at any point on the

(r,t) plane. Thus we can conclude that $P^2(\zeta,t) = S^2(t) + \frac{1}{\pi} \int_{\zeta}^{\zeta_s} \rho_0^{1-\gamma} (1 - \varepsilon) \varepsilon^\gamma \dot{S}^2(t') \}^{\frac{1}{\gamma}} P^{\frac{1}{\gamma}} d\zeta$

where ζ_s is the value of ζ at the shock at any time t . From the equation of momentum

we find that $\frac{\partial P}{\partial r} dr = \frac{1}{2\pi r} \frac{\partial^2 r}{\partial t^2} d\zeta$. Upon integration from the shock to any particle this

becomes $P(\zeta,t) = P_i(t) + \frac{1}{2\pi} \int_{\zeta_s(t)}^{\zeta} \frac{\partial^2 r(\zeta,t)}{\partial t^2} \frac{d\zeta}{r(\zeta,t)}$. Let us putting this information back into

equation of $P^2(\zeta,t)$ and nondimensionalize the quantities appearing in the equation as

$x = \frac{r}{a}, x_s = \frac{S}{a}, Z = \frac{\zeta}{\pi a^2}, \tau = \frac{t}{t_0}$. t_0 is chosen so that τ is in μs . Due to Schlieren images and

visible-radiation pictures taken with a high-speed camera before and after the maximum compression of plasma layer most interesting cases may be covered by assuming a parabolic shock trajectory as $x_s = 1 - \alpha\tau - \beta\tau^2$ in which a is nondimensional velocity of the shock and β is the shock's constant acceleration or deceleration toward the axis depending upon β is positive or negative (Sadowski & Sholz, 2008). Substituting this information into

equation of $P^2(\zeta,t)$, we find $x^2(Z,\tau) = x_s^2(\tau) + \varepsilon \int_Z^{Z_s=1-x_s^2(\tau)} \left[\frac{\alpha^2 + 4\beta(1 - \sqrt{1-Z})}{(\alpha + 2\beta)^2} \frac{1}{2(1-\varepsilon)} \int_Z^{x_s^2} \frac{x^2 dx}{\tau^2 x(Z,\tau)} dZ \right]^{\frac{1}{\gamma}} dZ$. this

equation implies that a particle Z is, at a time τ , at a position away from the shock by a distance equal to ϵ multiplied by an integral, the integrand of which consists of a numerator representing the isentropic condition and a denominator which is a constant fraction of the pressure. ϵ varies from 0 for $\gamma = 1$ to 0.25 for $\gamma = 1.667$. The piston trajectory is $x(0, t)$, that is the piston is always at the particle $Z = 0$. Since the right hand side of above equation involves a second derivative of the desired solution, a method of iteration must be used. Due to the difficulty in taking derivatives numerically we should search for an analytic solution that may be placed back into the equation. From this analytic a second approximation may be found numerically and compared to the first solution. As a first approximation, it may be assumed that the pressure doesn't change much between the

shock wave and CS. In effect we assume $\frac{1}{2(1-\epsilon)} \int_z^{z_s=1-x_s^2} \frac{\partial^2 x}{\partial \tau^2} \frac{dZ}{x} \ll (\alpha + 2\beta\tau)^2$. This relation

is good when two conditions are met. First we must not be too close to the axis. Otherwise x will be small. Secondly the second derivative of x with respect to τ should be small and the denominator of the integrand of the large integral may approach zero. These conditions are met at least in the beginning stage of piston's propagation. With this assumption, we

conclude that $x^2(Z, \tau) = x_s^2(\tau) + \frac{\epsilon}{(\alpha + 2\beta\tau)^{2/\gamma}} \int_z^{1-x_s^2(\tau)} (\alpha + 4\beta - 4\beta\sqrt{1-Z})^{\frac{1}{\gamma}} dZ$ which readily

integrates to a first approximation for x . The equation of CS path for the first approximation becomes $x_{first}^2 = x_s^2 + \epsilon(1 - x_s^2) - \epsilon Z$. In order to find the second approximation, we must substitute the second derivative of $x_{first}(Z, \tau)$ with respect to τ into the subintegral of

$x^2(Z, \tau)$ equation. Therefore we have $x_{second}^2 = x_s^2 + \epsilon \int_z^{1-x_s^2} \left(\frac{2\epsilon}{\Omega - \ln(\Gamma - Z) - \frac{\Sigma}{\Gamma - Z}} \right)^{\frac{1}{\gamma}} dz$ in which

$\Omega = 1 + \epsilon + \ln \frac{x_s^2}{\epsilon}, \Sigma = \frac{(1-\epsilon)x_s^2}{\epsilon}$ and $\Omega = 1 + \epsilon + \ln \frac{x_s^2}{\epsilon}, \Sigma = \frac{(1-\epsilon)x_s^2}{\epsilon}$ $\Gamma = \Sigma + 1$ (Drake, 2005 & Lister, 1960).

3. Simulation results and discussion

We invoked a program to perform a numerical integration of nondimensional equation obtained for $x^2(Z, \tau)$ to simulate path of CS respect to a parabolic strong shock wave trajectory [7]. The program was run for $A = 0.999$ and $B = 0.001$ and was also run for $A = 1$ and $B = 0$. The general philosophy of the program is first to determine what time steps to use, and then when the time steps are known, to calculate all quantities that are dependent only upon time for the first $\Delta\tau$. Then at the time under consideration the quantities that depend on Z are calculated. Specifically the value of the integrand for $Z = Z_s$ is first calculated. Then the values of Z for which we want the particle positions spelled out are determined. The program does this by taking Z_s' rounding it off to the next

lowest 0.05 and then using in steps of 0.05, the steps for which the two integrations are performed. The subintegral is found for the interval from Z_s to the next lower Z . With this value of the subintegral, the total integral may be found giving x for the rounded off Z and the time $\Delta\tau$. Using the next lower value of Z , the next portion of the subintegral is added to the value obtained above. Similarly the next portion of entire integral is added on to the part already found. This process is carried on until we reach the value $Z = 0$ which is the piston. At this time we proceed to the next time and repeat the entire process.

Figure 4 shows a constant velocity shock ($\beta = 0$) with the CS path computed for $\gamma = 1.667$ and $\gamma = 1.1$.

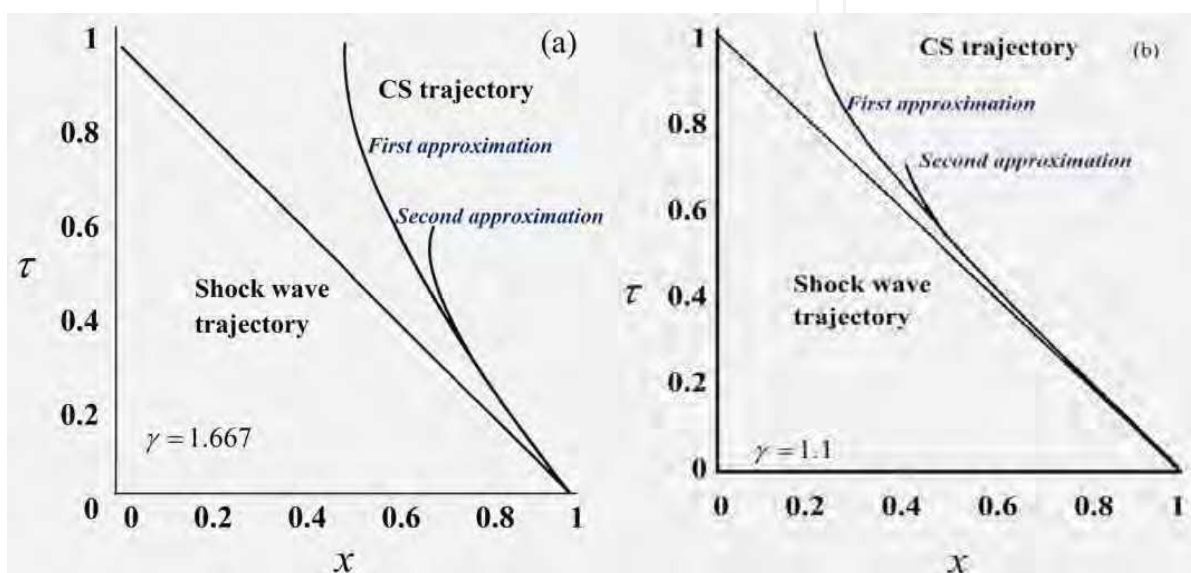


Fig. 4. CS trajectory for constant velocity shock

In Figure 5, CS trajectory for accelerating shock and in Figure 6, CS trajectory for decelerating shock ($\gamma = 1.667$, and 1.1) simulated.

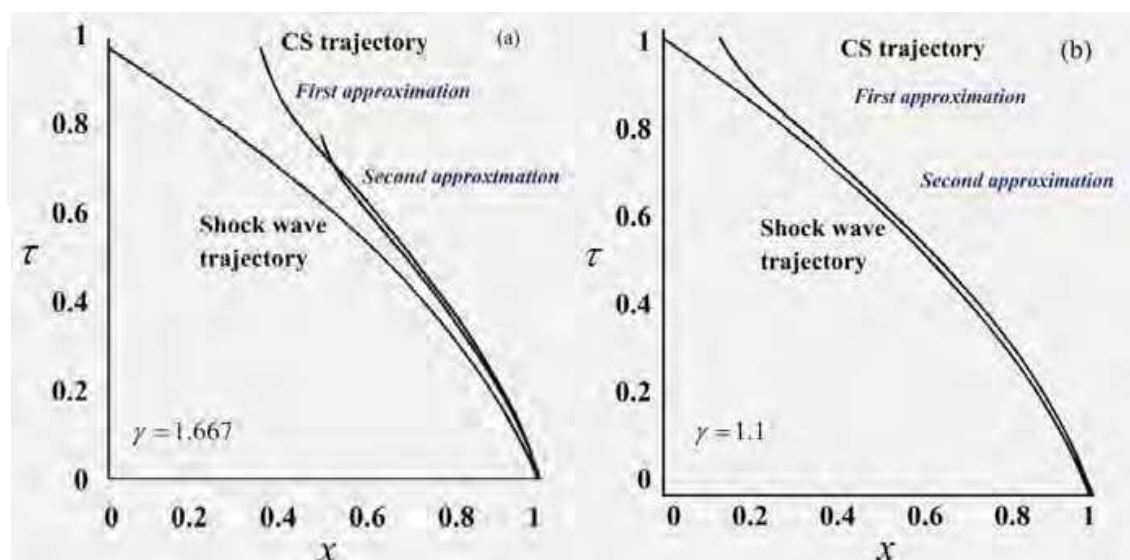


Fig. 5. CS trajectory for accelerating velocity shock

In the pinch phase of focused plasma much of the energy available is absorbed in the ionization process (Shan et al,2000). Here the real value of γ when argon is used as working gas may be expected would be closer to 1.1 than to 1.667 . As it shown in Figure 4, for $\gamma = 1.667$ it is seen that the first and second approximations for the CS trajectory are very close together until the CS reaches a radial position 0.75 . At this point the second approximation diverges from the first approximation and ultimately turns back toward its initial position. Physically a decrease in the denominator corresponds to a decrease in pressure at the CS. It is logical the pressure decrease from the shock to the CS at a given time because in this region of the flow, there is quasi-steady supersonic flow into a converging channel which implies a decrease in velocity and a corresponding adverse pressure gradient [12]. Since the conditions behind the shock are fixed, the pressure at the CS must be steadily decreasing as the gap between the shock and CS widens. As we see in Figure 5, for $\gamma = 1.1$ there is no difference large enough to be seen between the first and second approximations until the second approximation reaches the zero pressure limit. This occurs at $x(0,\tau) = 0.27$ that is much smaller than the final radius of the CS for the constant shock. This fact implies that the CS pushing an accelerating shock has control over the shock for a longer time than the CS pushing a constant velocity shock. For $\gamma = 1.667$, the accelerating shock has a piston path given by the second approximation that is closer to the center than the first approximation. That is, in order to accelerate the flow, the pressure at the piston must be greater than the pressure at the shock. For the first approximation, this pressure difference is neglected. In the second approximation it is included. This effect is also present for the $\gamma = 1.1$ case; however, it is so small that it cannot be seen on the scale of Figure 5. Figure 6 shows the case of a decelerating shock. There is little new on this graph except that the piston turns back even sooner than it does for the constant velocity shock.

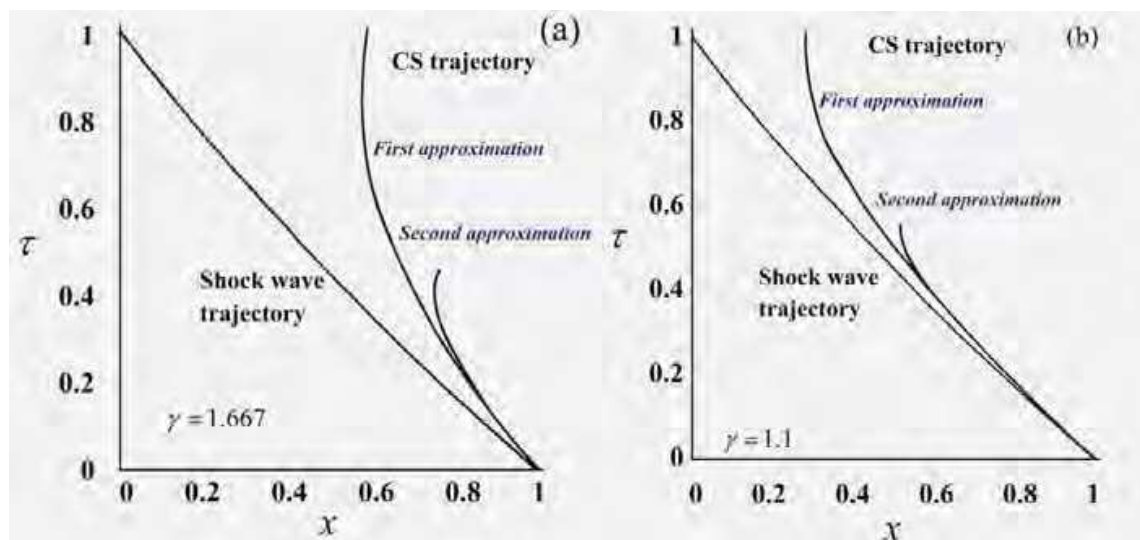


Fig. 6. CS trajectory for decelerating velocity shock

To compare the results with the three phase theory of plasma focus performance We solved the axial and radial phases' current sheath equations coupled with equivalent plasma focus

circuit equations (Habibi et al,2008). In Figure 7, shock wave and CS trajectories obtained by the three phase theory have been compared with the simulation result of the analytical model. As we see, there is a good agreement between two models especially when the CS moved toward the axis of central electrode and similarity of curves obtained by these models can illustrate validity of presented analytical model.

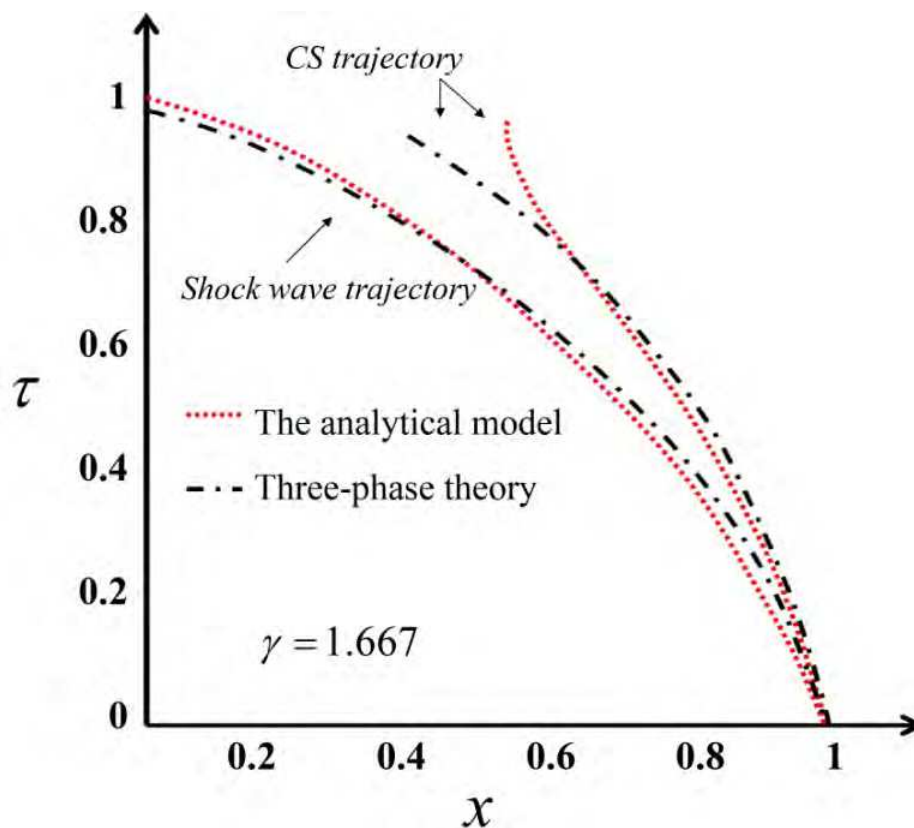


Fig. 7. trajectory of shock wave and CS based upon proposed analytical model and the three phase theory

4. Conclusion

In this paper we presented an analytical model to describe the CS path based upon the shock wave trajectory before dense pinch formation in a plasma focus system. When the results from the analytical method are compared to the results obtained by the three phase theory of plasma focus, it is seen that the trajectory of CS and shock wave were almost the same for accelerating velocity of shock wave. The results of first and second approximations for the straight shock wave don't have physical relevance. When this procedure is extended to the accelerating and decelerating shock trajectories, it would seem to imply that the first and second approximation are very close together and also an accelerating shock is controlled more by its piston than is a decelerating shock wave. Because of complexity of pinched plasma behavior, this approach can be applied to investigate the dynamic of accelerating plasma layer.

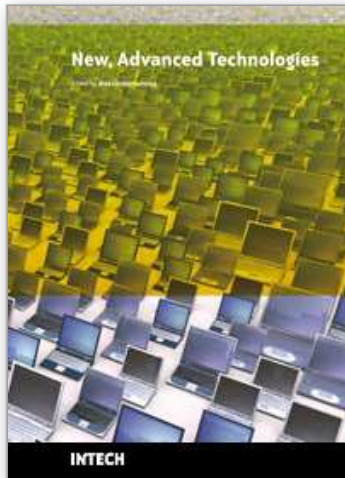
5. References

- Bates, D.R. , A.E. Kingston and R.W.P. McWhirter, (1962) *Proc. Roy. Soc., A* 267-297
- Drake, P., (2005) *High-Energy-Density Physics*, ISBN-10 3-540-29314-0 Springer Berlin Heidelberg New York
- Habibi M., Amrollahi R. , Attaran M., and Etaati R.,(2008) *Design, construction and the first experiments on the Amirkabir Plasma Focus (APF) facility*, *Plasma Devices and operations*, Vol. 16, No. 3, 163–169
- Lee S., T. Y. Tou, S. P. Moo, et al, (1988) *a simple facility for the teaching of plasma dynamics and plasma nuclear fusion*, *Am. J. Phys.* 56,1,62-68
- Leopoldo Soto, (2005) *New trends and future perspectives on plasma focus research*, *Plasma Physics and Control Fusion*, 47,361-381
- Lindemuth, I. R. , B. C. Freeman,(1982) *Shock dynamics and neutron production in an explosive generator driven dense plasma focus*, *Appl. Phys. Lett.* 40(6)
- Lister, M. (1960) "*The Numerical Solutions of Hyperbolic Partial Differential Equations by the Method of Characteristics*," Chap. 15 of *Mathematical Methods for Disital Computers*, John Wiley & Sons, Inc.
- Mather J. W., (1971) *Methods of Experimental Physics*, P. 187, 9B, Academic Press, New York
- Mather J. W., (1965) *Formation of high density deuterium plasma focus*, *the physics of fluid*, V.8, N.2
- Mathuthu M., Zengeni G., Gholap A. V. , (1997) *The Three-Phase Theory for Plasma Focus Devices*, *IEEE TRANSACTIONS ON PLASMA SCIENCE*, VOL. 25, NO. 6
- Sadowski, M. D. , M. Sholz, (2008) *The main issues of research on dense magnetized plasmas in PF discharges*, *Plasma Sources Sci. Technol.* 17 -024001
- Shan.B., M.H. Liu, R. S. Rawat, and S. Lee, *X-ray Emission Properties of Neon and Argon Plasmas in Conventional and Gas-puff Plasma Focus Devices*, *Proceedings of Regional Conference on Plasma Research in 21st Century, Thailand*, 75-79 (2000)
- Yanagidaira, T. , K. Shimoda, Y. Ono, and K. Hirano, (1999) "*Soft x-ray emission in the neon gas-puff plasma focus*", *Phys. of Plasmas*, 6-4679-4684

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