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Nonlocal Electromagnetic Media: A Paradigm for Material Engineering

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Abstract
In this paper, we provide a general Fourier transform formalism suitable for studying the electromagnetic response of material media. This approach can handle media that exhibit natural optical activity, magnetoelectric effects, spatial dispersion, etc. Moreover, it is a powerful method in addressing the impact of electromagentic systems on the spatial structure of the field, particulary at the nanoscale (e.g., near-field nano-optics, subwavelength imaging, etc.) The formalism is employed to analyze the localization of electromagnetic energy around radiating sources and also to provide a new paradigm for thinking about metamaterials.

1. Introduction

Traditionally, the research area known under the label "artificial materials," or what has become popular nowadays as "metamatrials," is based on the idea of mimicking the way natural media respond to an applied electromagnetic field. The mechanism responsible of the electromagnetic character of the medium, for example the optical properties, can be applied to repeat the whole process artificially in the sense that the atomic constituents of matter are individually manipulated and controlled in order to achieve a desired electromagnetic profile.
The conventional approach to describe material responses rely on assuming that the external field induce multipole electric and magnetic moments in the medium, giving rise to polarization and magnetization density vectors. This approach, as will be demonstrated in this paper, has its merits although theoretically problematic. It provides an extremely simple mathematical model that is adequate for a very wide range of applications. However, on the other hand, with the exploding progress in nanotechnology and experimental research, it is becoming increasingly pressing to employ a more general mathematical formalism that allows us to explore new dimensions in the material response that go beyond the traditional multipole description.
It is the vision of the present authors that a large proportion of the future research in the field of artificial and metamaterials should be invested in studying the *spatial* degrees of freedom of the medium response, a space hitherto unexplored in depth with few notable exceptions (1),

(2). The purely spatial effects, for example spatial dispersion, has been often neglected because natural materials happen to have very small interatomic spacing to operating wavelength ratio, which implies that when a macroscopic field measurement is employed, all microscopic spatial information are washed away. However, there is nothing in principle prohibiting designing artificial media with arbitrary spatial response profile.[1] The possibility of controlling wave propagation through a given device by manipulating both the temporal and spatial dispersion was already proposed in conjunction with realizing the so-called negative refraction metamaterials (4).

In this paper, we review a Fourier-space formalism suitable for modeling the spatial effects of a given natural or artificial medium. The formalism is compatible with the traditional multipole approach but is conceptually easier to understand. The Fourier transform method we introduce here is inspired by techniques developed in the physics community to attack plasma problems (1), (2).

There is a plethora of advantages in employing this particular point of view in this setting. The chief advantage is that the Fourier-space formalism is more general in its applicability to fluctuating fields with higher frequencies. Also, it naturally provides a complete characterization of the field in both space and time. Finally, being a spectral method, it allows for deeper understanding of localization phenomena and coupling mechanisms.

Some of the disadvantages is that it requires an additional mathematical background that is not usually part of the training of professional electromagnetic engineers. It also does not apply to static field problems. In general, the Fourier formalism does not conform with the conventional literature standards of notation and usage. For this reason, the current paper will provide, in a pedagogically illuminating way, a review of the space formalism starting from the ground, Maxwell's equations, and building up to advanced applications in the concluding section.

2. Maxwell's Equation

We start with the fundamental equations governing the free Maxwellian fields **B** and **E**. These are

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \left(1\Big/c^2\right) \frac{\partial \mathbf{E}}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}, \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{4}$$

where c is the speed of light and $\varepsilon_0 = 8.854 \times 10^{-}12\,\mathrm{F/m}$ and $\mu_0 = 4\pi \times 10^{-}7\,\mathrm{H/m}$ are the permittivity and permeability of free space, respectively.

We notice that these set of Maxwell's equations are complete since they capture everything related to electromagnetic interactions. However, in order to solve Maxwell's equations *in the presence of matter*, one has to supply suitable decompositions of the source terms appearing in (2) and (3) in the following manner

$$\rho = \rho_{\mathrm{ext}} + \rho_{\mathrm{ind}} \tag{5}$$

[1] The implementation of a particular solution of Maxwell's equations coupled with a suitable mechanical model is a technological problem, not a theoretical one. In this sense, the present paper should be viewed as a theoretical contribution.

and

$$\mathbf{J} = \mathbf{J}_{\text{ext}} + \mathbf{J}_{\text{ind}}, \tag{6}$$

where ρ_{ext} and $\mathbf{J}_{\text{ext}}$ are the imposed sources supplied externally. Matter will interact with the fields radiated by these sources and respond by generating induced sources ρ_{ind} and $\mathbf{J}_{\text{ind}}$. These induced sources cannot be deduced from Maxwell's equations themselves. They must be found upon constructing an appropriate mechanical model for matter in the radiation field.

2.1 The Continuity Equation and Energy Conservation

By imposing the conservation of electric charge density $\rho(t,\mathbf{r})$, the equation of continuity for electromagnetism takes the following form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0. \tag{7}$$

Energy conservation is already built into the structure of Maxwell's equations in continuous media. Indeed, it is possible to directly derive the following relation

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\varepsilon_0 |\mathbf{E}|^2 + \frac{1}{2}|\mathbf{B}|^2 \Big/ \mu_0\right) + \nabla \cdot \left(\frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}\right) = -\mathbf{J} \cdot \mathbf{E}. \tag{8}$$

Let us supplement this equation with Lorentz force law

$$\mathbf{F} = q\mathbf{E} + \mathbf{v} \times \mathbf{B}. \tag{9}$$

One can carefully build the interpretation of the terms appearing in the RHS of (8) starting from the basic law of force (9). As it turns out, the time rate of the volume density of the work done by the electric current $\mathbf{J}$ on the electric field $\mathbf{E}$ is given by $-\mathbf{J} \cdot \mathbf{E}$. This provides us with an interpretation of the RHS of (8). Now, in order to interpret (8) as a continuity equation, we observe that, in vacuum, the quantities $\varepsilon_0 |\mathbf{E}|^2 \Big/ 2$ and $|\mathbf{B}|^2 \Big/ 2\mu_0$ can straightforwardly be interpreted as volume densities of electric and magnetic energies, respectively, stored in free space. It follows then that the last term, that of $\mathbf{E} \times \mathbf{B}/\mu_0$, can be easily interpreted to stand for the volume density of the power flow, or the electromagnetic flux.

3. Fourier Transform Approach to the Greens Functions

3.1 Maxwell's Equations in the Spectral Domain

As we are going to formulate the entire problem in terms of Fourier transform, the usual spatio-temporal form of Maxwell's equations must be transformed into the spectral domain. In this section, we handle the problem of a source radiating in infinite isotropic and homogenous medium. Maxwell's equations (1)-(4) can be written in the Fourier transform domain as

$$\mathbf{k} \times \mathbf{E}(\omega,\mathbf{k}) = \omega \mathbf{B}(\omega,\mathbf{k}), \tag{10}$$

$$i\mathbf{k} \times \mathbf{B}(\omega,\mathbf{k}) = -i\omega \mathbf{E}(\omega,\mathbf{k}) \Big/ c^2 + \mu_0 \mathbf{J}(\omega,\mathbf{k}), \tag{11}$$

$$\mathbf{k} \cdot \mathbf{E}(\omega,\mathbf{k}) = -i\rho(\omega,\mathbf{k})/\varepsilon_0, \tag{12}$$

$$\mathbf{k} \cdot \mathbf{B}(\omega,\mathbf{k}) = 0. \tag{13}$$

The equation of continuity (7) can be also Fourier transformed into the form

$$\omega \rho(\omega,\mathbf{k}) = \mathbf{k} \cdot \mathbf{J}(\omega,\mathbf{k}). \tag{14}$$

The reader must notice that these equations cannot be used to describe static fields, which may be tackled on their own by applying the Coloumb gauge. Therefore, throughout this paper, we restrict ourselves to the case $\omega \neq 0$.
The program of performing calculations in electromagnetism using the Fourier transform method can be elucidated in the following manner

1. Express the magnetic field in terms of the electric field using Maxwell's equation (10)
$$\mathbf{B}(\omega,\mathbf{k}) = \mathbf{k} \times \mathbf{E}(\omega,\mathbf{k})/\omega. \tag{15}$$
2. Express the charge density in terms of the current density using the equation of continuity (14)
$$\rho(\omega,\mathbf{k}) = \frac{1}{\omega}\mathbf{k} \cdot \mathbf{J}(\omega,\mathbf{k}). \tag{16}$$
3. End up with a single equation in one unknown, $\mathbf{E}(\omega,\mathbf{k})$, and forcing term $\mathbf{J}(\omega,k)$; i.e., solve
$$\frac{\omega^2}{c^2}\mathbf{E}(\omega,\mathbf{k}) + \mathbf{k} \times [\mathbf{k} \times \mathbf{E}(\omega,\mathbf{k})] = -i\omega\mu_0 \mathbf{J}(\omega,\mathbf{k}). \tag{17}$$

Therefore, the program of solving Maxwell's equations reduces to solving a single algebraic equation in terms of the electric field $\mathbf{E}(\omega,\mathbf{k})$. All the other field and source components can obtained from the solution of the electric field together with the given form of the source.

3.2 The Greens Function Tensor in the Spectral Domain

To obtain the Greens function in the Fourier domain, we first put equation (17) in a suitable form. We use the following identity

$$\mathbf{A} \times \mathbf{B} = \epsilon_{ijk} A_j B_k, \tag{18}$$

where ϵ_{ijk} is the permutation tensor.[2] Therefore, we have

$$\mathbf{k} \times \mathbf{E} = \epsilon_{ijk} k_j E_k. \tag{19}$$

Iterating, we obtain

$$\mathbf{k} \times \mathbf{k} \times \mathbf{E} = \epsilon_{ijk} k_j \epsilon_{kj'k'} k_{j'} E_{k'} = \epsilon_{ijk}\epsilon_{kj'k'} k_j k_{j'} E_{k'}. \tag{20}$$

We use the following basic identity

$$\begin{aligned}\epsilon_{abc}\epsilon_{ijk} = \delta_{ai}\delta_{bj}\delta_{ck} + \delta_{ak}\delta_{bi}\delta_{cj} + \delta_{aj}\delta_{bk}\delta_{ci} \\ -\delta_{bi}\delta_{aj}\delta_{ck} - \delta_{ak}\delta_{ai}\delta_{cj} - \delta_{bj}\delta_{ak}\delta_{ci}.\end{aligned} \tag{21}$$

Therefore, we have

$$\epsilon_{iab}\epsilon_{ijk} = \delta_{aj}\delta_{bk} - \delta_{ak}\delta_{bj}. \tag{22}$$

Using this identity in (17), we arrive to

$$\left[\left(\frac{\omega^2}{c^2} - k^2\right)\delta_{nm} + k_n k_m\right] E_m(\omega,\mathbf{k}) = -i\omega\mu_0 J_n(\omega,\mathbf{k}). \tag{23}$$

[2] Throughout this paper, the Einstein (repeated) summation index is used. That is, whenever an index is repeated in a given expression, summation is implied with respect to these indices.

The Greens function tensor is defined to satisfy the following equation

$$\left[\left(\frac{\omega^2}{c^2}-k^2\right)\delta_{nm}+k_n k_m\right]G_{ml}(\omega,\mathbf{k})=-i\omega\mu_0\delta_{nl}(\omega,\mathbf{k}). \tag{24}$$

Therefore, by inverting the matrix operator appearing in the equation above, the Greens function tensor is readily obtained in the following compact closed form

$$G_{nm}(\omega,\mathbf{k})=\frac{-i\omega\mu_0}{\omega^2/c^2-k^2}\left(\delta_{nm}-\frac{c^2}{\omega^2}k_n k_m\right) \tag{25}$$

Finally, we notice that it is possible to separate the field into two components, one transverse to the direction of the wave vector $\mathbf{k}$ (transverse mode), and another perpendicular to this direction, which we call longitudinal mode. The longitudinal mode is not involved in the radiation and is related to the near field. It contributes directly to the structure of the field surrounding the source.

4. Review of the Traditional Description of Electromagnetic Materials in terms of Multipole Moments

The conventional old description of electromagnetic materials involves the introduction of two quantities to calculate the induced charge and current distributions. We review here the traditional view and show how it can be derived by a Fourier transform approach to the multipole expansion of the source.

The conventional idea is to assume that a given medium responds to both electric and magnetic fields by generating an induced *polarization* density $\mathbf{P}$ and *magnetization* $\mathbf{M}$. However, this description is strictly valid when both the electric and magnetic responses can be unambiguously separated from each other. This is possible only when the fields are static; otherwise, it should be viewed as an approximation. Indeed, if rapid field fluctuations at the microscopic scale are taken into consideration, then the separation becomes ill-defined and problematic.

Let us see how $\mathbf{P}$ and $\mathbf{M}$ arise from the Fourier transform perspective. Consider an arbitrary charge and current distribution

$$\rho(t,\mathbf{k})=\int d^3 r e^{-i\mathbf{k}\cdot\mathbf{r}}\rho(t,\mathbf{r}), \tag{26}$$

$$\mathbf{J}(t,\mathbf{k})=\int d^3 r e^{-i\mathbf{k}\cdot\mathbf{r}}\mathbf{J}(t,\mathbf{r}). \tag{27}$$

Expand the exponential in Taylor series

$$e^{-i\mathbf{k}\cdot\mathbf{r}}=1-i\mathbf{k}\cdot\mathbf{r}+\frac{1}{2}(i\mathbf{k}\cdot\mathbf{r})^2+\cdots. \tag{28}$$

Inserting (28) into (26), we obtain

$$\begin{aligned}\rho(t,\mathbf{k}) &= \int d^3r\rho(t,\mathbf{r})\left[1-i\mathbf{k}\cdot\mathbf{r}+\tfrac{1}{2}(i\mathbf{k}\cdot\mathbf{r})^2+\cdots\right]\\ &= \int d^3r\rho(t,\mathbf{r})-\int d^3ri\mathbf{k}\cdot\mathbf{r}\rho(t,\mathbf{r})+\tfrac{1}{2}\int d^3r(i\mathbf{k}\cdot\mathbf{r})^2\rho(t,\mathbf{r})+\cdots\\ &= -\int d^3rik_nr_n\rho(t,\mathbf{r})-\tfrac{1}{2}\int d^3rk_nr_nk_mr_m\rho(t,\mathbf{r})+\cdots\\ &= -ik_n\int d^3rr_n\rho(t,\mathbf{r})-\tfrac{1}{2}k_nk_m\int d^3rr_nr_m\rho(t,\mathbf{r})+\cdots\\ &= -i\mathbf{k}\cdot\mathbf{p}(t)-\tfrac{1}{2}k_nk_mq_{nm}(t)+\cdots,\end{aligned} \tag{29}$$

where

$$p_n(t)=\int d^3rr_n\rho(t) \tag{30}$$

and

$$q_{nm}(t)=\int d^3rr_nr_m\rho(t) \tag{31}$$

are the dipole and quadrable moments, respectively. We also used the assumption that the charge distribution is neutral $\int d^3r\rho(t,\mathbf{r})=0$. Similarly, by inserting (28) into (27), one obtains

$$\begin{aligned}J_n(t,\mathbf{k}) &= \int d^3rJ_n(t,\mathbf{r})\left[1-i\mathbf{k}\cdot\mathbf{r}+\tfrac{1}{2}(i\mathbf{k}\cdot\mathbf{r})^2+\cdots\right]\\ &= \int d^3rJ_n(t,\mathbf{r})-\int d^3r(i\mathbf{k}\cdot\mathbf{r})J_n(t,\mathbf{r})+\tfrac{1}{2}\int d^3r(i\mathbf{k}\cdot\mathbf{r})^2J_n(t,\mathbf{r})+\cdots\\ &= \underbrace{\int d^3rJ_n(t,\mathbf{r})}_{\mu_n(t)}-ik_m\underbrace{\int d^3rr_mJ_n(t,\mathbf{r})}_{\mu_{mn}(t)}-\tfrac{1}{2}k_lk_m\int d^3rr_lr_mJ_n(t,\mathbf{r})+\cdots\\ &= \tfrac{\partial}{\partial t}p_n(t)-ik_m\tfrac{1}{2}\tfrac{\partial}{\partial t}q_{mn}(t)-i\epsilon_{mns}k_mm_s(t)+\cdots\\ &= \tfrac{\partial}{\partial t}p_n(t)-ik_m\tfrac{1}{2}\tfrac{\partial}{\partial t}q_{mn}(t)+i\epsilon_{nms}k_mm_s(t)+\cdots,\end{aligned} \tag{32}$$

where equations (110) and (120) (see Appendix) were utilized in obtaining the fourth equality, and the relation $\epsilon_{nms}=-\epsilon_{mns}$ is employed in the writing last equality. By ignoring all quadrable and higher terms in (29) and (32), we find

$$\rho(t,\mathbf{k})=-i\mathbf{k}\cdot\mathbf{p}(t), \tag{33}$$

$$\mathbf{J}(t,\mathbf{k})=\frac{\partial}{\partial t}\mathbf{p}(t)+i\mathbf{k}\times\mathbf{m}(t). \tag{34}$$

Define the polarization and magnetization densities $\mathbf{P}$ and $\mathbf{M}$, respectively, by the following relations

$$\mathbf{p}(t)=\int d^3r\mathbf{P}(t,\mathbf{r}) \tag{35}$$

and

$$\mathbf{m}(t)=\int d^3r\mathbf{M}(t,\mathbf{r}). \tag{36}$$

Inserting (33) and (34) into (26) and (27), it follows

$$\rho(t,\mathbf{k})=\int d^3r\left[-i\mathbf{k}\cdot\mathbf{P}(t,\mathbf{r})\right], \tag{37}$$

$$\mathbf{J}(t,\mathbf{k}) = \int d^3r \left[\frac{\partial}{\partial t}\mathbf{P}(t,\mathbf{r}) + i\mathbf{k} \times \mathbf{M}(t,\mathbf{r}) \right]. \tag{38}$$

Therefore, by inverting the Fourier transforms (37) and (38), we obtain

$$\rho_{\text{ind}}(t,\mathbf{r}) = -\nabla \cdot \mathbf{P}(t,\mathbf{r}), \tag{39}$$

$$\mathbf{J}_{\text{ind}}(t,\mathbf{r}) = \frac{\partial}{\partial t}\mathbf{P}(t,\mathbf{r}) + \nabla \times \mathbf{M}(t,\mathbf{r}). \tag{40}$$

As can be seen now, this derivation ignores higher-order multipole without providing a clear-cut criterion for when and why this approximation is valid. Since we are attempting to construct a general theory for *both* near and far fields in the context of material response, it is important to employ a formulation that does not involve approximations that may not hold in certain media. Some other difficulties relate to the question of the convergence of the multipole expansion that is seldom addressed in literature. Finally, there is the incompleteness issue in the expansion (28), which includes only terms with zero trace.

5. Material Response Through the Fourier Transform Approach

We will now carefully introduce the equivalent representation of the electromagnetic material response in terms of the Fourier transform of the fields, not the actual field in space and time. There are several advantages in this approach that are worthy detailed considerations in themselves. First, notice that this approach does not apply to static fields, which are better addressed by the classical **P**-**M** approach. On other hand, certain complex electromagnetic effects, like spatial dispersion (nonlocality) magnetoelectric responses and optical activity, can be regarded as special case of spatial dispersion.

It appears to the authors that operating directly on material systems with a formalism tailored especially to handle spatial dispersion is very advantageous. Besides its ability to deal with complex media exhibiting phenomena like magnetoelectric effects and optical activity, it can also provide a natural window to probe near-field interactions. Although we are still trying to mathematically identify the meaning of the near field, remember that one of the most immediate features that come to mind when thinking about fields in the near zone (close to the radiator or the scatterer) is that they tend to be localized, or, equivalently, contain short wavelength components that contribute significantly to the field structure. In this case, one is looking naturally for a mathematical device that characterize electromagnetic wave phenomena in terms of the Fourier spatial modes, i.e., the **k**-component. Therefore, the formalism should look for information about the response of the system to particular wavevecotrs **k**. This is essentially the goal of integrating spatial dispersion in the theoretical description of material media.

Let us try to address in more details some of the difficulties in the traditional approach to electromagnetic material response. By Fourier transforming equation (40) in time, we obtain

$$\mathbf{J}_{\text{ind}}(\omega,\mathbf{k}) = -i\omega\mathbf{P}(\omega,\mathbf{k}) + i\mathbf{k} \times \mathbf{M}(\omega,\mathbf{k}). \tag{41}$$

The problem here is that there exists no general *a priori* method to tell how the individual contributions of the quantities **P** and **M** divide in forming the total induced current. In this sense, one can view these two vectors as mere calculational tools, auxiliary devices used to compute the actually observed induced current $\mathbf{J}_{\text{ind}}$. In particular, there seems to be no harm in just setting the magnetization density **M** to zero and considering only a polarization density **P** contributing to the induced charge and current densities.

As we have just observed in Section 3.1, the program of calculating the fields through Maxwell's equations can be reduced to the solution of a single equation, namely (17), which contains a single unknown, the electric field vector $\mathbf{E}(\omega,\mathbf{k})$ itself. If the relation between the induced current density $\mathbf{J}_{\text{ind}}(\omega,\mathbf{k})$ and the electric field is known, then this relation, together with the master equation (17), can be used to completely solve the problem of light-matter interaction. It seems natural then to introduce a *single* material response tensor

$$(J_{\text{ind}})_m(\omega,\mathbf{k}) = \sigma_{mn}(\omega,\mathbf{k})\, E_n(\omega,\mathbf{k}), \tag{42}$$

where the matrix $\sigma_{mn}(\omega,\mathbf{k})$ is called the *conductivity tensor*. After solving for the electric field, all the remaining quantities, the magnetic field $\mathbf{B}(t,\mathbf{r})$ and the charge density $\rho(t,\mathbf{r})$, can be calculated from the knowledge of the total current and the electric field.
One can replace the conductivity tensor by different equivalent representations that may turn out to be handy in some applications. In particular, we discuss here the polarization tensor $\alpha_{nm}(\omega,\mathbf{k})$ and the equivalent dielectric constant $\varepsilon^{\text{eq}}_{nm}(\omega,\mathbf{k})$, defined by the following equation

$$\begin{aligned}\varepsilon^{\text{eq}}_{nm}(\omega,\mathbf{k}) &= \delta_{nm} + \tfrac{i}{\omega\varepsilon_0}\sigma_{nm}(\omega,\mathbf{k}) \\ &= \delta_{nm} + \tfrac{1}{\omega^2\varepsilon_0}\alpha_{nm}(\omega,\mathbf{k}) \\ &= \delta_{nm} + \chi_{nm}(\omega,\mathbf{k}).\end{aligned} \tag{43}$$

The reader should notice that the equivalent dielectric function $\varepsilon_0\varepsilon^{\text{eq}}_{nm}(\omega,\mathbf{k})$ is *not* the same as the conventional dielectric function defined in terms of the polarization and magnetization densities appearing in equation (40). In terms of the new dielectric function $\varepsilon^{\text{eq}}_{nm}(\omega,\mathbf{k})$, we write

$$D_n(\omega,\mathbf{k}) = \sum_m \varepsilon_0\varepsilon^{\text{eq}}_{nm}(\omega,\mathbf{k})\, E_m(\omega,\mathbf{k}). \tag{44}$$

It follows that in the Fourier transform approach to the material response, we effectively kill the magnetization vector $\mathbf{M}$ and collect all relevant physical processes into a single vector, the effective polarization density $\mathbf{P}$.

6. Comparison between the Traditional Multipole and the Fourier Transform Approaches to the Material Response

Within the miltipole approach to the material response, two new fields are traditionally introduced, the electric induction $\mathbf{D}$ (the electric displacement vector), and the magnetic field strength $\mathbf{H}$. These are defined by the relations

$$\mathbf{D} \equiv \varepsilon_0\mathbf{E} + \mathbf{P}, \tag{45}$$

$$\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}. \tag{46}$$

The electric susceptibility χ^e and the magnetic susceptibility χ^m are defined by the following equations

$$\mathbf{P} = \varepsilon_0\chi^e\mathbf{E}, \tag{47}$$

$$\mathbf{M} = \frac{1}{\mu_0}\chi^m\mathbf{B}. \tag{48}$$

The effective dielectric constant, or electric permittivity ε, and the magnetic permeability μ, can now be defined in terms of the quantities above as

$$\mathbf{D}(\omega,\mathbf{k}) = \varepsilon\mathbf{E}(\omega,\mathbf{k}), \tag{49}$$

$$\mathbf{H}(\omega,\mathbf{k}) = \frac{1}{\mu}\mathbf{B}(\omega,\mathbf{k}). \tag{50}$$

We now proceed to derive the equivalence between this traditional approach and the Fourier formalism of Section 5. First, the current distribution is decomposed into two parts, one due to external (applied) sources, $\mathbf{J}_{\text{ext}}$, and the other, $\mathbf{J}_{\text{ind}}$ due to the interaction between the medium and the electromagnetic fields. We write

$$\mathbf{J}(t,\mathbf{r}) = \mathbf{J}_{\text{ext}}(t,\mathbf{r}) + \mathbf{J}_{\text{ind}}(t,\mathbf{r}). \tag{51}$$

The induced current is written using the conductivity tensor introduced in (42) and the result is substituted to the master equation (17). After simple re-arranging of terms, we find

$$\frac{\omega^2}{c^2}\mathbf{E}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) + i\omega\mu_0\bar{\bar{\sigma}}(\omega,\mathbf{k})\cdot\mathbf{E}(\omega,\mathbf{k}) = -i\omega\mu_0\mathbf{J}_{\text{ext}}(\omega,\mathbf{k}). \tag{52}$$

Now let us calculate by means of the ϵ-μ method. In this case, the induced current is written in terms of both the polarization and magnetization current densities $\mathbf{P}$ and $\mathbf{M}$ as shown in (40). Using (47) and (48) in (41), we find

$$\mathbf{J}_{\text{ind}}(\omega,\mathbf{k}) = -i\omega\varepsilon_0\chi^e\mathbf{E}(\omega,\mathbf{k}) + i\mathbf{k}\times\frac{\chi^m}{\mu_0}\mathbf{B}(\omega,\mathbf{k}). \tag{53}$$

But from Maxwell's equations in the Fourier domain, specifically (10), we know that

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \frac{i}{\omega}\mathbf{k}\times\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}). \tag{54}$$

The induced current in (53) becomes then

$$\mathbf{J}_{\text{ind}}(\omega,\mathbf{k}) = -i\omega\varepsilon_0\chi^e\mathbf{E}(\omega,\mathbf{k}) + i\frac{\chi^m}{\omega\mu_0}\mathbf{k}\times\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}). \tag{55}$$

Combining (51) and (55) and substituting the result into (17), we arrive after some rearranging to

$$\begin{aligned}&\tfrac{\omega^2}{c^2}\mathbf{E}(\omega,\mathbf{k}) + \mathbf{k}\times\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k})\\ &\quad+\tfrac{\omega^2}{c^2}\chi^e\mathbf{E}(\omega,\mathbf{k}) - \chi^m\mathbf{k}\times\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = -i\omega\mu_0\mathbf{J}_{\text{ext}}(\omega,\mathbf{k})\end{aligned} \tag{56}$$

By comparing (52) and (56), we conclude that we must have

$$i\omega\mu_0\bar{\bar{\sigma}}(\omega,\mathbf{k})\cdot\mathbf{E}(\omega,\mathbf{k}) = \frac{\omega^2}{c^2}\chi^e\mathbf{E}(\omega,\mathbf{k}) - \chi^m\mathbf{k}\times\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}). \tag{57}$$

In tensor form, equation (57) becomes

$$i\omega\mu_0\sigma_{nl}(\omega,\mathbf{k})E_l(\omega,\mathbf{k}) = \frac{\omega^2}{c^2}\chi^e E_n(\omega,\mathbf{k}) - \chi^m\left[k_nk_l - k^2\delta_{nl}\right]E_l(\omega,\mathbf{k}). \tag{58}$$

Since the equality holds for arbitrary E_l, we obtain

$$\sigma_{nl}(\omega,\mathbf{k}) = \frac{1}{i\omega\mu_0}\left\{\frac{\omega^2}{c^2}\chi^e\delta_{nl} - \chi^m\left[k_nk_l - k^2\delta_{nl}\right]\right\}. \tag{59}$$

From (43), we reach to

$$\begin{aligned}\varepsilon_{nl}^{\mathrm{eq}}(\omega,\mathbf{k}) &= \delta_{nl} + \tfrac{i}{\omega\varepsilon_0}\tfrac{1}{i\omega\mu_0}\left\{\tfrac{\omega^2}{c^2}\chi^e\delta_{nl} - \chi^m\left[k_nk_l - k^2\delta_{nl}\right]\right\} \\ &= \delta_{nl} + \chi^e\delta_{nl} - \tfrac{c^2}{\omega^2}\chi^m\left(k_nk_l - k^2\delta_{nl}\right) \\ &= \left(1+\chi^e\right)\delta_{nl} - \tfrac{c^2}{\omega^2}\chi^m\left(k_nk_l - k^2\delta_{nl}\right)\end{aligned} \tag{60}$$

Finally, we use the definitions (45), (46), (47), (48) to write

$$\varepsilon_{nm}^{\mathrm{eq}}(\omega,\mathbf{k}) = \left(\frac{\varepsilon}{\varepsilon_0}\right)\delta_{nm} - \frac{c^2}{\omega^2}\left(1-\frac{\mu_0}{\mu}\right)\left(k_nk_m - k^2\delta_{nm}\right). \tag{61}$$

This is the main equation we are looking for. It shows that a medium which is magnetic in the ε-μ approach translates into spatial dispersion in the Fourier approach. It follows also that the two dielectric constants are the same only if there is no spatial dispersion.

7. General Properties of the Material Response Tensors

The requirement that the electromagnetic fields should by themselves satisfy Maxwell's equations cannot fully specify how the very same fields will behave in a material environment. Such behavior is dictated by a more complex structure consisting of the mechanical response coupled with the electromagnetic fields. In this section, we survey and present rigorously the most important *non*-electromagnetic restrictions imposed on the material tensor. Such restrictions can be conveniently gathered under the heading 'General properties of the Material Tensor' since they involve quite broad characteristics that are wider than the particular dynamical laws encapsulated by the Maxwell's equations.
Our main equations will be the relation between the electric flux density vector and the electric field in both the spatio-temporal and spectral domain. These are, respectively,

$$D_n(\omega,\mathbf{k}) = \sum_m \varepsilon_0\varepsilon_{nm}^{\mathrm{eq}}(\omega,\mathbf{k})\,E_m(\omega,\mathbf{k}), \tag{62}$$

$$D_n(t,\mathbf{r}) = \varepsilon_0\int dt'\int d^3r'\sum_m \varepsilon_{nm}^{\mathrm{eq}}\left(t-t',\mathbf{r}-\mathbf{r}'\right)E_m\left(t',\mathbf{r}'\right). \tag{63}$$

These equations describe electromagnetic processes in homogenous, isotropic or anisotropic media. It is important to keep in mind that within the Fourier-space formalism the equivalent dielectric tensor is *inherently* a tensor; even when the medium under consideration is isotropic, the dielectric function is still generally a tensor. Also, the reader may notice from (63) that the field induced at particular time t and location $\mathbf{r}$ depends generally on the applied field at *different* times and locations. We say that the medium exhibit "memory" in both the temporal and spatial sense. The spatial sense of the this memory, which is going to be the main concern for us here, is called *nonlocality*.[3]

[3] Whenever there is no risk of confusion, we drop the superscript 'eq' from $\varepsilon_{nm}^{\mathrm{eq}}(\omega,\mathbf{k})$ and refer to the equivalent dielectric function as merely the *dielectric tensor*.

7.1 The Reality of the Fields

Since the fields appearing in equation (63) are all real, the properties of the Fourier transform dictate that the negative and positive frequencies appearing in the spectrum of the fields are both essentially equivalent to each other. Formally, we express this requirement in the following relation that any material tensor describing the responses of the medium to real quantities must satisfy

$$\varepsilon_{nm}^{*}(\omega,\mathbf{k}) = \varepsilon_{nm}(-\omega,-\mathbf{k}). \tag{64}$$

7.2 Dissipative and Non-Dissipative Processes

The material tensorial response is the Fourier transform of a real quantity and hence generally complex. The real part and the imaginary part of this tensor are usually interpreted as those responsible for dispersion and losses (dissipation), respectively. In this section, we provide the mathematical evidence in support of this interpretation.

We start be decomposing an arbitrary response tensor into hermitian and antihermitian parts

$$\varepsilon_{nm}(\omega,\mathbf{k}) = \varepsilon_{nm}^{\mathrm{H}}(\omega,\mathbf{k}) + \varepsilon_{nm}^{\mathrm{A}}(\omega,\mathbf{k}), \tag{65}$$

where

$$\varepsilon_{nm}^{\mathrm{H}}(\omega,\mathbf{k}) = \frac{1}{2}\left[\varepsilon_{nm}(\omega,\mathbf{k}) + \varepsilon_{mn}^{*}(\omega,\mathbf{k})\right], \tag{66}$$

$$\varepsilon_{nm}^{\mathrm{A}}(\omega,\mathbf{k}) = \frac{1}{2}\left[\varepsilon_{nm}(\omega,\mathbf{k}) - \varepsilon_{mn}^{*}(\omega,\mathbf{k})\right]. \tag{67}$$

It is obvious that the two parts satisfy

$$\varepsilon_{nm}^{\mathrm{H}*}(\omega,\mathbf{k}) = \varepsilon_{mn}^{\mathrm{H}*}(\omega,\mathbf{k}), \tag{68}$$

$$\varepsilon_{nm}^{\mathrm{A}*}(\omega,\mathbf{k}) = -\varepsilon_{mn}^{\mathrm{A}*}(\omega,\mathbf{k}). \tag{69}$$

We now recall our interpretation in Section 2.1 of the term $-\mathbf{J}\cdot\mathbf{E}$ as the density of the rate of energy transfer by the current $\mathbf{J}$ into the electric field $\mathbf{E}$. The current can be decomposed into external and induced parts as $\mathbf{J} = \mathbf{J}_{\mathrm{ex}} + \mathbf{J}_{\mathrm{ind}}$. Thus, the total work done by the *medium* on the electric field is given by integrating $-\mathbf{J}_{\mathrm{ind}}\cdot\mathbf{E}$ in both time and space as

$$-\int dt \int d^3r \mathbf{J}_{\mathrm{ind}}(t,\mathbf{r})\cdot\mathbf{E}(t,\mathbf{r}) = \int \frac{d\omega d^3k}{(2\pi)^4}\mathbf{J}_{\mathrm{ind}}(\omega,\mathbf{k})\cdot\mathbf{E}^{*}(\omega,\mathbf{k}), \tag{70}$$

where the power theorem of Fourier transforms was used in writing the equality. We now have

$$\begin{aligned}&\int \tfrac{d\omega d^3k}{(2\pi)^4}\mathbf{J}_{\mathrm{ind}}(\omega,\mathbf{k})\cdot\mathbf{E}^{*}(\omega,\mathbf{k})\\ &\quad= \int \tfrac{d\omega d^3k}{(2\pi)^4}\tfrac{1}{2}\left[\mathbf{J}_{\mathrm{ind}}^{*}(\omega,\mathbf{k})\cdot\mathbf{E}(\omega,\mathbf{k}) + \mathbf{J}_{\mathrm{ind}}(\omega,\mathbf{k})\cdot\mathbf{E}^{*}(\omega,\mathbf{k})\right]\end{aligned} \tag{71}$$

In deriving this, the integral was first divided into its negative and positive frequency parts, and then a transformation of variables was applied to the negative frequencies integral. Finally, the symmetry condition (reality condition) given in (64) was applied. Employing equation (42) in (71), we can write

$$\begin{aligned}&-\int \tfrac{d\omega d^3k}{(2\pi)^4}\mathbf{J}_{\mathrm{ind}}(\omega,\mathbf{k})\cdot\mathbf{E}^{*}(\omega,\mathbf{k})\\ &= \int \tfrac{d\omega d^3k}{(2\pi)^4}\tfrac{1}{2}\left[\sigma_{nm}(\omega,\mathbf{k})E_m(\omega,\mathbf{k})E_n^{*}(\omega,\mathbf{k}) + \sigma_{mn}^{*}(\omega,\mathbf{k})E_n^{*}(\omega,\mathbf{k})E_m(\omega,\mathbf{k})\right]\\ &= \int \tfrac{d\omega d^3k}{(2\pi)^4}\sigma_{nm}^{H}(\omega,\mathbf{k})E_m(\omega,\mathbf{k})E_n^{*}(\omega,\mathbf{k}).\end{aligned} \tag{72}$$

Therefore, it is the hermitian part of the conductivity tensor which contributes to the dissipation of energy by the medium. Equivalently, by considering the relation between the conductivity and the equivalent dielectric tensor (43), we find that it is the antihermitian part of the dielectric tensor that contributes to energy dissipation by the medium.

7.3 Onsager Relations

Since any material responses tensor is ultimately based on a mechanical model, of which the dynamical equations must satisfy certain space-time symmetry transformations, there exists certain general restrictions on the mathematical form of a physically realizable material tensor. In order to give the reader some idea about such requirement, we list the classical dynamical equation for the particle motion, namely the Lorentz force (9). By writing the force as $\mathbf{F} = d\mathbf{p}/dt$, where $\mathbf{p}$ is the linear momentum, it is an easy matter to verify that the equation of motion is invariant under the transformations

$$t \to -t,\ \mathbf{p} \to -\mathbf{p},\ \mathbf{B} \to -\mathbf{B} \tag{73}$$

The same conclusion can be obtained if the Lorentz force law is replaced by the Schrodinger equation.

Notice that a time-reversal corresponds to the substitution $\omega \to -\omega$ in the Fourier domain. The reversal of the sign of the momentum corresponds to reversing the sign of the wavevector. Finally, the reversal of the sign of the magnetic field is shown explicitly in the following standard form of the Onsager relations[4]

$$\varepsilon^{\mathrm{eq}}_{nm}(\omega,-\mathbf{k})\Big|_{-\mathbf{B}} = \varepsilon^{\mathrm{eq}}_{mn}(\omega,\mathbf{k})\Big|_{\mathbf{B}}. \tag{74}$$

The Onsager relations places severe restrictions on the physically allowable form of the material response. We discuss below particular examples of isotropic spatially dispersed media.

Let us focus on materials that don't respond to the magnetic field. In this case, the Onsager relations reduces to the situation in which the tensorial responses is required to be invariant under the transformation

$$\mathbf{k} \to -\mathbf{k},\ n \leftrightarrow m. \tag{75}$$

First, notice that in the Fourier transform approach, even when the medium is isotropic, the response is still described by a tensorial quantity, c.f. equation (61). For isotropic media that is spatially dispersive, we can analyze the situation by pure matrix theoretic arguments. Indeed, the only available vector in this case is k_m, while the only available tensors are δ_{nm} and ϵ_{nml}. It can be shown that the Onsager relations leads to the result that we can construct only three independent second-rank tensors. A popular choice in the condensed-matter physics literature is the following

$$\varepsilon^{\mathrm{eq}}_{nm}(\omega,\mathbf{k}) = \varepsilon^{L}(\omega,k)\,\kappa_n\kappa_m + \varepsilon^{T}(\omega,k)\,(\delta_{nm}-\kappa_n\kappa_m) + i\varepsilon^{R}(\omega,k)\,\epsilon_{nml}k_l, \tag{76}$$

where

$$\kappa_m = k_m/k,\quad k = |\mathbf{k}|. \tag{77}$$

Here, the quantities $\varepsilon^L(\omega,k)$, $\varepsilon^T(\omega,k)$, $\varepsilon^R(\omega,k)$ are the longitudinal, transverse, and rotational permittivities, respectively. The rotatory parts can be ignored in media that don't exhibit optical activity. Notice that for media in which both the longitudinal and transverse parts happen to be equal to each other, the equivalent dielectric tensor reduces to the scalar case.

[4] The symmetry relations (64) are used to simplify the final form.

7.4 The Kramers–Kronig Relations

The fact that the dielectric tensor is a response function imposes a restriction on the relationship between the real and imaginary part. This restriction is due to causality and can be rigourously derived by standard techniques in the theory of complex functions.[5] Kramers-Kronig relations say that the real and imaginary parts of the Fourier transform of a function that is causal (i.e., a function that its inverse Fourier transform is identically zero for a time interval in the form $-\infty < t < t_0$) satisfy

$$\varepsilon_{nm}^{\mathrm{eq,H}}(\omega,\mathbf{k}) - \delta_{nm} = \frac{i}{\pi}\wp\int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_{nm}^{\mathrm{eq,A}}(\omega',\mathbf{k}) - \delta_{nm}}{\omega-\omega'}, \tag{78}$$

$$\varepsilon_{nm}^{\mathrm{eq,A}}(\omega,\mathbf{k}) = \frac{i}{\pi}\wp\int_{-\infty}^{\infty} d\omega' \frac{\varepsilon_{nm}^{\mathrm{eq,H}}(\omega',\mathbf{k})}{\omega-\omega'}, \tag{79}$$

where $\wp$ symbolizes the Cauchy principal value.[6] Equations (78) and (79) show that if dissipation is known, then dispersion can be uniquely determined (and vice versa) by applying the Hilbert transform operator to the available data.

One can see that when spatial dispersion is present, then in the case of non-dissipative medium, i.e., a medium with negligible losses which, as can be seen from Section 7.2, corresponds to $\varepsilon_{nm}^{\mathrm{eq,A}}(\omega,\mathbf{k}) = 0$, the dispersion behavior dictated by $\varepsilon_{nm}^{\mathrm{eq,H}}(\omega,\mathbf{k})$ is restricted to only the class of functions of ω which has zero Hilbert transform. It can be shown that such functions take the basic form $1/(\omega-\omega_m)$ with constant ω_m. This explains partially why such basic form pops out very frequently in practice. However, they also demonstrate the power of Kramers-Kronig relations in being able to severely restrict the allowable functional form of the dispersion in lossless media.

The general lesson we learn from taking casuality into consideration when thinking about designing artificial media is that once the losses is neglected for the entire frequency range $-\infty < \omega < \infty$ (or the medium is designed to have small losses globally), the *global* form of dispersion is no more a free degree of freedom but, instead, takes a particular form. However, in practice we seldom achieve or require particular specifications of the losses and/or dispersion to hold for the entire frequency range. Noticing that the Hilbert transform relations in (78) and (79) are global operators, i.e., they involve integration over the entire frequency range in order to know the value at a single frequency (nonlocal or memory-dependent in frequency), we need just to restrict ourselves to a finite frequency and wavenumber range upon which the desired losses and dispersion characteristics are required to apply. By this restrictions, the Kramers–Kronig relations cannot impose a serious restriction on the design and analysis of artificial media.

8. Advanced Properties of the Material Tensor

In this Section, we look at the material tensor through the point of view of complex analysis. The motivation for such study is that certain characteristics of signals excited in media, like

[5] The causality restriction translates formally to the following setting. Imagine that the medium is excited by an applied electric field **E**. The material responses, for example through (47), will appear in the form of a forced (induced) quantity, here the polarization density **P**. If the applied field was zero for time $t < 0$, then causality implies that there must be no induced polarization in this time interval.

[6] These relations represent a Hilbert transform relation between the hermitian and antihermitian parts, which play the role of real and imaginary parts, respectively, in the case of matrices (linear operators).

short-term disturbances and damped waveforms are best understood analytically if viewed using the mathematical device of Laplace transform instead of the familiar Fourier transform, the latter being best suited ideally to analyze the steady-state behavior of a given system. As will be shown below, there are general restrictions on the mathematical form of the response functions when viewed in the complex plane. Knowledge of such global restrictions is vital in the theory and practice of meta-materials.

8.1 Stability Restrictions

From the physical point of view, a passive medium cannot generate energy and hence all propagating signals must be damped or decaying when the distance goes to infinity.[7] Mathematically, this translates to the requirement that all poles are located in the LHP. We call the following the statement of the stability condition of material media

$$\textit{All poles of the material tensor must be located in the LHP.} \tag{80}$$

To see why this should be the case, just (Laplace) invert a spectral component in the form $1/(\omega - \omega_0 + i\gamma/2)$ and notice the sign of the resulting exponential factor. For signals to exponentially decay, instead of growing, the algebraic sign of the factor γ must be positive.

8.2 Causality Restrictions

Although we have already looked at casuality in the study of Kramers -Kronig relations, we want to understand here this topic at a deeper level. Consider the Fourier transform of a causal function $f(t)$ given by

$$f(\omega) = \int_0^\infty dt\, f(t)\, e^{i\omega t}. \tag{81}$$

Let us study the asymptotic behavior of this function when $t \to \infty$. We first notice that when $\mathrm{Im}\{\omega\} > 0$, the integral in (81) has a finite value since the integrand approaches zero as t grows to infinity. Moreover, on repeatedly differentiating this integral, we conclude also that all derivatives of $f(\omega)$ are finite. Therefore, the function $f(\omega)$ is analytic in the upper half complex plane. We have then

$$\textit{A causal function is analytic in the UHP.} \tag{82}$$

An immediate corollary is that

$$\textit{A causal function has no poles or branch points in the UHP.} \tag{83}$$

This principle forms the mathematical background behind the derivation of Kramers-Kronig relations.

8.3 Landau Condition

The Laplace transform of a signal is defined as

$$F(s) \equiv \int_0^\infty dt\, f(t)\, e^{-st}. \tag{84}$$

[7] Notice that for a range that is bounded, both growing and decaying signals are possible. For example, consider a multilayered medium. In one intermediate layer both growing and decaying waves are permitted.

Therefor, the s-plane and the complex ω-plane are related by $s = i\omega$, which means that 'upper' and 'lower' in one plane translates into 'right' and 'left', respectively, in the other plane.
The inverse Laplace transform is given by the equation

$$f(t) = \frac{1}{2\pi i} \int_{\Gamma - i\infty}^{\Gamma + i\infty} ds\, F(s)\, e^{st}, \tag{85}$$

where Γ specifies how the integration contour should be chosen. Landau condition states that

The contour in (85) is to the left of all singularites in the s-plane. (86)

Therefore, the integration contour must be above all singularities in the complex ω-plane. It can be shown then that the resulting function does not depend on the particular path provided it satisfies the Landau condition.

9. Wave Propagation

9.1 Dispersion Relations

By wave modes or wave propagation we mean electromagnetic disturbances that can propagate in a source-free medium. In our case, the medium response is described by the nonlocal model of the Fourier approach.
Equation (52) is the *inhomogeneous* wave equation in our medium. From the definition (42), the induced current in terms of the vector potential (temporal gauge) is expressed as follows

$$J_{\text{ind},m}(\omega, \mathbf{k}) = \alpha_{mn}(\omega, \mathbf{k})\, A_n(\omega, \mathbf{k}). \tag{87}$$

In tensor form, we can write then (52) as

$$\Xi_{nm}(\omega, \mathbf{k})\, A_m(\omega, \mathbf{k}) = -\frac{\mu_0 c^2}{\omega^2} J_{\text{ext},n}(\omega, \mathbf{k}), \tag{88}$$

where

$$\Xi_{nm}(\omega, \mathbf{k}) = \frac{c^2}{\omega^2}\left(k_n k_m - k^2 \delta_{nm}\right) + \chi_{nm}(\omega, \mathbf{k}). \tag{89}$$

If the source term in (88) is set to zero, we obtain the *homogenous* wave equation describing the propagation of waves in a source-free environment, i.e., the eigenmodes. However, as we found in Section 7.2, the antihermitian part of the tensor $\Xi_{nm}(\omega, \mathbf{k})$ is responsible of dissipation or energy generation in the medium. Such term must be omitted from the final homogeneous equation describing pure wave propagation. The desired equation of motion is therefore given by

$$\Xi^{\mathrm{H}}_{nm}(\omega, \mathbf{k})\, A_m(\omega, \mathbf{k}) = 0, \tag{90}$$

where $\Xi^{\mathrm{H}}_{nm}(\omega, \mathbf{k})$ describes the hermitian part of the tensor $\Xi_{nm}(\omega, \mathbf{k})$. The reader should notice that there is a thermodynamic hypothesis implicit in the derivation of this fundamental equation. That is, dissipation is treated as equivalent to source, and so the antihermitian part is removed even when it describes only a passive medium. Such hypothesis, equivalence of source and sink, is an additional postulate that cannot be derived from Maxwell's equations and should be supplied by an external theory, in this case thermodynamics of continuous media.

Notice that (90) is a matrix equation. From linear algebra, the necessary and sufficient condition for the existence of a nontrivial solution is that the determinant of the hermitian matrix $\Xi^{\mathrm{H}}_{nm}(\omega,\mathbf{k})$ is identically zero. That is, the dispersion relation is given by

$$\det\left[\Xi^{\mathrm{H}}_{nm}(\omega,\mathbf{k})\right]=0. \tag{91}$$

In general, this dispersion relation has potentially many solutions, each is called a *mode* or *branch*. We write the solution of the lth mode of the dispersion equation (91) as

$$\omega=\omega_l(\mathbf{k}). \tag{92}$$

For each mode, there corresponds a vector A_n satisfying equation (90). Such vector is called the *polarization* of the wave mode. within the scheme of Fourier-space electromagnetics, there exists a detailed theory of how to obtain and classify polarization in various types of media, which is based on direct application of results from tensor calculus. However, we omit such details for the limitations of space.

9.2 The Greens Function

The solution of the inhomogeneous wave equation (88) can be formally written as

$$A_n(\omega,\mathbf{k})=-\frac{\mu_0 c^2}{\omega^2}G_{nm}(\omega,\mathbf{k})\,J_{\mathrm{ext},n}(\omega,\mathbf{k}), \tag{93}$$

where $G_{nm}(\omega,\mathbf{k})$ is the Greens function dyad in the spectral domain. From matrix theory, an expression of this dyad can be immediately written as

$$G_{nm}(\omega,\mathbf{k})=\frac{\mathrm{cof}_{nm}\left[\Xi_{n'm'}(\omega,\mathbf{k})\right]}{\det\left[\Xi_{nm}(\omega,\mathbf{k})\right]}, \tag{94}$$

where cof_{nm} is the cofactor matrix. In deriving this result, only the hermitian part of the operator $\Xi_{nm}(\omega,\mathbf{k})$ is used, and therefore the Greens dyad as it stands here is hermitian. However, when inverting the Fourier transform to calculate the fields in the spatio-temporal domain, a singularity in the spectral domain is encountered around $\omega=\omega_l(\mathbf{k})$. The traditional solution of this problem is to carefully enforce suitable causality conditions. Technically, the determinant is expanded in the following form

$$\det\left[\Xi_{nm}(\omega,\mathbf{k})\right]\approx\frac{\partial}{\partial\omega}\det\left[\Xi_{nm}(\omega,\mathbf{k})\right]\Big|_{\omega=\omega_l(\mathbf{k})}\{\omega-\omega_l(\mathbf{k})+i0\}, \tag{95}$$

where the expansion illustrated here is taken around the lth mode pole. By formally inverting the Fourier transform using the Dirac delta function, we obtain the following expression for the antihermitian part of the Greens function

$$D^{\mathrm{A}}_{nm}(\omega,\mathbf{k})=\sum_l -i\pi\omega_l(\mathbf{k})\,\mathbf{e}^*_{l,m}(\mathbf{k})\,\mathbf{e}_{l,n}(\mathbf{k})\,F_l\left[\Xi_{nm}(\omega,\mathbf{k})\right]\delta\left(\omega-\omega_l(\mathbf{k})\right), \tag{96}$$

where the $F_l\left[\Xi_{nm}(\omega,\mathbf{k})\right]$ can be directly determined by the dispersion profile of the medium, but its explicit expression is not of direct concern to us here. The unit vectors $\mathbf{e}_{l,n}(\mathbf{k})$ describe the polarization of the lth mode.[8] This derivation shows that although the antihermitian part

[8] Notice that for the case of transverse modes, the degeneracy of the eigenvalue problems requires a special treatment. Indeed, in this case one has to resort to the use of polarization matrices. We omit such details here.

was not originally taken into consideration in writing up the expression of the Greens dyad (94), causality considerations *forces* us to introduce an antihermitian part. As we will show in Section 10.1, it is precisely this antihermitian part that contributes to the radiated field.[9]

It is interesting to observe again the role played by thermodynamics in the solution of Maxwell's equations. Indeed, the ultimate origin of the causality consideration introduced above can be tracked back to the thermodynamic requirement that energy decays away from the source and toward the sink. Maxwell's equations themselves are blind to the direction of power flow; they can support both (temporally) forward and backward waves. However, thermodynamics appears to fix the sign of the pole contribution around the real ω-axis and hence effectively imposes a particular form of the solution of the field equations. The reader can appreciate better the subtlety of this fact by recalling that the very concepts of source and sink are thermodynamic in nature and cannot be based ultimately on Maxwell's equations. An impulsive excitation, say an ideal Dirac delta function, can be mathematically introduced to the theory in a straightforward manner, e.g., using generalized function theory. However, the choice of the signs of the imaginary part of the pole associated with source or sink depends on energetics and dissipation, a topic that is best described macroscopically by classical thermodynamics. Since the ultimate origin of the antihermitian part of the Greens function, as shown above in equation (96), is causality, and the particular form of this depends in turn on thermodynamic consideration, and knowing that it is this part of the Greens dyad that is responsible of radiation (see Section 10.1), we can claim that the ultimate answer to the question of why an antenna can radiate appears to be purely thermodynamic in nature.

10. Applications

In this section, we provide some applications for the general Fourier approach toward the characterization of the material responses to the electromagnetic fields as sketched above.

10.1 Localization of Electromagnetic Energy Radiated by Antennas in Complex Media

In this part, we perform an explicit calculation of the electromagnetic energy radiated by an arbitrary antenna in a medium described by a nonlocal response tensor. We will show that the Fourier approach described in this paper provides a direct method to understand the structure of the near-field surrounding the antenna, and therefore the possibility of localizing energy in complex artificial media.

The method relies on calculating the total energy of the radiated field using the Fourier integral. We start from the statement of energy conservation as stated in (8). The current $\mathbf{J}$ appearing at the RHS is replaced by the current distribution on the antenna, which is taken as an external current $\mathbf{J}_{\text{ext}}$. As discussed in Section 2.1, the energy density (work) transferred to the surrounding field by this current is given by $-\mathbf{J}_{\text{ext}} \cdot \mathbf{E}$. The trick in performing general calculation is to introduce a new quantity $U_l(\mathbf{k})$, which is defined as *the density in the* $\mathbf{k}$*-space of the energy added to the surrounding field by the antenna when radiating through the* lth *mode*. That is, by energy conservation, the time-averaged total energy added to the field by all modes is given by the following equation

$$\int_{-T/2}^{T/2} dt \int d^3r \mathbf{J}_{\text{ext}}(t,\mathbf{r}) \cdot \mathbf{E}(t,\mathbf{r}) = \sum_l \int \frac{d^3k}{(2\pi)^3} U_l(\mathbf{k}). \tag{97}$$

[9] The hermitian part will contribute to the non-propagating field (near field) surrounding the source.

Expressing the electric field in terms of the vector potential in the temporal gauge as $\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{A}(\omega,\mathbf{k})$, using the Greens function of (93), and employing the Pareseval (power) theorem of Fourier analysis, we write the LHS as

$$\begin{aligned}\int_{-T/2}^{T/2} dt \int d^3r \mathbf{J}_{\text{ext}}(t,\mathbf{r}) \cdot \mathbf{E}(t,\mathbf{r}) &= -\int \frac{d\omega d^3k}{(2\pi)^4} \mathbf{J}^*_{\text{ext}}(\omega,\mathbf{k}) \cdot \mathbf{E}(\omega,\mathbf{k}) \\ &= \int \frac{d\omega d^3k}{(2\pi)^4} \frac{i}{\varepsilon_0\omega} J^*_{\text{ext},m}(\omega,\mathbf{k}) G_{mn}(\omega,\mathbf{k}) J_{\text{ext},n}(\omega,\mathbf{k}) \\ &= \int \frac{d\omega d^3k}{(2\pi)^4} \frac{i}{\varepsilon_0\omega} J^*_{\text{ext},m}(\omega,\mathbf{k}) \left[G^{\text{A}}_{mn}(\omega,\mathbf{k}) + G^{\text{H}}_{mn}(\omega,\mathbf{k})\right] J_{\text{ext},n}(\omega,\mathbf{k})\end{aligned} \tag{98}$$

Due to the presence of the factor i in the integrand, together with the fact that the integral must be real, it follows from the basic properties of hermitian and antihermitian functions (operators) that only the antihermitian part of the Greens dyad contributes to the radiation field. Now, by plugging this part as given in (96) into (98), we finally arrive to the following expression of the energy density

$$U_l(\mathbf{k}) = \frac{F_l\left[\Xi_{nm}(\omega,\mathbf{k})\right]}{\varepsilon_0} \left|\mathbf{e}^*_l(\mathbf{k}) \cdot \mathbf{J}_{\text{ext}}(\omega_l(\mathbf{k}),\mathbf{k})\right|^2. \tag{99}$$

We will propose an interpretation for the physical meaning of $U_l(\mathbf{k})$. Consider the inverse Fourier transform

$$u_{F,l}(\mathbf{r}) = \Im^{-1}\left\{\sqrt{U_l(\mathbf{k})}\right\}. \tag{100}$$

Next, we use the Parseval theorem to write

$$\int d^3r\, u_{F,l}(\mathbf{r}) \left(u_{F,l}(\mathbf{r})\right)^* = \int \frac{d^3k}{(2\pi)^3} \sqrt{U_l(\mathbf{k})}\sqrt{U_l(\mathbf{k})}. \tag{101}$$

Since the RHS is by definition the total energy radiated by the antenna in the lth mode, it follows that the integrand of the LHS, namely $u_l(\mathbf{r}) \equiv \left|u_{F,l}(\mathbf{r})\right|^2$ can be interpreted as the *spatial* distribution of the energy density radiated by the antenna through the lth mode. We have

$$\begin{aligned}u_l(\mathbf{r}) = \int\int d^3k d^3k' \frac{\sqrt{F_l\left[\Xi_{nm}(\omega,\mathbf{k})\right] F_l\left[\Xi_{nm}(\omega,\mathbf{k}')\right]^*}}{\varepsilon_0} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} \\ \times \left|\mathbf{e}^*_l(\mathbf{k}) \cdot \mathbf{J}_{\text{ext}}(\omega_l(\mathbf{k}),\mathbf{k})\, \mathbf{e}^*_l(\mathbf{k}') \cdot \mathbf{J}_{\text{ext}}(\omega_l(\mathbf{k}'),\mathbf{k}')\right|.\end{aligned} \tag{102}$$

This new quantitative measure contains information about the spatial structure of the time-averaged energy surrounding a radiator specified by its externally enforced current distribution $\mathbf{J}_{\text{ext}}(t,\mathbf{r})$. For example, it can be directly used in studying the localization of the radiated energy in an antenna inside an artificial medium described by the dispersion profile $\chi_{nm}(\omega,\mathbf{k})$.

10.2 Nonlocal Electromagnetic Media

10.2.1 General Theoretical Background from the Field of Crystal Optics

By the term *nonlocal medium* we refer to a material described by response functions similar to (47). As we we noticed previously, it follows from this definition that the material exhibits a memory-like behavior in the sense that the response to a field excreted at a particular location appears to depend on the field values at *other* locations. We will show below that this phenomenon is very general and does not just refer to a particular physical process occurring in the crystal.

First, notice that we arrived to the definition (47) through a Fourier transform approach to the electromagnetic fields. Moreover, we were able to derive a relation connecting the traditional multipole approach and the Fourier approach. We found that nonlocality or spatial dispersion arises very naturally to account for nonmagnetic media. However, it is in the nature of the Fourier approach itself to introduce the spatial spectral variable $\mathbf{k}$ into the description of the material medium, and hence one can view nonlocality as a characteristics of the formalism itself, rather than a particular label given to an exotic physical process, for example exciotons-polaritons in crystal optics.

Let us start by providing a global qualitative look at the response of material media in classical and quantum optics. This view will serve as standard theoretical background upon which we measure our understanding of how to design artificial media.

Imagine that the material is composed of a system of uncoupled (hence, independent) oscillators. Each oscillator can interact with the applied electromagnetic fields by producing a dipole moment $\mathbf{p}$. From the basic picture of Lorentz models, we can express the functional dependence of this induced dipole moment on the temporal frequency ω in the broad Lorentzian form $\zeta/(\omega^2-\omega_0^2)$, where ω_0 is a constant called the eigenfrequency or the resonance frequency, and ζ the oscillator strength. In general, each independent oscillator will resonate with the applied field according to its own eigenfrequency and strength, and the medium overall response will be taken as the sum of all individual resonances. In this view, it is useful to think of each oscillator as representing an 'atom', even when its actual physical dimensions are much larger than real atoms. The essential idea in the art of artificial material design is taking this conceptual framework into its extreme by assuming that one can manipulate each atom individually in order to control and tailor the resulted material responses. The assumption that the atoms are *un*coupled will be translated to the fact that the resulted eigenfrequency ω_0 and oscillator strength ζ don't depend on wavelength, or equivalently on $\mathbf{k}$. For natural materials observed and studied through macroscopic electromagnetics, the atomic separation, for example in periodic structures like crystals, denoted here by a, is very small compared with the operating wavelength, i.e., we have $a/\lambda \ll 1$. In this case, all atoms appear to be in perfect phase synchronization and no significant coupling mechanism takes place.[10]

The situation is dramatically different in periodic structures, like photonic crystals and frequency selective surfaces, where, in this case, the operating wavelength can become appreciable compared with the characteristic spatial scale of the separation between the atoms (or unit cells), and hence interesting electromagnetic behavior can arise, like stopbands, localization, etc. It is still however possible to describe all these complex structures by employing an effective dielectric function that is *nonlocal*. Such function can contain the full information of the symmetry group of the periodic structure. Therefore, Maxwell's equations, written in terms of these equivalent responses functions, can be used to describe the electromagnetics of the medium without writing explicitly the set of the boundary conditions.[11] Aside from the economic advantage of such formulation, allowing the effective dielectric function to become nonlocal has the advantage of bringing the full power of the conceptual framework of effective medium theory right to the fore even though the artificial medium under consideration may not satisfy the natural condition of infinitesimally small atomic constituents.

[10] The fundamental pre-condition for this to be true is that the fields are averaged on a spatial scale much larger than this natural characteristic spatial scale, i.e., the atomic separation a.

[11] As an example for a concrete implementation of this general idea, see (6).

10.2.2 Negative Group Velocity Artificial Media

We now briefly demonstrate the above general theory by showing that a new genre of artificial (meta-) materials can be envisioned by thinking in terms of the Fourier-space formalism of this paper. Specifically, we consider an idealized isotropic and homogenous medium that exhibit both temporal and spatial dispersion.

The dispersion relation for the transverse waves is $\mathbf{k}\cdot\mathbf{k} = (\omega/c)^2 n^2(\omega,\mathbf{k})$, where we have *defined* the index of refraction as $n(\omega,\mathbf{k})^2 \equiv \varepsilon(\omega,\mathbf{k})\mu$. We will work with such general index of refraction given by $n = n(\omega,\mathbf{k})$. The group velocity is defined as $\mathbf{v}_g = \nabla_{\mathbf{k}}\omega$. It can be shown that an equation connecting the spatial and temporal dispersion such that the resulted medium supports negative group velocity propagation can be put in the following form

$$\frac{\omega}{c}\frac{\partial n(\omega,\mathbf{k})}{\partial k} - \frac{\gamma}{c}\left(1+\omega\frac{\partial}{\partial\omega}\right)n(\omega,\mathbf{k}) = 1. \tag{103}$$

where $\gamma = -\left|\mathbf{v}_g\right|$. We call this partial differential equation the dispersion engineering equation for negative group velocity. An exact solution for this equation can be attempted for interesting special cases.

Consider the boundary-value problem consisting of the partial differential equation (103) together with

$$\frac{\partial\gamma}{\partial\omega} = 0, n(\omega,k=k_1) = \phi(\omega), \tag{104}$$

where $\omega_1 < \omega < \omega_2, \omega_1 > 0, k_1 > 0$. Here, $k_1 < k_2$ and $\omega_1 < \omega_2$ are positive real numbers and $\phi(\omega)$ is a general function representing the boundary condition of the problem. In the interesting scenario where the group velocity is constant and negative, an exact solution was found to be (4)

$$n(\omega,k) = \frac{c(k-k_1)}{\omega} + \frac{1}{\omega}\left[\omega+\gamma(k-k_1)\right]\phi(\omega+\gamma(k-k_1)). \tag{105}$$

Such solution is then theoretically feasible. It demonstrates that there is rich degrees of freedom in the material design waiting for us to discover and exploit for novel and interesting applications. A more comprehensive theory for the negative group velocity metamaterial was developed by the authors in (4).

A. Magnetic Moments in Terms of Electric Moments

A.1 The Magnetic Moment Term

Multiply the equation of continuity (7) by r_l and integrate over all space to get

$$\int d^3r \frac{\partial\rho(t,\mathbf{r})}{\partial t} r_l = -\int d^3r\, r_l \nabla\cdot\mathbf{J}(t,\mathbf{r}). \tag{106}$$

Consider first the LHS of (106). By employing the definition of the dipole moment (30), we write immediately

$$\int d^3r \frac{\partial\rho(t,\mathbf{r})}{\partial t} r_l = \frac{\partial}{\partial t}\int d^3r\rho(t,\mathbf{r})\, r_l = \frac{\partial}{\partial t} p_l(t,\mathbf{r}). \tag{107}$$

Now we consider the RHS of (106). Write the divergence as $\nabla \cdot \mathbf{J}(t,\mathbf{r}) = \left(\partial/\partial r_s\right) J_s(t,\mathbf{r})$ and integrate by parts through the variable r_s to obtain

$$\begin{aligned}\int d^3r\, r_l \nabla \cdot \mathbf{J}(t,\mathbf{r}) &= \int d^3r\, r_l \tfrac{\partial}{\partial r_s} J_s(t,\mathbf{r}) \\ &= \int d^2r \int dr_s\, r_l \tfrac{\partial}{\partial r_s} J_s(t,\mathbf{r}) \\ &= \int d^2r \left[J_s(t,\mathbf{r})\, r_l \big|_{r_s=-\infty}^{r_s=+\infty} - \int dr_s\, \tfrac{\partial r_l}{\partial r_s} J_s(t,\mathbf{r}) \right] \\ &= \int d^2r \left[J_s(t,\mathbf{r})\, r_l \big|_{r_s=-\infty}^{r_s=+\infty} - \int dr_s\, \delta_s^l J_s(t,\mathbf{r}) \right] \\ &= \int d^2r \left[J_s(t,\mathbf{r})\, r_l \big|_{r_s=-\infty}^{r_s=+\infty} - \int dr_s\, J_l(t,\mathbf{r}) \right].\end{aligned} \tag{108}$$

Using the assumption that the surface current $J_s(t,\mathbf{r})$ vanishes on the surface of the integration volume, we obtain

$$\int d^3r\, r_l \nabla \cdot \mathbf{J}(t,\mathbf{r}) = -\int d^3r\, J_l(t,\mathbf{r}) \equiv -\mu_l(t,\mathbf{r}). \tag{109}$$

From (106), (107), and (109), we finally arrive to

$$\mu_l(t,\mathbf{r}) = \frac{\partial}{\partial t} p_l(t,\mathbf{r}). \tag{110}$$

A.2 The Magnetic Quadrable Term

Multiply the equation of continuity (7) by $r_l r_m$ and integrate over all space to get

$$\int d^3r \frac{\partial \rho(t,\mathbf{r})}{\partial t} r_n r_m = -\int d^3r\, r_n r_m \nabla \cdot \mathbf{J}(t,\mathbf{r}). \tag{111}$$

Consider first the LHS of (111). By employing the definition of the electric quadrable moment (31), we write immediately

$$\int d^3r \frac{\partial \rho(t,\mathbf{r})}{\partial t} r_n r_m = \frac{\partial}{\partial t} \int d^3r \rho(t,\mathbf{r})\, r_n r_m = \frac{\partial}{\partial t} q_{nm}(t). \tag{112}$$

Now let us take the RHS of (111). We first decompose the magnetic moment into the sum of symmetric and anti-symmetric parts as follows

$$x_n J_m = \frac{1}{2}(x_n J_m + x_m J_n) + \frac{1}{2}(x_n J_m - x_m J_n). \tag{113}$$

Now, write again the divergence as $\nabla \cdot \mathbf{J}(t,\mathbf{r}) = \left(\partial/\partial r_s\right) J_s(t,\mathbf{r})$ and integrate by parts through the variable r_s to obtain

$$\begin{aligned}\int d^3r\, r_n r_m \nabla \cdot \mathbf{J}(t,\mathbf{r}) &= \int d^3r\, r_n r_m \tfrac{\partial}{\partial r_s} J_s(t,\mathbf{r}) \\ &= \int d^2r \int dr_s\, r_n r_m \tfrac{\partial}{\partial r_s} J_s(t,\mathbf{r}) \\ &= \int d^2r \left[J_s(t,\mathbf{r})\, r_n r_m \big|_{r_s=-\infty}^{r_s=+\infty} - \int dr_s\, \tfrac{\partial}{\partial r_s}(r_n r_m) J_s(t,\mathbf{r}) \right] \\ &= \int d^2r \left[J_s(t,\mathbf{r})\, r_n r_m \big|_{r_s=-\infty}^{r_s=+\infty} - \int dr_s\, (r_n \delta_s^n + r_m \delta_s^m) J_s(t,\mathbf{r}) \right] \\ &= \int d^2r \left[J_s(t,\mathbf{r})\, r_n r_m \big|_{r_s=-\infty}^{r_s=+\infty} - \int dr_s\, \{r_n J_s(t,\mathbf{r}) + r_m J_s(t,\mathbf{r})\} \right]\end{aligned} \tag{114}$$

Using again the assumption that the surface current $J_s(t,\mathbf{r})$ vanishes on the surface of the integration volume, we obtain

$$\int d^3r\, r_n r_m \nabla \cdot \mathbf{J}(t,\mathbf{r}) = -\int d^3r\, [r_n J_s(t,\mathbf{r}) + r_m J_s(t,\mathbf{r})]. \tag{115}$$

From (111), (112), and (115), we reach

$$\int d^3r\, [r_n J_s(t,\mathbf{r}) + r_m J_s(t,\mathbf{r})] = \frac{\partial}{\partial t} q_{ns}(t). \tag{116}$$

The antisymmetrical part in (113) can be written readily in the form $1/2\epsilon_{lmn}\mu_{mn}(t)$, where magnetic quadrable moment μ_{mn} is defined as

$$\mu_{nm}(t) = \int d^3r\, r_m J_n(t,\mathbf{r}). \tag{117}$$

Therefore, one can express the *axial* vector as

$$\mathbf{m}(t) = \frac{1}{2}\int d^3r\, \mathbf{r} \times \mathbf{J}(t,\mathbf{r}). \tag{118}$$

It follows then

$$\begin{aligned} \epsilon_{lsn} m_n(t) &= \tfrac{1}{2}\int d^3r \epsilon_{lsn} \left(\mathbf{r} \times \mathbf{J}(t,\mathbf{r})\right)_n \\ &= \tfrac{1}{2}\int d^3r \epsilon_{lsn}\epsilon_{ns'n'} r_{s'} J_{n'}(t,\mathbf{r}) \\ &= \tfrac{1}{2}\int d^3r \left(\delta_{ss'}\delta_{nn'} - \delta_{sn'}\delta_{ns'}\right) r_{s'} J_{n'}(t,\mathbf{r}) \\ &= \tfrac{1}{2}\int d^3r \left[r_s J_n(t,\mathbf{r}) - r_n J_s(t,\mathbf{r})\right] \end{aligned} \tag{119}$$

where the the definition of the cross product (18) was used in the second equality, and the identity (22) was employed for the third equality. Thus, from (112), (113), (116), and (119) we finally arrive to

$$\mu_{ln}(t) = \frac{1}{2}\frac{\partial}{\partial t} q_{ln}(t) + \epsilon_{lns} m_s(t). \tag{120}$$

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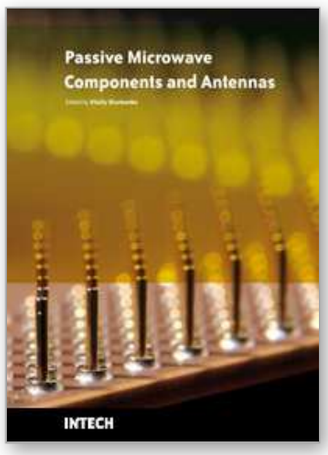

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Modelling and computations in electromagnetics is a quite fast-growing research area. The recent interest in this field is caused by the increased demand for designing complex microwave components, modeling electromagnetic materials, and rapid increase in computational power for calculation of complex electromagnetic problems. The first part of this book is devoted to the advances in the analysis techniques such as method of moments, finite-difference time- domain method, boundary perturbation theory, Fourier analysis, mode-matching method, and analysis based on circuit theory. These techniques are considered with regard to several challenging technological applications such as those related to electrically large devices, scattering in layered structures, photonic crystals, and artificial materials. The second part of the book deals with waveguides, transmission lines and transitions. This includes microstrip lines (MSL), slot waveguides, substrate integrated waveguides (SIW), vertical transmission lines in multilayer media as well as MSL to SIW and MSL to slot line transitions.

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