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# Direct visual servoing of planar manipulators using moments of planar targets

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## 1. Introduction

Visual servoing is a control strategy that uses a vision system with one or more cameras to establish the movement of a robotic system (Hutchinson et al., 1996) and emerges as a good alternative to close the control loop between the robot and its environment. Vision is a non-contact method that can relax the setup on an industrial robot and can give more characteristic of autonomy to advanced robots, which operate in adverse ambient or execute service tasks. Depending on the place where the cameras are located, visual servoing is classified by the fixed-camera and camera-in-hand configurations.

This chapter addresses the regulation of a planar manipulator by the visual servoing strategy in fixed-camera configuration. To simplify the problem, it is considered that the camera optical axis is perpendicular to the robot motion plane. The specification of the robot motion by the visual servoing strategy must be established through image features obtained from target objects in the scene or robot workspace. However, most of the developed theory in this area is based on the use of target objects with simple geometry like points, lines, cylinders or spheres (local features); or in more complex target objects but simplifying them to simple geometric objects through image processing techniques (Collewet & Chaumette, 2000; Benhimane & Malis, 2006). On the other hand, image moments represent global features of an arbitrary target object projected in image plane. The main objective of this chapter is to extend the use of simple target objects toward the use of a more complex target objects in the direct visual servoing of manipulators. Particularly, the target object will be a planar object with arbitrary shape and the image features will be computed through image moments combinations of this planar target.

The direct visual servoing term refers to the class of visual servo-controllers where the visual feedback is converted to joint torques instead to joint or Cartesian velocities; it does mean that the full nonlinear dynamic model of the robot is considered in the control analysis (Hager, 1997). The first explicit solution to the direct visual servoing problem is due to Miyazaki & Masutani (1990). We can find similar works in Espiau et al. (1992) and Kelly et al. (2000) where the problem is approached for a 6 degrees of freedom (6 d.o.f.) manipulator in camera-in-hand configuration. In Kelly (1996); Zergeroglu et al., (1999); Fang et al., (2002); Cheah et al., (2007); and Wang et al. (2008) we can see works that consider the dynamic

model of the manipulator in the control analysis and the robustness against to parametric uncertainties of the vision system.

The definition of image moments as image features for the visual servoing was expressed rigorously in Bien et al. (1993); although image moment combinations like the area, the orientation and the centroid were used to control 4 d.o.f. of a manipulator in an approximated manner. The analytic form of the time variation for the image moments was developed first in Tu & Fu (1995) and later in Chaumette (2004). This time variation for the image moments is expressed in terms of a matrix named image Jacobian (due to the image moments) which is essential for the design of a visual servoing scheme (Espiau et al., 1992; Hutchinson et al., 1996). This image Jacobian depends on the target object and the vision system 3D parameters.

In Chaumette (2004) is addressed a visual servo-control for the regulation of a 6 d.o.f. manipulator in camera-in-hand configuration by means of 6 image features that are based on combinations of image moments of a planar target with arbitrary shape; and in Tahri & Chamette (2005) is continued the later work with 6 image features based on image moments invariants to uncoupling the manipulator degrees of liberty and to achieve a better convergence domain with an adequate robot trajectory. It is worth noticing that these works do not belong to the direct visual servoing scheme because they just consider the cinematic model in the control analysis.

Specifically, in this chapter the controller is designed under the transpose Jacobian structure (Takegaki & Arimoto, 1981) and the robotic system stability and parametric robustness are analyzed in the Lyapunov sense. Also, we propose an alternative method for the determination of the time variation of the image moments based on the transformation of moments.

This chapter is organized as follows. Section 2 presents some definitions and the transformation between the two-dimensional Cartesian moments of a planar object and their corresponding image moments. Section 3 describes the formulation of the control problem. The design of the visual servo-controller is developed in Section 4 together with a robustness analysis. Section 5 proposes the selection of acceptable image features. To corroborate the performance of a robotic system with the designed controller, Section 6 presents some simulation data. Finally, the concluding remarks are exposed in Section 7.

## 2. Transformation of moments

### 2.1 Definitions

Consider a dense planar object  $\mathcal{O}_s$  placed on the plane  $S_1 - S_2$  of a frame  $\Sigma_s = \{S_1, S_2, S_3\}$ , also consider that the object is compound by a set of closed contours; then the two-dimensional Cartesian moments  ${}^s m_{ij}$  of  $\mathcal{O}_s$  (respect to  $\Sigma_s$ ), of order  $i + j$  ( $i, j \in \{0, 1, 2, \dots\}$ ), are defined as (Prokop & Reeves, 1992)

$${}^s m_{ij} = \iint_{\mathcal{O}_s} S_1^i S_2^j f(S_1, S_2) dS_1 dS_2 \quad (1)$$

where  $f(S_1, S_2)$  is the density distribution function of  $\mathcal{O}_s$ .

A special class of moments are determined placing the object  $\mathcal{O}_s$  such that its centroid  $S_g = [S_{g_1} \ S_{g_2}]^T = [{}^S m_{10}/{}^S m_{00} \ {}^S m_{01}/{}^S m_{00}]^T$  matches with the origin of the plane  $S_1 - S_2$ , these moments are called Cartesian centered moments  ${}^S \mu_{ij}$  of the object  $\mathcal{O}_s$ ; which are computed by

$${}^S \mu_{ij} = \iint_{\mathcal{O}_s} [S_1 - S_{g_1}]^i [S_2 - S_{g_2}]^j f(S_1, S_2) dS_1 dS_2 \quad (2)$$

where  $f(S_1, S_2)$  is the density distribution function of  $\mathcal{O}_s$ .

There exist the next relations between the regular moments and the centered moments:

$$\begin{aligned} {}^S \mu_{ij} &= \sum_{k=0}^i \sum_{l=0}^j \binom{i}{k} \binom{j}{l} [-S_{g_1}]^{i-k} [-S_{g_2}]^{j-l} {}^S m_{kl} \\ {}^S m_{kl} &= \sum_{m=0}^k \sum_{n=0}^l \binom{k}{m} \binom{l}{n} S_{g_1}^{k-m} S_{g_2}^{l-n} {}^S \mu_{mn} \end{aligned} \quad (3)$$

where

$$\binom{i}{k} = \frac{i!}{k!(i-k)!}.$$

## 2.2 The transformation

In this subsection is detailed the transformation between the two-dimensional Cartesian moments of a planar object with arbitrary shape  $\mathcal{O}_o$  (respect to a plane  $O_1 - O_2$ ) and the image moments computed from the projection of this object  $\mathcal{O}_o$  over  $\mathcal{O}_y$  in the image plane  $y_1 - y_2$ . The used camera model corresponds to the thin lens model one and it is considered that the camera optical axis is perpendicular to the object plane.

Figure 1 shows a view of the object perpendicular to the camera optical axis. Notice that it has been placed several Cartesian frames: the object frame  $\Sigma_o$  attached, precisely, to the object; the world frame  $\Sigma_w$  fixed somewhere in the scene or workspace; the camera frame  $\Sigma_c$ ; and the image plane  $y_1 - y_2$ . The variable  $\theta$  denotes the orientation of the object frame respect to the world frame,  $\psi$  represents also the orientation of the object frame but now respect to the image plane and  $\phi$  is the rotation of the camera frame respect to  $W_3$ ; in such a way that  $\theta = \psi + \phi$ .

A point  $x_o = [x_{o_1} \ x_{o_2} \ x_{o_3}]^T$  in the object, respect to  $\Sigma_o$ , can be transformed to  $\Sigma_w$  by means of

where the vector  $O_w^o \in \mathbb{R}^3$  denotes the position of the origin of  $\Sigma_o$  respect to  $\Sigma_w$  and

$$x_w = R_w^o(\theta) x_o + O_w^o \quad (4)$$

represents the rotation matrix of the frame  $\Sigma_o$  respect to  $\Sigma_w$ .

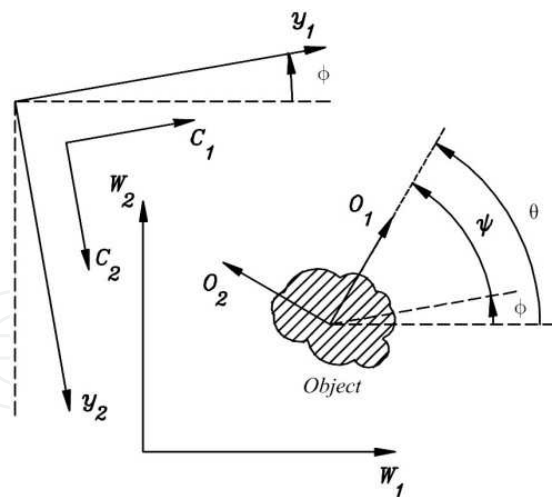


Fig. 1. View of the object perpendicular to the camera optical axis

$$R_W^O(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In its turn, the vector  $x_W = [x_{w_1} \ x_{w_2} \ x_{w_3}]^T$  can be transformed to camera frame coordinates through

$$x_C = R_W^C(\phi) [x_W - O_W^C] \quad (5)$$

where  $O_W^C \in \mathbb{R}^3$  is the position vector of  $\Sigma_C$  origin respect to  $\Sigma_W$  and

$$R_W^C(\phi) = \begin{bmatrix} \cos(\phi) & \sin(\phi) & 0 \\ \sin(\phi) & -\cos(\phi) & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (6)$$

is the rotation matrix of the camera frame respect to  $\Sigma_W$ .

For this particular case, the components of the vector  $x_C$ , after the substitution of (4) in (5) and simplifying, are

$$\begin{aligned} x_{C_1} &= \cos(\psi)x_{O_1} - \sin(\psi)x_{O_2} + c_1 \\ x_{C_2} &= -\sin(\psi)x_{O_1} - \cos(\psi)x_{O_2} + c_2 \\ x_{C_3} &= -[O_{W_3}^O - O_{W_3}^C] \end{aligned} \quad (7)$$

where

$$\begin{aligned} c_1 &= \cos(\phi)[O_{W_1}^O - O_{W_1}^C] + \sin(\phi)[O_{W_2}^O - O_{W_2}^C] \\ c_2 &= \sin(\phi)[O_{W_1}^O - O_{W_1}^C] - \cos(\phi)[O_{W_2}^O - O_{W_2}^C]. \end{aligned}$$

In this way, the mapping of a point  $x_o$  in the object (respect to  $\Sigma_o$ ) to the image plane is obtained through the coordinate transformations (4) and (5) and the next thin lens camera model (Hutchinson et al., 1996; Kelly, 1996):

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{\alpha \lambda}{x_{C_3} - \lambda} \begin{bmatrix} x_{C_1} \\ x_{C_2} \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$$

where  $\alpha$  is a conversion factor from meters to pixels,  $\lambda$  is the focal length of the lens, the vector  $[u_0 \ v_0]^T$  denotes the image center and  $x_C = [x_{C_1} \ x_{C_2} \ x_{C_3}]^T$  is the position vector of the point  $x_o$  respect to the camera frame; which is expressed in (7).

Observe that the depth  $x_{C_3}$  of all the points in the  $O_1 - O_2$  plane is the same for this adopted camera configuration. For the sake of notation, define

$$\gamma = \frac{\alpha \lambda}{x_{C_3} - \lambda},$$

which, therefore, is a constant. Hence, the imaging model, for the adopted camera configuration, can be expressed as

$$y = \gamma \begin{bmatrix} x_{C_1} \\ x_{C_2} \end{bmatrix} + \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}. \quad (8)$$

Now, according to (1), the two-dimensional Cartesian moments  ${}^o m_{ij}$  of the object  $\mathcal{O}_o$  respect to  $O_1 - O_2$  are computed through

$${}^o m_{ij} = \iint_{\mathcal{O}_o} x_{O_1}^i x_{O_2}^j dO_1 dO_2 \quad (9)$$

where, in this case,  $f(O_1, O_2) = 1$  for a point  $x_o$  in  $\mathcal{O}_o$  and null elsewhere. And the image moments  ${}^y m_{ij}$  of the object  $\mathcal{O}_y$  respect to the image plane  $y_1 - y_2$  in a binarized image  $f(y_1, y_2)$  are defined as

$${}^y m_{ij} = \iint_{\mathcal{O}_y} y_1^i y_2^j dy_1 dy_2 \quad (10)$$

where  $f(y_1, y_2) = 1$  for a point  $y = [y_1 \ y_2]^T$  inside the object  $\mathcal{O}_y$  and null elsewhere.

The purpose is to find the relation between the Cartesian moments of the object and their respective image moments. To this end, consider the following theorem for the change of variables with multiple integrals (Swokowski, 1988).

**Theorem 1.** If  $S_1 = g(U_1, U_2)$  and  $S_2 = h(U_1, U_2)$  is a coordinate transformation from  $\Sigma_U$  to  $\Sigma_S$ , then

$$\iint_{\mathcal{O}_S} f(S_1, S_2) dS_1 dS_2 = \pm \iint_{\mathcal{O}_U} f(g(U_1, U_2), h(U_1, U_2)) \left| \frac{\partial(S_1, S_2)}{\partial(U_1, U_2)} \right| dU_1 dU_2$$

where  $\mathcal{O}_U$  is the object respect to  $\Sigma_U$ ,  $\mathcal{O}_S$  is the transformed object from  $\Sigma_U$  to  $\Sigma_S$  and

$$\left| \frac{\partial(S_1, S_2)}{\partial(U_1, U_2)} \right| = \begin{vmatrix} \frac{\partial S_1}{\partial U_1} & \frac{\partial S_1}{\partial U_2} \\ \frac{\partial S_2}{\partial U_1} & \frac{\partial S_2}{\partial U_2} \end{vmatrix}$$

is the determinant of the transformation Jacobian.

It is chosen the positive or the negative sign depending on the fact that, if when a point  $[U_1 \ U_2]^T$  in the frontier of  $\mathcal{O}_U$  travels in the positive sense ( $\mathcal{O}_U$  always remains at the left side), the corresponding point  $[S_1 \ S_2]^T$  in  $\mathcal{O}_S$  travels in the positive or the negative sense, respectively.

Therefore, following Theorem 1 and using (7)-(10), we have that

$$\begin{aligned} {}^y m_{ij} &= \gamma^2 \iint_{\mathcal{O}_O} [\gamma x_{C_1} + u_0]^i [\gamma x_{C_2} + v_0]^j dO_1 dO_2 \\ &= \gamma^2 \iint_{\mathcal{O}_O} [\gamma \cos(\psi) x_{O_1} - \gamma \sin(\psi) x_{O_2} + \gamma c_1 + u_0]^i \cdot \\ &\quad [-\gamma \sin(\psi) x_{O_1} - \gamma \cos(\psi) x_{O_2} + \gamma c_2 + v_0]^j dO_1 dO_2 \end{aligned} \quad (11)$$

where the determinant of the transformation Jacobian is

$$\left| \frac{\partial(y_1, y_2)}{\partial(O_1, O_2)} \right| = \begin{vmatrix} -\gamma \sin(\psi) & -\gamma \cos(\psi) \\ -\gamma \cos(\psi) & \gamma \sin(\psi) \end{vmatrix} = -\gamma^2.$$

Note that it has been selected the negative sign in the application of the theorem because the axis  $C_3$  of the camera frame and the  $O_3$  axis of the object frame point in opposite directions and this provokes a negative sense in the motion of a point in the object frontier.

According to the multinomial theorem and the distributive law for multiple sums (Graham et al., 1989), (11) can be expressed as

$$\begin{aligned}
{}^y m_{ij} &= \gamma^2 \iint_{\mathcal{O}_0} \left[ \sum_{k_1, k_2, k_3} \frac{i!}{k_1! k_2! k_3!} [\gamma \cos(\psi) x_{O_1}]^{k_1} [-\gamma \sin(\psi) x_{O_2}]^{k_2} [\gamma c_1 + u_0]^{k_3} \right. \\
&\quad \left. \sum_{l_1, l_2, l_3} \frac{j!}{l_1! l_2! l_3!} [-\gamma \sin(\psi) x_{O_1}]^{l_1} [-\gamma \cos(\psi) x_{O_2}]^{l_2} [\gamma c_2 + v_0]^{l_3} \right] dO_1 dO_2 \\
&= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} [\gamma c_1 + u_0]^{k_3} \cdot \\
&\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} [\gamma c_2 + v_0]^{l_3} \iint_{\mathcal{O}_0} x_{O_1}^{k_1+l_1} x_{O_2}^{k_2+l_2} dO_1 dO_2 \\
&= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} [\gamma c_1 + u_0]^{k_3} \cdot \\
&\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} [\gamma c_2 + v_0]^{l_3} {}^O m_{k_1+l_1, k_2+l_2}
\end{aligned} \tag{12}$$

where  $k_1, k_2, k_3, l_1, l_2$  and  $l_3$  are nonnegative integers such that  $k_1 + k_2 + k_3 = i$  and  $l_1 + l_2 + l_3 = j$ . Notice that in the last step it has been used (9).

In this way, it is shown that by means of (12) the image moments of a planar object with arbitrary shape, in the adopted camera configuration, can be computed from the previous knowledge of the Cartesian moments of this object respect to its plane of definition. Though, it may be necessary to know camera parameters and the posture of the object frame  $\Sigma_O$  respect to the camera frame  $\Sigma_C$ .

Particularly, the object centroid in image plane  $y_g$  is related to the corresponding centroid in the object plane  $x_{g_O} = [x_{g_{O_1}} \ x_{g_{O_2}}]^T$  by

$$\begin{aligned}
y_g &= \begin{bmatrix} y_{g_1} \\ y_{g_2} \end{bmatrix} \\
&= \gamma \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ -\sin(\psi) & -\cos(\psi) \end{bmatrix} x_{g_O} + \begin{bmatrix} \gamma c_1 + u_0 \\ \gamma c_2 + v_0 \end{bmatrix}.
\end{aligned} \tag{13}$$

Following similar arguments we can find the next relation for the centered moments:

$$\begin{aligned}
{}^y \mu_{ij} &= \gamma^2 \sum_{k_1, k_2} \sum_{l_1, l_2} \frac{i!}{k_1! k_2!} \frac{j!}{l_1! l_2!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} \cdot \\
&\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} {}^O \mu_{k_1+l_1, k_2+l_2}
\end{aligned} \tag{14}$$

where  $k_1, k_2, l_1$  and  $l_2$  are nonnegative integers such that  $k_1 + k_2 = i$  and  $l_1 + l_2 = j$ .



### 3. Formulation

Consider a planar manipulator of  $n$  d.o.f. with the next dynamic model (Sciavicco & Siciliano, 2000):

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (15)$$

where  $q \in \mathbb{R}^n$  is the vector of joint displacements,  $\tau \in \mathbb{R}^n$  is the vector of applied joint torques,  $M(q) \in \mathbb{R}^{n \times n}$  is the symmetric and positive definite inertia matrix,  $C(q, \dot{q}) \in \mathbb{R}^{n \times n}$  is the matrix associated with the centrifugal and Coriolis torques, and  $g(q) \in \mathbb{R}^n$  is the vector of the gravitational torques.

Two important properties of the dynamics of a manipulator are as follows (Spong & Vidyasagar, 1989):

**Property 1.** The matrix  $C(q, \dot{q})$  and the time derivative  $\dot{M}(q)$  of the inertia matrix satisfy

$$\dot{q}^T \left[ \frac{1}{2} \dot{M}(q) - C(q, \dot{q}) \right] \dot{q} = 0, \quad \forall q, \dot{q} \in \mathbb{R}^n.$$

**Property 2.** The matrix  $C(q, \dot{q})$  satisfies

$$C(q, 0) = 0, \quad \forall q \in \mathbb{R}^n.$$

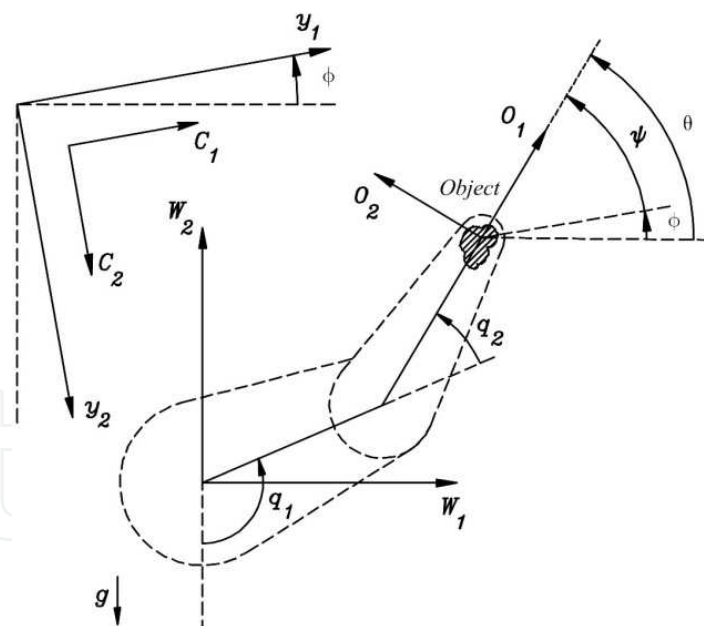


Fig. 2. View of the planar manipulator in fixed-camera configuration

Now, consider also a fixed camera observing the manipulator with a planar target object of arbitrary shape attached in its end-effector. Figure 2 presents a view of this robotic system. Also, in this figure, it can be observed the world frame  $\Sigma_w$  in the manipulator base, the camera frame  $\Sigma_c$  fixed somewhere in such a way that the planar target object can always be projected in the image plane  $y_1 - y_2$ , and the object frame  $\Sigma_o$  attached to the manipulator

end-effector where the target object is located. Notice that it is used the imaging model (8) where the camera optical axis is perpendicular to the manipulator motion plane; in this way the planes  $W_1 - W_2$ ,  $O_1 - O_2$ ,  $C_1 - C_2$ , and  $y_1 - y_2$  are parallel (this keep relation with the system already described in the previous section). The angle  $\phi$  denotes the rotation of the camera frame respect to  $W_3$ ,  $\theta$  is the orientation of the plane  $O_1 - O_2$  respect to  $W_1$  and  $\psi = \theta - \phi$  is the orientation of the plane  $O_1 - O_2$  in the image plane. The position of the manipulator end-effector respect to  $\Sigma_W$  is expressed trough the vector  $O_W^O \in \mathbb{R}^3$  because the frame  $\Sigma_O$  is attached to the end-effector.

Observe that if the planar manipulator is rotational, that is, with only revolute joints, then  $\theta = \sum_{i=1}^n q_i - \pi/2$  and  $\psi = \sum_{i=1}^n q_i - \pi/2 - \phi$ . Otherwise, just the revolute joints would contribute to the sum. If such contribution is denoted by  $\Sigma_{q_r}$ , then

$$\theta = \Sigma_{q_r} - \pi/2 \quad \text{and} \quad \psi = \Sigma_{q_r} - \pi/2 - \phi.$$

Now define an image feature vector  $s \in \mathbb{R}^r$  ( $r \geq m$ , where  $m$  is the dimension of the operational space) in function of the image moments  ${}^y m_{ij}$  of the projection in the image plane of the target object. It is worth noticing that also the centered image moments  ${}^y \mu_{ij}$  can be used since there exist the relation (3). According to (12) and (14) the image moments are in function of the variables  $\psi$  and  $O_W^O$ , which in their turn are in function of the vector of joint displacements  $q$ ; this means that

$$\begin{aligned} s &= s({}^y m_{ij}(q)) \\ &= s(q). \end{aligned}$$

Thus, the time variation of the image feature vector  $\dot{s}$  can be determined by

$$\begin{aligned} \dot{s} &= \frac{\partial s(q)}{\partial q} \dot{q} \\ &= J_s(q, {}^y m_{ij}) \dot{q} \end{aligned}$$

where

$$J_s(q, {}^y m_{ij}) = \frac{\partial s(q)}{\partial q}.$$

If the next factorization for the image feature vector  $s$  is valid:

$$s = \gamma A(\phi) f(q), \quad (16)$$

with  $A(\phi) \in \mathbb{R}^{r \times r}$  an orthogonal and constant matrix, then

$$\begin{aligned} \dot{s} &= \gamma A(\phi) \frac{\partial f(q)}{\partial q} \dot{q} \\ &= \gamma A(\phi) J(q, {}^y m_{ij}) \dot{q} \end{aligned} \quad (17)$$

where

$$J(q, {}^y m_{ij}) = \frac{\partial f(q)}{\partial q}. \quad (18)$$

On the other hand, denote with  $s_d \in \mathbb{R}^r$  the desired vector of image features which is supposed constant. Also, it is supposed that there exist at least one vector of joint displacements  $q_d$ , unknown but isolated, where the manipulator end-effector satisfies  $s_d$ . One way to establish such a reference  $s_d$  is by means of the teach-by-showing strategy (Weiss et al., 1987).

Finally, define the error vector of image features  $\tilde{s}$  as

$$\tilde{s} = s_d - s.$$

If  $s = \gamma A(\phi)f(q)$ , then

$$\tilde{s} = \gamma A(\phi)[f(q_d) - f(q)]. \quad (19)$$

In short, the control problem is to design a control law for the system just described such that determines the torques  $\tau$  to move the manipulator in such a way that the image feature vector  $s$  reaches the constant desired image feature vector  $s_d$  established previously; that is, the control objective is to drive asymptotically to zero the image feature error vector  $\tilde{s}$ , which is expressed by

$$\lim_{t \rightarrow \infty} \tilde{s}(t) = 0. \quad (20)$$

#### 4. Control design

The designed controller corresponds to the transpose Jacobian structure, which was originally introduced by Takegaki & Arimoto (1981) and applied to the direct visual servoing in the case of punctual image features in Kelly (1996). Assuming that the image feature vector meets  $s = \gamma A(\phi)f(q)$ , which is described in (16); the controller is expressed by

$$\tau = J(q, {}^y m_{ij})^T K_p A(\phi)^T \tilde{s} - K_v \dot{q} + g(q) \quad (21)$$

where  $K_p \in \mathbb{R}^{r \times r}$  is a symmetric and positive definite matrix called proportional gain and  $K_v \in \mathbb{R}^{n \times n}$  is another symmetric and positive definite matrix named derivative gain.

It is worth noticing that the controller needs the measures of the joint positions  $q$  and the joint velocities  $\dot{q}$ , the knowledge of the gravitational torques vector  $g(q)$  and the computation of the Jacobian  $J(q, {}^y m_{ij})$ . This Jacobian, depending on the selection of the image feature vector  $s$ , requires the direct measure in the image plane of certain image moments and the previous knowledge of some 3D parameters from the target object, the camera and the manipulator. However, it is not necessary to solve the inverse kinematics of the robotic system.

The closed-loop system corresponds to a nonlinear autonomous differential equation and it is obtained substituting in the manipulator dynamic model (15) the controller (21):

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = J(q, {}^y m_{ij})^T K_p A(\phi)^T \tilde{s} - K_v \dot{q};$$

which, in terms of the state vector  $\begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}^T \in \mathbb{R}^{2n}$ , it is expressed by means of

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M(q)^{-1} [J(q, {}^y m_{ij})^T K_p A(\phi)^T \tilde{s} - K_v \dot{q} - C(q, \dot{q}) \dot{q}] \end{bmatrix}. \quad (22)$$

The equilibrium points of the closed-loop system (22) satisfy

$$\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} q_e \\ 0 \end{bmatrix}$$

where  $q_e \in \mathbb{R}^n$  is the solution of

$$J(q, {}^y m_{ij})^T K_p A(\phi)^T \tilde{s}(q) = 0.$$

Suppose that the Jacobian  $J(q, {}^y m_{ij})$  is continuously differentiable with respect to each element of  $q$  and that it is of full range in  $q = q_d$ ; then the equilibrium

$$\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} q_d \\ 0 \end{bmatrix}$$

is a isolated equilibrium of (22), since it is also supposed that  $s(q) = 0$  has isolated solution in  $q = q_d$ .

The stability analysis will be held trough the direct method of Lyapunov (see Vidyasagar (1993) for example). In this way, consider the next Lyapunov function candidate:

$$V(q_d - q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \frac{1}{2} \gamma [f(q_d) - f(q)]^T K_p [f(q_d) - f(q)], \quad (23)$$

which is a locally definite positive function because in the first term,  $M(q) = M(q)^T > 0$ ; and in the second term  $\gamma > 0$ ,  $K_p = K_p^T > 0$  by design and it is supposed that  $s(q) = \gamma A(\phi)[f(q_d) - f(q)] = 0$  has an isolated solution in  $q = q_d$ . Notice that (19) is used. The time derivative of (23) yields

$$\dot{V}(q_d - q, \dot{q}) = \dot{q}^T M(q) \ddot{q} + \frac{1}{2} \dot{q}^T \dot{M}(q) \dot{q} + \gamma [\dot{f}(q_d) - \dot{f}(q)]^T K_p [f(q_d) - f(q)]. \quad (24)$$

Notice that  $\dot{f}(q_d) = 0$  and, according to (18),  $\dot{f}(q) = J(q, {}^y m_{ij}) \dot{q}$ ; in this way, substituting the later and the closed-loop system (22) in (24), (23) can be simplified to

$$\begin{aligned} \dot{V}(q_d - q, \dot{q}) &= \dot{q}^T [J(q, {}^y m_{ij})^T K_p A(\phi)^T \tilde{s} - \gamma J(q, {}^y m_{ij})^T K_p [f(q_d) - f(q)] - K_v \dot{q}] + \\ &\quad \dot{q}^T \left[ \frac{1}{2} \dot{M}(q) - C(q, \dot{q}) \right] \dot{q} \end{aligned}$$

Now, by means of Property 1 of the manipulator dynamic model and the fact that related with (19):  $f(q_d) - f(q) = \frac{1}{\gamma} A(\phi)^T \tilde{s}$ , since  $A(\phi)$  is an orthogonal matrix; the time derivative of (23) finally yields

$$\dot{V}(q_d - q, \dot{q}) = -\dot{q}^T K_v \dot{q}.$$

And, because  $K_v = K_v^T > 0$  by design, then  $\dot{V}(q_d - q, \dot{q})$  is a globally negative semidefinite function. Therefore, according to the direct Lyapunov method, the equilibrium

$$\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} q_d \\ 0 \end{bmatrix}$$

of the closed-loop system (22) is a stable equilibrium.

As mentioned, the closed-loop system is an autonomous one, hence it can be studied the asymptotic stability of the equilibrium by LaSalle's theorem (see Vidyasagar (1993) for example). For this purpose, in the region

$$\Omega = \left\{ \begin{bmatrix} q \\ \dot{q} \end{bmatrix} : \dot{V}(q_d - q, \dot{q}) = 0 \right\},$$

it is obtained the invariant set over the close-loop system (22) as  $\dot{q} = 0$  and  $q \in \mathbb{R}^n : J(q, {}^y m_{ij})^T K_p A(\phi)^T \tilde{s}(q) = 0$ . And according to the assumptions imposed on  $J(q, {}^y m_{ij})$  and  $\tilde{s}(q)$ , the later is satisfied in  $q = q_d$ . Hence, by LaSalle's theorem it is demonstrated that the equilibrium point

$$\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} q_d \\ 0 \end{bmatrix}$$

is asymptotically stable, it means that  $\lim_{t \rightarrow \infty} [q_d - q(t)] = 0$  and  $\lim_{t \rightarrow \infty} \dot{q}(t) = 0$  provided that  $q_d - q(0)$  and  $\dot{q}(0)$  are sufficiently small. Now, since  $q_d - q = 0$  implies that  $f(q_d) - f(q) = 0$ , then, according to (20),  $f(q_d) - f(q) = 0$  is true if and only if  $\tilde{s} = 0$ ; therefore, the control objective (21) is satisfied.

#### 4.1 Robustness analysis

Based on the results found in Kelly (1996), here it is analyzed the robustness of the controller (21) against uncertainties in 3D parameters of the target object and the camera. To this end, it will be used the first Lyapunov method instead of the direct Lyapunov method (see Vidyasagar (1993) for example).

Basically, it will be analyzed the uncertainty to  $\phi$  and to parameters in the Jacobian  $J(q, {}^y m_{ij})$ ; therefore there will be only an estimate  $\hat{\phi}$  of the angle  $\phi$  and an estimate  $\hat{J}(q, {}^y m_{ij})$  of the Jacobian  $J(q, {}^y m_{ij})$ . This modifies the control law (21) to the next:

$$\tau = \hat{J}(q, {}^y m_{ij})^T K_p A(\hat{\phi})^T \tilde{s} - K_v \dot{q} + g(q). \quad (25)$$

The closed-loop system with (25) as control law in terms of the state vector  $[\tilde{q}^T \quad \dot{q}^T]^T$ , where  $\tilde{q} = q_d - q$ , can be written as

$$\frac{d}{dt} \begin{bmatrix} \tilde{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -\dot{q} \\ M(q)^{-1} [\hat{J}(q, {}^y m_{ij})^T K_p A(\hat{\phi})^T \tilde{s} - K_v \dot{q} - C(q, \dot{q})\dot{q}] \end{bmatrix}, \quad (26)$$

which is an autonomous system with the origin as an equilibrium point.

To linearize the system (26), it will be applied the next lemma (Kelly, 1996):

**Lemma 1:** Consider the nonlinear system

$$\dot{x} = D(x)x + E(x)h(x) \quad (27)$$

where  $x \in \mathbb{R}^n$ ,  $D(x)$  and  $E(x)$  are  $n \times n$  nonlinear functions of  $x$  and  $h(x)$  is a  $n \times 1$  nonlinear function of  $x$ . Suppose that  $h(0) = 0$ , hence  $x = 0 \in \mathbb{R}^n$  is an equilibrium point of system (27). Then, the linearized system of (27) around the equilibrium point  $x = 0$  is given by

$$\dot{z} = \left[ D(0) + E(0) \frac{\partial h}{\partial x}(0) \right] z \quad (28)$$

where  $z \in \mathbb{R}^n$ .

Following Lemma 1, defining  $x = [\tilde{q}^T \quad \dot{q}^T]^T$  and using Property 2 of the manipulator dynamic model; the system (26) linearized around the origin results

$$\dot{z} = \begin{bmatrix} 0 & -I \\ \gamma M(q_d)^{-1} \hat{J}(q_d, {}^y m_{ij}^*)^T K_p A(\hat{\phi})^T A(\phi) J(q_d, {}^y m_{ij}^*) & -M(q_d)^{-1} K_v \end{bmatrix} z \quad (29)$$

where  ${}^y m_{ij}^*$  is  ${}^y m_{ij}$  evaluated in  $q = q_d$  and  $I$  is the identity matrix.

Now, define the error matrix of the Jacobian estimation  $\tilde{J}(q, {}^y m_{ij})$  as

$$\tilde{J}(q, {}^y m_{ij}) = \hat{J}(q, {}^y m_{ij}) - J(q, {}^y m_{ij}), \quad (30)$$

and the error of angle estimation  $\tilde{\phi}$  as

$$\tilde{\phi} = \hat{\phi} - \phi.$$

In what follows, to simplify the development, it will be considered a 2 d.o.f. manipulator (nonredundant) with the dimension of the image feature space equal to the dimension of the operational space, that is,  $n = m = r = 2$ . Also, it will be considered  $K_p = k_p I \in \mathbb{R}^{2 \times 2}$  (with  $k_p > 0$ ) and  $A(\phi) \in \mathbb{R}^{2 \times 2}$  an elementary rotation matrix (see Sciavicco & Siciliano (2000) for example), so that

$$\begin{aligned} A(\hat{\phi})^T A(\phi) &= A(\tilde{\phi})^T = A(-\tilde{\phi}) \\ \frac{A(\tilde{\phi})^T - A(\tilde{\phi})}{2} &= \sin(\tilde{\phi}) I_{a_2} \\ \frac{A(\tilde{\phi})^T + A(\tilde{\phi})}{2} &= \cos(\tilde{\phi}) I_2 \end{aligned} \quad (31)$$

where  $I_{a_2}$  is a  $2 \times 2$  skew symmetric matrix with unitary norm and  $I_2$  is the  $2 \times 2$  identity matrix. The last equation implies that if  $\cos(\tilde{\phi}) > 0$  then  $A(\tilde{\phi}) > 0$ .

Observing that  $z = [z_1^T \ z_2^T]^T$ , (29) can be rewritten as

$$\frac{d}{dt} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 & -I \\ M(q_d)^{-1}[F + G] & -M(q_d)^{-1}K_v \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad (32)$$

where

$$F = \gamma k_p J(q_d, {}^y m_{ij}^*)^T A(\tilde{\phi})^T J(q_d, {}^y m_{ij}^*) \quad (33)$$

and

$$G = \gamma k_p \tilde{J}(q_d, {}^y m_{ij}^*)^T A(\tilde{\phi})^T J(q_d, {}^y m_{ij}^*). \quad (34)$$

The stability analysis of (32) will be held through the next Lyapunov function candidate proposed in Kelly (1996):

$$V(z_1, z_2) = \frac{1}{2} [\varepsilon z_1 - z_2]^T M(q_d) [\varepsilon z_1 - z_2] + \frac{1}{2} z_1^T [F + \varepsilon K_v - \varepsilon^2 M(q_d)] z_1 \quad (35)$$

where<sup>1</sup>

$$\varepsilon = \frac{\lambda_m \{K_v\}}{2\lambda_M \{M(q_d)\}}$$

---

<sup>1</sup>The notations  $\lambda_m \{A\}$  and  $\lambda_M \{A\}$  indicate the smallest and largest eigenvalues of a matrix  $A$ , respectively.

is a positive constant, since both  $K_v$  and  $M(q_d)$  are symmetric and positive definite matrices. This also means that  $\lambda_m\{K_v\} > \varepsilon\lambda_M\{M(q_d)\}$ , which implies that the matrix  $K_v - \varepsilon M(q_d)$  is positive definite. Finally, suppose that  $A(\tilde{\phi}) > 0$ , if this is the case then the matrix  $F$  will be positive definite (due to the full range assumption for  $J(q_d, {}^y m_{ij})$ ). Hence, the Lyapunov function candidate, under the previous implications, is globally positive definite function. The time derivative of the Lyapunov function candidate (35) along the trajectories of the system (32) after some algebraic manipulations, results (eliminating the obvious arguments for simplicity)

$$\begin{aligned} \dot{V} = & -\gamma k_p \varepsilon z_1^T \left[ \frac{\hat{J}^T A(\tilde{\phi})^T \hat{J} + \tilde{J}^T A(\tilde{\phi}) \tilde{J}}{2} \right] z_1 + \gamma k_p \varepsilon z_1^T \hat{J}^T A(\tilde{\phi})^T \tilde{J} z_1 - \\ & z_2^T [K_v - \varepsilon M(q_d)] z_2 + \gamma k_p z_2^T \left[ \tilde{J}^T A(\tilde{\phi})^T [\hat{J} - \tilde{J}] \right] z_1 - \\ & \frac{1}{2} \gamma k_p z_1^T \left[ [\hat{J} - \tilde{J}]^T A(\tilde{\phi})^T [\hat{J} - \tilde{J}] - [\hat{J} - \tilde{J}]^T A(\tilde{\phi}) [\hat{J} - \tilde{J}] \right] z_2. \end{aligned} \quad (36)$$

Observe that for any matrix  $N \in \mathbb{R}^{2 \times 2}$ , the following is true:

$$N^T I_{a_2} N = \det\{N\} I_{a_2}.$$

According to the above and considering (31), (36) satisfies

$$\begin{aligned} \dot{V} \leq & -\gamma k_p \varepsilon \lambda_m\{\cos(\tilde{\phi}) \hat{J}^T \hat{J}\} \|z_1\|^2 + \gamma k_p \varepsilon \|\hat{J}\| \|\tilde{J}\| \|z_1\|^2 - \\ & \lambda_m\{K_v\} \|z_2\|^2 + \varepsilon \lambda_M\{M\} \|z_2\|^2 + \gamma k_p \|\hat{J}\| \|\tilde{J}\| \|z_1\| \|z_2\| + \gamma k_p \|\tilde{J}\|^2 \|z_1\| \|z_2\| + \\ & \gamma k_p |\sin(\tilde{\phi})| \left[ |\det\{\hat{J}\}| + 2\|\hat{J}\| \|\tilde{J}\| + |\det\{\tilde{J}\}| \right] \|z_1\| \|z_2\| \\ \leq & - \begin{bmatrix} \|z_1\| & \|z_2\| \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} \|z_1\| \\ \|z_2\| \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned} v_{11} &= \gamma k_p \varepsilon \lambda_m\{\cos(\tilde{\phi}) \hat{J}^T \hat{J}\} - \gamma k_p \varepsilon \|\hat{J}\| \|\tilde{J}\| \\ v_{12} = v_{21} &= -\frac{1}{2} \gamma k_p \left[ \|\hat{J}\| \|\tilde{J}\| + \|\tilde{J}\|^2 + |\sin(\tilde{\phi})| \left[ |\det\{\hat{J}\}| + 2\|\hat{J}\| \|\tilde{J}\| + |\det\{\tilde{J}\}| \right] \right] \\ v_{22} &= \frac{1}{2} \lambda_m\{K_v\}. \end{aligned}$$

Consequently, the time derivative of the Lyapunov function candidate is negative definite if  $v_{11} > 0$  and if  $v_{11}v_{22} - v_{12}^2 > 0$ . This leads to next inequalities:

$$\|\tilde{J}\| < \cos(\tilde{\phi}) \frac{\lambda_m\{\hat{J}^T \hat{J}\}}{\|\hat{J}\|} \quad (37)$$



$$\lambda_m^2\{K_v\} > \frac{\gamma k_p \lambda_M\{M\} \left[ \|\hat{J}\| \|\tilde{J}\| + \|\tilde{J}\|^2 + |\sin(\tilde{\phi})| \left[ |\det\{\hat{J}\}| + 2\|\hat{J}\| \|\tilde{J}\| + |\det\{\tilde{J}\}| \right] \right]^2}{\cos(\tilde{\phi}) \lambda_m\{\hat{J}^T \tilde{J}\} - \|\hat{J}\| \|\tilde{J}\|}. \quad (38)$$

The assumption  $A(\tilde{\phi}) > 0$  implies that  $\cos(\tilde{\phi}) > 0$ . In conclusion, if the inequality (37) is satisfied, then the inequality (38) indicates that there will be a symmetric and positive definite matrix  $K_v$  sufficiently large such that the equilibrium point  $[z_1^T \ z_2^T]^T = 0 \in \mathbb{R}^4$  of the linearized system (32) be asymptotically stable. This means, according to the first Lyapunov method, that the equilibrium point  $[\tilde{q}^T \ \dot{\tilde{q}}^T]^T = 0 \in \mathbb{R}^4$  of the original closed-loop system (26) is asymptotically stable and by the implications at the end of the previous subsection, then it is guaranteed the fulfillment of the control objective (20).

## 5. Selection of image features

In this section will be described two image feature vectors that are in function of image moments and that satisfy the requirements of the controllers (21) and (25). The first one is the object centroid in the image plane and the second one is a combination of image moments of order two.

### 5.1 Centroid

Equation (13) represents the mapping of the centroid  $x_{g_o}$  of the target object (respect to  $\Sigma_o$ ) located on the manipulator end-effector (see Figure 2) to the image plane. Note that  $\psi = \Sigma_{q_r} - \pi/2 - \phi$  and both  $c_1$  and  $c_2$  come from (7); also note that their time derivatives are expressed by

$$\begin{aligned} \dot{\psi} &= \dot{\Sigma}_{q_r} \\ \dot{c}_1 &= \cos(\phi) \dot{O}_{W_1}^o + \sin(\phi) \dot{O}_{W_2}^o \\ \dot{c}_2 &= \sin(\phi) \dot{O}_{W_1}^o - \cos(\phi) \dot{O}_{W_2}^o. \end{aligned} \quad (39)$$

Now, since it is a planar manipulator, the linear and angular velocities (respect to  $\Sigma_w$ ) denoted by  $v_w$  and  $w_w$ , respectively; can be expressed as

$$\begin{bmatrix} v_w \\ w_w \end{bmatrix} = \begin{bmatrix} \dot{O}_{W_1}^o & \dot{O}_{W_2}^o & 0 & 0 & 0 & \dot{\psi} \end{bmatrix}^T = J_{G_w}(q) \dot{q}$$

where  $J_{G_w}(q)$  is the geometric Jacobian of the manipulator; or simplifying

$$\begin{bmatrix} \dot{O}_{W_1}^o \\ \dot{O}_{W_2}^o \\ \dot{\psi} \end{bmatrix} = J_{G_{126W}}(q) \dot{q} \quad (40)$$

where  $J_{G_{126W}}(q)$  are the rows 1, 2 and 6 of the geometric Jacobian  $J_{G_W}(q)$ .

Consequently, the time derivative of (13) can be written as

$$\begin{aligned} \dot{y}_g &= \gamma \underbrace{\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}}_{A(\phi)} \underbrace{\begin{bmatrix} 1 & 0 & c_3(q, x_{g_o}) \\ 0 & -1 & -c_4(q, x_{g_o}) \end{bmatrix}}_{J(q, {}^y m_{ij})} J_{G_{126W}}(q) \dot{q} \\ &= \gamma A(\phi) J(q, {}^y m_{ij}) \dot{q} \end{aligned}$$

where

$$\begin{aligned} c_3(q, x_{g_o}) &= \cos(\Sigma_{q_r}) x_{g_{o1}} - \sin(\Sigma_{q_r}) x_{g_{o2}} \\ c_4(q, x_{g_o}) &= \sin(\Sigma_{q_r}) x_{g_{o1}} + \cos(\Sigma_{q_r}) x_{g_{o2}}; \end{aligned} \quad (41)$$

that is, the centroid  $y_g$ , which depends on the image moments of order one, fulfills the requirement of (16) and can be used in the controllers (21) or (25). It is worth noticing that the Jacobian  $J(q, {}^y m_{ij})$  depends on  $x_{g_o}$ , which is a 3D parameter of the target object; on the geometric Jacobian of the manipulator  $J_{G_{126W}}(q)$ ; and on the measures of the joint displacements  $q$ . If the centroid of the target object  $x_{g_o}$  coincides with the origin of the frame  $\Sigma_o$ , then we have the same robotic system as in Kelly (1996).

## 5.2 Image features in function of image moments of order two

Next, it is described an image feature vector in function of the image moments of order two that fulfills the requirements of the controllers (21) and (25). In this sense, it is necessary to compute the time variation of the image moments of order two.

Thus, from (49), (39) and (40):

$$\begin{aligned} {}^y \dot{m}_{11} &= \gamma \begin{bmatrix} \cos(\phi) & -\sin(\phi) \end{bmatrix} \begin{bmatrix} {}^y m_{01} & -{}^y m_{10} \\ -{}^y m_{10} & -{}^y m_{01} \end{bmatrix} \begin{bmatrix} 1 & 0 & c_3(q, x_{g_o}) \\ 0 & 1 & c_4(q, x_{g_o}) \end{bmatrix} J_{G_{126W}}(q) \dot{q} + \\ &\quad \begin{bmatrix} 0 & 0 & {}^y \mu_{02} - {}^y \mu_{20} \end{bmatrix} J_{G_{126W}}(q) \dot{q}. \end{aligned} \quad (42)$$

Now, consider  $s_{aux} = \frac{1}{2} [{}^y m_{02} - {}^y m_{20}]$ , in such a way that from (49), (39) and (40),  $\dot{s}_{aux}$  results

$$\begin{aligned} \dot{s}_{aux} &= \gamma \begin{bmatrix} \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} {}^y m_{01} & -{}^y m_{10} \\ -{}^y m_{10} & -{}^y m_{01} \end{bmatrix} \begin{bmatrix} 1 & 0 & c_3(q, x_{g_o}) \\ 0 & 1 & c_4(q, x_{g_o}) \end{bmatrix} J_{G_{126W}}(q) \dot{q} - \\ &\quad \begin{bmatrix} 0 & 0 & 2 {}^y \mu_{11} \end{bmatrix} J_{G_{126W}}(q) \dot{q}. \end{aligned} \quad (43)$$

The time variation of the centered image moments can be computed from (50), (39) and (40), yielding

$$\begin{aligned}
{}^y\dot{\mu}_{11} &= \begin{bmatrix} 0 & 0 & {}^y\mu_{02} - {}^y\mu_{20} \end{bmatrix} J_{G_{126W}}(q) \dot{q} \\
{}^y\dot{\mu}_{20} &= \begin{bmatrix} 0 & 0 & {}^y\mu_{11} \end{bmatrix} J_{G_{126W}}(q) \dot{q} \\
{}^y\dot{\mu}_{02} &= \begin{bmatrix} 0 & 0 & -{}^y\mu_{11} \end{bmatrix} J_{G_{126W}}(q) \dot{q}.
\end{aligned} \tag{44}$$

The proposed image feature vector in function of image moments of order two  $s_{m_2}$  is expressed as

$$s_{m_2} = \begin{bmatrix} {}^y m_{11} - {}^y \mu_{11} \\ \frac{1}{2} [{}^y m_{02} - {}^y m_{20}] + {}^y \mu_{20} - {}^y \mu_{02} \end{bmatrix},$$

which from (42)-(44) has the next time derivative:

$$\begin{aligned}
\dot{s}_{m_2} &= \gamma \underbrace{\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}}_{A(\phi)} \underbrace{\begin{bmatrix} {}^y m_{01} & -{}^y m_{10} \\ -{}^y m_{10} & -{}^y m_{01} \end{bmatrix} \begin{bmatrix} 1 & 0 & c_3(q, x_{g_o}) \\ 0 & 1 & c_4(q, x_{g_o}) \end{bmatrix}}_{J(q, {}^y m_{ij})} J_{G_{126W}}(q) \dot{q} \\
&= \gamma A(\phi) J(q, {}^y m_{ij}) \dot{q},
\end{aligned} \tag{45}$$

therefore  $s_{m_2}$  is another image feature vector that fulfills the conditions of the controllers (21) and (25).

Notice that

$$s'_{m_2} = {}^y m_{00}^p \begin{bmatrix} {}^y m_{11} - {}^y \mu_{11} \\ \frac{1}{2} [{}^y m_{02} - {}^y m_{20}] + {}^y \mu_{20} - {}^y \mu_{02} \end{bmatrix} \tag{46}$$

where  $p \in \mathfrak{R}$  is a constant, with time derivative

$$\begin{aligned}
\dot{s}'_{m_2} &= \gamma \underbrace{\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}}_{A(\phi)} \underbrace{{}^y m_{00}^p \begin{bmatrix} {}^y m_{01} & -{}^y m_{10} \\ -{}^y m_{10} & -{}^y m_{01} \end{bmatrix} \begin{bmatrix} 1 & 0 & c_3(q, x_{g_o}) \\ 0 & 1 & c_4(q, x_{g_o}) \end{bmatrix}}_{J(q, {}^y m_{ij})} J_{G_{126W}}(q) \dot{q} \\
&= \gamma A(\phi) J(q, {}^y m_{ij}) \dot{q},
\end{aligned}$$

is another acceptable image feature vector (since  ${}^y m_{00}$  is constant in the configuration of the robotic system considered).

6. Simulations

To illustrate the performance of the direct visual servoing just described, it will be presented simulations using the model of a 2 d.o.f. manipulator that is in the Robotics Laboratory of CICESE. A scheme of such manipulator can be seen in Figure 3.

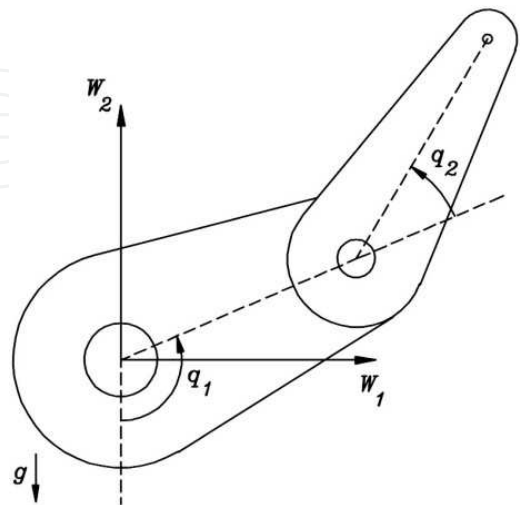


Fig. 3. Scheme of the manipulator

Respect to its dynamic model (15), their elements are expressed by

$$\begin{aligned} M(q) &= \begin{bmatrix} 0.3353 + 0.0244 \cos(q_2) & 0.0127 + 0.0122 \cos(q_2) \\ 0.0127 + 0.0122 \cos(q_2) & 0.0127 \end{bmatrix} \text{ [Nm sec}^2\text{/rad]} \\ C(q, \dot{q}) &= \begin{bmatrix} -0.0122 \sin(q_2) \dot{q}_2 & -0.0122 \sin(q_2) \dot{q}_1 - 0.0122 \sin(q_2) \dot{q}_2 \\ 0.0122 \sin(q_2) \dot{q}_1 & 0 \end{bmatrix} \text{ [Nm sec/rad]} \quad (47) \\ g(q) &= \begin{bmatrix} 11.5081 \sin(q_1) + 0.4596 \sin(q_1 + q_2) \\ 0.4596 \sin(q_1 + q_2) \end{bmatrix} \text{ [Nm]}. \end{aligned}$$

Description	Notation	Value	Units
Conversion factor ([m] to [pixels])	$\alpha$	72000	pixels/m
Focal length of the lens	$\lambda$	0.0075	m
Image center	$[u_0 \ v_0]^T$	$[160 \ 120]^T$	pixels
Camera frame position	$O_W^C$	$[0 \ 0 \ 3]^T$	m
Camera frame orientation	$\phi$	$10 \pi / 180$	rad

Table 1. Camera parameters

Now, its geometric Jacobian  $J_{G_{126W}}(q)$  is

$$J_{G_{126W}}(q) = \begin{bmatrix} l[\cos(q_1) + \cos(q_1 + q_2)] & l \cos(q_1 + q_2) \\ l[\sin(q_1) + \sin(q_1 + q_2)] & l \sin(q_1 + q_2) \\ 1 & 1 \end{bmatrix}$$

(48)

where  $l = 0.26$  [m].

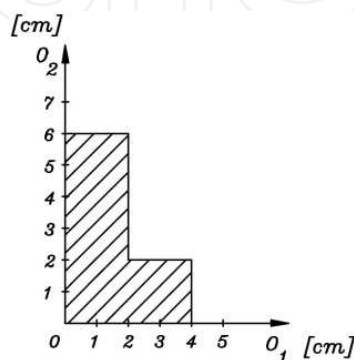


Fig. 4. The planar target object

Table 1 presents the intrinsic and extrinsic camera parameters, which correspond to the adopted camera configuration. Note that there are eight camera parameters, however, the controller only needs a estimation of the parameter  $\phi$ .

Figure 4 shows the planar target object which has a relative complex shape to facilitate the simulations, but the target can be a more sophisticated planar object like a photography for example. The two-dimensional Cartesian moments of this target object to order two, respect to the plane  $O_1 - O_2$ , are concentrated in Table 2.

Moment	Value	Units
${}^O m_{00}$	$1.6 \times 10^{-3}$	$\text{m}^2$
${}^O m_{10}$	$2.4 \times 10^{-5}$	$\text{m}^3$
${}^O m_{01}$	$4 \times 10^{-5}$	$\text{m}^3$
$x_{g_o}$	$[1.5 \ 2.5]^T \times 10^{-2}$	$\text{m}$
${}^O m_{11}$	$4.8 \times 10^{-7}$	$\text{m}^4$
${}^O m_{20}$	$5.3333 \times 10^{-7}$	$\text{m}^4$
${}^O m_{02}$	$1.4933 \times 10^{-6}$	$\text{m}^4$
${}^O \mu_{11}$	$-1.2 \times 10^{-7}$	$\text{m}^4$
${}^O \mu_{20}$	$1.7333 \times 10^{-7}$	$\text{m}^4$
${}^O \mu_{02}$	$4.9333 \times 10^{-7}$	$\text{m}^4$

Table 2. Two-dimensional Cartesian moments of the target object respect to  $O_1 - O_2$

Respect to the 3D parameters of the target object, the controller only needs a estimation of the object centroid  $x_{g_o}$ .

The image feature vector selected corresponds to (46) with  $p = -1$ , denote with  $s_a$  this image feature vector expressed with

$$s_a = \frac{1}{{}^y m_{00}} \left[ \frac{1}{2} [{}^y m_{02} - {}^y m_{20}] + {}^y \mu_{20} - {}^y \mu_{02} \right],$$

with Jacobian  $J(q, {}^y m_{ij})$  described by

$$J(q, {}^y m_{ij}) = \begin{bmatrix} y_{g_2} & -y_{g_1} \\ -y_{g_1} & -y_{g_2} \end{bmatrix} \begin{bmatrix} 1 & 0 & c_3(q, x_{g_o}) \\ 0 & 1 & c_4(q, x_{g_o}) \end{bmatrix} J_{G_{126W}}(q).$$

The initial condition is the manipulator at rest with position vector  $q(0) = 0$  [rad]. By means of the teach-by-showing method it is computed the desired image feature vector  $s_{a_d}$  on the desired manipulator configuration, such that  $q_d = [45\pi/180 \ 90\pi/180]^T$  [rad], obtaining

$$s_{a_d} = \begin{bmatrix} 2.8408 \\ -1.7333 \end{bmatrix} \times 10^4 \text{ [pixels}^2\text{]}.$$

The controller (25) was tuned with the gains:

$$K_p = 4I_2 \times 10^{-6} \quad \text{and} \quad K_v = 0.8I_2;$$

but with the next estimations:

$$\begin{aligned} \hat{\phi} &= 0.5 \phi \\ \hat{x}_{g_{o1}} &= 0.5 x_{g_{o1}} \\ \hat{x}_{g_{o2}} &= 0.5 x_{g_{o2}} \\ \hat{l} &= 0.95 l. \end{aligned}$$

This satisfies the inequalities (37) and (38), since

$$\begin{aligned} \|\tilde{J}\| &< \cos(\tilde{\phi}) \frac{\lambda_m\{\hat{J}^T \hat{J}\}}{\|\hat{J}\|} \\ 7.8242 &< 15.8702 \\ \lambda_m^2\{K_v\} &> \frac{\gamma k_p \lambda_M\{M\} [\|\hat{J}\| \|\tilde{J}\| + \|\tilde{J}\|^2 + |\sin(\tilde{\phi})| [|\det\{\hat{J}\}| + 2\|\hat{J}\| \|\tilde{J}\| + |\det\{\tilde{J}\}|]^2]}{\cos(\tilde{\phi}) \lambda_m\{\hat{J}^T \hat{J}\} - \|\hat{J}\| \|\tilde{J}\|} \\ 0.6400 &> 0.5524. \end{aligned}$$

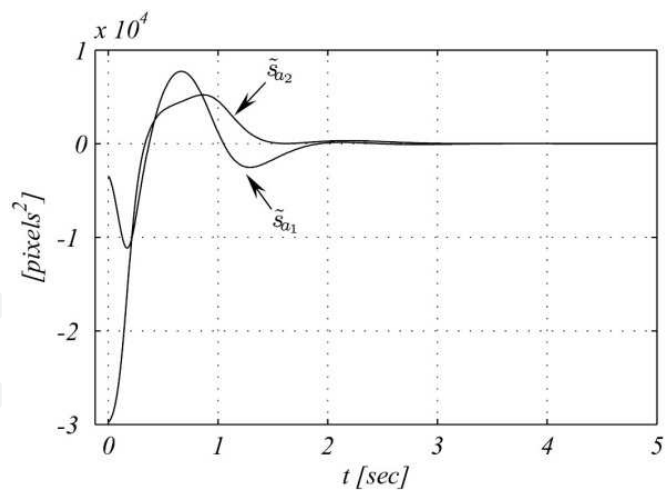


Fig. 5. Evolving respect to time of  $\tilde{s}_a$

Observe trough figures 5 and 6 that the performance of the robotic system is satisfactory. The error image feature vector is practically null in about 3 [sec], as can be seen in Figure 5; this shows that the control objective is reached. Likewise, the trace of the centroid  $y_g$  of the target object is reported in Figure 6 together with 3 snapshot of the manipulator and the target object configuration at the initial condition when  $t = 0$  [sec], when  $t = 0.2$  [sec] and when the simulation ends at  $t = 5$  [sec].

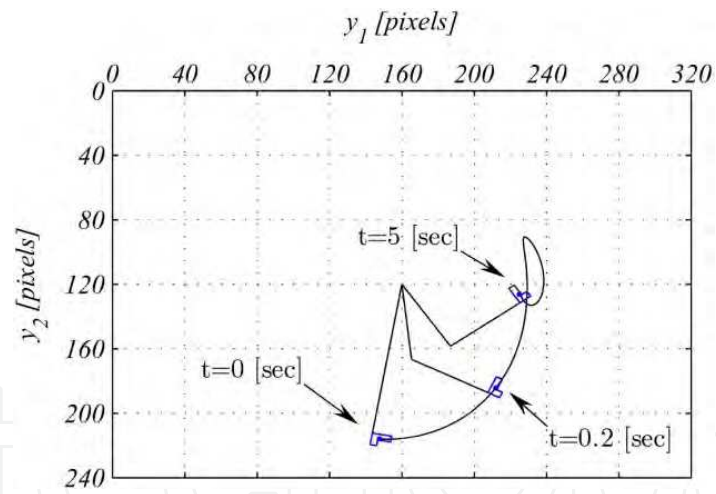


Fig. 6. Trace of the target object centroid  $y_g$

### 7. Conclusions

In this chapter is designed a direct visual servo control for planar manipulators in fixed-camera configuration, with the camera optical axis perpendicular to the robot motion plane. The target is a planar object with arbitrary shape, that is, it can be of complex geometry. It has been proposed a global image feature vector in function of the image moments of such a target object; and in base on the transformation of moments it is computed the time variation of this global image features. This represents an alternative development compared to the one described in Chaumette (2004).

The designed controller corresponds to the transpose Jacobian structure and, by means of the Lyapunov theory, it is demonstrated that the controller is robust against uncertainties in 3D parameters of the target object, the camera and the geometric Jacobian of the manipulator. Finally, simulations are presented to validate the fulfillment of the control objective.

## 8. Appendix

In this appendix is developed the analytic form of the time variation for the regular and centered image moments with the configuration of the system detailed in Section 2.

### 8.1 Regular image moments

The time variation of the regular image moments  ${}^y\dot{m}_{ij}$  can be computed by the time derivative of (12), therefore

$${}^y\dot{m}_{ij} = a_1 + a_2 + a_3 + a_4 + a_5 + a_6$$

where

$$\begin{aligned} a_1 &= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} k_1 [\gamma \cos(\psi)]^{k_1-1} [-\gamma \sin(\psi)]^{k_2} [\gamma c_1 + u_0]^{k_3} \cdot \\ &\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} [\gamma c_2 + v_0]^{l_3} m_{k_1+l_1, k_2+l_2} [-\gamma \sin(\psi)] \dot{\psi} \\ a_2 &= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} k_2 [-\gamma \sin(\psi)]^{k_2-1} [\gamma c_1 + u_0]^{k_3} \cdot \\ &\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} [\gamma c_2 + v_0]^{l_3} m_{k_1+l_1, k_2+l_2} [-\gamma \cos(\psi)] \dot{\psi} \\ a_3 &= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} k_3 [\gamma c_1 + u_0]^{k_3-1} \cdot \\ &\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} [\gamma c_2 + v_0]^{l_3} m_{k_1+l_1, k_2+l_2} [\gamma \dot{c}_1] \\ a_4 &= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} [\gamma c_1 + u_0]^{k_3} \cdot \\ &\quad l_1 [-\gamma \sin(\psi)]^{l_1-1} [-\gamma \cos(\psi)]^{l_2} [\gamma c_2 + v_0]^{l_3} m_{k_1+l_1, k_2+l_2} [-\gamma \cos(\psi)] \dot{\psi} \\ a_5 &= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} [\gamma c_1 + u_0]^{k_3} \cdot \\ &\quad [-\gamma \sin(\psi)]^{l_1} l_2 [-\gamma \cos(\psi)]^{l_2-1} [\gamma c_2 + v_0]^{l_3} m_{k_1+l_1, k_2+l_2} [\gamma \sin(\psi)] \dot{\psi} \\ a_6 &= \gamma^2 \sum_{k_1, k_2, k_3} \sum_{l_1, l_2, l_3} \frac{i!}{k_1! k_2! k_3!} \frac{j!}{l_1! l_2! l_3!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} [\gamma c_1 + u_0]^{k_3} \cdot \\ &\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} l_3 [\gamma c_2 + v_0]^{l_3-1} m_{k_1+l_1, k_2+l_2} [\gamma \dot{c}_2] \end{aligned}$$

and  $k_1, k_2, k_3, l_1, l_2$  and  $l_3$  are nonnegative integers such that  $k_1 + k_2 + k_3 = i$  and  $l_1 + l_2 + l_3 = j$ .



Simplifying,

$$\begin{aligned}a_3 &= i\gamma^y m_{i-1,j} \dot{c}_1 \\a_6 &= j\gamma^y m_{i,j-1} \dot{c}_2 \\a_1 + a_2 &= i \left[ {}^y m_{i-1,j+1} - [\gamma c_2 + v_0]^y m_{i-1,j} \right] \dot{\psi} \\a_4 + a_5 &= j \left[ -{}^y m_{i+1,j-1} + [\gamma c_1 + u_0]^y m_{i,j-1} \right] \dot{\psi}.\end{aligned}$$

Finally,

$$\begin{aligned}{}^y \dot{m}_{ij} &= i\gamma^y m_{i-1,j} \dot{c}_1 + j\gamma^y m_{i,j-1} \dot{c}_2 + i \left[ {}^y m_{i-1,j+1} - [\gamma c_2 + v_0]^y m_{i-1,j} \right] \dot{\psi} + \\&\quad j \left[ -{}^y m_{i+1,j-1} + [\gamma c_1 + u_0]^y m_{i,j-1} \right] \dot{\psi}.\end{aligned}\quad (49)$$

## 8.2 Centered image moments

The time variation of the centered image moments  ${}^y \dot{\mu}_{ij}$  can be computed by the time derivative of (14), hence

$${}^y \dot{\mu}_{ij} = b_1 + b_2 + b_3 + b_4$$

where

$$\begin{aligned}b_1 &= \gamma^2 \sum_{k_1, k_2} \sum_{l_1, l_2} \frac{i!}{k_1! k_2!} \frac{j!}{l_1! l_2!} k_1 [\gamma \cos(\psi)]^{k_1-1} [-\gamma \sin(\psi)]^{k_2} \cdot \\&\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} {}^{l_2} \mu_{k_1+l_1, k_2+l_2} [-\gamma \sin(\psi)] \dot{\psi} \\b_2 &= \gamma^2 \sum_{k_1, k_2} \sum_{l_1, l_2} \frac{i!}{k_1! k_2!} \frac{j!}{l_1! l_2!} [\gamma \cos(\psi)]^{k_1} k_2 [-\gamma \sin(\psi)]^{k_2-1} \cdot \\&\quad [-\gamma \sin(\psi)]^{l_1} [-\gamma \cos(\psi)]^{l_2} {}^{l_2} \mu_{k_1+l_1, k_2+l_2} [-\gamma \cos(\psi)] \dot{\psi} \\b_3 &= \gamma^2 \sum_{k_1, k_2} \sum_{l_1, l_2} \frac{i!}{k_1! k_2!} \frac{j!}{l_1! l_2!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} \cdot \\&\quad l_1 [-\gamma \sin(\psi)]^{l_1-1} [-\gamma \cos(\psi)]^{l_2} {}^{l_2} \mu_{k_1+l_1, k_2+l_2} [-\gamma \cos(\psi)] \dot{\psi} \\b_4 &= \gamma^2 \sum_{k_1, k_2} \sum_{l_1, l_2} \frac{i!}{k_1! k_2!} \frac{j!}{l_1! l_2!} [\gamma \cos(\psi)]^{k_1} [-\gamma \sin(\psi)]^{k_2} \cdot \\&\quad [-\gamma \sin(\psi)]^{l_1} l_2 [-\gamma \cos(\psi)]^{l_2-1} {}^{l_2} \mu_{k_1+l_1, k_2+l_2} [\gamma \sin(\psi)] \dot{\psi}\end{aligned}$$

and  $k_1, k_2, l_1$ , and  $l_2$  are nonnegative integers such that  $k_1 + k_2 = i$  and  $l_1 + l_2 = j$ .

Simplifying,

$$\begin{aligned}b_1 + b_2 &= i {}^y \mu_{i-1,j+1} \dot{\psi} \\b_3 + b_4 &= -j {}^y \mu_{i+1,j-1} \dot{\psi}.\end{aligned}$$

Finally,

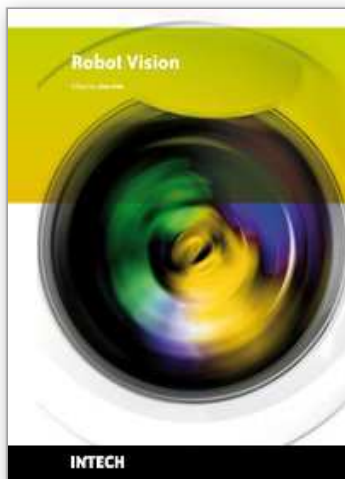
$${}^y \dot{\mu}_{ij} = [i {}^y \mu_{i-1,j+1} - j {}^y \mu_{i+1,j-1}] \dot{\psi}.\quad (50)$$

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The purpose of robot vision is to enable robots to perceive the external world in order to perform a large range of tasks such as navigation, visual servoing for object tracking and manipulation, object recognition and categorization, surveillance, and higher-level decision-making. Among different perceptual modalities, vision is arguably the most important one. It is therefore an essential building block of a cognitive robot. This book presents a snapshot of the wide variety of work in robot vision that is currently going on in different parts of the world.

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