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## Motion Control

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### 1. Introduction

This chapter presents an introduction to the various methods of controlling the motion of rigid manipulators. Motion control of robotic manipulators has been the subject of considerable research, and many control schemes have been evolved. Typical and recently proposed motion control strategies are introduced and the strengths and weaknesses of each control scheme are also described in this chapter. We assume that the robotic manipulators are rigid, that is, the manipulators do not have flexible links and elastic joints.

In this chapter, we discuss some useful properties of the robot dynamic equations, which are used in deriving robot control schemes in section 2. Proportional-integral-derivative (PID) control schemes, which are widely used in robotic manipulator control, are introduced in section 3. Computed torque control is described in section 4. A modified computed-torque control scheme which overcomes some disadvantages of the conventional one is also introduced in the section. To compensate for parametric uncertainties in the robot dynamic equations, various adaptive strategies for the control of robotic manipulators are introduced in section 5. We discuss the robust control that is capable of compensating for both structured and unstructured uncertainties in section 6 and conclude the chapter in section 7.

### 2. Robot Dynamic Equation

In the absence of friction and other disturbances, the dynamic equation of an  $n$ -link robot manipulator can be written as:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau, \quad (1)$$

where  $q$  is the  $n \times 1$  vector of joint variables,  $\tau$  is the  $n \times 1$  vector of input torques,  $M(q)$  is the  $n \times n$  symmetric positive-definite manipulator inertia matrix,  $C(q, \dot{q})\dot{q}$  is the  $n \times 1$  vector of centrifugal and Coriolis torques and  $g(q)$  is the  $n \times 1$  vector of gravitational torques. The control schemes that will be introduced in this chapter are based on some important properties of dynamic equation (1).

*Property of Inertia Matrix:*

The inertia matrix  $M(q)$  is symmetric positive-definite and bounded as

$$\mu_1 I \leq M(q) \leq \mu_2 I, \quad (2)$$

where  $\mu_1$  and  $\mu_2$  are the positive scalars that may be computed for any given arm.

*Property of Centrifugal and Coriolis Vector:*

The matrix

$$\dot{M}(q) - 2C(q, \dot{q}) \quad (3)$$

is skew-symmetric. This property implies that  $\dot{M}(q) = C(q, \dot{q}) + C^T(q, \dot{q})$ . The matrix  $C(q, \dot{q})$  is quadratic in  $\dot{q}$  and bounded so that  $\|C(q, \dot{q})\| \leq c_0 \|\dot{q}\|$  for some positive constant  $c_0$ .

*Property of Linearity in the Parameters:*

(Craig, 1988) exploited a property that the equation (1) is linear in the inertia parameters. This is important, because some or all of the parameters may be unknown; thus the dynamics are linear in the unknown terms:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = W(q, \dot{q}, \ddot{q})\varphi, \quad (4)$$

where  $\varphi$  is a vector of unknown constant parameters and  $W(q, \dot{q}, \ddot{q})$  is a known matrix of robot functions.

Those properties are used in deriving robot control schemes in subsequent sections.

### 3. PID Control

The conventional proportional-derivative (PD) and PID controllers are general feedback control mechanisms that are widely used in industrial control systems. These controllers have a strong point in that they are simple to implement and control.

#### 3.1 PD control

PD control is useful for fast-response controllers that do not need a steady-state error of zero. Fundamentally, PD control is a position and velocity feedback control that gives good closed-loop properties when applied to a double integrator system.

First, consider the regulation problem of the robot manipulator described by (1). Because the desired joint velocity  $\dot{q}_d = 0$ , the control law of the PD controller with gravity compensation is

$$\tau = -K_v \dot{q} + K_p e + g(q), \quad (5)$$

where  $K_v$  and  $K_p$  are positive-definite gain matrices and  $e = q_d - q$ . Because this control law has no feed-forward term, it can never achieve zero steady-state error. A common

modification is to add an integral term to eliminate steady-state errors. This introduces additional complications because care must be taken to maintain stability and to avoid integrator windup. When the control law (5) is applied to (1), the closed-loop system becomes

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + K_v\dot{q} - K_p e = 0. \quad (6)$$

Now, we investigate the stability achieved by PD control with gravity compensation. We choose the Lyapunov function candidate,

$$V(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \frac{1}{2} e^T K_p e. \quad (7)$$

The function  $V$  is positive-definite; and has a derivative that is negative semi-definite using property (3):

$$\begin{aligned} \dot{V}(q, \dot{q}) &= -\dot{q}^T K_v \dot{q} + \frac{1}{2} \dot{q}^T (\dot{M}(q) - 2C(q, \dot{q})) \dot{q} \\ &= -\dot{q}^T K_v \dot{q} \\ &\leq -\lambda_{\min}\{K_v\} \|\dot{q}\|^2 \end{aligned} \quad (8)$$

where  $\lambda_{\min}\{\cdot\}$  denotes the smallest eigenvalue. By the Lyapunov stability theory and LaSalle's theorem (Khalil, 2002), the regulation error converges to zero asymptotically. In case of PD control without gravity compensation

$$\tau = -K_v \dot{q} + K_p e, \quad (9)$$

and, the closed-loop dynamic equation becomes

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) + K_v\dot{q} - K_p e = 0. \quad (10)$$

Let us choose Lyapunov function candidate,

$$V(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} + \frac{1}{2} e^T K_p e + U(q) + U_0 \quad (11)$$

where  $U(q)$  is the potential energy generating gravity forces and  $U_0$  is a suitable constant. Taking the time derivative of  $V$  along the closed-loop dynamics (10) gives the same result (8) as the PD control with gravity compensation. In this case, the control system must be stable in the sense of Lyapunov, but we cannot conclude that the regulation error will converge to zero by LaSalle's theorem.

Next, consider tracking control. The control law of PD control with gravity compensation is

$$\tau = M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q) + K_v\dot{e} + K_p e. \quad (12)$$

Then the closed-loop system is

$$M(q)\ddot{e} + C(q, \dot{q})\dot{e} - K_v\dot{e} - K_p e = 0. \quad (13)$$

To show the stability, we chose the Lyapunov function candidate

$$V(e, \dot{e}, t) = \frac{1}{2}\dot{e}^T M(q)\dot{e} + \frac{1}{2}e^T K_p e + \varepsilon e^T M(q)\dot{e}, \quad (14)$$

where  $\varepsilon$  is a positive small constant. The derivative of the function  $V$  becomes

$$\dot{V}(e, \dot{e}, t) = -\dot{e}^T (K_v - \varepsilon M)\dot{e} - \varepsilon e^T K_p e + \varepsilon e^T (-K_v + \frac{1}{2}\dot{M})\dot{e}. \quad (15)$$

Choosing  $\varepsilon$  sufficiently small insures that  $\dot{V}$  is negative-definite and hence that the system is exponentially stable by LaSalle's theorem. It is notable that asymptotic tracking requires exact cancelation of gravity and disturbance forces and relies on accurate models of these quantities as well as the manipulator inertia matrix. Therefore, in practical implementations, modeling errors and disturbances result in tracking errors.

### 3.2 PID control

We have seen that PD control makes the system exponentially stable. However, in practical implementation, in the presence of constant disturbance (from the local point of view), PD control gives a nonzero steady-state error. Consequently, adding an integral action to the controller can compensate for the constant disturbance. The PID controller has the form

$$\begin{aligned} \tau &= K_p e + K_i \int e \, dt + K_v \dot{e} \\ &= K_p e + K_v \dot{e} + v \\ \dot{v} &= K_i e, \quad v(0) = v_0, \end{aligned} \quad (16)$$

where  $K_i$  is a positive-definite gain matrix. Choose any positive diagonal matrix  $K'_p$  and let

$$K_p := K'_p + \frac{1}{\varepsilon} K_i, \quad (17)$$

where  $\varepsilon$  is a positive small constant to be determined. Then the error dynamics become

$$\begin{aligned} M(q)\ddot{q} + C(q, \dot{q})\dot{q} + K_v\dot{q} - K'_p e &= \frac{1}{\varepsilon} K_i e + \tilde{v} \\ \dot{\tilde{v}} &= -K_i e, \end{aligned} \quad (18)$$

where  $\tilde{v}$  denotes  $v - g(q_d)$ . To analyze the stability of the closed loop system, we choose the following Lyapunov function candidate with cross terms (Loria et al., 2000)

$$V = \frac{1}{2} \dot{q}^T M \dot{q} + U(q) - U(q_d) + e^T g(q_d) + \frac{1}{2} e^T K'_p e + \frac{\varepsilon}{2} \left( \frac{1}{\varepsilon} K_i e + \tilde{v} \right)^T K_i^{-1} \left( \frac{1}{\varepsilon} K_i e + \tilde{v} \right) - \varepsilon e^T M \dot{q}. \quad (19)$$

Using property of inertia matrix (2), rewriting  $e^T K'_p e = (\lambda_1 + \lambda_2 + \lambda_3) e^T K'_p e$  and  $\dot{q}^T M(q) \dot{q} = (\lambda_1 + \lambda_2 + \lambda_3) \dot{q}^T M(q) \dot{q}$  with  $0 < \lambda_i < 1$ , one can show that if

$$\lambda_{\min} \{K'_p\} \geq \max \left\{ \frac{c_g}{\lambda_1}, \frac{\varepsilon^2 \mu_2}{\lambda_1 \lambda_2} \right\}, \quad (20)$$

then the function  $V$  satisfies the inequality

$$V \geq \frac{\lambda_3}{2} e^T K'_p e + \frac{\lambda_2 + \lambda_3}{2} \dot{q}^T M(q) \dot{q}. \quad (21)$$

Hence,  $V$  is positive-definite and radially unbounded. Next, using the property of the matrix  $C(q, \dot{q})$  and the inequality  $\|g(q_d) - g(q)\| \leq c_g \|e\|$ , the time derivative of  $V$  along the trajectories of (16) satisfies

$$\dot{V} \leq - \left( \lambda_{\min} \{K_v\} - \frac{\varepsilon}{2} \lambda_{\max} \{K_v\} - \varepsilon c_0 \|e\| - \varepsilon \mu_2 \right) \|\dot{q}\|^2 - \varepsilon \left( \lambda_{\min} \{K'_p\} - c_g - \frac{1}{2} \lambda_{\max} \{K_v\} \right) \|e\|^2, \quad (22)$$

which is negative semi-definite if

$$\begin{aligned} \lambda_{\min} \{K_v\} &> \varepsilon (\lambda_{\max} \{K_v\} + 2\mu_2) \\ \lambda_{\min} \{K_p\} &> c_g + \frac{1}{2} \lambda_{\max} \{K_v\} \\ \|e\| &\leq \frac{\lambda_{\min} \{K_v\}}{2\varepsilon c_0}, \end{aligned} \quad (23)$$

where  $\lambda_{\max} \{\cdot\}$  denotes the largest eigenvalue. Then the local asymptotic stability of the origin  $x=0$  follows by LaSalle's theorem.

(Qu & Dorsey, 1991) proposed a similar proof for the uniform ultimate boundedness of the error in the trajectory tracking problem. (Rocco, 1996) proposed a stability analysis method different from other approaches. The proof is based on a formulation of the robot dynamic model where the nominal, decoupled and linear closed loop system is emphasized, whereas the nonlinear terms are split into terms dependent on the control parameters and other norm-bounded terms. However, PID control lacks a global asymptotic stability proof. Moreover, to ensure local stability, the gain matrices must satisfy complicated inequalities.

### 3.3 Saturated PID control

In implementing PID control on any actual robot manipulator, one effect can cause serious problems: any real robot arm will have limits on the voltages and torques of its actuators. These limits may or may not cause a problem with PD control, but are virtually guaranteed to cause problems with integral control due to a phenomenon known as integrator windup (Lewis, 1992).

To account for bounded control torques, i.e.,

$$|\tau_i| \leq \tau_{i,\max}, \quad i = 1, \dots, n, \quad (24)$$

the actual control torque equipped with a saturation function is defined as

$$\tau = \text{Sat}\left(\hat{g}(q_d) + K_p e + K_v \dot{e} + K_i \int e \, dt, \tau_{\max}\right), \quad (25)$$

where  $\tau_{\max} = [\tau_{1,\max}, \dots, \tau_{n,\max}]^T$ , and  $\text{Sat}(\cdot, \tau_{\max})$  is a strictly increasing saturation function with upper limit  $+\tau_{\max}$  and lower limit  $-\tau_{\max}$ . The assumption that the saturation function dominates over gravitational torques should be considered. The assumption becomes a necessary condition for the manipulator to be stabilizable at any desired equilibrium configuration  $q_d \in \mathbb{R}^n$ . In the presence of uncertainty in the gravitational force vector  $g(q)$ ,  $\tau_{i,\max}$  should be chosen such that it is acceptably lower than the maximum torque. Under this assumption, if  $\|K_p\|$  and  $\|K_v\|$  are large enough, and  $\|K_i\|$  is small enough, the saturated PID control (25) yields semi-global asymptotic stabilization of the robot dynamics at any desired position  $q_d \in \mathbb{R}^n$  (Alvarez-Ramirez et al., 2008). (Sun et al., 2009) presented global stability of a saturated nonlinear PID controller with a new class of saturated function.

### 3.4 Summary

In this section, we have presented various PID control methods. Although the success of industrial applications has proven the effectiveness of the PD and PID controllers for complex nonlinear robotic manipulators, PID control is cannot cope with highly nonlinear systems for tracking problems. To overcome these limitations, several types of modified PID controllers were introduced subsequently. These are described in the next section.

## 4. Computed-Torque Control

A special application of the feedback linearization of nonlinear systems is computed-torque control, which consists of an inner nonlinear compensation loop and an outer feedback loop (Fig. 1). In this section, we cover computed-torque and computed-torque with a compensation control scheme which is a dynamic controller.

### 4.1 Computed-torque control

The computed-torque control law with a PD outer-loop controller is given by

$$\tau = M(q)(\ddot{q}_d - u) + C(q, \dot{q})\dot{q} + g(q), \quad (26)$$

where the auxiliary control signal  $u = -K_v \dot{e} - K_p e$ , which is of the PD feedback. Computed-torque control is a model-based motion control approach created for manipulators, that is, in which one makes explicit use of the knowledge of the matrices  $M(q)$ ,  $C(q, \dot{q})\dot{q}$  and  $g(q)$ . Furthermore, the control action (26) is computed using the desired trajectory of motion  $q_d(t)$ , and its derivatives  $\dot{q}_d(t)$  and  $\ddot{q}_d(t)$ , as well as the position and velocity measurements  $q(t)$  and  $\dot{q}(t)$ .

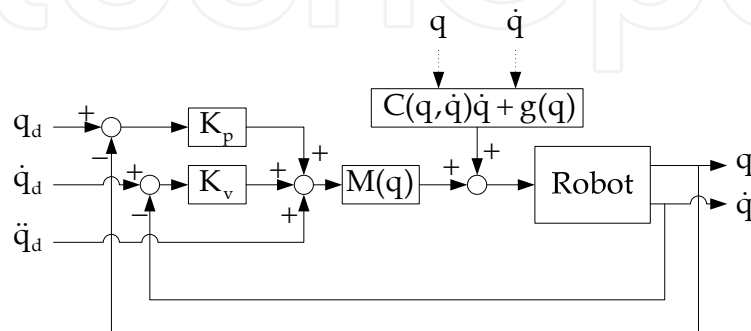


Fig. 1. Computed-torque control

The closed-loop error dynamics of the system (Fig. 1) have the form

$$\ddot{e} + K_v \dot{e} + K_p e = 0. \quad (27)$$

The error dynamics (27) can also be rewritten in state-space form as

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \triangleq A \begin{bmatrix} e \\ \dot{e} \end{bmatrix}. \quad (28)$$

Because the error dynamics (28) is linear, its solutions may be obtained in closed form and used to assess the stability of the origin.

We start by introducing the small constant  $\varepsilon$  satisfying

$$\lambda_{\min}\{K_v\} > \varepsilon > 0. \quad (29)$$

Multiplying by  $x^T x$  where  $x \in \mathbb{R}^{2n}$  is any nonzero vector yields  $\lambda_{\min}\{K_v\} x^T x > \varepsilon x^T x$ .

Because  $K_v$  is a symmetric and positive-definite matrix,  $x^T K_v x \geq \lambda_{\min}\{K_v\} x^T x$  and therefore,

$$x^T [K_v - \varepsilon I] x > 0 \quad \forall x \neq 0 \in \mathbb{R}^{2n}. \quad (30)$$

This means that the matrix  $K_v - \varepsilon I$  is positive-definite. Considering all this, we conclude that



$$K_p + \varepsilon K_v - \varepsilon^2 I > 0. \quad (31)$$

We choose the total energy of the system as Lyapunov function,

$$\begin{aligned} V(e, \dot{e}) &= \frac{1}{2} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}^T \begin{bmatrix} K_p + \varepsilon K_v & \varepsilon I \\ \varepsilon I & I \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} \\ &= \frac{1}{2} \dot{e}^T \dot{e} + \frac{1}{2} e^T [K_p + \varepsilon K_v] e + \varepsilon e^T \dot{e}. \end{aligned} \quad (32)$$

Taking the derivative and applying (26) yields,

$$\begin{aligned} \dot{V}(e, \dot{e}) &= \ddot{e}^T \dot{e} + e^T [K_p + \varepsilon K_v] \dot{e} + \varepsilon \dot{e}^T \dot{e} + \varepsilon e^T \ddot{e} \\ &= -\dot{e}^T [K_v - \varepsilon I] \dot{e} - \varepsilon e^T K_p e. \end{aligned} \quad (33)$$

Because  $K_v - \varepsilon I$  is positive-definite (30), the function  $\dot{V}(e, \dot{e})$  is globally negative-definite. By the Barbashin-Krasovskii theorem (Khalil, 2002), we conclude that the origin  $[e^T, \dot{e}^T]^T$  of the closed-loop equation is globally uniformly asymptotically stable.

#### 4.2 Computed-torque control with compensation

The computed-torque method is an approach that makes direct use of the complete dynamic model of the manipulator. Therefore, we have to know accurate parameters of the model. To compensate modeling errors, we introduce a computed-torque controller with compensation which consists of the computed-torque control law (26), and dynamic terms. The control law (Kelly et al., 2005) is

$$\tau = M(q)(\ddot{q}_d + K_v \dot{e} + K_p e) + C(q, \dot{q})\dot{q} + g(q) - C(q, \dot{q})v, \quad (34)$$

where  $v$  represents the filtered errors of the position and velocity. We choose  $v$  as

$$v = -\frac{bp}{p + \lambda} \dot{e} - \frac{b}{p + \lambda} (K_v \dot{e} + K_p e), \quad (35)$$

where  $p$  is the differential operator  $d/dt$ ;  $\lambda$  and  $b$  are positive design constants. For simplicity, and without loss of generality, we take  $b=1$ . Due to the presence of the vector  $v$  the computed-torque with compensation control law is dynamic, that is, the control action  $\tau$  depends not only on the actual values of the state vector formed by  $q$  and  $\dot{q}$ , but also on its past values. As a consequence of this fact, additional state variables are defined as (37) to characterize the control law completely. The state space realization of (35) is a linear autonomous system given by

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} -\lambda I & 0 \\ 0 & -\lambda I \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + \begin{bmatrix} K_p & K_v \\ 0 & -\lambda I \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}, \quad (36)$$

$$v = \begin{bmatrix} -I & -I \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} - \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}, \quad (37)$$

where  $\xi_1, \xi_2 \in \mathbb{R}^n$  are the new state variables.

To derive the closed-loop equation, we substitute the control law (34) into (1).

$$M(q)(\ddot{e} + K_v \dot{e} + K_p e) - C(q, \dot{q})v = 0 \quad (38)$$

In terms of the state vector  $[e^T, \dot{e}^T, \xi_1^T, \xi_2^T]^T \in \mathbb{R}^{4n}$ , equations (36) to (38) can be used to obtain the closed-loop equation

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \\ \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} = \begin{bmatrix} 0 & I & 0 & 0 \\ -K_p & -M^{-1}(q)C(q, \dot{q}) - K_v & -M^{-1}(q)C(q, \dot{q}) & -M^{-1}(q)C(q, \dot{q}) \\ K_p & K_v & -\lambda I & 0 \\ 0 & -\lambda & 0 & -\lambda I \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \\ \xi_1 \\ \xi_2 \end{bmatrix} \quad (39)$$

of which the origin  $[e^T, \dot{e}^T, \xi_1^T, \xi_2^T]^T = 0$  is an equilibrium point.

To analyze the control system we first write it in a different but equivalent form. For this, notice that the expression for  $v$  given in (35) allows one to derive

$$\dot{v} + \lambda v = -(\ddot{e} + K_v \dot{e} + K_p e). \quad (40)$$

Substituting (40) into (38) yields

$$M(q)(\dot{v} + \lambda v) + C(q, \dot{q})v = 0. \quad (41)$$

Equation (40) is the starting point in the following stability analysis. Consider the Lyapunov function

$$V(v, e) = \frac{1}{2} v^T M(q) v. \quad (42)$$

The derivative of  $V$  with respect to time is given by

$$\begin{aligned} \dot{V}(v, e) &= v^T M(q) \dot{v} + \frac{1}{2} v^T \dot{M}(q) v \\ &= -v^T \lambda M(q) v \leq 0. \end{aligned} \quad (43)$$

Considering  $V$  and  $\dot{V}$

$$\dot{V}(v, e) = -2\lambda V(v, e), \quad (44)$$

which implies that

$$V(v(t), e(t)) = V(v(0), e(0)) \exp(-2\lambda t). \quad (45)$$

From the property of inertia matrix (2),

$$\begin{aligned} \mu_1 v^T v &\leq v^T M(q) v = 2V(v(t), e(t)) \\ v^T v &\leq \frac{2V(v(0), e(0))}{\mu_1} \exp(-2\lambda t). \end{aligned} \quad (46)$$

This means that  $v(t)$  tends to zero exponentially as time  $t$  is increasing. Equation (40) may also be written as

$$e = -(p + \lambda)(p^2 I + pK_v + K_p)^{-1} v. \quad (47)$$

The input to the linear system (47) is  $v$  which tends to zero exponentially, and the output is  $e$ . Because system (47) is a strictly proper linear system which is asymptotically stable, we invoke the fact that a stable strictly proper filter with an exponentially decaying input produces an exponentially decaying output, that is,

$$\lim_{t \rightarrow \infty} e(t) = 0, \quad (48)$$

which means that the motion control objective is verified.

We need an accurate dynamic model or must calculate the control input in real time because computed-torque methods are an approach that makes direct use of the complete dynamic model of the manipulator. To avoid these conditions, various kind of modified computed-torque control schemes are introduced in the following section.

## 5. Adaptive Control

Adaptive controllers are formulated by updating controller parameters on-line and are adequate for systems that have structured uncertainties. Designing an adaptive controller is to develop an estimation algorithm, called the adaptation law, that guarantees convergence of the controller parameters as well as stability.

### 5.1 Adaptive computed-torque control

Because the computed-torque method needs exact dynamic model of the manipulator, performance and stability of the system cannot be guaranteed when parametric mismatches

exist. One way to solve the problem of parameter uncertainties is to use the computed-torque controller with estimates of the unknown parameters in place of the actual parameters. Based on computed-torque control law (26), the adaptive computed-torque controller has the form

$$\tau = \hat{M}(q)(\ddot{q}_d + K_v \dot{e} + K_p e) + \hat{C}(q, \dot{q})\dot{q} + \hat{g}(q), \quad (49)$$

where  $\hat{M}(q)$ ,  $\hat{C}(q, \dot{q})$  and  $\hat{g}(q)$  are the estimations of  $M(q)$ ,  $C(q, \dot{q})$  and  $g(q)$ . The adaptive controller is based on the fact that the parameters appear linearly in the robot model as (4). By utilizing (4), control law (49) can be written as

$$\tau = \hat{M}(q)(\ddot{e} + K_v \dot{e} + K_p e) + W(q, \dot{q}, \ddot{q})\hat{\phi}, \quad (50)$$

where  $\hat{\phi}$  is an  $r \times 1$  vector that represents a time-varying estimate of the unknown constant parameters. Using (1) and (4), we have the tracking error system

$$\ddot{e} + K_v \dot{e} + K_p e = \hat{M}^{-1}(q)W(q, \dot{q}, \ddot{q})\tilde{\phi}, \quad (51)$$

where  $\tilde{\phi} = \phi - \hat{\phi}$  is the parameter error. To obtain an adaptive control law, (51) can be rewritten in the state-space form

$$\dot{e} = \begin{bmatrix} 0_n & I_n \\ -K_p & -K_v \end{bmatrix} e + \begin{bmatrix} 0_n \\ I_n \end{bmatrix} M^{-1}(q)W(q, \dot{q}, \ddot{q})\tilde{\phi} \triangleq A e + B M^{-1}(q)W(q, \dot{q}, \ddot{q})\tilde{\phi}, \quad (52)$$

where the tracking error vector  $e = [e^T, \dot{e}^T]^T$ . We select the Lyapunov function

$$V = e^T P e + \tilde{\phi}^T \Gamma^{-1} \tilde{\phi}, \quad (53)$$

where  $P$  is a  $2n \times 2n$  positive symmetric matrix, and  $\Gamma$  is a diagonal positive-definite  $r \times r$  matrix. The derivative of (53) is

$$\begin{aligned} \dot{V} &= e^T \dot{P} e + \dot{e}^T P e + 2\tilde{\phi}^T \Gamma^{-1} \dot{\tilde{\phi}} \\ &= e^T P (A e + B \hat{M}^{-1}(q)W(q, \dot{q}, \ddot{q})\tilde{\phi}) + (A e + B \hat{M}^{-1}(q)W(q, \dot{q}, \ddot{q})\tilde{\phi})^T P e + 2\tilde{\phi}^T \Gamma^{-1} \dot{\tilde{\phi}} \\ &= -e^T Q e + 2\tilde{\phi}^T (\Gamma^{-1} \dot{\tilde{\phi}} + W^T(q, \dot{q}, \ddot{q})\hat{M}^{-1}(q)B^T P e) \end{aligned} \quad (54)$$

where  $Q$  is the positive-definite symmetric matrix that satisfies the Lyapunov equation

$$A^T P + P A = -Q. \quad (55)$$

To have  $\dot{V}$  negative semi-definite, the adaptive update rule is chosen as

$$\dot{\tilde{\phi}} = -\Gamma W^T(q, \dot{q}, \ddot{q}) \hat{M}^{-1}(q) B^T P e, \quad (56)$$

which implies that  $\dot{V} = -e^T Q e$ . Equation (56) gives the adaptive update rule for the parameter estimate vector  $\hat{\phi}$  because  $\dot{\phi}$  is equal to zero. Substituting  $\tilde{\phi} = \phi - \hat{\phi}$  into (56) gives the adaptive update rule:

$$\dot{\hat{\phi}} = \Gamma W^T(q, \dot{q}, \ddot{q}) \hat{M}^{-1}(q) B^T P e \quad (57)$$

for the parameter estimate vector  $\hat{\phi}$ .

Detailed stability analysis (Craig, 1988) shows that the tracking error vector  $e$  approaches to zero asymptotically. The adaptive computed-torque controller has some restrictions required for the implementation. That is, the controller needs to measure accurate acceleration  $\ddot{q}$  and to ensure that  $\hat{M}^{-1}(q)$  exists. To avoid these restrictions, other adaptive control schemes are introduced in following sections.

## 5.2 Adaptive inertia-related control

(Slotine & Li, 1987) proposed an adaptive inertia-related control scheme that does not need to measure joint acceleration and ensure inversion of the estimated inertia matrix.

Consider the control input

$$\tau = \hat{M}(q)(\ddot{q}_d + \Lambda \dot{e}) + \hat{C}(q, \dot{q})(\dot{q}_d + \Lambda e) + \hat{g}(q) + K_v r, \quad (58)$$

where the auxiliary signal  $r$  is defined as  $r = \Lambda e + \dot{e}$ , with  $\Lambda$  being an  $n \times n$  positive-definite diagonal matrix. Using  $\dot{q} = \dot{q}_d + \Lambda e - r$ ,  $\ddot{q} = \ddot{q}_d + \Lambda \dot{e} - \dot{r}$  and property (4), the robot dynamic equation (1) can be rewritten as

$$\tau = Y(\cdot)\phi - M(q)\dot{r} - C(q, \dot{q})r, \quad (59)$$

where

$$Y(\cdot)\phi = M(q)(\ddot{q}_d + \Lambda \dot{e}) + C(q, \dot{q})(\dot{q}_d + \Lambda e) + g(q), \quad (60)$$

and  $Y(\cdot)$  is an  $n \times r$  matrix of known time functions. Equation (60) is the same type of parameter separation that was used in the formulation of the adaptive computed-torque controller. However, here  $Y(\cdot)$  is independent of the joint acceleration  $\ddot{q}$ . Similar to the formulation (60), we also have

$$\hat{M}(q)(\ddot{q}_d + \Lambda \dot{e}) + \hat{C}(q, \dot{q})(\dot{q}_d + \Lambda e) + \hat{g}(q) = Y(\cdot)\hat{\phi}. \quad (61)$$

To form the error system, substituting the control input (58) into the equation of motion (1) yields

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \hat{M}(q)(\ddot{q}_d + \Lambda \dot{e}) + \hat{C}(q, \dot{q})(\dot{q}_d + \Lambda e) + \hat{g}(q) + K_v r. \quad (62)$$

Substituting  $\ddot{q} = \ddot{q}_d - \ddot{e}$  and  $\dot{q} = \dot{q}_d - \dot{e}$  into (62), and using (60) and (61), the equation (62) can be rewritten as

$$M(q)\dot{r} + C(q, \dot{q})r + K_v r = Y(\cdot)\tilde{\phi}, \quad (63)$$

where  $\tilde{\phi} = \phi - \hat{\phi}$  is the parameter error. To show the convergence of the tracking error to zero, (Slotine & Li, 1987) selected the inertia-related Lyapunov-like function that is a function of the tracking error and the parameter error:

$$V = \frac{1}{2}r^T M(q)r + \frac{1}{2}\tilde{\phi}^T \Gamma^{-1} \tilde{\phi}, \quad (64)$$

where  $\Gamma$  is defined as in (53). Differentiating (64) with respect to time yields

$$\begin{aligned} \dot{V} &= r^T (Y(\cdot)\tilde{\phi} - K_v r) + r^T \left( \frac{1}{2} \dot{M}(q) - C(q, \dot{q}) \right) r + \tilde{\phi}^T \Gamma^{-1} \dot{\tilde{\phi}} \\ &= r^T (Y(\cdot)\tilde{\phi} - K_v r) + \tilde{\phi}^T \Gamma^{-1} \dot{\tilde{\phi}} \\ &= -r^T K_v r + \tilde{\phi}^T (\Gamma^{-1} \dot{\tilde{\phi}} + Y^T(\cdot)r). \end{aligned} \quad (65)$$

By selecting adaptive update rule as

$$\dot{\hat{\phi}} = \Gamma Y^T(\cdot)(\Lambda e + \dot{e}), \quad (66)$$

(65) becomes  $\dot{V} = -r^T K_v r$ , which is negative semi-definite. Detailed analysis (Slotine & Li, 1987) shows that the tracking error  $e$  and  $\dot{e}$  are asymptotically stable.

### 5.3 Adaptive control based on passivity

To unify many adaptive control schemes that have different torque control laws or adaptive update rules, an adaptive control scheme has been developed based on the passivity approach. It requires neither feedback of joint accelerations nor inversion of the estimated inertia matrix. First, we define an auxiliary filtered tracking error variable  $r(s)$  that is similar to that defined for the adaptive inertia-related controller:

$$r(s) = H^{-1}(s)e(s), \quad (67)$$

where

$$H^{-1}(s) = \left[ sI_n + \frac{1}{s}K(s) \right], \quad (68)$$

and  $s$  is the Laplace transform variable. The  $n \times n$  gain matrix  $K(s)$  is chosen such that  $H(s)$  is a strictly proper, stable transfer function matrix with relative degree 1. As in the preceding schemes, the adaptive control strategies require that the known time functions can be separated from the unknown constant parameters. Therefore, using  $\dot{q} = \dot{q}_d + (1/s)K(s)e - r$ ,  $\ddot{q} = \ddot{q}_d + K(s)e - \dot{r}$  and the property (4), the robot dynamic equation (1) can be rewritten as

$$\tau = Z(\cdot)\phi - M(q)\dot{r} - C(q, \dot{q})r, \quad (69)$$

where

$$Z(\cdot)\phi = M(q)(\ddot{q}_d + K(s)e) + C(q, \dot{q})\left(\dot{q}_d + \frac{1}{s}K(s)e\right) + g(q), \quad (70)$$

and  $Z(\cdot)$  is a known  $n \times r$  regression matrix. The equation (70) can be arranged such that  $Z$  and  $r$  do not depend on the measurements of the joint acceleration  $\ddot{q}$ . The adaptive control scheme given here is called the passivity approach because the mapping of  $-r \rightarrow Z(\cdot)\tilde{\phi}$  is constructed to be a passive mapping (Ortega & Spong, 1988). That is, we construct an adaptive update rule such that

$$\int_0^t -r^T(\sigma)Z(\sigma)\tilde{\phi}(\sigma) d\sigma \geq -\beta \quad (71)$$

is satisfied for all time and for some positive scalar constant  $\beta$ . We use the concept of passivity to analyze the stability of a class of adaptive controllers. For this class of adaptive controllers, the torque control is given by

$$\begin{aligned} \tau &= \hat{M}(q)(\ddot{q}_d + K(s)e) + \hat{C}(q, \dot{q})\left(\dot{q}_d + \frac{1}{s}K(s)e\right) + \hat{g}(q) + K_v r \\ &= Z(\cdot)\hat{\phi} + K_v r \end{aligned} \quad (72)$$

Similar to the formulation from (62) to (63), the tracking error system can be expressed in terms of the tracking error variable  $r$  and regression matrix  $Z(\cdot)$  as

$$M(q)\dot{r} + C(q, \dot{q})r + K_v r = Z(\cdot)\tilde{\phi}. \quad (73)$$

To analyze the stability of this system, we choose the Lyapunov-like function

$$V = \frac{1}{2}r^T M(q)r + \beta - \int_0^t r^T(\sigma)Z(\sigma)\tilde{\phi}(\sigma) d\sigma, \quad (74)$$

and note from (71) that  $V \geq 0$ . Differentiating  $V$  and substituting (73) into (75) give

$$\begin{aligned}
 \dot{V} &= \mathbf{r}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{r}} + \frac{1}{2} \mathbf{r}^T \dot{\mathbf{M}}(\mathbf{q}) \mathbf{r} - \mathbf{r}^T \mathbf{Z}(\cdot) \hat{\boldsymbol{\phi}} \\
 &= -\mathbf{r}^T \mathbf{K}_v \mathbf{r} + \frac{1}{2} \mathbf{r}^T (\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})) \mathbf{r} \\
 &= -\mathbf{r}^T \mathbf{K}_v \mathbf{r},
 \end{aligned} \tag{75}$$

which is negative semi-definite. The passivity approach gives a general class of torque control laws. (Lewis et al., 2003) showed the type of stability for the tracking error which is asymptotically stable and some examples that unify some of the research in adaptive control.

## 6. Robust Control

Robust control is a control of fixed structure that guarantees stability and performance in uncertain systems. Its design only requires some knowledge about bounding functions on the largest possible size of the uncertainties. This limited requirement implies that robust control is capable of compensating for both structured and unstructured uncertainties, and this is one of the major advantages of robust control over adaptive control. Compared to adaptive control, other advantages of robust control are computational simplicity in implementation, better compensation for time-varying parameters and for unstructured nonlinear uncertainties, and guaranteed stability.

### 6.1 Passivity-based approach

First, we present controllers that rely directly on the passive structure of rigid robots. Based on the passivity theorem (Ortega & Spong, 1988), if one can show the passivity of the system which maps control input  $\tau$  to a new vector  $\mathbf{r}$  which is a filtered version of  $\mathbf{e}$ , then a controller which closes the loop between  $-\mathbf{r}$  and  $\tau$  will guarantee the asymptotic stability of both  $\mathbf{e}$  and  $\dot{\mathbf{e}}$ . Consider the following controller (Abdallah et al., 1991)

$$\tau = \mathbf{M}(\mathbf{q})(\ddot{\mathbf{q}}_d + \mathbf{K}(s)\mathbf{e}) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\left(\dot{\mathbf{q}}_d + \frac{1}{s}\mathbf{K}(s)\mathbf{e}\right) + \mathbf{g}(\mathbf{q}) + \mathbf{K}_v \mathbf{r}, \tag{76}$$

where  $\mathbf{K}(s)$  and  $\mathbf{r}$  are defined in (67) and (68). Substituting (76) into (1), yields the tracking error system in terms of the tracking error variable  $\mathbf{r}$  as

$$\mathbf{M}(\mathbf{q})\dot{\mathbf{r}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{r} + \mathbf{K}_v \mathbf{r} = \mathbf{0}. \tag{77}$$

Then it may be shown that both  $\mathbf{e}$  and  $\dot{\mathbf{e}}$  are asymptotically stable. This passivity-based approach was introduced in section 5.3, but its modification in the design of robust controllers when  $\mathbf{M}(\mathbf{q})$ ,  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  and  $\mathbf{g}(\mathbf{q})$  are not exactly known is not obvious.



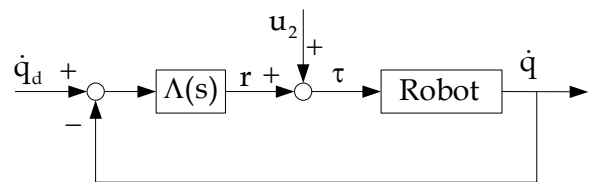


Fig. 2. Passivity-based control

The passivity-based control input is given by

$$\tau = \Lambda(s)\dot{e} + u_2,$$

(78)

where  $\Lambda(s)$  is a strictly proper and stable transfer function, and the external input  $u_2$  is bounded in the  $L_2$  norm. Using the control law (78), we get from Fig. 2,

$$r = \Lambda(s)\dot{e}.$$

(79)

By an appropriate choice of  $\Lambda(s)$  and  $u_2$ , we can apply the passivity theorem and deduce that  $\dot{e}$  and  $r$  are bounded in the  $L_2$  norm. Because  $\Lambda^{-1}(s)$  is strictly proper and stable function, we can conclude that  $\dot{e}$  is asymptotically stable because

$$\dot{e} = \Lambda^{-1}(s)r.$$

(80)

This implies that the position error  $e$  is bounded but the asymptotic stability is not guaranteed in case of trajectory tracking problem. However, in the regulation problem, the asymptotic stability of  $e$  can be guaranteed using LaSalle’s theorem.

6.2 Variable-structure controllers

The variable-structure theory has been applied to the control of many nonlinear processes. One of the main features of this approach is that one only needs to drive the error to a switching surface, after which the system is in sliding mode and will not be affected by any modeling uncertainties and disturbances. The first application of this theory to robot control seems to be in (Young, 1978), where the set-point regulation problem was solved using the following controller:

$$\tau_i = \begin{cases} \tau_i^+, & \text{if } r_i > 0 \\ 0, & \text{if } r_i = 0, \\ \tau_i^-, & \text{if } r_i < 0 \end{cases},$$

(81)

where  $i = 1, \dots, n$  for an  $n$ -link robot. The switching planes  $r_i$  are defined as

$$r = \Lambda e + \dot{e},$$

(82)

where  $\Lambda = \text{diag}\{\lambda_1 \cdots \lambda_n\}$  with  $\lambda_i > 0$ . Let the control input be

$$\tau = M(q)(\lambda \dot{e} - \ddot{q}_d + K \text{sgn}(r)) - C(q, \dot{q})\dot{q} - g(q), \quad (83)$$

where  $K = \text{diag}\{k_1 \cdots k_n\}$  with  $k_i > 0$  and

$$\text{sgn}(r_i) = \begin{cases} +1, & \text{if } r_i > 0 \\ -1, & \text{if } r_i < 0 \end{cases}. \quad (84)$$

Choose  $V = (1/2)r^T r$  as a Lyapunov candidate. Differentiating  $V$  and using (1) and (82),

$$\begin{aligned} \dot{V}(r) &= r^T \dot{r} \\ &= r^T (M^{-1}(q)C(q, \dot{q})\dot{q} + M^{-1}(q)g - M^{-1}\tau + \ddot{q}_d + \Lambda \dot{e}). \end{aligned} \quad (85)$$

Substituting the control input (83) into (85) yields

$$\begin{aligned} \dot{V}(r) &= r^T (-K \text{sgn}(r)) \\ &= -\sum_{i=1}^n k_i |r_i| \leq 0. \end{aligned} \quad (86)$$

According to the Lyapunov stability theorem, the origin is stable equilibrium point. When  $r=0$  in the sliding mode, the tracking error  $e$  decays at an exponential rate. Therefore, the control system is asymptotically stable with the switching function (82) and the control law (83).

For most of these schemes, the control effort is discontinuous along  $s_i = 0$ ; this causes chattering which may excite unmodeled high-frequency dynamics. (Slotine, 1985) modified the variable-structure controller. (Chen et al., 1990) introduced a variable-structure controller which avoided the need to invert of the inertia matrix.

### 6.3 Adaptive robust control

Robust controls ensure robust stability for robotic systems. Robust controls can be defined in terms of a bounding function; determination of this function requires information on the bound of the uncertainties, such as maximum load variation. Without specifying applications, this size information may be difficult to obtain. While under-estimation is not permitted when considering robustness, and over-estimating the maximum size of uncertainties can potentially give robust control an unnecessarily large magnitude and gain, and consequently put too many requirements on the actuators. One approach to maintaining robustness while reducing conservatism is to introduce an adaptive scheme into robust control, that is, to design a so-called adaptive robust control that estimates on-line the size of the uncertainties.

Suppose that the dynamic equation given by

$$w = M(q)(\ddot{q}_d + \dot{e}) + C(q, \dot{q})(\dot{q}_d + e) + g(q), \quad (87)$$

represents the uncertainty for a given robot controller. It is assumed that a positive scalar function  $\rho$  can be used to bound the uncertainty as follows (Lewis et al., 2003)

$$\rho = \delta_0 + \delta_1 \|e\| + \delta_2 \|e\|^2 \geq \|w\|, \quad (88)$$

where  $e = [e^T, \dot{e}^T]^T$  is the tracking error vector and the values of  $\delta_i$  represent the positive bounding constants that are based on such quantities as the largest possible payload mass, link mass and disturbances. The adaptive robust controller learns these bounding constants on-line as the manipulator moves. In the control implementation, knowledge of the bounding constants is not required; only the existence of the bounding constants defined in (88) is required.

The adaptive robust controller has the following form

$$\tau = K_v r + \frac{r \hat{\rho}^2}{\hat{\rho} \|r\| + \varepsilon} \quad (89)$$

where the filtered tracking error  $r = e + \dot{e}$  and

$$\dot{e} = -k_e e, \quad e(0) > 0, \quad (90)$$

where  $k_e$  is a positive scalar control constant.  $\hat{\rho}$  is a scalar function defined as

$$\hat{\rho} = \hat{\delta}_0 + \hat{\delta}_1 \|e\| + \hat{\delta}_2 \|e\|^2, \quad (91)$$

and the values of  $\hat{\delta}_i$  are the dynamic estimates of the corresponding bounding constants  $\delta_i$ . (91) can be rewritten in the matrix form

$$\hat{\rho} = S\hat{\theta} = \begin{bmatrix} 1 & \|e\| & \|e\|^2 \end{bmatrix} \begin{bmatrix} \hat{\delta}_0 & \hat{\delta}_1 & \hat{\delta}_2 \end{bmatrix}^T. \quad (92)$$

Similar to (92), the actual bounding function  $\rho$  can be also be written as

$$\rho = S\theta = \begin{bmatrix} 1 & \|e\| & \|e\|^2 \end{bmatrix} \begin{bmatrix} \delta_0 & \delta_1 & \delta_2 \end{bmatrix}^T. \quad (93)$$

Then the bounding estimates defined in (92) are updated on-line by the update rule

$$\dot{\hat{\theta}} = \gamma S^T \|r\|, \quad (94)$$

where  $\gamma$  is a positive scalar control constant and the filtered tracking error  $r = e + \dot{e}$ .

Because  $\delta_i$  are constants, (94) can be rewritten as

$$\dot{\tilde{\theta}} = -\gamma S^T \|r\|, \quad (95)$$

where  $\tilde{\theta} = \theta - \hat{\theta}$ .

Substituting the adaptive robust controller (89) into the robot dynamic equation (1) gives the error system:

$$M(q)\dot{r} + C(q, \dot{q})r + K_v r + \frac{r\hat{\rho}^2}{\hat{\rho}\|r\| + \varepsilon} - w = 0. \quad (96)$$

Now, let us investigate the stability of the corresponding error system (96) for the adaptive robust controller (89). We choose the Lyapunov function candidate as follows

$$V = \frac{1}{2} r^T M(q) r + \frac{1}{2} \tilde{\theta}^T \gamma^{-1} \tilde{\theta} + k_\varepsilon^{-1} \varepsilon. \quad (97)$$

The time derivative of (97) is

$$\begin{aligned} \dot{V} &= \frac{1}{2} r^T \dot{M}(q) r + r^T M(q) \dot{r} + \tilde{\theta}^T \gamma^{-1} \dot{\tilde{\theta}} + k_\varepsilon^{-1} \dot{\varepsilon} \\ &= -r^T K_v r - S\tilde{\theta}\|r\| + r^T \left( w - \frac{r\hat{\rho}^2}{\hat{\rho}\|r\| + \varepsilon} \right) + k_\varepsilon^{-1} \dot{\varepsilon}. \end{aligned} \quad (98)$$

Using (88), we can place an upper bound on  $\dot{V}$  in the following manner:

$$\begin{aligned} \dot{V} &\leq -r^T K_v r - S\tilde{\theta}\|r\| + S\hat{\theta}\|r\| - \frac{r^T r \hat{\rho}^2}{\hat{\rho}\|r\| + \varepsilon} + k_\varepsilon^{-1} \dot{\varepsilon} \\ &= -r^T K_v r + S\hat{\theta}\|r\| - \frac{r^T r \hat{\rho}^2}{\hat{\rho}\|r\| + \varepsilon} - \varepsilon \\ &= -r^T K_v r - \varepsilon + \frac{\varepsilon S\hat{\theta}\|r\|}{S\hat{\theta}\|r\| + \varepsilon}. \end{aligned} \quad (99)$$

Because the sum of the last two terms in (99) is always less than zero, we can place the new upper bound on  $\dot{V}$ :

$$\dot{V} \leq -r^T K_v r, \quad (100)$$

which is negative semi-definite and used to know the type of stability for the tracking error which is asymptotically stable (Corless & Leitmann, 1983).

In the conventional adaptive robust control methods, however, the explicit quantitative

relation between the tracking error bound and the design parameters is not clear. The tracking error precision is explicitly specified based on the quantitative relation between the control error and the design parameters (Imura et al., 1994). The conventional adaptive robust control has been extended so that the bounding function can be parameterized in terms of time varying parameters defined by exogenous systems (Qu, 2000).

#### 6.4 Disturbance observer based control

Disturbance observer based (DOB) control is one of the most useful robot control techniques. The DOB method was designed to estimate the sum of the disturbance torques due to the combined modeling error, nonlinear terms in the dynamic equation and unknown external torque. In DOB control, the difference between the actual output and the output of the nominal model is regarded to be an equivalent disturbance applied to the nominal model. For a multi-link robotic system, the DOB control regards the coupling torques from other links as an unknown external torque; this enables independent joint control for a multi-link robotic manipulator. Hence, a simple controller can be designed for the independent nominal model.

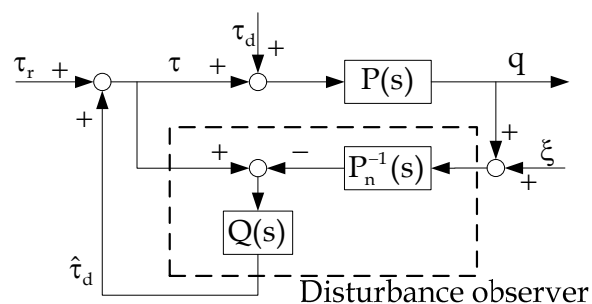


Fig. 3. Disturbance observer

In the classical disturbance observer structure (Fig. 3),  $P(s)$  represents the linear time invariant plant to be controlled,  $P_n^{-1}(s)$  is inverse of nominal plant model  $P_n(s)$ ,  $Q(s)$  is a low pass filter,  $\tau_d$  represents disturbance torque and  $\xi$  represents measurement noise. From Fig. 2, the input-output relation is obtained as follows:

$$q = G_{rq}(s)\tau_r + G_{dq}(s)\tau_d + G_{\xi q}(s)\xi, \quad (101)$$

where

$$\begin{aligned} G_{rq}(s) &= \frac{q}{\tau_r} = \frac{P(s)P_n(s)}{P_n(s) + Q(s)(P(s) - P_n(s))} \\ G_{dq}(s) &= \frac{q}{\tau_d} = \frac{P(s)P_n(s)(1 - Q(s))}{P_n(s) + Q(s)(P(s) - P_n(s))} \\ G_{\xi q}(s) &= \frac{q}{\xi} = \frac{P(s)Q(s)}{P_n(s) + Q(s)(P(s) - P_n(s))}. \end{aligned} \quad (102)$$

For low frequencies  $Q(s) \approx 1$ ,  $G_{rq}(s) \approx P_n(s)$ ,  $G_{dq}(s) \approx 0$  and  $G_{\xi q}(s) \approx 1$ . For high frequencies  $Q(s) \approx 0$ ,  $G_{rq}(s) \approx P(s)$ ,  $G_{dq}(s) \approx P(s)$  and  $G_{\xi q}(s) \approx 0$ . This implies that the disturbance observer rejects low-frequency disturbances and high-frequency measurement noise. Selection of a low-pass filter  $Q(s)$  is an important factor for designing the disturbance observer, because this selection constitutes a design trade-off between disturbance rejection versus noise rejection and robust stability. Because the disturbance observer uses a low-pass-filter to reduce the measurement noise of the output and to make the transfer function  $Q(s)P^{-1}(s)$  proper, the performance of the observer mainly depends on the designed filter. Hence, many studies have dealt with design methods of robust disturbance observer and of  $Q(s)$ . However most of these studies of design and analysis are based on linear system techniques (Umeno et al., 1993; Choi et al., 2003; Kobayashi et al., 2007). Those techniques are not applicable when the system does not work as a nominal linear plant. For this reason, nonlinear disturbance observers for nonlinear dynamics of the system have been proposed to overcome the limitation of analysis based on linear system (Chen, 2004; Liu & Svoboda, 2006).

## 7. Conclusion

In this chapter, various motion control schemes for rigid robotic manipulator were introduced. The first control schemes were conventional PD and PID control which have simple structures. However, PID control methods have limitations for nonlinear robotic manipulators. To overcome these disadvantages, modified PID control and computed-torque control were introduced as a special application of the feedback linearization of nonlinear systems.

To handle uncertainties in the robotic manipulator, adaptive and robust control methods were discussed. Adaptive controllers are formulated by updating controller parameters on-line; these controllers are suited for systems with structured uncertainties. In robust control of fixed structure, the stability and performance in uncertain systems is guaranteed. Robust control schemes can be combined with adaptive control techniques effectively. The bounds of uncertainties are estimated by adding an adaptive scheme to the robust controller. Some disturbance observer based control schemes are also shown to be robust control methods. By compensating for all disturbances which consist of system uncertainties and disturbances, DOB control can be applied to practical robot manipulators effectively.

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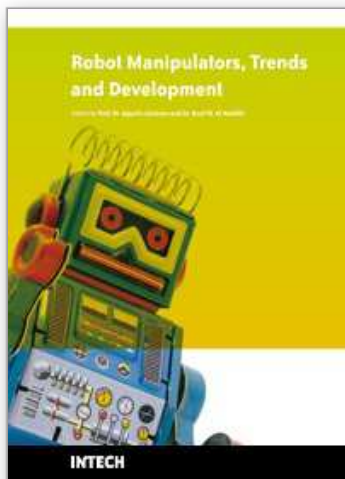


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## **Robot Manipulators Trends and Development**

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This book presents the most recent research advances in robot manipulators. It offers a complete survey to the kinematic and dynamic modelling, simulation, computer vision, software engineering, optimization and design of control algorithms applied for robotic systems. It is devoted for a large scale of applications, such as manufacturing, manipulation, medicine and automation. Several control methods are included such as optimal, adaptive, robust, force, fuzzy and neural network control strategies. The trajectory planning is discussed in details for point-to-point and path motions control. The results in obtained in this book are expected to be of great interest for researchers, engineers, scientists and students, in engineering studies and industrial sectors related to robot modelling, design, control, and application. The book also details theoretical, mathematical and practical requirements for mathematicians and control engineers. It surveys recent techniques in modelling, computer simulation and implementation of advanced and intelligent controllers.

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