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# Application of Higher Order Derivatives to Helicopter Model Control

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## 1. Introduction

Control of a helicopter model is a problem of both theoretical and practical interest. With the proliferation of autonomous unmanned aerial vehicles (UAVs) (Castillo et al., 2005; Valavanis, 2007) autopilot modes have become very important. Dynamic properties of a controlled helicopter depend on both its structure and aerodynamic qualities as well as on the control law applied. The problem of output regulation has received much attention and especially during the last decade, its nonlinear version has been intensively developed (Isidori & Byrnes, 1990), (Slotine & Li, 1991). The well known approach to decoupling problem solution based on the Non-linear Inverse Dynamics (NID) method (Balas et al., 1995) may be used if the parameters of the plant model and external disturbances are exactly known. Usually, incomplete information about systems in real practical tasks takes place. In this case adaptive control methods (Astrom & Wittenmark, 1994) or control systems with sliding mode (Utkin, 1992) may be used for solving this control problem. A crucial feature of the sliding mode techniques is that in the sliding phase the motion of the system is insensitive to parameter variation and disturbances in the system. A way of the algorithmic solution of this problem under condition of incomplete information about varying parameters of the plant and unknown external disturbances is the application of the Localization Method (LM) (Vostrikov & Yurkevich, 1993), which allows to provide the desired transients for nonlinear time-varying systems. A development of LM is applied in the present paper, and proposed in (Błachuta et al., 1999; Czyba & Błachuta, 2003; Yurkevich, 2004), method which based on two ideas. The first – the use of high gain in feedback to suppress the disturbances and varying parameters; the second – the use of higher order output derivatives in the feedback loop. The high gain and "dynamics" of the controller are separated by means of the summing junction with set point signal placed between them. This structure is the implementation of the model reference control with the reference model transfer function which is equal to the inverse of the controller "dynamics". It becomes that the proposed structure and method is insensitive to plant parameters changes and external disturbances, and works well both lineal, nonlinear, stationary and nonstationary objects. In the present paper, the proposed method is applied to control of the helicopter model, which is treated as a multivariable system.

In general, the goal of the design of a helicopter model control system is to provide decoupling, i.e. each output should be independently controlled by a single input, and to provide desired output transients under assumption of incomplete information about varying parameters of the plant and unknown external disturbances. In addition, we require that transient processes have desired dynamic properties and are mutually independent.

The paper is part of a continuing effort of analytical and experimental studies on aircraft control (Czyba & Błachuta, 2003), and BLDC motor control (Szafrański & Czyba, 2008). The main aim of this research effort is to examine the effectiveness of a designed control system for real physical plant – laboratory model of the helicopter. The paper is organized as follows. First, a mathematical description of the helicopter model is introduced. Section 3 includes a background of the discussed method and the method itself are summarized. The next section contains the design of the controller, and finally the results of experiments are shown. The conclusions are briefly discussed in the last section.

## 2. Helicopter model

The CE150 helicopter model was designed by Humusoft for the theoretical study and practical investigation of basic and advanced control engineering principles. The helicopter model (Fig.1) consists of a body, carrying two propellers driven by DC motors, and massive support. The body has two degrees of freedom. The axes of the body rotation are perpendicular as well as the axes of the motors. Both body position angles, i.e. azimuth angle in horizontal and elevation angle in vertical plane are influenced by the rotating propellers simultaneously. The DC motors for driving propellers are controlled proportionally to the output signals of the computer. The helicopter model is a multivariable dynamical system with two manipulated inputs and two measured outputs. The system is essentially nonlinear, naturally unstable with significant crosscouplings.
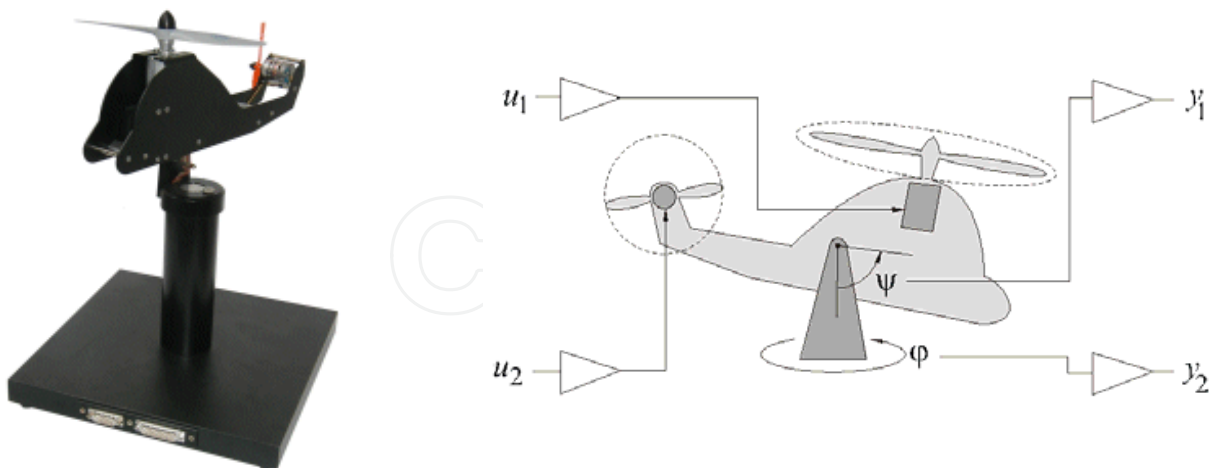


Fig. 1.  CE150 Helicopter model (Horacek, 1993)

In this section a mathematical model by considering the force balances is presented (Horacek, 1993). Assuming that the helicopter model is a rigid body with two degrees of freedom, the following output and control vectors are adopted:

$$\overline{Y} = [\psi, \varphi]^T \tag{1}$$

$$\overline{u} = [u_1, u_2]^T \tag{2}$$

where: $\psi$ - elevation angle (pitch angle); $\varphi$ - azimuth angle (yaw angle); $u_1$ - voltage of main motor; $u_2$ - voltage of tail motor.

## 2.1 Elevation dynamics

Let us consider the forces in the vertical plane acting on the vertical helicopter body, whose dynamics are given by the following nonlinear equation:

$$I_\psi \psi^{(2)} = \tau_1 + \tau_{\varphi^{(1)}} - \tau_{f1} - \tau_m + \tau_G \tag{3}$$

with $\qquad\qquad \tau_1 = k_{\omega_1} \omega_1^2 \tag{4}$

$$\tau_{\varphi^{(1)}} = \frac{1}{2} m l \left( \varphi^{(1)} \right)^2 \sin 2\psi \tag{5}$$

$$\tau_{f1} = C_\psi \, sign\psi^{(1)} + B_\psi \psi^{(1)} \tag{6}$$

$$\tau_m = mgl \sin \psi \tag{7}$$

$$\tau_G = K_G \omega_1 \varphi^{(1)} \cos \psi \tag{8}$$

where:

$I_\psi$ - moment of inertia around horizontal axis

$\tau_1$ - elevation driving torque

$\tau_{\varphi^{(1)}}$ - centrifugal torque

$\tau_{f1}$ - friction torque (Coulomb and viscous)

$\tau_m$ - gravitational torque

$\tau_G$ - gyroscopic torque

$\omega_1$ - angular velocity of the main propeller

$m$ - mass

$g$ - gravity

$l$ - distance from z-axis to main rotor

$k_{\omega_1}$ - constant for the main rotor

$K_G$ - gyroscopic coefficient

$B_\psi$ - viscous friction coefficient (around y-axis)

$C_\psi$ - Coulomb friction coefficient (around y-axis)

### 2.2 Azimuth dynamics

Let us consider the forces in the horizontal plane, taking into account the main forces acting on the helicopter body in the direction of $\varphi$ angle, whose dynamics are given by the following nonlinear equation:

$$I_\varphi \varphi^{(2)} = \tau_2 - \tau_{f2} - \tau_r \tag{9}$$

with

$$I_\varphi = I_\psi \sin\psi \tag{10}$$

$$\tau_2 = k_{\omega_2} \omega_2^2 \tag{11}$$

$$\tau_{f2} = C_\varphi sign\varphi^{(1)} + B_\varphi \varphi^{(1)} \tag{12}$$

where:

$\quad I_\varphi$   - moment of inertia around vertical axis

$\quad \tau_2$   - stabilizing motor driving torque

$\quad \tau_{f2}$   - friction torque (Coulomb and viscous)

$\quad \tau_r$   - main rotor reaction torque

$\quad k_{\omega_2}$   - constant for the tail rotor

$\quad \omega_2$   - angular velocity of the tail rotor

$\quad B_\varphi$   - viscous friction coefficient (around z-axis)

$\quad C_\varphi$   - Coulomb friction coefficient (around z-axis)

### 2.3 DC motor and propeller dynamics modeling

The propulsion system consists two independently working DC electrical engines. The model of a DC motor dynamics is achieved based on the following assumptions:

*Assumption1:* The armature inductance is very low.

*Assumption2:* Coulomb friction and resistive torque generated by rotating propeller in the air are significant.

*Assumption3:* The resistive torque generated by rotating propeller depends on $\omega$ in low and $\omega^2$ in high rpm.

Taking this into account, the equations are following:

$$I_j \omega_j^{(1)} = \tau_j - \tau_{cj} - B_j \omega_j - \tau_{pj} \tag{13}$$

with

$$\tau_j = K_{ij} i_j \tag{14}$$

$$i_j = \frac{1}{R_j}\left(u_j - K_{bj}\omega_j\right) \tag{15}$$

$$\tau_{cj} = C_j sign\left(\omega_j\right) \tag{16}$$

$$\tau_{pj} = B_{pj}\omega_j + D_{pj}\omega_j^2 \qquad (17)$$

where:

$j = 1, 2$   - motor number (1- main, 2- tail)

$I_j$   - rotor and propeller moment of inertia

$\tau_j$   - motor torque

$\tau_{cj}$   - Coulomb friction load torque

$\tau_{pj}$   - air resistance load torque

$B_j$   - viscous-friction coefficient

$K_{ij}$   - torque constant

$i_j$   - armature current

$R_j$   - armature resistance

$u_j$   - control input voltage

$K_{bj}$   - back-emf constant

$C_j$   - Coulomb friction coefficient

$B_{pj}$   - air resistance coefficient (laminar flow)

$D_{pj}$   - air resistance coefficient (turbulent flow)

Block diagram of nonlinear dynamics of a complete system is to be assembled from the above derivations and the result is in Fig.2.
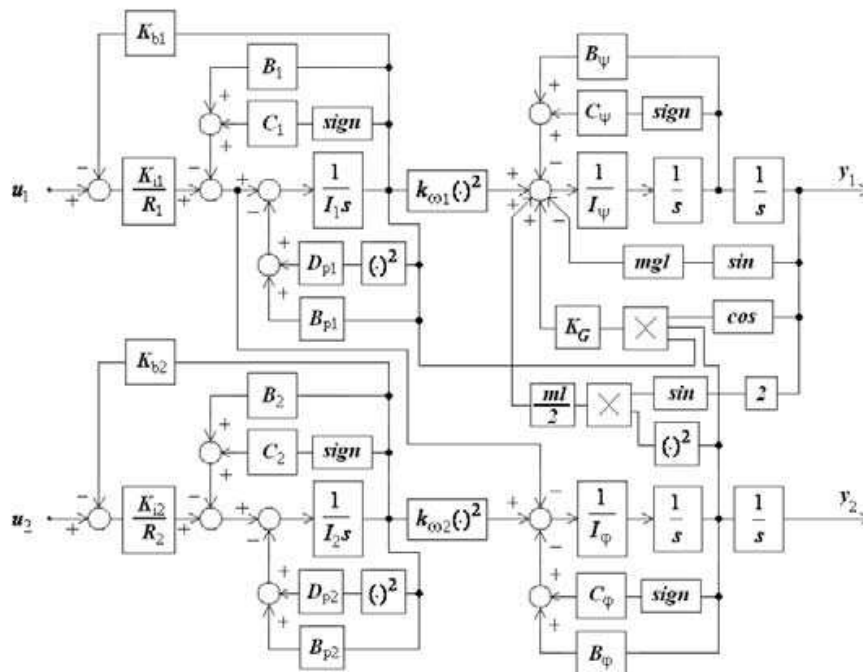


Fig. 2.  Block diagram of a complete system dynamics

## 3. Control scheme

Let us consider a nonlinear time-varying system in the following form:

$$x^{(1)}(t) = h(x(t), u(t), t), \quad x(0) = x_0 \tag{18}$$

$$y(t) = g(t, x(t)) \tag{19}$$

where $x(t)$ is $n$–dimensional state vector, $y(t)$ is $p$–dimensional output vector and $u(t)$ is $p$-dimensional control vector. The elements of the $f(t, x(t))$, $B(t, x(t))$ and $g(t, x(t))$ are differentiable functions.

Each output $y_i(t)$ can be differentiated $m_i$ times until the control input appears. Which results in the following equation:

$$y^{(m)}(t) = f(t, x(t)) + B(t, x(t)) u(t) \tag{20}$$

where:  $y^{(m)}(t) = \left[ y_1^{(m_1)}, y_2^{(m_2)}, ..., y_p^{(m_p)} \right]$,

$\left| f_i(t, x) \right| \le f_i^{\max}, \qquad i = 1, 2, ....., p$,

$\det(B(t, x(t))) \ne 0$.

The value $m_i$ is a relative order of the system (18), (19) with respect to the output $y_i(t)$ (or so called the order of a relative higher derivative). In this case the value $y_i^{(m_i)}$ depends explicitly on the input $u(t)$.

The significant feature of the approach discussed here is that the control problem is stated as a problem of determining the root of an equation by introducing reference differential equation whose structure is in accordance with the structure of the plant model equations. So the control problem can be solved if behaviour of the $y_i^{(m_i)}$ fulfills the reference model which is given in the form of the following stable differential equation:

$$y_{iM}^{(m_i)}(t) = F_{iM}\left( \overline{y}_{iM}(t), r_i(t) \right) \tag{21}$$

where: $F_{iM}$ is called the desired dynamics of $y_i(t)$, $\overline{y}_{iM}(t) = \left[ y_{iM}, y_{iM}^{(1)}, ..., y_M^{(m_i-1)} \right]^T$, $r_i(t)$ is the reference value and the condition $y_i = r_i$ takes place for an equilibrium point.

Denote the tracking error as follows:

$$\Delta(t) = r(t) - y(t). \tag{22}$$

The task of a control system is stated so as to provide that

$$\Delta(t) = 0 . \atop t \to \infty \tag{23}$$

Moreover, transients $y_i(t)$ should have the desired behavior defined in (21) which does not depend either on the external disturbances or on the possibly varying parameters of system in equations (18), (19). Let us denote

$$\Delta^F = F_M \left( \overline{y}(t), r(t) \right) - y^{(m)}(t) \tag{24}$$

where: $\Delta^F$ is the error of the desired dynamics realization, $F_M = \left[ F_{1\,M}, F_{2\,M}, ..., F_{p\,M} \right]^T$ is a vector of desired dynamics.

As a result of (20), (21), (24) the desired behaviour of $y_i(t)$ will be provided if the following condition is fulfilled:

$$\Delta^F \left( x(t), \overline{y}(t), r(t), u(t), t \right) = 0 . \tag{25}$$

So the control action $u(t)$ which provides the control problem solution is the root of equation (25). Above expression is the insensitivity condition of the output transient performance indices with respect to disturbances and varying parameters of the system in (18), (19).

The solution of the control problem (25) bases on the application of the higher order output derivatives jointly with high gain in the controller. The control law in the form of a stable differential equation is constructed such that its stable equilibrium is the solution of equation (25). Such equation can be presented in the following form (Yurkevich, 2004)

$$\mu_i^{q_i} v_i^{(q_i)} + \sum_{j=0}^{q_i-1} \mu_i^j d_{i,j} v_i^{(j)} = k \Delta_i^F \tag{26}$$

$$v_i(0) = v_{i,0}$$

where:

$i = 1, ..., p$ ,

$v_i(t) = \left[ v_i, v_i^{(1)}, ..., v_i^{(q_i-1)} \right]^T$ - new output of the controller,

$\mu_i$ - small positive parameter $\mu_i > 0$,

$k$ - gain,

$d_{i,0}, ..., d_{i,q_i-1}$ - diagonal matrices.

To decoupling of control channel during the fast motions let us use the following output controller equation:

$$u(t) = K_0 K_1 v(t) \tag{27}$$

where:

$K_1 = diag(k_1, k_2, ..., k_p)$ is a matrix of gains,

$K_0$ is a nonsingular matching matrix (such that $BK_0$ is positive definite).

Let us assume that there is a sufficient time-scale separation, represented by a small parameter $\mu_i$, between the fast and slow modes in the closed loop system. Methods of singularly perturbed equations can then be used to analyze the closed loop system and, as a result, slow and fast motion subsystems can be analyzed separately. The fast motions refer to the processes in the controller, whereas the slow motions refer to the controlled object.

*Remark 1:* It is assumed that the relative order of the system (18), (19), determined in (20), and reference model (21) is the same $m_i$.

*Remark 2:* Assuming that $q_i \geq m_i$ (where $i = 1, 2, ..., p$), then the control law (26) is proper and it can be realized without any differentiation.

*Remark 3:* The asymptotically stability and desired transients of $v_i(t)$ are provided by choosing $\mu_i, k, d_{i,0}, d_{i,1}, ..., d_{i,q_i-1}$.

*Remark 4:* Assuming that $d_{i,0} = 0$ in equation (26), then the controller includes the integration and it provides that the closed-loop system is type I with respect to reference signal.

*Remark 5:* If the order of reference model (21) is $m_i - 1$, such that the relative order of the open loop system is equal one, then we obtain sliding mode control.

## 4. Helicopter controller design

The helicopter model described by equations (1)–(17), will be used to design the control system that achieves the tracking of a reference signal. The control task is stated as a tracking problem for the following variables:

$$\lim_{t \to \infty} \left[ \psi_0(t) - \psi(t) \right] = 0 \tag{28}$$

$$\lim_{t \to \infty} \left[ \varphi_0(t) - \varphi(t) \right] = 0 \tag{29}$$

where $\psi_0(t)$, $\varphi_0(t)$ are the desired values of the considered variables.

In addition, we require that transient processes have desired dynamic properties, are mutually independent and are independent of helicopter parameters and disturbances.

The inverse dynamics of (18), (19) are constructed by differentiating the individual elements of $y$ sufficient number of times until a term containing $u$ appears in (20). From equations of helicopter motion (3)–(17) it follows that:

$$\psi^{(3)} = f_1 + \frac{K_{i1}\left(2k_{\omega_1}\omega_1 + K_G\varphi^{(1)}\cos\psi\right)}{I_1 R_1 I_\psi} u_1 \tag{30}$$

$$\varphi^{(3)} = f_2 + \frac{K_{i1}\left(K_{b1} + B_1\right)}{I_1 R_1 I_\psi \sin\psi} u_1 + \frac{2K_{i2}k_{\omega_2}\omega_2}{I_2 R_2 I_\psi \sin\psi} u_2 \tag{31}$$

Following (20), the above relationship becomes:

$$\begin{bmatrix} \psi^{(3)} \\ \varphi^{(3)} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + B \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{32}$$

where values of $f_1, f_2$ are bounded, and the matrix $B$ is given in the following form

$$B = \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix}. \tag{33}$$

In normal flight conditions we have $\det\left(B\big(t, x(t)\big)\right) \neq 0$. This is a sufficient condition for the existence of an inverse system model to (18), (19).

Let us assume that the desired dynamics are determined by a set of mutually independent differential equations:

$$\tau_\psi^3 \psi^{(3)} = -3\tau_\psi^2 \alpha_\psi \psi^{(2)} - 3\tau_\psi \alpha_\psi^2 \psi^{(1)} - \psi + \psi_0 \tag{34}$$

$$\tau_\varphi^3 \varphi^{(3)} = -3\tau_\varphi^2 \alpha_\varphi \varphi^{(2)} - 3\tau_\varphi \alpha_\varphi^2 \varphi^{(1)} - \varphi + \varphi_0 \tag{35}$$

Parameters $\tau_i$ and $\alpha_i$ $(i = \psi, \varphi)$ have very well known physical meaning and their particular values have to be specified by the designer.

The output controller equation from (27) is as follows:

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = K_0 K_1 \begin{pmatrix} v_\psi \\ v_\varphi \end{pmatrix} \tag{36}$$

where $K_1 = diag(k_\psi, k_\varphi)$ and assume that $K_0 = (B)^{-1}$ because matrix $BK_0$ must be positive definite. Moreover $BK_0 \cong I$ assures decoupling of fast mode channels, which makes controller's tuning simpler.

The dynamic part of the control law from (26) has the following form:

$$\mu_\psi^3 v_\psi^{(3)} + 3\mu_\psi^2 d_{\psi,2} v_\psi^{(2)} + 3\mu_\psi d_{\psi,1} v_\psi^{(1)} + d_{\psi,0} v_\psi =$$
$$= k\left(-\tau_\psi^3 \psi^{(3)} - 3\tau_\psi^2 \alpha_\psi \psi^{(2)} - 3\tau_\psi \alpha_\psi^2 \psi^{(1)} - \psi + \psi_0\right) \tag{37}$$

$$\mu_\varphi^3 v_\varphi^{(3)} + 3\mu_\varphi^2 d_{\varphi,2} v_\varphi^{(2)} + 3\mu_\varphi d_{\varphi,1} v_\varphi^{(1)} + d_{\varphi,0} v_\varphi =$$
$$= k\left(-\tau_\varphi^3 \varphi^{(3)} - 3\tau_\varphi^2 \alpha_\varphi \varphi^{(2)} - 3\tau_\varphi \alpha_\varphi^2 \varphi^{(1)} - \varphi + \varphi_0\right) \tag{38}$$

The entire closed loop system is presented in Fig.3.
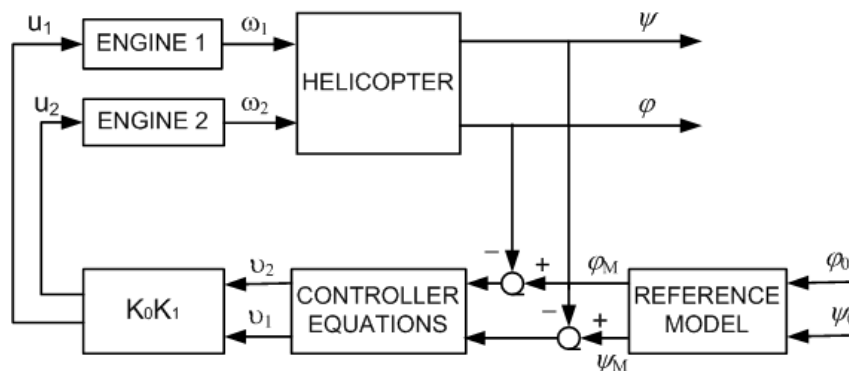


Fig. 3.  Closed-loop system

## 5. Results of control experiments

In this section, we present the results of experiment which was conducted on the helicopter model HUMUSOFT CE150, to evaluate the performance of a designed control system. As the user communicates with the system via Matlab Real Time Toolbox interface, all input/output signals are scaled into the interval <-1,+1>, where value "1" is called Machine Unit and such a signal has no physical dimension. This will be referred in the following text as MU.

The presented maneuver (experiment 1) consisted in transition with predefined dynamics from one steady-state angular position to another. Hereby, the control system accomplished a tracking task of reference signal. The second experiment was chosen to expose a robustness of the controller under transient and steady-state conditions. During the experiment, the entire control system was subjected to external disturbances in the form of a wind gust. Practically this perturbation was realized mechanically by pushing the helicopter body in required direction with suitable force. The helicopter was disturbed twice during the test: $t_1 = 130 \, [s]$, $t_2 = 170 \, [s]$.

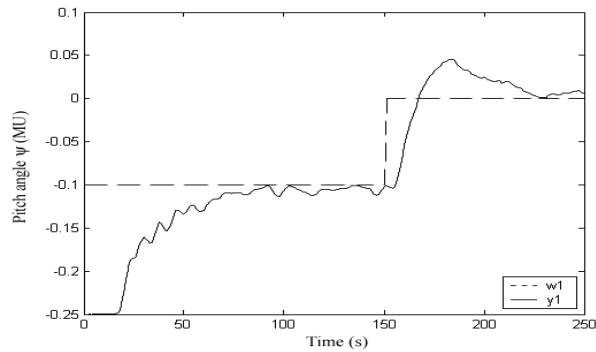## 5.1 Experiment 1 – tracking of a reference trajectory
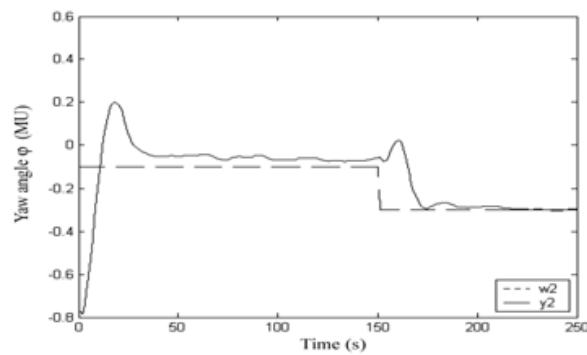


Fig. 4. Time history of pitch angle $\psi$



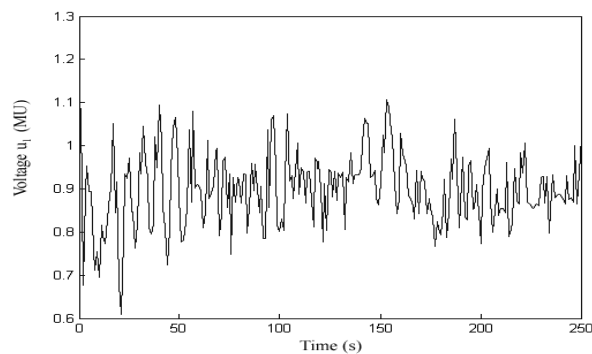Fig. 5. Time history of yaw angle $\varphi$



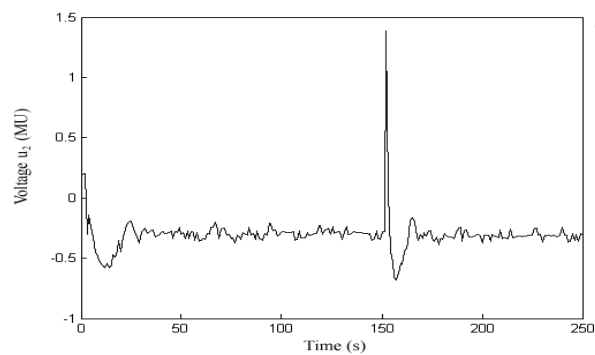Fig. 6. Time history of main motor voltage $u_1$



Fig. 7. Time history of tail motor voltage $u_2$

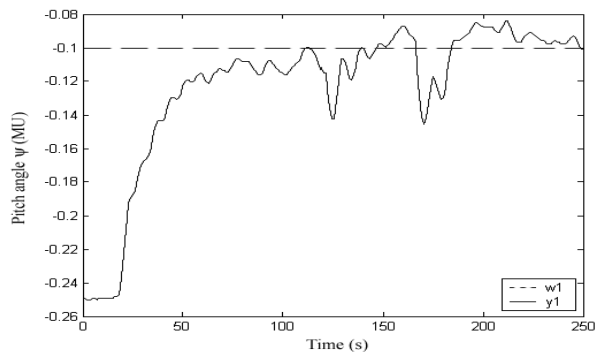## 5.2 Experiment 2 – influence of a wind gust in vertical plane
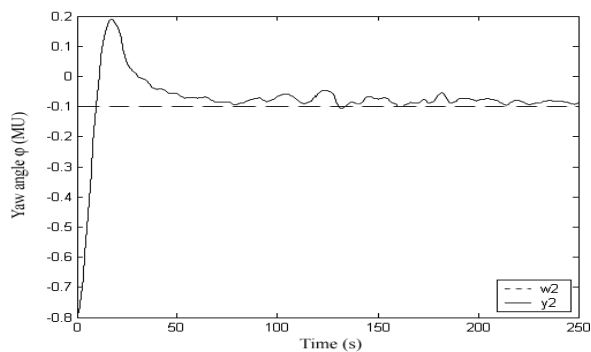


Fig. 8.  Time history of pitch angle $\psi$
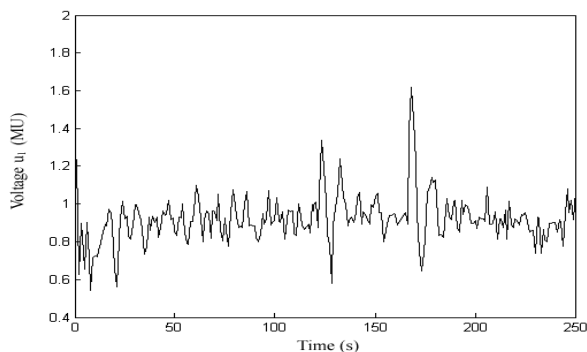


Fig. 9.  Time history of yaw angle $\varphi$



Fig. 10.  Time history of main motor voltage $u_1$
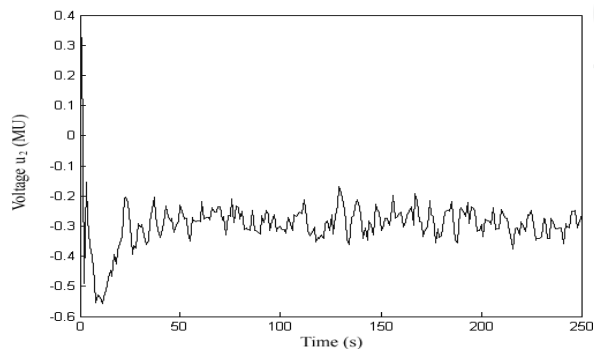


Fig. 11.  Time history of tail motor voltage $u_2$

## 6. Conclusion

The applied method allows to create the expected outputs for multi-input multi-output nonlinear time-varying physical object, like an exemplary laboratory model of helicopter, and provides independent desired dynamics in control channels. The peculiarity of the propose approach is the application of the higher order derivatives jointly with high gain in the control law. This approach and structure of the control system is the implementation of the model reference control. The resulting controller is a combination of a low-order linear dynamical system and a matrix whose entries depend non-linearly on some known process variables. It becomes that the proposed structure and method is insensitive to external disturbances and also plant parameter changes, and hereby possess a robustness aspects. The results suggest that the approach we were concerned with can be applied in some region of automation, for example in power electronics.

## 7. Acknowledgements

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**Mechatronic Systems Simulation Modeling and Control**

Edited by Annalisa Milella Donato Di Paola and Grazia Cicirelli

This book collects fifteen relevant papers in the field of mechatronic systems. Mechatronics, the synergistic blend of mechanics, electronics, and computer science, integrates the best design practices with the most advanced technologies to realize high-quality products, guaranteeing at the same time a substantial reduction in development time and cost. Topics covered in this book include simulation, modelling and control of electromechanical machines, machine components, and mechatronic vehicles. New software tools, integrated development environments, and systematic design methods are also introduced. The editors are extremely grateful to all the authors for their valuable contributions. The book begins with eight chapters related to modelling and control of electromechanical machines and machine components. Chapter 9 presents a nonlinear model for the control of a three-DOF helicopter. A helicopter model and a control method of the model are also presented and validated experimentally in Chapter 10. Chapter 11 introduces a planar laboratory testbed for the simulation of autonomous proximity manoeuvres of a uniquely control actuator configured spacecraft. Integrated methods of simulation and Real-Time control aiming at improving the efficiency of an iterative design process of control systems are presented in Chapter 12. Reliability analysis methods for an embedded Open Source Software (OSS) are discussed in Chapter 13. A new specification technique for the conceptual design of self-optimizing mechatronic systems is presented in Chapter 14. Chapter 15 provides a general overview of design specificities including mechanical and control considerations for micro-mechatronic structures. It also presents an example of a new optimal synthesis method to design topology and associated robust control methodologies for monolithic compliant microstructures.

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