We are IntechOpen,
the world's leading publisher of
Open Access books
Built by scientists, for scientists

4,800
Open access books available

122,000
International authors and editors

135M
Downloads

Our authors are among the

154
Countries delivered to

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

CLARIVATE ANALYTICS
BOOK CITATION INDEX
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Electromagnetic Models for Remote Sensing of Layered Rough Media

Pasquale Imperatore, Antonio Iodice and Daniele Riccio
University of Naples "Federico II",
Department of Biomedical, Electronic and Telecommunication Engineering,
Napoli, Italy

1. Introduction

Each region of the Earth's crust can be morphologically modeled as a suitable layered structure, in which some amount of roughness is presented by every interface. Actually, propagation in stratified soil, sand cover of arid regions, forest canopies, urban buildings, snow blanket, snow cover ice, sea ice and glaciers, oil flood on sea surface, and other natural scenes can be modeled referring to most likely discrete (piecewise-constant) systems, rather than continuous, with some amount of roughness presented by every interface. Moreover, a key issue in remote sensing of other Planets is to reveal the content under the surface illuminated by the sensors: also in this case a layered model is usually employed.

The aim of this chapter is to provide a structured presentation of the main theoretical and conceptual foundations for the problem of the electromagnetic wave interaction with layered rough media. In the first part, special emphasis is on the analytical models obtainable in powerful framework of the perturbation approach. The comprehensive scattering model based on the *Boundary Perturbation Theory* (BPT), which permits to systematically analyze the bi-static scattering patterns of 3D multilayered rough media, is then presented highlighting the formal connections with all the previously existing simplified perturbative models, as well as its wide relevance in the remote sensing applications scenario. The *polarimetric Scattering Matrix* of a multilayered medium with an arbitrary number of rough interfaces is also provided. The second part is devoted to a mathematical description which connects the concepts of *local* scattering and *global* scattering. Consequently, a functional decomposition of the BPT global scattering solution in terms of basic single-scattering local processes is rigorously established. The *scattering decomposition* gives insight into the BPT analytical results, so enabling a relevant physical-revealing interpretation involving ray-series representation. Accordingly, in first-order limit, the way in which the character of the local scattering processes emerges is dictated by the nature of the structural filter action, which is inherently governed by the series of coherent interactions with the medium boundaries. As a result, the phenomenologically successful BPT model opens the way toward new techniques for solving the inverse problem, for designing SAR processing algorithms, and for modelling the time-domain response of layered structures.
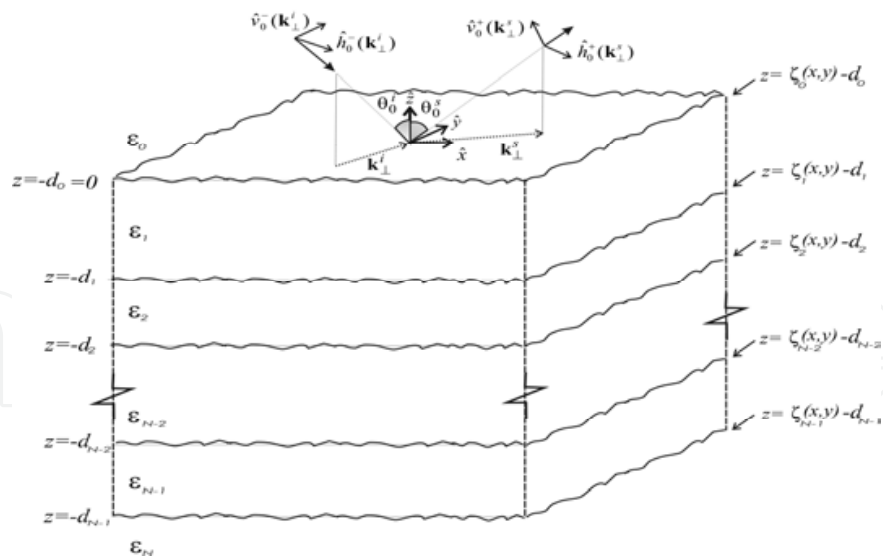
Fig. 1. Geometry for an N-rough boundaries layered medium

2. Problem definition

When stratified media with rough interfaces are concerned, the possible approaches to cope with the EM scattering problem fall within three main categories. First, the numerical approaches do not permit to attain a comprehensive understanding of the general functional dependence of the scattering response on the structure parameters, as well as do not allow capturing the physics of the involved scattering mechanisms. Layered structures with rough interfaces have been also treated resorting to *radiative transfer theory* (RT). However, coherent effects are not accounted for in RT theory and could not be contemplated without employing full wave analysis, which preserves phase information. Another approach relies on the full-wave methods. Although, to deal with the electromagnetic propagation and scattering in complex random layered media, several analytical formulation involving some idealized cases and suitable approximations have been conducted in last decades, the relevant solutions usually turn out to be too complicated to be generally useful in the remote sensing scenario, even if simplified geometries are accounted for. The proliferation of the proposed methods for the simulation of wave propagation and scattering in a natural stratified medium and the continuous interest in this topic are indicative of the need of appropriate modelling and interpretation of the complex physical phenomena that take place in realistic environmental structures. Indeed, the availability of accurate, sound physical and manageable models turns out still to be a strong necessity, in perspective to apply them in retrieving of add-valued information from the data acquired by microwave sensors. For instance, such models are high desirable for dealing with the inversion problem as well as for the effective design of processing algorithms and simulation of Synthetic Aperture Radar signals. Generally speaking, an exact analytical solution of *Maxwell* equations can be found only for a few idealized problems. Subsequently, appropriate approximation methods are needed. Regarding the perturbative approaches, noticeable progress has been attained in the investigation on the extension of the classical SPM (small perturbation method) solution for the scattering from rough surface to specific layered configurations. Most of previous existing works analyze different layered configurations in the first-order limit, using procedures, formalisms and final solutions that can appear of

difficult comparison (Yarovoy et al., 2000), (Azadegan and Sarabandi, 2003), (Fuks, 2001). All these formulations, which refer to the case of a single rough interface, have been recently unified in (Franceschetti et al, 2008). On the other hand, solution for the case of two rough boundaries has also been proposed in (Tabatabaeenejad and Moghaddam, 2006).

Methodologically, we underline that all the previously mentioned existing perturbative approaches, followed by different authors in analyzing scattering from simplified geometry, imply an inherent analytical complexity, which precludes the treatment to structures with more than one (Fuks, 2001) (Azadegan et al., 2003) (Yarovoy et al., 2000) or two (Tabatabaeenejad er al., 2006) rough interfaces.

The general problem we intend to deal with here refers to the analytical evaluation of the electromagnetic scattering by layered structure with an arbitrary number of rough interfaces (see Figure 1). As schematically shown in Figure 1, an arbitrary polarized monochromatic plane wave

$$\mathbf{E}_0^i(\mathbf{r}) = [E_0^{ih}\hat{h}_0^-(\mathbf{k}_\perp^i) + E_0^{iv}\hat{v}_0^-(\mathbf{k}_\perp^i)] \, e^{j(\mathbf{k}_\perp^i \cdot \mathbf{r}_\perp - k_{z0}^i z)} \tag{1}$$

is considered to be incident on the layered medium at an angle θ_0^i relative to the $\hat{z}$ direction from the upper half-space, where in the field expression a time factor *exp(-jωt)* is understood, and where, using a spherical frame representation, the incident vector wave direction is individuated by θ_0^i, φ_0^i:

$$k_0\hat{k}_0^i = \mathbf{k}_0^i = \mathbf{k}_\perp^i - \hat{z}k_{z0}^i = k_0(\hat{x}\sin\theta_0^i\cos\varphi_0^i + \hat{y}\sin\theta_0^i\sin\varphi_0^i - \hat{z}\cos\theta_0^i), \tag{2}$$

with

$$\hat{h}_0^-(\mathbf{k}_\perp^i) = \frac{\hat{k}_0^i \times \hat{z}}{\left|\hat{k}_0^i \times \hat{z}\right|} = \sin\varphi_0^i\hat{x} - \cos\varphi_0^i\hat{y},$$

$$\hat{v}_0^-(\mathbf{k}_\perp^i) = \hat{h}_0^-(\mathbf{k}_\perp^i) \times \hat{k}_0^i = (\hat{x}\cos\varphi_0^i + \hat{y}\sin\varphi_0^i)\cos\theta_0^i + \hat{z}\sin\theta_0^i,$$

where $\mathbf{k}_\perp^i = k_x^i\hat{x} + k_y^i\hat{y}$ is the two-dimensional projection of incident wave-number vector on the plane *z=0*. The parameters pertaining to layer *m* with boundaries $-d_{m-1}$ and $-d_m$ are distinguished by a subscript *m*. Each layer is assumed to be homogeneous and characterized by arbitrary and deterministic parameters: the *dielectric relative permittivity ε_m*, the *magnetic relative permeability μ_m* and the *thickness $\Delta_m = d_m - d_{m-1}$*. With reference to Figure 1, it has been assumed that in particular, $d_0 = 0$. In the following, the symbol $\perp$ denotes the projection of the corresponding vector on the plan *z=0*. Here $\mathbf{r} = (\mathbf{r}_\perp, z)$, so we distinguish the transverse spatial coordinates $\mathbf{r}_\perp = (x, y)$ and the longitudinal coordinate *z*. In addition, each *m*th rough interface is assumed to be characterized by a zero-mean two-dimensional *stochastic process* $\zeta_m = \zeta_m(\mathbf{r}_\perp)$ with normal vector $\hat{n}_m$. No constraints are imposed on the degree to which the rough interfaces are correlated.

A general methodology has been developed by *Imperatore et al.* to analytically treat EM bistatic scattering from this class of layered structures that can be described by small changes with respect to an idealized (unperturbed) structure, whose associated problem is

exactly solvable. A thorough analysis of the results of this theoretical investigation (BPT), which is based on perturbation of the boundary condition, will be presented in the following, methodologically emphasizing the development of the several inherent aspects.
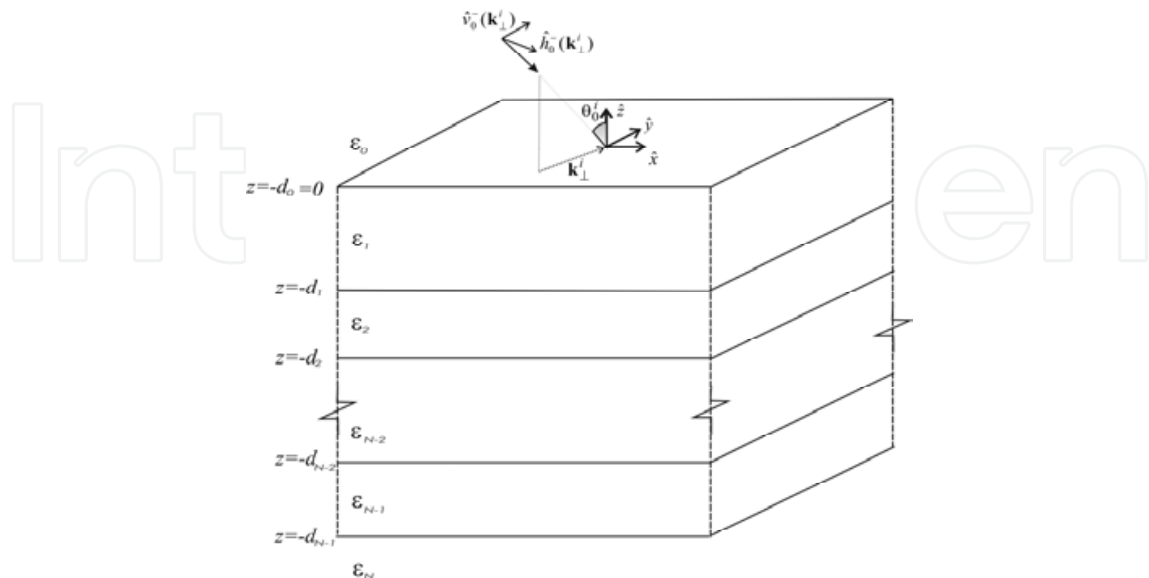

Fig. 2. Geometry for a flat boundaries layered medium

3. Preliminary notation and definitions

This section is devoted preliminary to introduce the formalism used in the following of this chapter. The Flat Boundaries layered medium (*unperturbed structure*) is defined as a stack of parallel slabs (Figure 2), sandwiched in between two half-spaces, whose structure is shift invariant in the direction of *x* and *y* (infinite lateral extent in *x-y* directions). With the notations $T^p_{m-1|m}$ and $R^p_{m-1|m}$, respectively, we indicate the *ordinary transmission* and *reflection coefficients* at the interface between the regions (*m-1*) and *m*, with the superscript $p \in \{v, h\}$ indicating the polarization state for the incident wave and may stand for *horizontal* (*h*) or *vertical* (*v*) polarization (Tsang et al., 1985) (Imperatore et al. 2009a). In addition, we stress that:

$$T^p_{i|j} = 1 + R^p_{i|j} \qquad\qquad R^p_{i|j} = -R^p_{j|i} \qquad\qquad i=j\pm1 \qquad (3)$$

The *generalized reflection coefficients* $\Re^p_{m-1|m}$, for the *p*-polarization (TE or TM), at the interface between the regions (*m-1*) and *m* are defined as the ratio of the amplitudes of *upward-* and *downward*-propagating waves immediately above the interface, respectively. They can be expressed by recursive relations as in (Chew W. C., 1997) (Imperatore et al. 2009a):

$$\Re^p_{m-1|m} = \frac{R^p_{m-1|m} + \Re^p_{m|m+1}\, e^{j2k_{zm}\Delta_m}}{1 + R^p_{m-1|m}\, \Re^p_{m|m+1}\, e^{j2k_{zm}\Delta_m}}. \qquad (4)$$

Likewise, at the interface between the regions (*m+1*) and *m*, $\Re^p_{m+1|m}$ is given by:

$$\Re^p_{m+1|m} = \frac{R^p_{m+1|m} + \Re^p_{m|m-1}e^{j2k_{zm}\Delta m}}{1 + R^p_{m+1|m}\Re^p_{m|m-1}e^{j2k_{zm}\Delta m}}, \tag{5}$$

where

$$k_{zm} = \sqrt{k_m^2 - |\mathbf{k}_\perp|^2} = k_m \cos\theta_m, \tag{6}$$

where $k_m = k_0\sqrt{\mu_m\varepsilon_m}$ is the *wave number* for the electromagnetic medium in the mth layer, with $k_0 = \omega/c = 2\pi/\lambda$, and where $\mathbf{k}_\perp = k_x\hat{x} + k_y\hat{y}$ is the two-dimensional projection of vector wave-number on the plane *z=0*. It should be noted that the factors

$$\vec{M}^p_m(k_\perp) = 1 - R^p_{m|m-1}\Re^p_{m|m+1}e^{j2k_{zm}\Delta m}, \tag{7}$$

$$\bar{M}^p_m(k_\perp) = 1 - \Re^p_{m|m-1}R^p_{m|m+1}e^{j2k_{zm}\Delta m}, \tag{8}$$

$$\vec{M}^p_m(k_\perp) = 1 - \Re^p_{m|m-1}\Re^p_{m|m+1}e^{j2k_{zm}\Delta m}, \tag{9}$$

take into account the multiple reflections in the *m*th layer. On the other hand, the *generalized transmission coefficients* in *downward* direction can be defined as:

$$\mathfrak{J}^p_{0|m}(k_\perp) = \exp\left[j\sum_{n=1}^{m-1}k_{zn}\Delta_n\right]\prod_{n=0}^{m-1}T^p_{n|n+1}\left[\prod_{n=1}^{m}\vec{M}^p_n\right]^{-1}, \tag{10}$$

where $p \in \{v, h\}$. The *generalized transmission coefficients* in *upward* direction are then given by:

$$\mathfrak{J}^p_{m|0} = \begin{cases} \mathfrak{J}^p_{0|m}\dfrac{\mu_0 k_{zm}}{\mu_m k_{z0}} & \text{for } p = h \\[2mm] \mathfrak{J}^p_{0|m}\dfrac{\varepsilon_0 k_{zm}}{\varepsilon_m k_{z0}} & \text{for } p = v \end{cases} \tag{11}$$

which formally express the *reciprocity* of the generalized transmission coefficients for an arbitrary flat-boundaries layered structure (Imperatore et al. 2009b). In addition, with reference to a layered slab sandwiched between two half-space, we consider the *generalized transmission coefficients* in *upward* direction for the layered *slab* between two half-spaces (*m*,0), which are defined as

$$\mathfrak{J}^{p(slab)}_{m|0}(k_\perp) = \exp\left[j\sum_{n=1}^{m-1}k_{zn}\Delta_n\right]\prod_{n=0}^{m-1}T^p_{n+1|n}\left[\prod_{n=1}^{m-1}\bar{M}^p_n\right]^{-1} \tag{12}$$

Note also that

$$\mathfrak{J}^p_{m|0}(k_\perp) = \mathfrak{J}^{p(slab)}_{m|0}(k_\perp)[\vec{M}^p_m(k_\perp)]^{-1}. \tag{13}$$

The *generalized transmission coefficients* in *downward* direction for the layered *slab* between two half-spaces (*m*,0), can be defined as

$$\mathfrak{I}_{0|m}^{p(slab)}(k_\perp) = \exp\left[j\sum_{n=1}^{m-1} k_{zn}\Delta_n \right] \prod_{n=0}^{m-1} T_{n|n+1}^p \left[\prod_{n=1}^{m-1} \vec{M}_n^p \right]^{-1} . \qquad (14)$$

On the other hand, it should be noted that the $\mathfrak{I}_{0|m}^{p}$ are distinct from the coefficients $\mathfrak{I}_{0|m}^{p(slab)}$, because in the evaluation of $\mathfrak{I}_{0|m}^{p}$ the effect of all the layers under the layer *m* is taken into account, whereas $\mathfrak{I}_{0|m}^{p(slab)}$ are evaluated referring to a different configuration in which the intermediate layers *1...m* are bounded by the half-spaces *0* and *m*. In the following, we shown how the employing the generalized reflection/transmission coefficient notions not only is crucial in obtaining a compact closed-form perturbation solution, but it also permit us to completely elucidate the obtained analytical expressions from a physical point of view, highlighting the role played by the *equivalent reflecting interfaces* and by *the equivalent slabs*, so providing the inherent connection between local and global scattering responses.

4. Spectral Representation of the Stochastic Geometry Description

In this section, the focus is on stochastic description for the geometry of the investigated structure, and the notion of wide-sense stationary process is detailed. First of all, when the description of a rough interface by means of deterministic function $\zeta_m(\mathbf{r}_\perp)$ is concerned, the corresponding *ordinary* 2-D *Fourier Transform* pair can be defined as

$$\tilde{\zeta}_m(\mathbf{k}_\perp) = (2\pi)^{-2} \iint d\mathbf{r}_\perp \, e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \zeta_m(\mathbf{r}_\perp), \qquad (15)$$

$$\zeta_m(\mathbf{r}_\perp) = \iint d\mathbf{k}_\perp \, e^{j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \tilde{\zeta}_m(\mathbf{k}_\perp). \qquad (16)$$

Let us assume now that $\zeta_m(\mathbf{r}_\perp)$, which describes the generic (*m*th) rough interface, is a 2-D stochastic process satisfying the conditions

$$< \zeta_m(\mathbf{r}_\perp) > = 0, \qquad (17)$$

$$< \zeta_m(\mathbf{r}_\perp + \boldsymbol{\rho}) \zeta_m(\mathbf{r}_\perp) > = B_{\zeta_m}(\boldsymbol{\rho}), \qquad (18)$$

where the *angular bracket* denotes statistical ensemble averaging, and where $B_{\zeta_m}(\boldsymbol{\rho})$ is the interface *autocorrelation* function, which quantifies the similarity of the spatial fluctuations with a displacement $\boldsymbol{\rho}$. Equations (17)-(18) constitute the basic assumptions defining a *wide sense stationary* (*WSS*) stochastic process: the statistical properties of the process under consideration are invariant to a spatial shift. Similarly, concerning two mutually correlated random rough interfaces ζ_m and ζ_n, we also assume that they are *jointly WSS*, i.e.

$$< \zeta_m (\mathbf{r}_\perp + \boldsymbol{\rho}) \zeta_n (\mathbf{r}_\perp) > = B_{\zeta_m \zeta_n} (\boldsymbol{\rho}) , \tag{19}$$

where $B_{\zeta_m \zeta_n} (\boldsymbol{\rho})$ is the corresponding *cross-correlation function* of the two random processes. It can be readily derived that

$$B_{\zeta_m \zeta_n} (\boldsymbol{\rho}) = B_{\zeta_n \zeta_m} (-\boldsymbol{\rho}) . \tag{20}$$

The integral in (15) is a *Riemann* integral representation for $\zeta_m(\mathbf{r}_\perp)$, and it exists if $\zeta_m(\mathbf{r}_\perp)$ is piecewise continuous and *absolutely integrable*. On the other hand, when the spectral analysis of a stationary random process is concerned, the integral (15) does not in general exist in the framework of theory of the ordinary functions. Indeed, a WSS process describing an interface $\zeta_m(\mathbf{r}_\perp)$ of infinite lateral extension, for its proper nature, is not *absolutely integrable*, so the conditions for the existence of the Fourier Transform are not satisfied. In order to obtain a spectral representation for a *WSS* random process, this difficulty can be circumvented by resorting to the more general *Fourier-Stieltjes* integral (Ishimaru, 1978); otherwise one can define space-truncated functions. When a finite patch of the rough interface with area A is concerned, the space-truncated version of (15) can be introduced as

$$\widetilde{\zeta}_m (\mathbf{k}_\perp ; A) = (2\pi)^{-2} \iint_A d\mathbf{r}_\perp \, e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \zeta_m (\mathbf{r}_\perp) , \tag{21}$$

subsequently, $\widetilde{\zeta}_m (\mathbf{k}_\perp) = \lim_{A \to \infty} \widetilde{\zeta}_m (\mathbf{k}_\perp ; A)$ is not an ordinary function. Nevertheless, we will use again the (15)-(16), regarding them as symbolic formulas, which hold a rigorous mathematical meaning beyond the ordinary function theory (generalized Fourier Transform). We underline that by virtue of the condition (17) directly follows also that $< \widetilde{\zeta}_m (\mathbf{k}_\perp) > = 0$. Let us consider

$$< \zeta_m (\mathbf{r}'_\perp) \zeta_n^* (\mathbf{r}''_\perp) > = \iint d\mathbf{k}'_\perp \iint d\mathbf{k}''_\perp \, e^{j(\mathbf{k}'_\perp \cdot \mathbf{r}'_\perp - \mathbf{k}''_\perp \cdot \mathbf{r}''_\perp)} < \widetilde{\zeta}_m (\mathbf{k}'_\perp) \widetilde{\zeta}_n^* (\mathbf{k}''_\perp) > , \tag{22}$$

where the asterisk denotes the complex conjugated, and where the operations of average and integration have been interchanged. When *jointly WSS* processes ζ_m and ζ_n are concerned, accordingly to (19), the RHS of (22) must be a function of $\mathbf{r}'_\perp - \mathbf{r}''_\perp$ only; therefore, it is required that

$$< \widetilde{\zeta}_m (\mathbf{k}'_\perp) \widetilde{\zeta}_n^* (\mathbf{k}''_\perp) > = W_{mn} (\mathbf{k}'_\perp) \delta (\mathbf{k}'_\perp - \mathbf{k}''_\perp) , \tag{23}$$

where $\delta(\cdot)$ is the *Dirac* delta function, and where $W_{mn}(\boldsymbol{\kappa})$ is called the (spatial) *cross power spectral density* of two interfaces ζ_m and ζ_n, for the spatial frequencies of the roughness. Equation (23) states that the different spectral components of the two considered interfaces must be uncorrelated. This is to say that the (generalized) Fourier transform of jointly WSS processes are *jointly non stationary* white noise with average power $W_{mn}(\mathbf{k}'_\perp)$. Indeed, by using (23) into (22), we obtain

$$< \zeta_m(\mathbf{r}_\perp') \zeta_n(\mathbf{r}_\perp'') > = \iint d\mathbf{k}_\perp'' \, e^{j \cdot \mathbf{k}_\perp'' \cdot (\mathbf{r}_\perp' - \mathbf{r}_\perp'')} W_{mn}(\mathbf{k}_\perp'') , \qquad (24)$$

where the RHS of (24) involves an (ordinary) 2D Fourier Transform. Note also that as a direct consequence of the fact that $\zeta_n(\mathbf{r}_\perp)$ is real we have the relation $\widetilde{\zeta}_n(\mathbf{k}_\perp) = \widetilde{\zeta}_n^*(-\mathbf{k}_\perp)$. Therefore, setting $\boldsymbol{\rho} = \mathbf{r}_\perp' - \mathbf{r}_\perp''$ in (24), we have

$$B_{\zeta_m \zeta_n}(\boldsymbol{\rho}) = \iint d\boldsymbol{\kappa} \, e^{j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} \, W_{mn}(\boldsymbol{\kappa}) . \qquad (25)$$

The *cross-correlation function* $B_{\zeta_m \zeta_n}(\boldsymbol{\rho})$ of two interfaces ζ_m and ζ_n, is then given by the (inverse) 2D Fourier Transform of their *(spatial) cross power spectral density*, and Equation (25) together with its Fourier inverse

$$W_{mn}(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho} \, e^{-j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} B_{\zeta_m \zeta_n}(\boldsymbol{\rho}) , \qquad (26)$$

may be regarded as the (generalized) *Wiener-Khinchin* theorem. In particular, when $n = m$, (23) reduces to

$$< \widetilde{\zeta}_m(\mathbf{k}_\perp') \widetilde{\zeta}_m^*(\mathbf{k}_\perp'') > = W_m(\mathbf{k}_\perp') \delta(\mathbf{k}_\perp' - \mathbf{k}_\perp'') , \qquad (27)$$

where $W_m(\boldsymbol{\kappa})$ is called the *(spatial) power spectral density* of nth corrugated interface ζ_m and can be expressed as the *(ordinary)* 2D Fourier transform of n-corrugated interface autocorrelation function, i.e., satisfying the transform pair:

$$W_m(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho} \, e^{-j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} B_{\zeta_m}(\boldsymbol{\rho}) , \qquad (28)$$

$$B_{\zeta_m}(\boldsymbol{\rho}) = \iint d\boldsymbol{\kappa} \, e^{j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} \, W_m(\boldsymbol{\kappa}) , \qquad (29)$$

which is the statement of the classical *Wiener-Khinchin* theorem. We emphasize the physical meaning of $W_m(\boldsymbol{\kappa}) d\boldsymbol{\kappa} = W_m(\kappa_x, \kappa_y) d\kappa_x d\kappa_y$: it represents the power of the spectral components of the mth rough interface having spatial wave number between κ_x and $\kappa_x + d\kappa_x$ and κ_y and $\kappa_y + d\kappa_y$, respectively, in x and y direction. Furthermore, from (20) and (26) it follows that

$$W_{mn}(\boldsymbol{\kappa}) = W_{nm}^*(\boldsymbol{\kappa}). \qquad (30)$$

This is to say that, unlike the power spectral density, the cross power spectral density is, in general, neither real nor necessarily positive. Furthermore, it should be noted that the *Dirac*'s delta function can be defined by the integral representation

$$\delta(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho} \, e^{-j \boldsymbol{\kappa} \cdot \boldsymbol{\rho}} = \lim_{A \to \infty} \delta(\boldsymbol{\kappa}; A). \qquad (31)$$

By using in (27) and (23) the relation $\delta(0; A) = A/(2\pi)^2$, we have, respectively, that the (spatial) power *spectral density* of nth corrugated interface can be also expressed as

$$W_m(\kappa) = (2\pi)^2 \lim_{A \to \infty} \frac{1}{A} < \left| \widetilde{\zeta}_m(\kappa; A) \right|^2 >, \tag{32}$$

and the (spatial) *cross power spectral density* of two interfaces ζ_m and ζ_n is given by

$$W_{mn}(\kappa) = (2\pi)^2 \lim_{A \to \infty} \frac{1}{A} < \widetilde{\zeta}_m(\kappa; A) \widetilde{\zeta}_n^*(\kappa; A) >. \tag{33}$$

It should be noted that the domain of a rough interface is physically limited by the illumination beamwidth. Note also that the different definitions of the Fourier transform are available and used in the literature: the sign of the complex exponential function are sometimes exchanged and a multiplicative constant $(2\pi)^{-2}$ may appear in front of either integral or its square root in front of each expression (15)-(16). Finally, we recall that the theory of random process predicts only the averages over many realizations.

5. Perturbative Field Formulation

With reference to the geometry of Figure 1, in order to obtain a solution valid in each region of the structure, we have to enforce the continuity of the tangential fields:

$$[\hat{n}_m \times \Delta \mathbf{E}_m]_{z=\zeta_m(\mathbf{r}_\perp)-d_m} = 0, \tag{34}$$

$$[\hat{n}_m \times \Delta \mathbf{H}_m]_{z=\zeta_m(\mathbf{r}_\perp)-d_m} = 0, \tag{35}$$

where $\Delta \mathbf{E}_m = \mathbf{E}_{m+1} - \mathbf{E}_m$, $\Delta \mathbf{H}_m = \mathbf{H}_{m+1} - \mathbf{H}_m$, and the surface normal vector is given by:

$$\hat{n}_m = \frac{\hat{z} - \boldsymbol{\gamma}_m}{\sqrt{1 - \gamma_m^2}}, \tag{36}$$

with the slope vector $\boldsymbol{\gamma}_m$:

$$\boldsymbol{\gamma}_m = \nabla_\perp \zeta_m = \left[\frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} \right] \zeta_m, \tag{37}$$

and where $\nabla_\perp$ is the *nabla* operator in the *x-y* plane. In order to study the fields $\mathbf{E}_m$ and $\mathbf{H}_m$ within the generic mth layer of the structure, we assume then that, for each mth rough interface, the deviations and slopes of surface with respect to the reference mean plane $z=-d_m$ are small enough in the sense of (Ulaby et al, 1982) (Tsang et al., 1985), so that the fields can

be expanded about the reference mean plane. The fields expansion can be then injected into the boundary conditions (34)-(35), so that, retaining only up to the first-order terms, the following *nonuniform boundary conditions* can be obtained (Imperatore et al. 2009a)

$$\hat{z} \times \Delta \mathbf{E}_m^{(1)}\bigg|_{z=-d_m} = \nabla_{\perp} \zeta_m \times \Delta \mathbf{E}_m^{(0)}\bigg|_{z=-d_m} - \zeta_m \, \hat{z} \times \frac{\partial \Delta \mathbf{E}_m^{(0)}}{\partial z}\bigg|_{z=-d_m} \, , \qquad (38)$$

$$\hat{z} \times \Delta \mathbf{H}_m^{(1)}\bigg|_{z=-d_m} = \nabla_{\perp} \zeta_m \times \Delta \mathbf{H}_m^{(0)}\bigg|_{z=-d_m} - \zeta_m \, \hat{z} \times \frac{\partial \Delta \mathbf{H}_m^{(0)}}{\partial z}\bigg|_{z=-d_m} \, , \qquad (39)$$

where the field solution has been formally represented as:

$$\mathbf{E}_m(\mathbf{r}_{\perp}, z) \approx \mathbf{E}_m^{(0)} + \mathbf{E}_m^{(1)} + \mathbf{E}_m^{(2)} + ... \, , \qquad (40)$$

$$\mathbf{H}_m(\mathbf{r}_{\perp}, z) \approx \mathbf{H}_m^{(0)} + \mathbf{H}_m^{(1)} + \mathbf{H}_m^{(2)} + ... \, . \qquad (41)$$

Therefore, the boundary conditions from each *m*th rough interface can be transferred to the associated equivalent flat interface. In addition, the right-hand sides of Eqs. (38) and (39) can be interpreted as effective magnetic ($\mathbf{J}_{Hm}^{p(1)}$) and electric ($\mathbf{J}_{Em}^{p(1)}$) surface current densities, respectively, with *p* denoting the incident polarization; so that we can identify the first-order fluctuation fields as being excited by these effective surface current densities imposed on the unperturbed interfaces. Accordingly, the geometry randomness of each corrugated interfaces is then translated in random current sheets imposed on each reference mean plane (z=-d_m), which radiate in an unperturbed (flat boundaries) layered medium. As a result, within the first-order approximation, the field can be than represented as the sum of an unperturbed part $\mathbf{E}_n^{(0)}, \mathbf{H}_n^{(0)}$ and a random part, so that $\mathbf{E}_n(\mathbf{r}_{\perp}, z) \approx \mathbf{E}_n^{(0)} + \mathbf{E}_n^{(1)}$, $\mathbf{H}_n(\mathbf{r}_{\perp}, z) \approx \mathbf{H}_n^{(0)} + \mathbf{H}_n^{(1)}$. The first is the primary field, which exists in absence of surface boundaries roughness (flat-boundaries stratification), detailed in (Imperatore et al. 2009a); whereas $\mathbf{E}_n^{(1)}, \mathbf{H}_n^{(1)}$ can be interpreted as the superposition of single-scatter fields from each rough interface. In order to perform the evaluation of perturbative development, the scattered field is then represented as the sum of *up-* and *down-going* waves, and the first-order scattered field in each region of the layered structure can be represented in the form:

$$\mathbf{E}_m^{(1)} = \mathbf{E}_m^{-(1)} + \mathbf{E}_m^{+(1)} \, , \qquad (42)$$

$$\mathbf{H}_m^{(1)} = \mathbf{H}_m^{-(1)} + \mathbf{H}_m^{+(1)} \, , \qquad (43)$$

With

$$\mathbf{E}_m^{\pm(1)} = \sum_{q=h,v} \iint d\mathbf{k}_{\perp} e^{j\mathbf{k}_{\perp} \cdot \mathbf{r}_{\perp}} \hat{q}_m^{\pm}(\mathbf{k}_{\perp}) S_m^{\pm q(1)}(\mathbf{k}_{\perp}) e^{\pm j k_{zm} z} \, , \qquad (44)$$

$$\mathbf{H}_m^{\pm(1)} = \sum_{q=h,v} \frac{1}{Z_m} \iint d\mathbf{k}_{\perp} e^{j\mathbf{k}_{\perp} \cdot \mathbf{r}_{\perp}} \hat{k}_m^{\pm} \times \hat{q}_m^{\pm}(\mathbf{k}_{\perp}) S_m^{\pm q(1)}(\mathbf{k}_{\perp}) e^{\pm j k_{zm} z} \, , \qquad (45)$$

Therefore, a solution valid in each region of the structure can be obtained from (42)-(45) taking into account the non uniform boundary conditions (38)-(39). In order to solve the

scattering problem in terms of the unknown expansion coefficients $S_m^{\pm q(1)}(\mathbf{k}_\perp)$, we arrange their amplitudes in a single vector according to the notation:

$$\mathbf{S}_m^{q(1)}(\mathbf{k}_\perp, d_m) = \begin{bmatrix} S_m^{+q(1)}(\mathbf{k}_\perp)e^{-jk_{zm}d_m} \\ S_m^{-q(1)}(\mathbf{k}_\perp)e^{+jk_{zm}d_m} \end{bmatrix}. \tag{46}$$

Subsequently, the *nonuniform boundary conditions* (38)-(39) can be formulated by employing a suitable matrix notation, so that for the ($q=h$) *horizontal* polarized scattered wave we have (Imperatore et al. 2008a) (Imperatore et al. 2009a):

$$\mathbf{S}_m^{h(1)}(\mathbf{k}_\perp, d_m) + \mathbf{\Theta}_m^p(\mathbf{k}_\perp, \mathbf{k}_\perp^i) = N_{m|m+1}^h(k_\perp)\mathbf{S}_{m+1}^{h(1)}(\mathbf{k}_\perp, d_m), \tag{47}$$

where

$$\mathbf{\Theta}_m^p(\mathbf{k}_\perp, \mathbf{k}_\perp^i) = \begin{bmatrix} -\dfrac{k_0 Z_0 \mu_m}{2k_{zm}}(\hat{k}_\perp \times \hat{z}) \cdot \widetilde{\mathbf{J}}_{Em}^{p(1)} + \dfrac{1}{2}\hat{k}_\perp \cdot \widetilde{\mathbf{J}}_{Hm}^{p(1)} \\ +\dfrac{k_0 Z_0 \mu_m}{2k_{zm}}(\hat{k}_\perp \times \hat{z}) \cdot \widetilde{\mathbf{J}}_{Em}^{p(1)} + \dfrac{1}{2}\hat{k}_\perp \cdot \widetilde{\mathbf{J}}_{Hm}^{p(1)} \end{bmatrix} \tag{48}$$

is the term associated with the *effective* source distribution, wherein the expressions of the effective currents $\widetilde{\mathbf{J}}_{Em}^{p(1)}$ and $\widetilde{\mathbf{J}}_{Hm}^{p(1)}$, imposed on the (flat) unperturbed boundary $z = -d_m$, for an incident polarization $p \in \{v, h\}$ are detailed in (Imperatore et al. 2009a); and where Z_0 is the intrinsic impedance of the vacuum. Furthermore, the fundamental *transfer matrix operator* is given by:

$$N_{m-1|m}^q(k_\perp) = \frac{1}{T_{m-1|m}^q}\begin{bmatrix} 1 & R_{m-1|m}^q \\ R_{m-1|m}^q & 1 \end{bmatrix}, \tag{49}$$

with the superscripts $q \in \{v, h\}$ denoting the polarization. Moreover, it should be noted that on a (kth) flat interface Eq. (47) reduces to the *uniform boundary conditions*, thus getting:

$$\mathbf{S}_k^{h(1)}(\mathbf{k}_\perp, d_k) = N_{k|k+1}^h(k_\perp)\mathbf{S}_{k+1}^{h(1)}(\mathbf{k}_\perp, d_k). \tag{50}$$

We emphasize that Eqs. (47) states in a simpler form the problem originally set by Eqs. (38)-(39): as matter of fact, solving Eq. (47) $\forall m$ implies dealing with the determination of unknown scalar amplitudes $S_m^{\pm q(1)}(\mathbf{k}_\perp)$ instead of working with the corresponding vector unknowns $\mathbf{E}_m^{(1)}, \mathbf{H}_m^{(1)}$. Therefore, the scattering problem in each mth layer is reduced to the algebraic calculation of the unknown expansion scattering coefficients vector (46). As a result, when a structure with rough interfaces is considered, the enforcement of the original non uniform boundary conditions through the stratification ($m=0, ..., N-1$) can be addressed by writing down a linear system of equations with the aid of the matrix formalism (47)-(48) with $m=0, ..., N-1$. As a result, the formulation of *non-uniform boundary conditions* in matrix

notation (47)-(48) enables a systematic method for solving the scattering problem: For the N-layer stratification of Figure 1, we have to find $2N$ unknown expansion coefficients, using N vectorial equations (47), i.e., $2N$ scalar equations. It should be noted that, for the considered configuration, the relevant scattering coefficients $S_N^{+q(1)}(\mathbf{k}_\perp)$, $S_0^{-q(1)}(\mathbf{k}_\perp)$ are obviously supposed to be zero. The scattering problem, therefore, results to be reduced to a formal solution of a linear equation system. We finally emphasize that here we are interested in the scattering from the stratification; therefore, the determination of the unknown expansion coefficients $S_0^{+q(1)}(\mathbf{k}_\perp)$ of the scattered wave into the upper half-space is required only. Full expressions for these coefficients are reported in (Imperatore et al. 2009a).

6. BPT Closed-form Solution

The field scattered upward in the upper half-space in the first-order limit can be written in the form (see (42)-(45)):

$$\mathbf{E}_0^{(1)}(\mathbf{r}) = \sum_{q=h,v} \iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \hat{q}_0^+(\mathbf{k}_\perp) S_0^{+q(1)}(\mathbf{k}_\perp) e^{jk_{z0}z} \tag{51}$$

By employing the *method of stationary phase,* we evaluate the integral (51) in the *far field* zone, obtaining:

$$\mathbf{E}_0^{(1)}(\mathbf{r}) \cdot \hat{q}_0^+(\mathbf{k}_\perp^s) \cong -j2\pi k_0 \cos\theta_0^s \frac{e^{jk_0 r}}{r} S_0^{+q(1)}(\mathbf{k}_\perp^s) \tag{52}$$

with $q \in \{v, h\}$ is the polarization of the scattered field. The scattering *cross section* of a generic (nth) rough interface embedded in the layered structure can be then defined as

$$\widetilde{\sigma}_{qp,n}^0 = \lim_{r\to\infty} \lim_{A\to\infty} \frac{4\pi r^2}{A} < \left| \mathbf{E}_0^{(1)}(\mathbf{r}) \cdot \hat{q}_0^+(\mathbf{k}_\perp^s) \right|^2 >, \tag{53}$$

where < > denotes ensemble averaging, where $q \in \{v, h\}$ and $p \in \{v, h\}$ denote, respectively, the polarization of scattered field and the polarization of incident field, and where A is the illuminated surface area. The estimate of the mean power density can be obtained by averaging over an ensemble of statistically identical interfaces. Therefore, considering that the (spatial) *power spectral density* $W_n(\mathbf{\kappa})$ of nth corrugated interface is defined as in (32), the *scattering cross section* relative to the contribution of the nth corrugated interface, according to the formalism used in [Franceschetti et al. 2008], can be expressed as:

$$\widetilde{\sigma}_{qp,n}^0 = \pi k_0^4 \left| \widetilde{\alpha}_{qp}^{n,n+1}(\mathbf{k}^s, \mathbf{k}^i) \right|^2 W_n(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i), \tag{54}$$

with $p, q \in \{v, h\}$ denoting, respectively, the incident and the scattered polarization states, which may stand for *horizontal* polarization (h) or *vertical* polarization (v); the coefficients $\widetilde{\alpha}_{qp}^{m,m+1}$ are relative to the p-polarized incident wave impinging on the structure from upper

half space 0 and to the q-polarized scattering contribution from structure into the upper half space, originated from the rough interface between the layers m, $m+1$:

$$\tilde{\alpha}_{hh}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{z0}^s}{k_{zm}^s} (\hat{k}_\perp^s \cdot \hat{k}_\perp^i) \tag{55}$$

$$e^{jk_{zm}^s \Delta_m} \mathfrak{I}_{m|0}^h(k_\perp^s)[1 + \mathfrak{R}_{m|m+1}^h(k_\perp^s)] e^{jk_{zm}^i \Delta_m} \mathfrak{I}_{0|m}^h(k_\perp^i)[1 + \mathfrak{R}_{m|m+1}^h(k_\perp^i)],$$

$$\tilde{\alpha}_{vh}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{z0}^s}{k_0} \hat{z} \cdot (\hat{k}_\perp^i \times \hat{k}_\perp^s) \tag{56}$$

$$e^{jk_{zm}^s \Delta_m} \mathfrak{I}_{m|0}^v(k_\perp^s)[1 - \mathfrak{R}_{m|m+1}^v(k_\perp^s)] e^{jk_{zm}^i \Delta_m} \mathfrak{I}_{0|m}^h(k_\perp^i)[1 + \mathfrak{R}_{m|m+1}^h(k_\perp^i)]$$

$$\tilde{\alpha}_{hv}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{z0}^s k_{zm}^i}{k_0 k_{zm}^s \varepsilon_m} \hat{z} \cdot (\hat{k}_\perp^i \times \hat{k}_\perp^s) \tag{57}$$

$$e^{jk_{zm}^s \Delta_m} \mathfrak{I}_{m|0}^h(k_\perp^s)[1 + \mathfrak{R}_{m|m+1}^h(k_\perp^s)] e^{jk_{zm}^i \Delta_m} \mathfrak{I}_{0|m}^v(k_\perp^i)[1 - \mathfrak{R}_{m|m+1}^v(k_\perp^i)],$$

$$\tilde{\alpha}_{vv}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{z0}^s}{k_0^2 k_{zm}^s \varepsilon_m} e^{jk_{zm}^s \Delta_m} \mathfrak{I}_{m|0}^v(k_\perp^s) e^{jk_{zm}^i \Delta_m} \mathfrak{I}_{0|m}^v(k_\perp^i)$$

$$\{[1 + \mathfrak{R}_{m|m+1}^v(k_\perp^s)][1 + \mathfrak{R}_{m|m+1}^v(k_\perp^i)] \frac{\varepsilon_m}{\varepsilon_{m+1}} k_\perp^i k_\perp^s \tag{58}$$

$$- [1 - \mathfrak{R}_{m|m+1}^v(k_\perp^s)][1 - \mathfrak{R}_{m|m+1}^v(k_\perp^i)] k_{zm}^s k_{zm}^i (\hat{k}_\perp^s \cdot \hat{k}_\perp^i)\},$$

where $\mathfrak{I}_{m|0}^p(k_\perp)$ are the generalized transmission coefficients in upward direction (see (11)).

Furthermore, we stress when the backscattering case ($\hat{k}_\perp^s \times \hat{k}_\perp^i = 0$) is concerned, our cross-polarized scattering coefficients (55)-(58) evaluated in the plane of incidence vanish, in full accordance with the classical first-order SPM method for a rough surface between two different media (Ulaby et al, 1982) (Tsang et al., 1985).

We now show that the solution, given by the expression (55)-(58), is susceptible of a straightforward generalization to the case of arbitrary stratification with N-rough boundaries. Taking into account the contribution of each nth corrugated interface, the global *bi-static scattering cross section* of the N-rough interface layered media can be expressed as:

$$\tilde{\sigma}_{qp}^0 = \pi k_0^4 \sum_{n=0}^{N-1} \left| \tilde{\alpha}_{qp}^{n,n+1}(\mathbf{k}^s, \mathbf{k}^i) \right|^2 W_n(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) + \pi k_0^4 \sum_{i \neq j} \mathrm{Re}\left\{ \tilde{\alpha}_{qp}^{i,i+1} \left(\tilde{\alpha}_{qp}^{j,j+1} \right)^* \right\} W_{ij}(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) \tag{59}$$

with $p, q \in \{v, h\}$, where the asterisk denotes the complex conjugated, where $\tilde{\alpha}_{qp}^{i,i+1}$ are given by (55)-(58), and where the *cross power spectral density* W_{ij}, between the interfaces i and j, for the spatial frequencies of the roughness is given by (33). As a result, the scattering from the

rough layered media is sensitive to the correlation between rough profiles of different interfaces. In fact, a real layered structure will have interfaces cross-correlation somewhere between two limiting situations: perfectly correlated and uncorrelated roughness. Consequently, the degree of correlation affects the phase relation between the fields scattered from each rough interface. Obviously, when the interfaces are supposed to be uncorrelated, the second term in (59) vanishes and, in the first-order approximation, the total scattering from the structure arises from the incoherent superposition of radiation scattered from each interface.

As a result, an elegant closed form solution is established, which takes into account parametrically the dependence of scattering properties on structure (geometric and electromagnetic) parameters. In addition, as it will be shown in the following, the proposed global solution turns out to be completely interpretable with basic physical concepts, clearly discerning the physics of the involved scattering mechanisms.

7. Generalized Scattering Matrix

In this section, to emphasize the polarimetric character of the BPT solution, we introduce the *generalized bistatic scattering matrix* of the layered rough media, which can be then formally expressed by:

$$\begin{bmatrix} E_0^{sv} \\ E_0^{sh} \end{bmatrix} = \pi k_0^2 \frac{e^{jk_0 r_0}}{r_0} \sum_{m=0}^{N-1} \widetilde{\zeta}_m(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i)\widetilde{\boldsymbol{\alpha}}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) \begin{bmatrix} E_0^{iv} \\ E_0^{ih} \end{bmatrix}. \tag{60}$$

In particular,

$$\boldsymbol{\mathcal{S}}^{m|m+1}(\mathbf{k}^s, \mathbf{k}^i) = \pi k_0^2 \widetilde{\zeta}_m(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i)\widetilde{\boldsymbol{\alpha}}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) \tag{61}$$

characterizes the polarimetric response of the generic (*m*th) rough interface of the layered structure, for a plane wave incident in direction $\mathbf{k}^i$ and for a given observation direction $\mathbf{k}^s$, with

$$\widetilde{\boldsymbol{\alpha}}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) = \begin{bmatrix} \widetilde{\alpha}_{vv}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) & \widetilde{\alpha}_{vh}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) \\ \widetilde{\alpha}_{hv}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) & \widetilde{\alpha}_{hh}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) \end{bmatrix}, \tag{62}$$

and wherein

$$\widetilde{\alpha}_{vv}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) = (\varepsilon_{m+1} - \varepsilon_m)\left[\frac{\varepsilon_m}{\varepsilon_{m+1}} \frac{k_\perp^s}{k_0 \varepsilon_m}\xi_{0\to m}^{+v}(k_\perp^s) \frac{k_\perp^i}{k_0 \varepsilon_m}\xi_{0\to m}^{+v}(k_\perp^i) \right.$$
$$\left. - \hat{k}_\perp^s \cdot \hat{k}_\perp^i \frac{k_{zm}^s}{k_0 \varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^s) \frac{k_{zm}^i}{k_0 \varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^i) \right], \tag{63}$$

$$\widetilde{\alpha}_{vh}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) = (\varepsilon_{m+1} - \varepsilon_m)\hat{z}\cdot(\hat{k}_\perp^i \times \hat{k}_\perp^s)\frac{k_{zm}^s}{k_0 \varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^s)\xi_{0\to m}^{+h}(k_\perp^i), \tag{64}$$

$$\widetilde{\alpha}_{hv}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) = (\varepsilon_{m+1} - \varepsilon_m)\hat{z}\cdot(\hat{k}_\perp^i \times \hat{k}_\perp^s)\xi_{0\to m}^{+h}(k_\perp^s)\frac{k_{zm}^i}{k_0 \varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^i), \tag{65}$$

$$\tilde{\alpha}_{hh}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_{m+1} - \varepsilon_m)\hat{k}_{\perp}^s \cdot \hat{k}_{\perp}^i \, \xi_{0\to m}^{+h}(k_{\perp}^s)\,\xi_{0\to m}^{+h}(k_{\perp}^i)\,, \tag{66}$$

where we have introduced the notation

$$\xi_{0\to m}^{\pm p}(k_{\perp}) = \mathfrak{I}_{0|m}^{p}(k_{\perp})e^{jk_{zm}\Delta_m}[1 \pm \mathfrak{R}_{m|m+1}^{p}(k_{\perp})]. \tag{67}$$

It should be noted that (63)-(67) are obtained directly by (55)-(58) by making use of (11). Note also that the coefficients $\tilde{\alpha}_{qp}^{m,m+1}$ are expressible in a direct closed-form and depend parametrically on the unperturbed structure parameters. We also emphasize that the scattering configuration we have considered is compliant with the classical *Forward Scattering Alignment* (FSA) convention adopted in radar polarimetry.

Denoting with the superscript *T* the transpose, it can be verified that the scattering matrix satisfies the following relationship (Imperatore et al. 2009b)

$$\mathcal{S}^{m|m+1}(\mathbf{k}^s,\mathbf{k}^i) = \left[\mathcal{S}^{m|m+1}(-\mathbf{k}^i,-\mathbf{k}^s)\right]^T, \tag{68}$$

which concisely expresses the *reciprocity* principle of the electromagnetic theory, as expected. This is to say that the scattering experiment is invariant for interchanging the role of transmitter and receiver. Note that the inversion of the projections on the *z=0* plane $\mathbf{k}_{\perp}^i$ and $\mathbf{k}_{\perp}^s$ are directly related to the inversion of the incident and scattered vector wave $\mathbf{k}^i = \mathbf{k}_{\perp}^i - k_{z0}^i\hat{z}$ and $\mathbf{k}^s = \mathbf{k}_{\perp}^s + k_{z0}^s\hat{z}$, respectively.

As a result, the presented closed-form solution permits the *polarimetric* evaluation of the scattering for a *bi-static* configuration, once the *three-dimensional* layered structure's parameters (shape of the roughness spectra, layers thickness and complex permittivities), the incident field parameters (frequency, polarization and direction) and the observation direction are been specified. Therefore, our formulation leads to a direct functional dependence (no integral evaluation is required) and, subsequently, allows us to show that the scattered field can be parametrically evaluated considering a set of parameters: some of them refer to an unperturbed structure configuration, i.e. intrinsically the physical parameters of the smooth boundary structure, and others which are determined exclusively by (random) deviations of the corrugated boundaries from their reference position. Procedurally, once the *generalized reflection/transmission coefficients* are recursively evaluated, the (63-67) can be than directly computed, so that the scattering cross section (59) or the generalized scattering matrix (60) of a structure with rough interfaces can be finally predicted. Finally, it should be noted that the method to be applied needs only the classical gently-roughness assumption, without any further approximation.

8. Unifying Perspective on Perturbation Approaches

In this section, we first summarize and discuss the previous existing scattering models introduced to cope with simplified layered geometry with only one (or two) rough interface, whose derivation methods belong to the class of perturbative methods. In the perspective of

providing a unifying insight for the different perturbative formulations, the aim is to reconsider the state of art in an organized mathematical framework, analytically demonstrating the formal consistency of *BPT* general scattering solution, which permits to deal with layered media with an arbitrary number of rough interfaces, with the previous existing perturbative models, whose relevant first-order solutions can appear already of difficult mutual comparison (Franceschetti et al 2003) (Franceschetti et a. 2008).

In (Fuks, 2001) a model to calculate scattering from a rough surface on top of a stratified medium (see the geometry of Figure 3a) has been proposed. Expressions for scattering bi-static cross section were obtained by using the plane wave expansion of scattered EM fields and an equivalent current method, without using to the Green's function formalism. With reference to the geometry pictured in Figure 3b, an analytical small-perturbation-based model was developed to deal with a slightly rough interface boundary covered with a homogeneous dielectric layer (Azadegan et al., 2003) (Sarabandi et al, 2000). Starting from a perturbation series expansion and by employing the Green's function formalism, a solution to predict the first order bi-static scattering coefficients was found. On the other hand, the backscattering problem from the two-middle layer structure with one embedded corrugation, as schematized in Figure 3c, was investigated in (Yarovoy et al., 2000) in the first-order approximation, by using the small perturbation method combined with the Green's function approach. This approach leads to some cumbersome analytical expressions for backscattering coefficients.

As matter of fact, all these models, which refer to different simplified geometry, employ different perturbative procedures and different notations in the relative analytical derivation, so that the resulting solutions turn out mutually of difficult comparison. Besides, the finding of the connection between these existing functional forms is not a trivial task. With regard to these models, in (Franceschetti et al., 2008) it was essentially demonstrated the equivalence of the relevant analytical procedures and the consistency of the respective solutions. It should be mentioned that the models in (Fuks, 2001) and (Azadegan et al., 2003) (Sarabandi et al, 2000) are derived for a bi-static configuration. Conversely, the solution derived in (Yarovoy et al., 2000) with reference to the geometry of Figure 3c, which is even relatively more general since contemplates flat-boundaries stratification above and under the roughness, is given only in backscattering case. On the other hand, none of the pertinent configurations of these simplistic considered models is directly applicable to an actual remote sensing scenario. In fact, the natural stratified media are definitely constituted by corrugated interfaces, each one exhibiting a certain amount of roughness, whereas the flatness is an idealization which does not occur in natural media. More specifically, it can occur that, for a given roughness, one might consider an operational EM wavelength for which the roughness itself can be reasonably neglected. However, in principle, there is no defensible motivation, beyond the relevant limitation of the involved analytical difficulties, for considering the effect of only one interfacial roughness, neglecting the other relevant ones. This poses not only a conceptual limitation. In fact, in the applications perspective of retrieving geo-physical parameters by scattering measurements, whether there is a dominant interfacial roughness, and, in case, which the dominant one is, should be established after the remote sensing data are analyzed and, conversely, they cannot constitute *a priori* assumptions.

Each of the existing first-order models referring to a simplified geometry with one (Fuks, 2001) (Azadegan et al., 2003) (Sarabandi et al, 2000) (Yarovoy et al., 2000) or two (Tabatabaeenejad et al., 2006) rough interfaces, can be rigorously regarded as a particular case of our general model. Indeed, it can be analytically demonstrated that when the general

geometry reduces to each simplified one, the consistency of the relevant solutions formally holds. In fact, when the (63)-(66) are specialized for the case of Figure 3a, the factors $\Im_{0|m}^{p}(k_\perp)\exp(jk_{zm}\Delta_m)$ turn out to be unitary and the general solution formally reduces to the one in (Fuks, 2001). Similarly, by specializing the solution to the configuration of Figure 3b (Azadegan et al., 2003), the computation is reduced to only $\widetilde{\alpha}_{qp}^{1,2}$, in which

$$\xi_{0\rightarrow1}^{\pm p}(k_\perp) = T_{0|1}^{p}(k_\perp)[1 + R_{0|1}^{p}(k_\perp)R_{1|2}^{p}(k_\perp)e^{j2k_{z1}\Delta_1}]^{-1}e^{jk_{z1}\Delta_1}[1 \pm R_{1|2}^{p}(k_\perp)],$$ so the equivalent solution

in (Azadegan et al., 2003) (Franceschetti et al., 2008) is formally obtained. Finally, specializing the general solution to the geometry of Figure 3c, and considering the backscattering case ($\mathbf{k}_\perp^{i} = -\mathbf{k}_\perp^{s}$), the computation is reduced to only $\widetilde{\alpha}_{qp}^{1,2}$ in which

$$\xi_{0\rightarrow1}^{\pm p}(k_\perp^{i}) = T_{0|1}^{p}(k_\perp^{i})[1 + R_{0|1}^{p}(k_\perp^{i})\Re_{1|2}^{p}(k_\perp^{i})e^{j2k_{z1}^{i}\Delta_1}]^{-1}e^{jk_{z1}\Delta_1}[1 \pm \Re_{1|2}^{p}(k_\perp^{i})],$$ so we formally obtain the

equivalent solution in (Yarovoy et al., 2000) (Franceschetti et al., 2008).

Analytically speaking, this allows us to obtain, in a unitary formal framework, a comprehensive insight into the first-order perturbation solutions formalism for scattering from stratified structure with rough interfaces.

Finally, the *Boundary Perturbation Theory* results can be also regarded as a generalization of the classical SPM for rough surface (Ulaby et al., 1982) (Tsang et al., 1985) to layered media with rough interfaces.
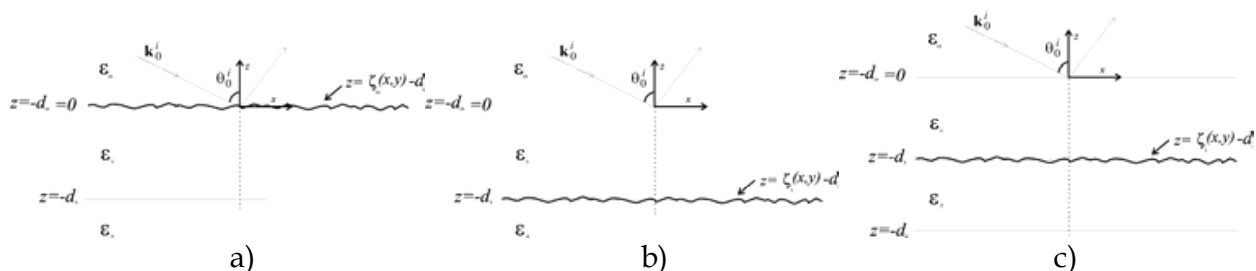

Fig. 3. Simplified geometry considered by other Authors

9. Wave Scattering Decomposition

In this section, the focus is on the intrinsic significance of the global BPT scattering solution, getting more concrete insight into the physics of the problem of the scattering from rough interfaces of a layered media. In order to be able to express the solution in terms of readable basic physical phenomena, a key point is to exploit the *local scattering* concept, differently from (Yarovoy et al., 2000) and (Fuks, 1998) wherein the authors resort to the *radar contrast*.

It should be noted that the exact *analytic decomposition* of the solution in terms of local interactions is rigorously feasible, since, in the first-order perturbative approximation, the scattering amplitude can be written as a single space integral with a kernel that depends only on the rough interface height and on its first-order derivatives at a given point. Moreover, since in the limit of first-order BPT solution the global response of a structure with all rough interfaces can be directly obtained considering the superposition of the response from each interface, we firstly focus our attention to a generic embedded rough interface. Afterwards, the general interpretation for a layered structure with an arbitrary number of rough interfaces can be addressed.

To focus formally on the relations among *local* and *global* scattering concepts, the identified *Wave Scattering Decomposition* (Imperatore et al 2008c) (Imperatore et al. 2009c), for the global scattering response of the structure in terms of the four types of local interactions, can be expressed with a compact notation as:

$$\widetilde{\alpha}_{qp}^{m,m+1} = \mathbf{P}_m^{qp}(k_\perp^s, k_\perp^i) \cdot \mathbf{\Psi}_{qp}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) \qquad (69)$$

Wherein

$$\mathbf{\Psi}_{qp}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i) = [\alpha_{qp}^{m,m+1} \quad \beta_{qp}^{m+1,m} \quad \beta_{qp}^{m,m+1} \quad \alpha_{qp}^{m+1,m}]^T \qquad (70)$$

captures the local response of the mth rough interface between two layer of permittivity $\varepsilon_m, \varepsilon_{m+1}$ respectively, and the *transfer vector* $\mathbf{P}_m^{qp}$ is related to the coherent propagation inside the stratification (Imperatore et al. 2009c). Specifically, four distinct types of local interaction with an embedded rough interface can be distinguished: two of them identifiable as local scattering through the relevant interfacial roughness and other ones as local scattering from the roughness. We emphasize that the corresponding coefficients $\alpha_{qp}^{m,m+1}$ and $\alpha_{qp}^{m+1,m}$ refer to cases in which both the observation and incidence directions are, respectively, above and under the roughness; whereas $\beta_{qp}^{m,m+1}$ and $\beta_{qp}^{m+1,m}$ concern the local scattering contribution that cross the roughness in opposite directions. In addition, we stress that the local scattering coefficients are formally identical to the classical ones relative to a rough surface between two half-spaces (Ulaby et al., 1982) (Tsang et al., 1985). On the other hand, we emphasize that the *transfer vector,* which measures the influence of the stratification on the local scattering, whatever the roughness is, can be expressed in terms of the *generalized transmission/reflection coefficients* (Imperatore et al. 2009c). Once the local nature has been recognized, the solution can be suitably expanded, so that it can be expressed as a ray series or optical geometric series, where each term of the series is susceptible of a powerful physical interpretation (Imperatore et al. 2008c) as illustrated in the next section.

10. Physical Interpretation

In this section, we show in detailed the physical meaning of the wave scattering decomposition obtained in the previous section. The analytical solution (69), after suitable expansion of the elements of the *transfer vector*, is then susceptible of an expression in terms of an infinite sum of contributions (geometric power series). Consequently, the suitably expanded solution can be expressed as an *optical geometric series,* where each term of the series is susceptible of a direct physical interpretation. In particular, each individual term of the *absolutely summable* infinite series can be physically identified as a wave propagating in the structure that experiences a single-scattering local interaction with the roughness.

To this purpose, we introduce the following useful notations

$$\vec{\Lambda}_m^h(k_\perp) = R_{m|m-1}^h(k_\perp)\Re_{m|m+1}^h(k_\perp)e^{j2k_{zm}\Delta_m} \,, \tag{71}$$

$$\ddot{\Lambda}_m^h(k_\perp) = \Re_{m|m-1}^h(k_\perp)\Re_{m|m+1}^h(k_\perp)e^{j2k_{zm}\Delta_m} \,, \tag{72}$$

and recognize that these factors correspond to a complete roundtrip in the intermediate layer with coherent reflections at the layer boundaries. Moreover, in order to provide a symmetrical expansion, it is possible to explicit the factor $\vec{M}_m^h(k_\perp^i)$, which is associated with the multiple round trip in the mth layer and is included in $\Im_{0|m}^h(k_\perp^i)$; so we can write:

$$\Im_{0|m}^h(k_\perp^i) = [\vec{M}_m^h(k_\perp^i)]^{-1}\Im_{0|m}'^h(k_\perp^i) \tag{73}$$

It should be noted that the $\Im_{0|m}'^p$ are distinct from the coefficients $\Im_{0|m}^{p(slab)}$, because in the evaluation of the former the effect of all the layers under the layer m is taken into account, whereas the latter is evaluated referring to a different configuration in which the intermediate layers $1...m$ are bounded by the half-spaces 0 and m.

Furthermore, we focus our attention on the two layers just above (m) and under ($m+1$) the generic roughness. In Figure 4, the remaining part of the structure is visualized condensed in two equivalent slabs constituted, respectively, by the intermediate layers $m+2,...,N-1$ (under the ($m+1$)th layer) and $1,...,m-1$ (above the mth layer). Without loss of generality, since analogous considerations hold for the other polarization combinations, the analysis can be conducted for the hh case only. Consequently, four distinct families of rays can be recognized; each one associated to one type of local interaction, so that each term of the expansion of the (69) can be readily identified as follows (Imperatore et al. 2009c):

a) *Local upward scattered waves from rough interface*: each of these waves (see Figure 4.a) undergoes a coherent transmission into mth layer ($\Im_{0|m}'^h(k_\perp^i)\exp(jk_{zm}^i\Delta_m)$), through the intermediate layers $1,...,m-1$, then j_1 complete round-trips ($[\vec{\Lambda}_m(k_\perp^i)]^{j_1}$) in the mth layer with coherent reflections at the incident angle ($k_\perp^i$), then an incoherent local scattering from the rough interface ($\alpha_{hh}^{m,m+1}(\mathbf{k}_\perp^s,\mathbf{k}_\perp^i)$), upward within the observation plane ($\mathbf{k}_\perp^s$), subsequently j_2 complete round-trips ($[\ddot{\Lambda}_m^h(k_\perp^s)]^{j_2}$) in the mth layer with coherent reflections at the scattering angle ($k_\perp^s$), and finally a coherent transmission ($\exp(jk_{zm}^s\Delta_m)\Im_{m|0}^{h(slab)}(k_\perp^s)$) in the upper half-space through the intermediate layers $m-1,...,1$.

b) *Local upward scattered waves through rough interface*: each of these waves (see Figure 4.b) undergoes a coherent transmission into the mth layer ($\Im_{0|m}'^h(k_\perp^i)\exp(jk_{zm}^i\Delta_m)$), through the intermediate layers $1,...,m-1$, then j_1 complete round-trips ($[\vec{\Lambda}_m(k_\perp^i)]^{j_1}$) in the mth layer with coherent reflections at the incident angle ($k_\perp^i$), subsequently a coherent transmission ($T_{m|m+1}^h(k_\perp^i)$) followed by n_1 complete round-trips in the ($m+1$)th layer ($[\vec{\Lambda}_{m+1}^h(k_\perp^i)]^{n_1}$) and by

further bounce on the (*m+1*)th flat interface ($\Re^h_{m+1|m+2}(k^i_\perp)\exp(j2k^i_{z(m+1)}\Delta_{m+1})$) at the incident angle ($k^i_\perp$), and after that an incoherent local scattering through the rough interface ($\beta^{m+1,m}_{hh}(\mathbf{k}^s_\perp,\mathbf{k}^i_\perp)$), upward within the observation plane ($\mathbf{k}^s_\perp$), subsequently j_2 complete round-trips ($[\vec{\Lambda}^h_m(k^s_\perp)]^{j_2}$) in the *m*th layer with coherent reflections at the scattering angle ($k^s_\perp$), and finally a coherent transmission ($\exp(jk^s_{zm}\Delta_m)\mathfrak{I}^{h(slab)}_{m|0}(k^s_\perp)$) in the upper half-space through the intermediate layers *m-1,...,1.*

c) Local downward scattered waves through rough interface: each of these waves (see Figure 4.c) undergoes a coherent transmission into the *m*th layer ($\mathfrak{I}'^h_{0|m}(k^i_\perp)\exp(jk^i_{zm}\Delta_m)$), through the intermediate layers *1,...,m-1*, then j_1 complete round-trips ($[\Lambda_m(k^i_\perp)]^{j_1}$) in the *m*th layer with coherent reflections at the incident angle ($k^i_\perp$), then an incoherent local scattering through the rough interface ($\beta^{m,m+1}_{hh}(\mathbf{k}^s_\perp,\mathbf{k}^i_\perp)$) downward in the observation plane ($\mathbf{k}^s_\perp$) followed by further bounce on the (*m+1*)th flat interface ($\Re^h_{m+1|m+2}(k^s_\perp)\exp(j2k^s_{z(m+1)}\Delta_{m+1})$) with subsequently n_2 complete round-trips in the (*m+1*)th layer ($[\vec{\Lambda}^h_{m+1}(k^s_\perp)]^{n_2}$) at the scattering angle ($k^s_\perp$), next a coherent transmission ($T^h_{m+1|m}(k^s_\perp)$) followed by subsequently j_2 complete round-trips ($[\vec{\Lambda}^h_m(k^s_\perp)]^{j_2}$) in the *m*th layer with coherent reflections at the scattering angle ($k^s_\perp$), and finally a coherent transmission ($\exp(jk^s_{zm}\Delta_m)\mathfrak{I}^{h(slab)}_{m|0}(k^s_\perp)$) in the upper half-space through the intermediate layers *m-1,...,1.*

d) Local downward scattered waves from rough interface: each of these waves (see Figure 4.d) undergoes a coherent transmission into the *m*th layer ($\mathfrak{I}'^h_{0|m}(k^i_\perp)\exp(jk^i_{zm}\Delta_m)$), through the intermediate layers *1,...,m-1*, then j_1 complete round-trips ($[\vec{\Lambda}_m(k^i_\perp)]^{j_1}$) in the *m*th layer with coherent reflections at the incident angle ($k^i_\perp$), next a coherent transmission ($T^h_{m|m+1}(k^i_\perp)$) followed by n_1 complete round-trips in the (*m+1*)th layer ($[\vec{\Lambda}^h_{m+1}(k^i_\perp)]^{n_1}$) and by further bounce on the *(m+1)*th flat interface ($\Re^h_{m+1|m+2}(k^i_\perp)\exp(j2k^i_{z(m+1)}\Delta_{m+1})$) at the incident angle ($k^i_\perp$), and after that an incoherent local scattering from the rough interface ($\alpha^{m+1,m}_{hh}(\mathbf{k}^s_\perp,\mathbf{k}^i_\perp)$), downward in the observation plane ($\mathbf{k}^s_\perp$), followed by further bounce on the (*m+1*)th flat interface ($\Re^h_{m+1|m+2}(k^s_\perp)\exp(j2k^s_{z(m+1)}\Delta_{m+1})$) with subsequently n_2 complete round-trips in the (*m+1*)th layer ($[\vec{\Lambda}^h_{m+1}(k^s_\perp)]^{n_2}$) at the scattering angle ($k^s_\perp$), then a coherent transmission ($T^h_{m+1|m}(k^s_\perp)$) followed by subsequently j_2 complete round-trips ($[\vec{\Lambda}^h_m(k^s_\perp)]^{j_2}$) in the *m*th layer with coherent reflections at the scattering angle ($k^s_\perp$), and finally a coherent transmission ($\exp(jk^s_{zm}\Delta_m)\mathfrak{I}^{h(slab)}_{m|0}(k^s_\perp)$) in the upper half-space through the intermediate layers *m-1,...,1.*

Note also that when an arbitrary layered structure with all rough interfaces is concerned, since in the first-order limit the multiple scattering contributions are neglected, the relative physical interpretation can be obtained effortless by superposition of the several ray contributions obtained considering separately each rough interface.
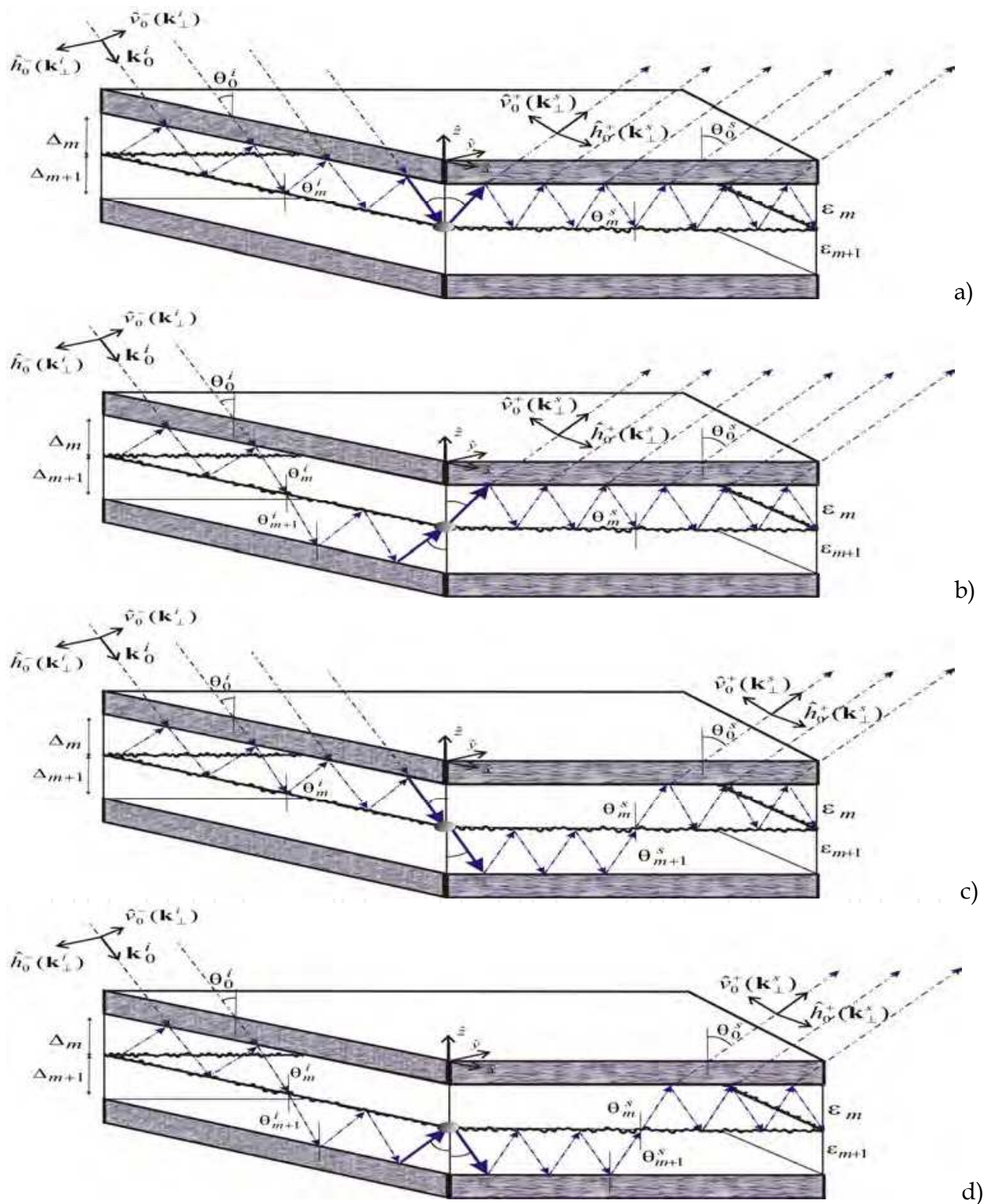

a)

b)

c)

d)

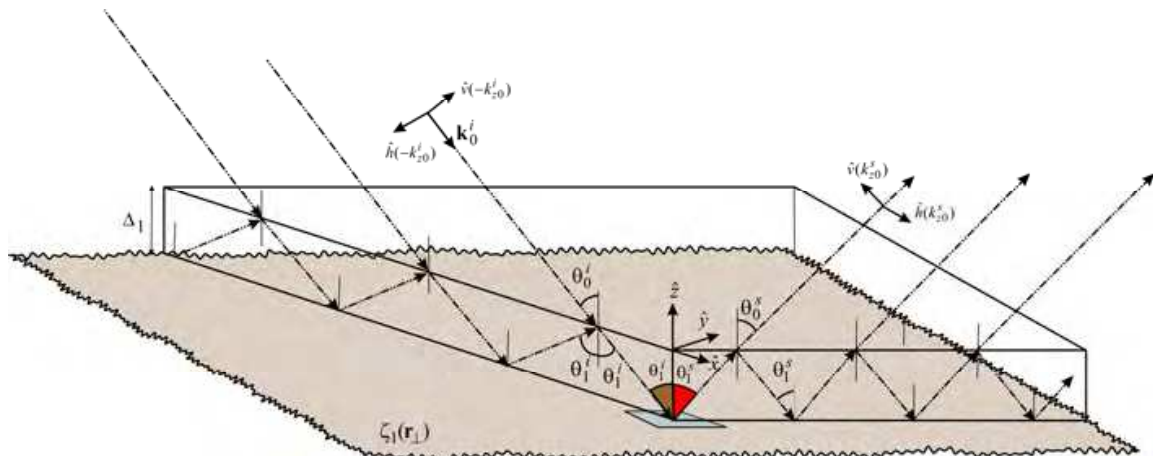Fig. 4. Physical interpretation for the scattering from an arbitrary layered structure with an embedded rough interface.

Fig. 5. Physical Interpretation: bistatic configuration
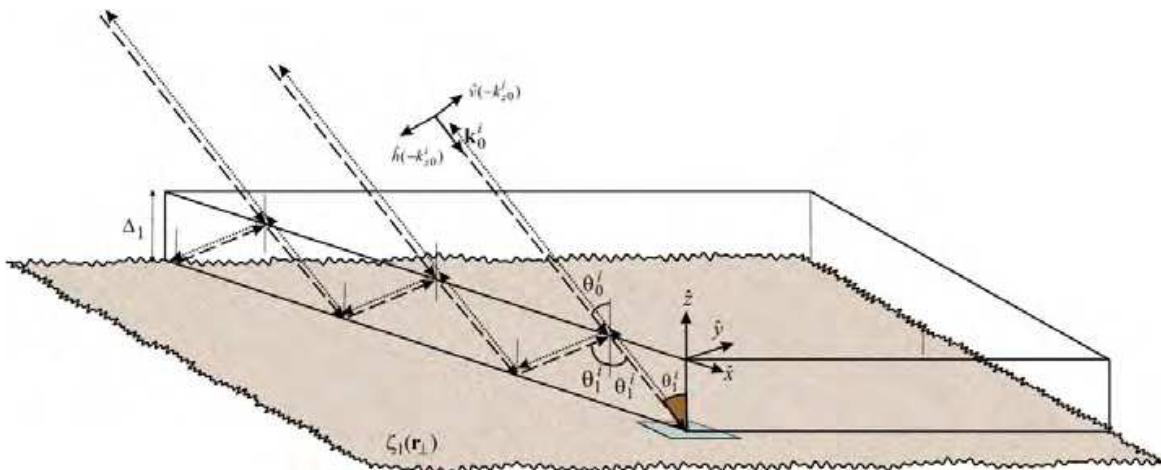
Fig. 6. Physical Interpretation: mono-static configuration

As a result, the obtained interpretation (Figure 4) enables the *global* scattering phenomenon involved to be visualized as a superposition of *local* interactions, emphasizing the role of the interference effects in the structure as well (Imperatore et al. 2009c). It should be also noted that, despite the expansion is attained rigorously without any further approximation with respect to the solution proposed (see (54)-(58)), the resulting interpretation turns out to be extremely intuitive and surprisingly simple. In particular, when the configuration reduces to a rough interface covered by a dielectric layer, as the reader can easily verify, we obtain the interpretation (Franceschetti et al., 2008) for the bistatic and monostatic configuration illustrated in Figure 5 and Figure 6, respectively.

11. Scattering patterns computation

In this section, we present some numerical examples aimed at studying scattering coefficients (59). To this purpose, we consider the canonical layered media with three rough interfaces pictured in Figure 8, which is representative of several situations of interest. In common with classical theoretical studies of the scattering of waves from random surfaces, we assume that the interfaces constitute Gaussian 2D random processes with *Gaussian correlations*, whose spectral representation is given by

$$W_n(\mathbf{\kappa}) = (\sigma_n^2 l_n^2 / 4\pi)\exp(-|\mathbf{\kappa}|^2 l_n^2 / 4) \qquad (74)$$

where, with regard to the nth interface, σ_n and l_n are the surface height standard deviation and correlation length, respectively. In order to perform a consistent comparison, we refer to interfaces with the same roughness. In addition, we suppose no correlation between the interfaces. For instance, we analyze the layered medium with three rough interfaces schematized in Figure 7, which can be parametric characterized as follows. We assume $k_0 l_n=1.5$, $k_0\sigma_n=0.15$ for $n= 0, 1, 2$. In addition, the considered vertical profile is characterized by the following parameters: $\varepsilon_0=1$, $\varepsilon_1=3.0+j0.0$, $\varepsilon_2=5.5+j0.00055$, $\varepsilon_3=10.5+j1.55$; $\Delta_1/\lambda=1.50$, $\Delta_2/\lambda=2.80$ (see Table 1). Once this reference structure has been characterized, we study the scattering cross section of the structure as a function of the scattering direction in the upper half-space, assuming fixed the incident direction. It should be noted that, also considering a limited number of layers, the number of parameters involved by the model makes difficult the jointly visualization of the multi-variables dependency. As matter of fact, once the structure has been parametrically defined and incident direction has been fixed, it is possible to visualize the scattering cross section of the structure as a function of the scattering direction in the upper half-space (Imperatore et al. 2008c). Therefore, to characterize the re-irradiation pattern of the structure in three-dimensional space, scattering cross-section distributions are represented (Figure 8) as function of zenithal and azimuthal angles and are treated as three-dimensional surfaces. To save space, only the case hh is considered. In addition, we assume fixed the incidence angle $\theta_0^i = 45°$ ($\hat{k}_\perp^i = \hat{x}$). Therefore, to evaluate the effect on the global response of each rough interface, the several single contributions are shown in Figure 8a, Figure 8b, and Figure 8c, respectively. In addition, the total contribution is also pictured (Figure 8.d). It should be noted that to visualize the patterns an offset of +40dB has been considered for the radial amplitude, so that scattering coefficients less than -40dB are represented by the axes origin.
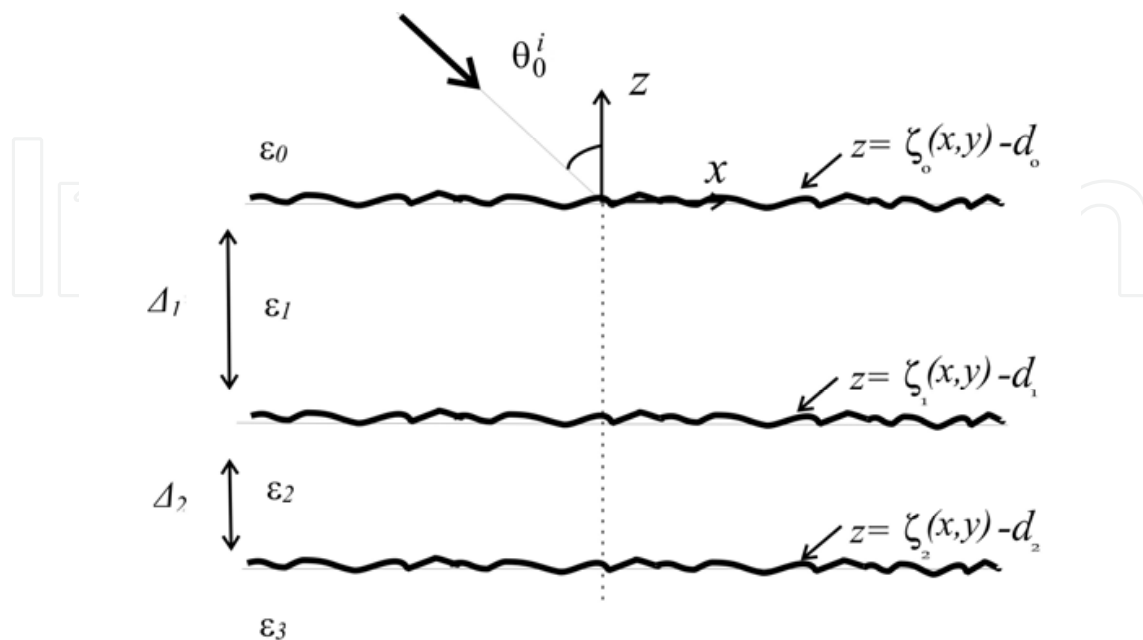

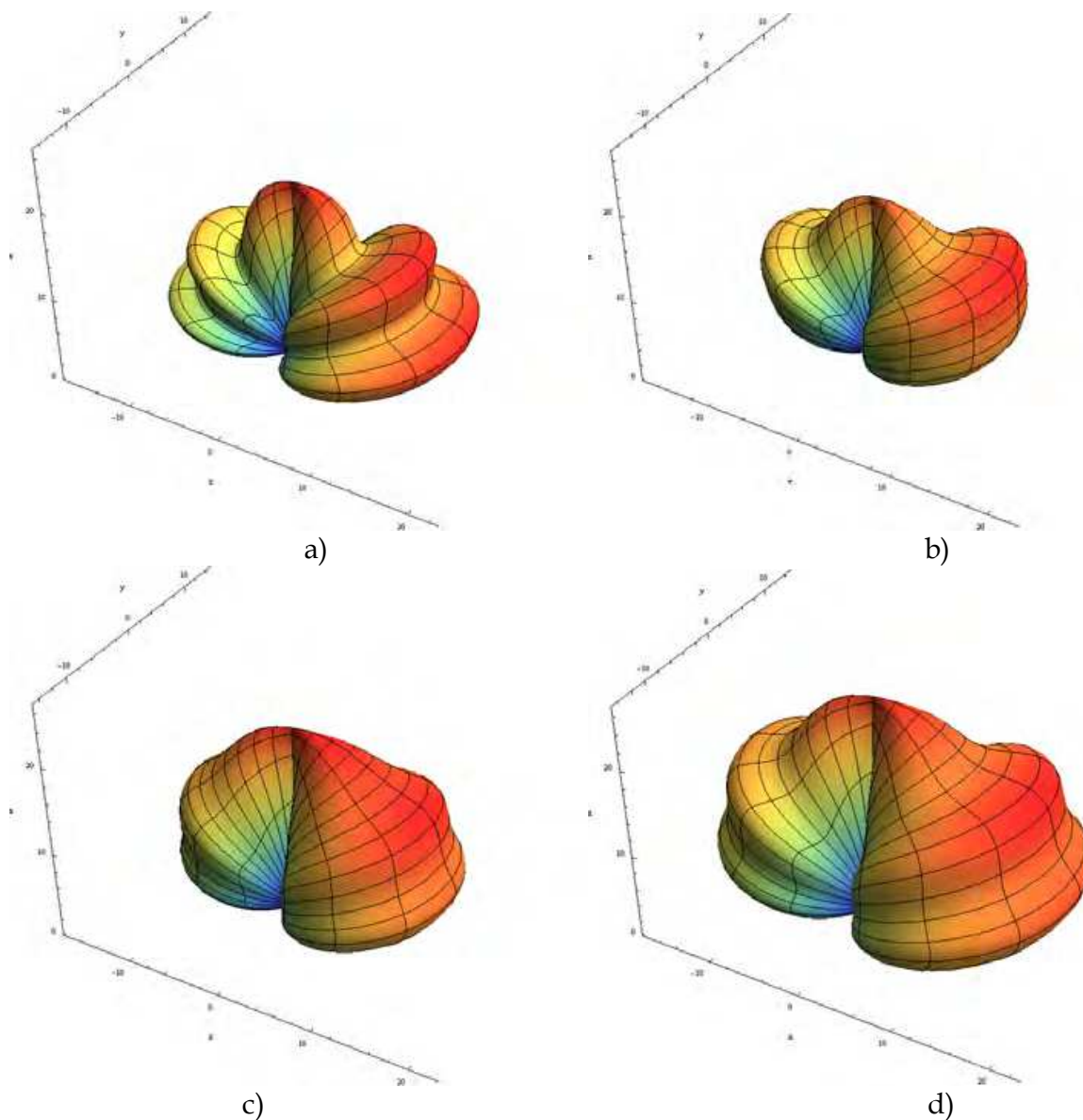Fig. 7. Three rough interfaces layered media

a)   b)

c)   d)

Fig. 8. Bi-static scattering coefficients *hh* for a three rough interfaces layered media: ζ_0 contribution (a), ζ_1 contribution (b), ζ_2 contribution (c), total contribution (d) (note that scattering coefficients values less than -40 dB are represented by the axes origin).

$\theta_0^{\,i}$	45°	$k_0\sigma_0$	0.15
Δ_1/λ	1.50	$k_0\sigma_1$	0.15
Δ_2/λ	2.80	$k_0\sigma_2$	0.15
f	1.0 GHz	k_0l_0	1.5
ε_1	3.0	k_0l_1	1.5
ε_2	5.5+j 0.00055	k_0l_2	1.5
ε_3	10.5+j 1.55		

Table 1. Parametric characterization of the layered media of Figure 7

12. Conclusion

A quantitative mathematical analysis of wave propagation in three-dimensional layered rough media is fundamental in understanding intriguing scattering phenomena in such structures, especially in the perspective of remote sensing applications. The results of the *Boundary Perturbation Theory* (BPT), as introduced by P. Imperatore and his coauthors in many different papers, essentially constitutes the content of this chapter in which the theoretical body of results is presented in organized manner, emphasizing the applications perspective. These formally symmetric and physically revealing analytical results are crucial and will contribute to innovatory applications in microwave remote sensing. For instance, they open the way toward new techniques for solving the inverse problem, for designing SAR processing algorithms, and for modelling the time-domain response of complex layered structures.

13. References

Franceschetti G., Imperatore P., Iodice A., Riccio D., and Ruello G. (2003), Scattering from a Layered Medium with One Rough Interface: Comparison and Physical Interpretation of Different Methods, *Proceedings of IEEE International Geoscience and Remote Sensing Symposium (IGARSS 2003), Toulouse, France,* pp. 2912–2914, Jul. 2003.

Franceschetti G., Imperatore P., Iodice A., Riccio D., and Ruello G. (2008). Scattering form Layered Strucutures with One rough Interface: A Unified Formulation of Perturbative Solutions. *IEEE Trans. on Geoscience and Remote Sensing*, vol.46, no.6, June 2008, pp.1634-1643.

Imperatore P., Iodice A., Riccio D. (2008a), Perturbative Solution for the Scattering from Multilayered Structure with Rough Boundaries, *Proceedings of Microwave Radiometry and Remote Sensing of the Environment* (MICRORAD 2008), pp.1-4, 11-14 March 2008, Florence, Italy.

Imperatore P., Iodice A., Riccio D. (2008b), Small Perturbation Method for Scattering From Rough Multilayers, *Proceedings of IEEE International Geoscience and Remote Sensing Symposium*, 2008 (IGARSS 2008), vol.5, pp.V-271-V-274, Boston 7-11 July 2008.

Imperatore P., Iodice A., Riccio D. (2008c), Interpretation of Perturbative Solution for the Scattering From Layered Structure With Rough Interfaces, *Proceedings of IEEE International Geoscience and Remote Sensing Symposium*, 2008 (IGARSS 2008), vol.4, pp. IV -1141-IV-1144, Boston 7-11 July 2008.

Imperatore P., Iodice A., Riccio D. (2009a). Electromagnetic Wave Scattering From Layered Structures with an Arbitrary Number of Rough Interfaces. *IEEE Trans. on Geoscience and Remote Sensing,* vol.47, no.4, pp.1056-1072, April 2009.

Imperatore P., Iodice A., Riccio D. (2009b). Transmission Through Layered Media With Rough Boundaries: First-Order Perturbative Solution. *IEEE Transaction on Antennas and Propagation*, vol.57, no.5, pp.1481-1494, May 2009.

Imperatore P., Iodice A., Riccio D. (2009c). Physical Meaning of Perturbative Solutions for Scattering from and Through Multilayered Structure with Rough Interfaces. *submitted to IEEE Trans. on Antennas and Propagation*.

Sarabandi K., Azadegan R. (2000), Simulation of interferometric SAR response to rough surfaces covered with a dielectric layer, *Proceedings* IEEE *IGARSS*, 24-28 July 2000 pp.1739 - 1741 vol.4.

Azadegan R. and Sarabandi K. (2003), Analytical formulation of the scattering by a slightly rough dielectric boundary covered with a homogeneous dielectric layer, *Proceedings of IEEE AP-S Int. Symp.*, Columbus, OH, Jun. 2003, pp. 420–423.

Fuks I. M.(1998), Radar contrast polarization dependence on subsurface sensing, *Proceedings of IEEE IGARSS'98*, vol. 3, Seattle, WA, USA, July 6–10, 1998, pp. 1455–1459.

Fuks I.M. (2001). Wave diffraction by a rough boundary of an arbitrary plane-layered medium. *IEEE Transaction on Antennas and Propagation*, vol. 49, no.4, Apr. 2001. pp.630–639.

Yarovoy A.G., de Jongh R.V., Ligthard L.P. (2000). Scattering properties of a statistically rough interface inside a multilayered medium. *Radio Science*, vol.35, n.2, 2000, pp. 445–462.

Tabatabaeenejad A., Moghaddam M. (2006). Bistatic scattering from three-dimensional layered rough surfaces. *IEEE Trans. on Geoscience and Remote Sensing*, vol. 44, no. 8, pp. 2102–2114, Aug. 2006.

Chew W. C. (1997). *Waves and Fields in Inhomogeneous Media*. IEEE Press, 1997.

Ishimaru A. (1978). *Wave Propagation and Scattering in Random Media*, vol.2. Academic Press, New York, 1978.

Tsang L., Kong J. A, and Shin R. T. (1985). *Theory of Microwave Remote Sensing*. New York: Wiley, 1985.

Ulaby F. T., Moore R. K., and Fung A. K. (1982). *Microwave Remote Sensing*, vol. I, II, II. Reading, MA: Addison-Wesley, 1982.

Geoscience and Remote Sensing New Achievements

Edited by Pasquale Imperatore and Daniele Riccio

Our planet is nowadays continuously monitored by powerful remote sensors operating in wide portions of the electromagnetic spectrum. Our capability of acquiring detailed information on the environment has been revolutionized by revealing its inner structure, morphology and dynamical changes. The way we now observe and study the evolution of the Earth's status has even radically influenced our perception and conception of the world we live in. The aim of this book is to bring together contributions from experts to present new research results and prospects of the future developments in the area of geosciences and remote sensing; emerging research directions are discussed. The volume consists of twenty-six chapters, encompassing both theoretical aspects and application-oriented studies. An unfolding perspective on various current trends in this extremely rich area is offered. The book chapters can be categorized along different perspectives, among others, use of active or passive sensors, employed technologies and configurations, considered scenario on the Earth, scientific research area involved in the studies.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

INTECH
open science | open minds