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### Method to Estimate the Basin of Attraction and Speed Switch Control for the Underactuated Biped Robot

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#### 1. Introduction

The biped robots have higher mobility than conventional wheeled robots, especially when moving on rough terrains, up and down slopes and in environments with obstacles. The geometry of the biped robot is similar to the human beings, so it is easy to adapt to the human life environment and can help the human beings to finish the complex work. With the development of the society, the needs for robots to assist human beings with activities in daily environments are growing rapidly. Therefore, a large number of researches have been done on the bipedal walking.

The dynamic system of the biped robot is a nonlinear hybrid dynamic system, which consists of continuous differential equations and discrete events dynamic maps. Therefore, this system is a complex nonlinear system. The most effective way of analyzing the global properties of the nonlinear system is probably the straightforward numerical evaluation to compute the motions and then to infer some global properties from the numerical results. It has been reported that the passive biped robot has weak tolerance for large disturbances. The basin of attraction is widely used as a measure for the disturbance rejection for the biped robots, and it is a total set of state variables from which the walker can walk successfully (Ning, L. et. al., 2007). The larger the size of the basin of attraction is, the stronger the stability is. Therefore, more and more researchers have studied the methods to compute the basin of attraction for the biped robot. The cell mapping method was proposed to compute the basin of attraction for the simplest walking model with point feet and the planar model with round feet (Schwab, A.L. & Wisse, M., 2001); (Ning, L. et. al., 2007). The results of experiments show that this method is effective; however, it is time-consuming for multidimensional state space (Zhang, P., et. al., 2008). Based on the bionics study, most humanoid robot control methods are in terms of the basic principles and characteristics of hominine gait. A robotic simulacrum potentially can be very useful. The passive biped robot can walk down along the slope only by inertial and gravitational force. But this passive walking has weak robust and stability. The basin of attraction of the simplest walker can only tolerate a deviation of 2% from the fixed point (Schwab, A.L. & Wisse, M., 2001). In

order to improve the stability of the biped robot and expand the size of the basin of attraction, most researchers have designed the controller on the biped robot. Powered robots based on the concept of the passive biped robot can walk on a level floor by ankle push-off (Tedrake, R., et. al., 2004) or hip actuation(Wisse, M., 2004). S.H. Collins exploited the robot with only ankles actuators (Collins, S.H & Ruina A., 2005). M. Wisse exploited the robot with pneumatic actuators (Wisse, M. & Frankenhuyzen, J.van, 2003). Ono proposed the selfexcited control with hip joint (Ono, K. et. al., 2004). Since the number of the input torques of these robots is less than the freedom degree, they are called the underactuated biped robot. Compared with traditional biped robots such as Asimo, the underactuated biped robot has higher energy efficiency (Garcia, M., et. al., 1998). Goswami have carried out the extensive simulation analyses of the stability of the underactuated biped walker. However, the biped robots are expected not only to walk steadily, but also to walk fast. How to accelerate the biped walking has attracted a number of researchers during the last years. Energy shaping control law was proposed by Mark Spong for the non-linear hybrid system. J.K. Holm and others applied the law to two passive-dynamic bipeds: the compass-like biped and a simple biped with torso (Jonathan, K. Holm, et. al., 2007). As the compensation for the self-gravity effects, the robot can get different speeds and different stable limit cycles. The angular velocity is changed with changing the gravity compensation coefficient; but step length can not be changed. Based on this study, we eventually develop a method to accelerate the speed of the kneed biped robot and analyze the changes of potential energy and kinetic energy during this process.

The chapter is organized as follows. In section 2, poincaré-like-alter-cell-to-cell mapping method is presented for estimating the basin of attraction of the biped robot. This method is based on the theories of the cell mapping and the point-to-point mapping. Based on the theory of the cell mapping, a method to find the fixed point of Poincaré map for the biped robot is proposed. The basin of attraction for the biped robot with knees is estimated with this method. The effects of parameters variation on the basin of attraction are discussed. Simulations and experiments will be introduced. In section 3, the speed switch control is introduced for the biped robot with knees, and the transformation of potential energy and kinetic energy is analyzed in the control process. The relationship between the control parameters and the forward speed is obtained by simplifying and analyzing the model of the kneed passive walker. In section 4, the conclusion will be presented.

#### 2. Method to estimate the basin of attraction for the biped robot

In this section, we introduce a new method to estimate the basin of attraction of the biped robot. This method is called Poincaré-like-alter-cell-to-cell mapping method, which is guided by the method proposed by (Liu, L. et. al., 2008). Poincaré-like-alter-cell-to-cell mapping method can not only be used to estimate the basin of attraction of the biped robot, but also can be used to estimate the fixed point of the Poincaré map. And then, the effects of the configurable parameters on the basin of attraction are discussed. In experiments, a kneed biped robot with point feet is used; and the effect on the basin of attraction is obtained with the variation of the mass ratio between the thigh and the shank. Results show that the size of the basin of attraction is enlarged with increasing the ratio.

#### 2.1 Cell mapping

We will introduce some concepts and terminology of the cell mapping. Firstly, a domain of interest  $\Omega \subset \mathbb{R}^{N}$  in the state space is chosen. Let the coordinate axis of a state variable  $x_i$  (*i*=1,2,...,*N*) of this domain be divided into a large number of intervals with an interval size  $h_i$ . The total number of the intervals on the  $x_i$ -axis of the domain of interest is denoted by  $n_i$ . The interval  $Z_i$  of the  $x_i$ -axis is defined to be one which contains all  $x_i$  satisfying  $a_i + (z_i - 1) \cdot h_i \le x_i \le a_i + Z_i \cdot h_i$ , where  $a_i$  is the smallest value of the  $x_i$ -axis of this domain.  $Z_i$  is a positive integer. An N-tuple  $Z_i$  (*i*=1,2,...,*N*) is called a cell vector of the state space and is denoted by z. All the cell vectors constitute a cell space, and the total number of cells is  $\prod_{i=1}^{N} n_i$ . A point  $x = (x_1, x_2, \dots, x_N)$  belongs to a cell  $z = (Z_1, Z_2, \dots, Z_N)$ , if  $x_i$  belongs to a cell  $Z_i$  for all i. Each cell is now considered as an entity and the state space is regarded as a collection of cells. So with this procedure, the continuous state space is replaced by a discrete cell space. The evolution of the cell z(n) can be described by z(n+1) = C(z(n)), that is  $Z_i(n+1) = C(z(n))$  (*i*=1,2,...,N), where C map a set of positive integers to a set of positive integers. Obviously, this mapping is called a cell-to-cell mapping, or a cell mapping.

Let  $C^m$  denote the cell mapping C applied m times with  $C^0$  understood to be the identity mapping. A sequence of K distinct cells  $z^*(j)(j=1,2,\dots,K)$  that satisfy

$$\begin{cases} z^*(m+1) = C^m(z^*(1)), \ m = 1, 2, \cdots, K-1 \\ z^*(1) = C^K(z^*(1)) \end{cases}$$
(1)

are said to form a periodic motion of period K. They are called a P-K motion. And each of its elements  $z^*(j)$  is called a periodic cell of period K or simple a P-K cell.

A cell  $z^*$  is called equilibrium cell, if it satisfies  $z^* = C(z^*)$ .

A cell *z* is said to be transient cell which is "r-step removed from a P-*K* motion", if r is the minimum positive integer such that  $C^r(z) = z^*(j)$ , where  $z^*(j)$  is one of the P-*K* cells of that P-*K* motion. In other words, *z* is mapped after r-steps into one of the P-*K* cells of the P-*K* motion and any further mapping will lock the evolution of the system in the P-*K* motion. The set of all cells which are r-steps removed from a P-*K* motion is called the "r-step domain of attraction" for that P-*K* motion.

For most physical problems once the state variable exceeds a certain scope of the domain of interest, none is interested in the further evolution of the variable. If the range of the state variable exceeds the ones that are interested, then the cell lying in it is called sink cell. Once the cell is sink cell, none is interested in its further evolution. That is to say that the region outside the domain of interest constitutes a collection of the sink cells.

It is obviously that the total number of cells is always finite, although the total number could be usually huge. The evolution of the system starting with any regular cell *z* can lead to only three possible outcomes: periodic cell, sink cell and transient cell (Hsu, C.S., 1980).

#### 2.2 Method to find the fixed point of Poincaré map for the biped robot

On the basis of the theory of the fixed point of the Poincaré map, we have analyzed the stability of the biped robot. Starting from the given point, if the gaits converge to the limit cycle that starts from the fixed point, we can say that the given point is a stable initial state and the robot can walk stably. But it is difficult to find the fixed point of the Poincaré map. The common method is Newton-Raphson iteration method. However, the initial values of the iteration have to be guessed in experience. If the initial values are not close enough to the fixed point, the iteration can not converge. Coleman and Garcia have made failure in finding the stable fixed point for 3D model with the Newton-Raphson iteration method (Garcia, M., 1999; Coleman, M.J., 1998).

Based on the theory of cell mapping, we propose a new method to find the fixed point of the Poincaré map. Firstly, we choose an initial condition state space at random. And the initial condition state space will be subdivided into cell states with feasible interval sizes. Then we obtain periodic cells under the cell mapping. All center points  $x^{(d)}$  of the periodic cells are selected together. Each center point  $x^{(d)}$  of the periodic cell is looked as the initial point of iteration. The iteration evolution of  $x^{(d)}$  is as follows: let  $x' = x^{(d)}$ , x'' = P(x'), where *P* is a Poincaré map. Let x' = x'', and calculate the above Poincaré map repeatedly. x'' = P(x') is a fixed point of the Poincaré map of the biped robot, if  $|P(x') - x'| \le \varepsilon$ . That is to say x'' = P(x'). The effective initial value of iteration can be obtained easily with this method. This method is still effective in the multidimensional state space.

#### 2.3 Poincaré-like-alter-cell-to-cell mapping method

The system that is considered in this section is

$$\begin{cases} \dot{x} = f(x) & x \notin S \\ x^+ = H(x) & x \in S \\ S = \{x | r(x) = 0\} \end{cases}$$

The steps of estimating the basin of attraction for the biped robot are listed as follows: **Step1. The state space is divided into a discrete cell space.** 

A domain of interest  $\Omega \subset \mathbb{R}^N$  in the state space is chosen. The coordinate axis of the state variable  $x_i$  (*i*=1,2,...,*N*) of this domain is divided into a large number of intervals with an interval size  $h1_i$ . The total number of the intervals is denoted by  $n_i$ . The interval  $Z_i$  of the  $x_i$ -axis is defined to be one which contains all  $x_i$  satisfying

$$a_{i} + (Z_{i} - 1) \cdot h \mathbf{1}_{i} \le x_{i} < a_{i} + Z_{i} \cdot h \mathbf{1}_{i}$$
(2)

Where  $a_i$  is the smallest value of the  $x_i$ -axis of this domain. So the cell vector z of this domain is denoted by an N-tuple  $Z_i$  (*i*=1,2,···,*N*). All the cell vectors constitute a cell space, and the total number of cells is  $\prod_{i=1}^{N} n_i$ .

#### Step2. The cell space is classified by the evolution of the cell.

(1)Let n = 1, choosing a cell z(n) ( $Z_i(n)$  ( $i=1,2,\dots,N$ )),  $x^{(d)}(n)$  is its center point, where  $x_i^{(d)}(n) = a_i + (Z_i(n)-1/2) \cdot h l_i$ . z(n) is called an original cell.

(2) Let  $x_0 = x^{(d)}(n)$  is the initial point of  $\dot{x} = f(x)$  at t = 0. Then the trajectory of this equation is calculated with initial point  $x_0$ . The original cell is a sink cell, if  $x(t_e)$  satisfies  $r(x(t_e)) = \emptyset$ , and the evolution of cell is stopped. If  $r(x(t_e)) = 0$ ,  $x^+ = H(x(t_e))$ . Then z(n+1) = C(z(n)), that is

$$Z_{i}(n+1) = C_{i}(z_{i}(n)) = Int\left\{\frac{\left(x_{i}^{+} - x_{i}\right)}{h_{i}} + \frac{1}{2}\right\}$$
(3)

where Int(y) denotes the largest integer which is not bigger than y. One can obtain the new cell z(n+1).

(3) If the state value in which z(n+1) lies exceeds the domain of interest of the state space, the original cell is a sink cell and the evolution of cell is stopped. Otherwise z(n) and z(n+1) are compared. The evolution of the cell is ended, if z(n) = z(n+1), and the original cell is an equilibrium cell. When  $z(n) \neq z(n+1)$ , let  $x_0 = x^+$  and one does repetitive operation of (2) and (3), until the new cell is equal to one of the cells that get from the evolution of z(n) previously. In the end, the original cell is a periodic cell or a transient cell that is r-step removed from a P-K motion.

Every cell of the cell space must carry out the procedure of the evolution.

#### Step3. Almost basin of attraction is obtained

Every sink cell is divided into many intervals with an interval size  $h2_i$  (*i*=1,2,...,*N*). Every equilibrium cell and periodic cell and transient cell are divided into a lot of intervals with an interval size  $h3_i$  (*i*=1,2,...,*N*), where  $h3_i$  is much smaller than  $h2_i$ . Then the total number of cells in the cell space is much larger. All center points of the cells are picked out. They constitute an almost basin of attraction of the biped robot. **Step4. Filter step-obtaining the basin of attraction** 

Since the division of the cell space affects the accuracy of the results, we set this step. Let the fixed point be a reference point. Every point  $x^{(d)}$  of the almost basin of attraction is imposed x'' = P(x') repeatedly. The point belongs to the basin of attraction, if it is close to the fixed point under the calculations.

#### 2.4 Basin of attraction for the biped robot with knees

#### 2.4.1 Model of the biped robot with knees

In this section, the goal is to estimate the basin of attraction for the biped robot with knees with the Poincaré-like-alter-cell-to-cell mapping method. Here we focus on the biped robot which could go down incline by using potential energy. This robot does not have a torso and consists of two point feet and two legs that are connected at the hip joint. Each leg has a

thigh and a shank connected at a passive knee joint that has a knee stopper. By the knee stopper, an angle of the knee rotation is restricted like the human knee. The thigh and the shank of the swing leg are assumed to be kept straight by the knee stopper during a period from the knee collision to the end of the heelstrike. Fig. 1 shows the diagram of the model of the biped robot with knees (Vanessa, F.& Hsu Chen, 2007). Table 1 lists the physical parameters and the values in simulation.

Symbols	Parameter	Value in Simulation
mH	The hip mass [kg]	0.5
m1	The shank mass [kg]	0.05
m2	The thigh mass [kg]	0.5
a1	Length between the heel and the shank COG of the swing leg [m]	0.375
b1	Length between the knee and the shank COG of the swing leg [m]	0.125
a2	Length between the knee and the thigh COG of the stance leg [m]	0.175
b2	Length between the hip and the thigh COG of the stance leg [m]	0.325
L	Leg length [m]	0.5
φ	The slope of ground [radian]	0.0504
α	Interleg angle [radian]	0.4772

Table 1. The physical parameters and the values in simulation

The entire step cycle is divided into four processes:

- (1) The stance leg straightens out and the knee is locked, just like a single link. While the swing leg with unlock knee comes forward, just like two links connected by a frictionless joint. This stage is called unlocked swing stage.
- (2) When the swing leg straightens out, the knee of swing leg is locked. The kneestrike occures. The impact takes place instantaneously.
- (3) After the kneestrike, the knee of swing leg remains locked and the system switchs to the double-link pendulum dynamics. Therefore, this stage is just like the swing stage of the compass gait model. This stage is called locked swing stage.
- (4) The locked-knee swing leg hits the ground. The premises underlying this stage are that: the impact takes place instantaneously; the impact of the swing leg with the ground is assumed to be inelastic and without sliding; the tip of the support leg is assumed not to be slip, and the robot behaves as a ballistic double-pendulum.



Fig. 1. Model of the biped robot with knees (Vanessa, F. & Hsu Chen, 2007)

Figure 2 shows the diagram of the four stages of the entire step cycle. Equations of the entire step cycle are shown in (Zhang, P. et. al., 2009).



Fig. 2. Diagram of the four stages of the entire gait cycle

#### 2.4.2 Finding the fixed point of the Poincaré map for the biped robot with knees

A passive biped robot with knees is chosen to do experiment. The values of parameters are listed in Table 1 and all of the input torques are zero. The instant just after heelstrike is defined as the Poincar é section. Let the initial condition state spaces be respectively  $[0,0.8] \times [-2,0] \times [-2,0] \times [-5,10] \times [-3,0] \times [-3,0]$  and  $[-0.8,0] \times [-1,2] \times [-1,2] \times [-5,10] \times [-3,0] \times [-3,0]$ . Each state space is subdivided into five equal division. The fixed point is found to be [0.1882 - 0.2890 - 0.2890 - 1.1090 - 0.0571 - 0.0571] by using the method proposed in this chapter, though the initial condition state spaces are different. Figure 3 presents a limit cycle for the thigh of the swing leg starting from this fixed point. In figure 3, the instantaneous angle velocity changes from the kneestrike and heelstrike are expressed as the straight lines in the limit cycle, while the angles remain the same. Figure 4 shows that the gaits of the biped robot will converge to this limit cycle within a few steps, if the initial state starts slightly away from this fixed point. [0.1982 -0.2890 -0.2890 -0.0571 -0.0571] is marked as a blue star, and the fixed point is marked as a red star. Therefore, the biped robot with knees

can walk stably. This experiment shows that the method to find the fixed point of Poincaré map for the biped robot is effective and the result of the method does not rely on the initial condition state space. The initial state of the iterations is not to be guessed.



Fig. 3. Limit cycle of the thigh of the swing leg starting from [0.1882 -0.2890 -0.2890 -1.1090 -0.0571 -0.0571]



Fig. 4. Limit cycle of the thigh of the swing leg starting from [0.1982 -0.2890 -0.2890 -0.0590 -0.0571 -0.0571]

#### 2.4.3 Estimate the basin of attraction for the biped robot with knees

In simulations, the basin of attraction of the biped robot with knees is estimated with the Poincar é -like-alter-cell-to-cell mapping method. The instance just after heelstrike is set to be the Poincar é section, so the state space satisfies  $\theta_2 = \theta_3 \dot{\theta}_2 = \dot{\theta}_3$ . In order to reduct dimensions, the interleg angle is fixed to be the fixed point's interleg angle. That is to say  $\theta_1 - \theta_2$  be equal to the fixed point's interleg angle. The values of parameters in simulation are listed in table1. The feasible state space is set as  $\theta_1 \in [0, 0.4772], \theta_2 \in [-0.4772, 0], \dot{\theta}_1 \in [-4, 1], \dot{\theta}_2 \in [-5, 11]$ . Each state space is subdivided into 80000 cells. Every periodic cell and transient cell are divided into 20 cells, and every sink cell is divided into 4 cells. Figure 5 shows the sections of this basin of attraction. In order to ensure accuracy, time-consuming is inevitable for the cell mapping method. Therefore, compared with the cell mapping method, the advantages of Poincaré-like-alter-cell-to-cell mapping method are that this method is more accuracy and saves time.



Fig. 5. The sections of the basin of attraction of the biped robot with knees

#### 2.4.4 Effect on the basin of attraction with parameters variation

In this section, we will do research in the effect on the basin of attraction of the biped robot with knees, when the mass ratio in each leg is varied. Let total mass of each leg be 0.55 [kg], and  $\mu$  denotes the ratio between the mass of the thigh and the mass of the shank. Figure 6 shows the variations of the basin of attraction of the biped robot with knees, when  $\mu$  is increased. It presents that the size of the basin of attraction becomes larger with increasing  $\mu$ . Most of the points lying in the basin of attraction assemble in the neighborhood of the fixed points. Since the basin of attraction of the biped robot with knees is a collection of the initial state points that lead to the perpetual walking, the size of the basin of attraction determines the disturbance rejection of the stable gaits. From the results of this simulation, it is proved that the greater  $\mu$  is, the stronger roboustness is. And it further proved that the greater the ratio between the mass of the thigh and the mass of the shank was, the more stable the walker became ((Vanessa, F.& Hsu Chen, 2007).





Fig. 6. Variations of the basin of attraction of the biped robot with knees with  $\mu$  ( $\mu$  is set to be 1, 15, 40, 300 respectively)

#### 3. Speed switch control for the biped robot

Now that the basins of attraction in the stable limited cycle is obtained in the last part, control methods based on calculation of the basin of attraction can be carried into execution. In this section, a speed switch control algorithm for the biped robot model is designed, based on the energy shaping theory and the estimate of the basin of attraction, to accelerate the dynamic walking and regulating the speed of walking when the parameters are varied. In order to keeping the gaits stable in accelerating process, we design a switch rule based on distinguishing the position of the switch point in the phase space. If the switch point lies in

the common pats of the basins of attraction which is belong to the stable limited cycle in different speeds, the walking speed can be adjusted only by changing the grave parameter to objective value directly. Otherwise, a transition function based on the equational constraint condition is necessary to be constructed.

First of all, the functional relationship between the control parameters and the forward speed is analyzed, which is used to construct the speed switch control algorithm. Then the speed swith control algorithm is introduced in detail and the effects on the forward speed and the walking gaits under control are studied. In the end, the trends of the kinetic energy and the potential energy are analyzed with control parameters variations during the controlled walking.



Fig. 7. Respective postures of the biped robot with knees during controlled walking stage

#### 3.1 Stable walking and gravity parameter

#### 3.1.1 Relationship between forward speed and gravity parameter

Energy shaping control law was proposed by Mark Spong for the non-linear hybrid system. Holm and others applied the law to the simple biped with torso (Jonathan K. Holm. et. al., 2007).

Based on this method, we calculate the relationship between the average speed  $\overline{v}$  and gravity parameter *f* , and showed in the equation (4)

$$\vec{v} \approx \sqrt{\frac{f g_0}{L}} \frac{L(\sin(\theta_1(0)) + \sin(\theta_2(0)))}{\pi}$$
(4)

where *f* is defined as the gravity parameter, which is the proportional coefficient of the gravity:  $f = g / g_0$ ,  $g_0 = 9.8m/s^2$ ,  $\overline{v}$  is the average speed of each gait cycle,  $\theta_1(0)$  is the initial angle values of the stance leg and  $\theta_2(0)$  is the initial angle values of the swing leg. Also we

can get the different stable limit cycles that means different walking speeds due to the different initial values with the compensation for the self-gravity effects as below.

Fig. 9 shows that the limit cycle stretches in the vertical direction with increasing the value of *f*. This experiment indicates that the angular velocity is changed, but the step length holds in line when changing *f*. Fig.10 shows the curve of  $(\theta_1, \theta_2, \theta_3)$  and  $(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3)$  for the different value of *f*.



Fig. 9. The limit cycle corresponding to different value of f (f is 1, 3, 5, 7, 9 respectively)



Fig. 10. Angular displacement and angular velocity corresponding to different value of f (f is 1, 3, 5, 7 respectively)

#### 3.1.2 Energy analysis for different gravity parameter

Expressions of the kinetic energy and the potential energy for the biped robot with knees are showed as the equation (5) and (6).

$$P(\theta) = [(m_{H} + m_{1} + m_{2})gL\cos\theta_{1} + m_{1}ga_{1}\cos\theta_{1} + m_{2}g(l_{1} + a_{2})\cos\theta_{1} - (m_{2}b_{2} + m_{1}l_{2})g\cos\theta_{2} - m_{1}gb_{1}\cos\theta_{3}]f(t)$$
(5)

$$K(\theta, \dot{\theta}) = \frac{1}{2} m_{H} L^{2} \dot{\theta}_{1}^{2} + \frac{1}{2} (m_{1}a_{1}^{2} + m_{2}(l_{1} + a_{2})^{2}) \dot{\theta}_{1}^{2} + \frac{1}{2} m_{1} (b_{1}^{2} \theta_{3}^{2} + l_{2}^{2} \dot{\theta}_{2}^{2} + L^{2} \dot{\theta}_{1}^{2}) + m_{1} b_{1} l_{2} \dot{\theta}_{3} \dot{\theta}_{2} \cos(\theta_{3} - \theta_{2}) - m_{1} b_{1} L \dot{\theta}_{3} \dot{\theta}_{1} \cos(\theta_{1} - \theta_{3}) - m_{1} l_{2} L \dot{\theta}_{1} \dot{\theta}_{2} \cos(\theta_{1} - \theta_{2}) + \frac{1}{2} m_{2} (b_{2}^{2} \theta_{2}^{2} + L^{2} \dot{\theta}_{1}^{2} - 2 b_{2} L \dot{\theta}_{1} \dot{\theta}_{2} \cos(\theta_{1} - \theta_{2}))$$

$$(6)$$

With the analysis of energy evaluation, it can be seen that the mechanical energy as well as the kinetic and potential energy changes in a similar manner. From Fig. 11, at the swing stage, mechanical energy is conserved, showd nearly as level lines in the fingure. The curves of kinetic energy with different f have similar variation, which simply decrease to the minimum value and then rise up, compensating the energy loss at kneestrike. The same trend can also be seen in the figure of potential energy.

![](_page_13_Figure_3.jpeg)

Fig. 11. The variation of potential energy and the variation of kinetic energy corresponding to different value of f (f is 1, 3, 5, 7 respectively)

#### 3.2 Construction of the control law

From the last part, we can get the basin of the attraction for different stable limit cycles, which represents different walking speeds. If we want to switch the gait from one stable limit cycle into another, we should determine whether the switch point (the initial position under control) lies in the common part of the two basins of attraction or not. In general, the point of heelstrike is chosen as the switch point and the changes of touch sensors are examined as the controlling signals. Also this piont is the fixed point in the stable limited cycle as we know from section 2.

Since different speeds corresponds to the different gravity parameters f, the speed switch control method is mainly based on the gravity parameter. The speed switch controller is designed as

$$J_r^T l_r + B(q)u = (1 - f(t))g(q) = (1 - f(t))(g_1, g_2, g_3)^T$$
(7)

When the controller is carried out, we only need to adjust the value of f.

Let  $x_0 = (q_0, \dot{q}_0)^T$ , the basin of attraction related to  $f_i$  is denoted as follows (Liu, L., et. al., 2008)

$$U_{i} = \left\{ x_{0} \middle| \begin{array}{c} \exists T_{0} > 0, \\ \text{st} , P(T_{0}, 0, x_{0}, f_{i}) = x^{*} \end{array} \right\} (i=1,2)$$
(8)

Where  $f_i$  is the gravity parameter, and  $x^*$  is the fixed point of the Poincaré return map for the biped robot with knees, that is  $P(x^*) = x^*$ .

(1) If the switch point  $x_0$  lies in the common part of  $U_1$  and  $U_2$ , f can be turn to  $f_2$  from  $f_1$  directly. Since the switch point in the basin of attraction  $U_2$ , the stable gait in the high speed related to  $f_2$  will be obtained again after a few steps.

(2) If the switch point  $x_0$  does not lie in the common part of  $U_1$  and  $U_2$ , we construct a transition function f(t) to finish the switch. Considering  $f(t) = a_0 + a_1t + a_2t^2$ ,  $a_0$   $a_1$  and  $a_2$  are the coefficients parameters. Table 2 gives some necessary signals to estimate the coefficient parameters.

$f_i$	The initial value of $f$	
$f_{e}$	The final value of $f$	
$t_{f}$	The transition time	
Т	The stable gaits period	

Table 2. The signals used to deduce f(t)

The transition function f(t) should satisfy the conditions as follows

$$\begin{cases} f_t(0) = f_i \\ f_t(t_{-f}) = f_e \\ \int f_t(0)dt = 0 \\ \int f_t(t_{-f})dt = T \end{cases}$$

$$(9)$$

In our simulation experiment, the coefficient of f(t) are listed as follows  $(f_i = f_1, f_e = f_2)$ 

$$a_{0} = f_{1}$$

$$a_{1} = \frac{1}{t_{-f}} \left( f_{2} - a_{0} - a_{2}t_{-f}^{2} \right)$$

$$a_{2} = \frac{1}{2 \times t_{-f}^{3}} \left( 6T - 6a_{0}t_{-f} - 3a_{1}t_{-f}^{2} \right)$$

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The transition function of f(t) is designed as follows:

$$f(t) = \left\{9\left(\frac{f_1 + f_2}{t_{-f}} - \frac{2T}{t_{-f}^3}\right)^2 t^4 + 12\left(\frac{3T}{t_{-f}^2} - \frac{2f_1 + f_2}{t_{-f}}\right)\left(\frac{f_1 + f_2}{t_{-f}} - \frac{2T}{t_{-f}^3}\right)t^3 + \left[4\left(\frac{3T}{t_{-f}^2} - \frac{2f_1 + f_2}{t_{-f}}\right)^2 + 6\left(\frac{f_1 + f_2}{t_{-f}} - \frac{2T}{t_{-f}^3}\right)\tau_1\right]t^2 + \left(\frac{3T}{t_{-f}^2} - \frac{2f_1 + f_2}{t_{-f}^2}\right)f_1t^2 + f_1^2\right\}^{\frac{1}{2}}\right\}$$

$$(10)$$

#### 3.3 Simulation experiments

In order to verify the validity of the speed switch control for the biped robot with knees, a model has been described by visual software. Fig. 12 shows the simulation model. The values of the structure parameters have been given in Table 1.

![](_page_15_Figure_5.jpeg)

Fig. 12. Visual simulation model of dynamic biped robot with knees

We choose the fixed ponit of the gait cycle as the switch initial position of our algorithm. Fig.13 shows the basins of the attraction in the fixed points of the limit cycles, which are obtained by the method when the gravity parameter is equal to 2 and 3 respectively. We named the basin (red part )  $U_2$  when f = 2, and the basin (blue part )  $U_3$  when f = 3. We can see the common parts of the two basins clearly. And the green points is the fixed points, which are used for the beginning points of the switch process.

![](_page_15_Figure_8.jpeg)

Fig. 13. The basins of attraction for the biped robots with different gravity parameters

The parameters of transition function are set as  $f_1 = 2$ ,  $f_2 = 3$ ,  $f_{-f} = 0.5$  and T=1.1, which means the speed can change from 1.3466 times to 2.1130 times of the original speed within 0.5s, and the variable of transition function begins with  $f_1 = 2$  and ends with  $f_2 = 3$ .

Fig. 14 lists the variation of the hip angular velocity in one step. Fig. 15 shows the change process of the hip angular velocity with changing f(t), which indicates the angular velocity began to accelerate in the second step, and then the biped robot walked fast and stably after the fifth step.

![](_page_16_Figure_3.jpeg)

Fig. 14. The hip angular velocity with f(t) compare to the hip angular velocity without f(t) in one step

![](_page_16_Figure_5.jpeg)

Fig. 15. The hip angular velocity corresponding to f(t) in several steps (begin with  $f_1 = 2$  and end with  $f_2 = 3$ )

![](_page_17_Figure_1.jpeg)

Fig. 16. The variation of potential energy and kinetic energy using the transition function f(t) (f(t) transits from 2 to 3)

The energy transformation is different from original when the forward speed is controlled. Fig.16 show the potential energy and kinetic energy with different value of f. Obviously, with increasing of the values of control parameter, the speed of the kinetic energy and the potential energy transformation raises proportionally. However, the variable range of the kinetic energy is larger than potential energy, because that the energy supplied by the control torques compensates the kinetic energy and accelerates the dynamic walking. With the gravity potential energy shaping, the kneed biped trajectory converges to a high energy limit cycle in one to three steps depending on the transition function f(t). Fig.16 present the trajectory after each step moves closer to the limit cycle on which the energy is high and stable.

#### 4. Conclusion

In this chapter, we introduce a new method to estimate the basin of attraction for the biped robot and a speed switch control algorithm to change its walking speed. The method which is called Poincaré-like-alter-cell-to-cell mapping proposed in this chapter can also be used as a tool to estimate the basin of attraction of the hybrid system. Then a research about the law of controlling the walking speed based on energy shaping and the estimate of the basin of attraction for the underactuated kneed dynamic walker is carried out, which can change the walking speed smoothly. Simulation tests are done to verify the validity of these methods.

We make a brief conclusion about the main works as follows

1) The robustness of the biped robot can be justified by the size of the basin of attraction that can be obtained by Poincaré-like-alter-cell-to-cell mapping method.

2) Poincaré-like-alter-cell-to-cell mapping method has more accuracy, and it is not time-consuming.

3) Fixed point of the Poincaré map for the biped robot can be easily found with our method which is also used as an initial parameter in the proposed control algorithm.

4) Based on this speed switch control, the speed of the dynamic walking robot can be adjusted easily.

5) The description of the kinetic energy and potential energy and the visual simulation model can be used as effective tools for the analysis and control of our model.

The work in our near future study as below

1) Using this method to estimate the stability of more complex dynamic models, especially the dynamic walking robot whose locomotion is in the three dimensional space. And obtaining its stable initial state is a tough work.

2) Optimizing the speed switch algorithm and the switch function to obtain the timeoptimized and energy-optimized trajectory in the controlled gait.

3) Researching for new mathematical tools to give the basin of the attraction more elaborated description. Meanwhile reducing the computational work in solving process is also a pressing problem.

#### 5. Acknowledgment

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![](_page_21_Picture_0.jpeg)

**Climbing and Walking Robots** Edited by Behnam Miripour

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Nowadays robotics is one of the most dynamic fields of scientific researches. The shift of robotics researches from manufacturing to services applications is clear. During the last decades interest in studying climbing and walking robots has been increased. This increasing interest has been in many areas that most important ones of them are: mechanics, electronics, medical engineering, cybernetics, controls, and computers. Today's climbing and walking robots are a combination of manipulative, perceptive, communicative, and cognitive abilities and they are capable of performing many tasks in industrial and non- industrial environments. Surveillance, planetary exploration, emergence rescue operations, reconnaissance, petrochemical applications, construction, entertainment, personal services, intervention in severe environments, transportation, medical and etc are some applications from a very diverse application fields of climbing and walking robots. By great progress in this area of robotics it is anticipated that next generation climbing and walking robots will enhance lives and will change the way the human works, thinks and makes decisions. This book presents the state of the art achievments, recent developments, applications and future challenges of climbing and walking robots. These are presented in 24 chapters by authors throughtot the world The book serves as a reference especially for the researchers who are interested in mobile robots. It also is useful for industrial engineers and graduate students in advanced study.

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![](_page_21_Picture_8.jpeg)

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![](_page_23_Picture_1.jpeg)

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