Performance Evaluation Methods to Study IEEE 802.11 Broadband Wireless Networks under the PCF Mode

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1. Introduction

In the Chapter, we consider design concepts and protocols for metropolitan area wireless networks, realization methods for adaptive dynamic polling in these networks and investigation of their main performance characteristics by means of stochastic polling models.
Polling mechanism is widely used in the wireless metropolitan area networks (WMANs). In the wireless networks with PCF (Point Coordination Function), a base station polls subscriber stations accordingly to a polling table describing the order of polling. For IEEE 802.11 Wi-Fi networks, the polling is an option; for WiMAX networks (IEEE 802.16), it is basic. Using the PCF in the MANs allows to avoid the problem of hidden stations, efficiently schedule an order of station access to the wireless channel, flexibly control the radio cell operation and change its parameters correspondingly to the current situation by adjusting only the base station.
The methods to form and keep up the polling table are not specified in the standard thus the wireless network developers can freely decide on how to realize it. The specific polling mechanism and its parameters are the main factors determining the efficiency of the broadband wireless MAN with centralized control. In the Section, we give the description of the IEEE 802.11 protocols and the main directions of their development, including the recent versions IEEE 802.11n and IEEE 802.11 VHT. The much attention is given to development and modelling of the algorithms to poll subscriber stations, the schemes of adaptive dynamic polling (ADP). The adaptive dynamic polling is proposed to cut down the expenses of polling the empty subscriber stations and stations that stopped working for some reason. The adaptive dynamic polling is the prospective direction of developing the IEEE 802.11 broadband wireless networks.
We present the models of adaptive polling system and the system with threshold polling. With the adaptive polling order, the order of queue visit is cyclic but a server does not visit queues that were empty at the instant of polling in the previous cycle. Under the threshold polling, a queue is served only of its length exceeds the given threshold. Such service discipline is a possible way to assign a priority to a queue depending on the threshold value, and it allows server to give more attention to queues with high traffic intensity rather than spend time in queues with low traffic.

2. Performance evaluation of the broadband wireless networks

Polling mechanism is widely used in the metropolitan area wireless transmission networks. In the wireless networks with PCF (Point Coordination Function), a base station polls subscriber stations accordingly to a polling table describing the order of polling. For IEEE 802.11 Wi-Fi networks, the polling is an option; for WiMAX networks (IEEE 802.16), it is basic. In this Section, we consider design concepts and protocols for metropolitan area wireless networks, realization methods for adaptive dynamic polling in these networks.

2.1 Development of the broadband wireless networks: state of the art and prospects

In the recent years, the wireless transmission networks become the main direction of the network industry development. It was provided by both the rapid Internet development and the adoption of new progressive methods for coding, modulation and wireless data transmission. Recently, it is obvious that broadband wireless networks are without a rival with their efficiency of deployment, portability, price and area of potential applications.

Wireless technologies displace the wired one almost in all places where they can provide high-quality data transmission. The tendency is evidently continuing to the future since the wireless world is more comfortable. Nowadays, the wireless data transmission technologies have become ingrained in everyday life of millions of people and enterprises. The modern wireless networks allows solving variety of problems from the indoor network management to the distributed wireless networks within a city, a region or a country. Low cost, efficiency of deployment, wide performance capabilities to transmit data, IP telephony and video streams, all these make the wireless technologies the most rapidly developing telecommunication area.

Rapid growth of the broadband wireless networks often called the "wireless revolution" in the field of data transmission networks is explained by a number of their own distinctive features, such as

- flexibility of a network topology enabling the dynamic change of the topology without time loss when mobile users connect to the network, move or disconnect;
- high data transmission rate (up to 54 Mbit/sec);
- rapidity of designing and realization which is significant because of the strict technical conditions to network construction;
- high unauthorized access protection level;
- high-priced laying or rent of the fiber optic or copper cable are not needed.

Recently, the wireless technologies provide effective solution of the following problems:

- mobile access to the Internet;
- organization of the wireless radiocommunication between workstations of a local area network (organization of the wireless access to a local area network resources);
- unification of local area networks and workstations into a single data transmission network and providing the remote access to the Internet for local area networks;
- last mile problem solution;
- interconnecting the automatic telephone systems with wireless channels of up to 54 Mbit/sec rate;
- creation of the land cellular radio modem data networks.

The mentioned features of the wireless technologies are substantially brought about by the fact that wireless networks operating within the range 2.4-6.4 GHz are based on the technology of broadband, or noise-type signal. The technology was initially used for the military purposes and recently it is efficiently used for civil radio networks.
The broadband wireless technologies use two radically different methods of frequency band utilization, they are Direct Sequence Spread Spectrum (DSSS) and Frequency Hopping Spread Spectrum (FHSS). Both methods imply the frequency band division into n subchannels. Under DSSS method, each data bit is coded as a sequence of n bits, and all those n bits are transmitted simultaneously through all n subchannels, and the coding algorithm is individual for each pair 'transmitter-receiver' so as to provide the transmission security. Under the FHSS method, a station transmits only through one of n subchannels at each time moment periodically changing the subchannel. Those change-overs (hops) happen simultaneously for both a transmitter and a receiver, and their sequence is pseudo-random and is known only for 'transmitter-receiver' pair that provide the transmission security as well.
Undoubtedly, each method has its own advantages and disadvantages. The DSSS method allows reaching the maximal throughput and due to the n-modular redundancy it, first, provides the narrow-band interference immunity, and, second, gives the opportunity to use the low power signal so as not to interfere with ordinary radio devises. On the other hand, the FHSS equipment is considerably simpler and cheaper and has the broadband interference immunity.
To work with the wireless networks, we need the special MAC (Media Access Control) protocols due to the fundamental differences from the cable medium, namely the lack of complete connection (the stations can be hidden from each other), the wireless medium is not protected from the outside signals and its signal propagation properties are asymmetric and variable by times. In order to provide the effective wireless medium access, the international standards, protocols and recommendations are developed which specify the physical and MAC layers of wireless networks: Bluetooth, ETSI Hiperlan and IEEE 802.11 for local area networks (LAN); IEEE 802.11 using the necessary amplifiers and parabolic antennae for metropolitan area networks (MAN), and finally, IEEE 802.16 and callular telephony technologies modified for data and video images transmission (GPRS, UMTS and CDMA-2000) for MANs (Vishnevsky et al., 2009).
Among the LAN and MAN developers, the IEEE 802.11 protocol is very popular (referred to as Radio-Ethernet as well) adopted as an international standard in 1997 and having the following features:

- it can be used in both LANs and MANs;
- both DSSS and FHSS methods of the broadband wireless network deployment are regulated;
- the huge number of software and hardware of the large companies (such as CISCO Aironet, Lucent Technologies, Alvarion, etc.) in the world markets support the standard.

The IEEE 802.11 protocol determines two network development topology, they are topologies with infrastructure and so called ad-hoc-topology. With infrastructure topology, a wireless network has a single access point (or base station). An access point provides the synchronization and coordination for stations within the range, transmits broadcast packets and, what is of the great importance, can be a portal into the global network. Such topology is referred to as the Basic Service Set (BSS). To cover the wide area, it is possible to set up several access points working in different frequency channels and connected to the joint wired or wireless backbone. Besides, the subscriber stations can be provided with a roaming between the access points. Such topology is called the Extended Service Set (ESS). The ad-hoc topology called the Independent Basic Service Set (IBSS) is the network performance scheme under which the numerous stations are connected directly avoiding connection to the special access point. This regime is effective when the wireless network infrastructure is not constructed (e.g., conference hall), or can not be constructed by some reason.

The IEEE 802.11 protocol specifies the data transmission rate equal to 1 or 3 Mbit/sec, with this the packet header and the service information can be transmitted at 1 Mbit/sec. Note that the transmission rate did not satisfy the users even when the protocol was adopted and approved. In order to make the wireless technology popular, cheap and, above all, to satisfy the modern strict conditions of business applications, the developers had to set up new standards which were the extensions of IEEE 802.11. Consider them in brief.

IEEE 802.1a. The IEEE 802.11a protocol exploits the radio frequency band of 5 GHz (5150-5250 MHz, 5250-5350 MHz and 5725-5850 MHz). In contrast to IEEE 802.11, the protocol applies not the spectrum broadening technologies but Orthogonal Frequency Division Multiplexing (OFDM), also referred to as multiple carrier modulation, which uses several carrier signals of different frequencies each transmitting a number of bites. This technology allows reaching the following data transmission rates: 6, 9, 12, 18, 24, 36, 48 and 54 Mbit/sec.

IEEE 802.11b. The IEEE 802.11b protocol involves the changes within the IEEE 802.11 physical layer. The network operates in 2.4 GHz radio frequency band. But the other signal modulation technology, Complementary Code Keying (CCK) allows reaching the rates 5.5 and 11 Mbit/sec and increases the connection stability in interference and multipath signal propagation conditions.

IEEE 802.11g. The IEEE 802.11g protocol as long as IEEE 802.11b operates in 2,4 GHz radio frequency band but applies the orthogonal frequency division multiplexing (OFDM) allowing to reach the data transmission rate equivalent to IEEE 802.11a (up to 54 Mbit/sec). Nevertheless, the protocol enables stations to get back to rates 1, 2, 5.5 and 11 Mbit/sec, i.e to CCK modulation. Therefore, the devices 802.11b and 802.11g are compatible within a segment of broadband wireless network.

Recently, the standard IEEE 802.11n describing the networks with data transmission rate 100Mbit/sec on the base of antenna system technology MIMO is going to be finished. The mobile version of the standard (IEEE 802.11p) and the addition IEEE 802.11e to provide the guaranteed quality of service (QoS).

In 2007, the generalized standard (see IEEE Std. 802.11-2007) was approved involving all standards finished before June, 2007. They are IEEE 802.11a/b/g mentioned above and additions IEEE802.11e/h/i/j.

The standard IEEE 802.11 is constantly being improved and developed to provide new customer services and to increase the data transmission rate and its quality. In 2009, it is planned to release a number of new standards being developed from 2003-2004. First of all, they are IEEE 802.11n and IEEE 802.11s. Though those standards are being finished, many companies have started production of devices and provide the wireless network operation based on the draft versions of those standards.

The other standards to be approved in 2009 are:

- standard IEEE 802.11u describing communications between IEEE 802.11 networks and outer networks;
- standard IEEE 802.11r regulating procedures of switching between subscriber stations for delay sensible applications like IP-telephony, etc.;

- standard IEEE 802.11p for operation in dynamic environment, and for fast moving wireless devices, in particular;
- standard IEEE 802.11v describing the wireless network control protocols;
- standard IEEE 802.11w regulating the methods to protect supervisory frames in a wireless network;
- standard IEEE802.11z describing the protocol for direct data exchanging between stations without using an access point.

The standard IEEE 802.11k should be also mentioned but it is not included into the generalized standard IEEE 802.11-2007 since its final version was released at the end of 2007 only. The standard regulates the mechanisms to exchange information about radio resource, radiochannel performance and load, noise level, etc.

The persistent growth of the transmission data volume, release of new applications, e.g. high definition video, impose heavy demands on wireless network throughput.

In spite of high data transmission rate in third generation mobile networks based on LTE technology and in networks IEEE 802.11n (up to 300 Mbit/sec), work on new technologies creation in the framework of IEEE 802.11 is continued. From 2007, the standard IEEE 802.11 VNT (Very High Troughput) has been started providing a base for very high throughput local wireless networks with nominal speed up to 500Mbit/sec within the frequency range 6 GHz. The standard is planned to be finished in 2012.

High network throughput is obtained by the MIMO technology application with 8 spaced antennas on both transmitting and receiving sides, by band enhancement up to 80 MHz through multiplexing of four channels of width 20 MGz, also by using OFDMA to organize frequency division multiple access as in IEEE 802.16. The developed standard supports compatibility with devices working under IEEE 802.11a/b/g/n.

In Russian Federation, the new technology and both hardware and software for very high throughput mesh-networks operating in the frequency range 60GHz (Vishnevsky & Frolov, 2009). As compared to existing mesh-networks, the proposed approach provides transmission rate up to 1000 Mbit/sec and makes the frequency planning and operating in duplex mode unnecessary.

In the 802.11 protocol, the fundamental mechanism to access the medium is called Distributed Coordination Function (DCF). This is a random access scheme, based on the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) protocol. Retransmission of collided packets for each station is managed according to binary exponential backoff rules (Section 2.2). The alternative access mechanism as an option specified in IEEE 802.11 is point coordination function (PCF) under which the coordinator station manages the centralized polling of other stations (Section 2.3).

2.2 Medium access layer in IEEE 802.11. Distributed Coordination Function (DCF)

The IEEE 802.11 protocol is the part of IEEE 802 protocols for local area networks (LANs)and metropolitan area networks (MANs) involving the well-known protocols IEEE 802.3 (Ethernet LAN) and IEEE 802.5 (Token Ring LAN). The majority of IEEE 802 protocols determines the physical and data link layers of the Open Systems Interconnection (OSI) seven-level reference model of the International Organization for Standardization (ISO). Furthermore, the data link layer is represented as two sub-layers, the Logical Link Control (LLC) and Medium Access Control (MAC). Such division is conditioned by the fact that under the same LLC, the mechanisms providing MAC can be different. The IEEE 802.11 involves the functional description of both MAC and PHY layers. Both layers possess significant features, e.g. the high packet loss rate due to noise and collisions, and the fact that wireless data transmission can suffer from the unauthorized access. Logical link control is not considered in the protocol since it is the same as in IEEE 802.2.

All questions on regulation of the wireless medium sharing by the network stations are determined on MAC-layer. The necessity of such regulation rules is quite obvious. Imagine the situation when each station of the wireless network sends data to the medium without observing any rules. As a result of such signals interference, the destination stations can not receive the data, and even understand the data were destined for them. Therefore, the stringent regulating rules are essential to determine the wireless medium multiple access. The multiple access rules can be compared to the rules of the road which regulate the road sharing by road users.

As it is mentioned above, there are four types of IEEE 802.11 MAC layer multiple access to wireless medium, they are Distributed Coordination Function (DCF), its extension Extended DCF (EDCF), Point Coordination Function (PCF) and Hybrid Coordination Function (HCF). Below, we consider these mechanisms in details.

The DCF is a method to organize peer-to-peer access to the wireless medium. The function is based on the Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA). With this access, each station before sending a packet listens to the medium trying to detect the carrier signal and starts transmission only when the channel is idle. But in this case the probability that a packet collides with another one upon its transmission is high enough, when two or more stations find out the channel is idle and start transmission at the moment when some station is transmitting. In order to decrease the collision probability, the Collision Avoidance (CA) mechanism is applied. The mechanism is described as follows. A station detecting the channel idle waits for the prespecified time interval before it starts transmission. The time interval is random given by two intervals: the Distributed InterFrame Space (DIFS) and the random Backoff time. Consequently, each station waits for a random time before starting data transmission that essentially decreases the collision probability since the probability that at least two stations have the same backoff is negligible.

MAC-layer of the IEEE 802.11 protocol, its modification described in IEEE 802.11e thereof, specifies five types of a time interval between consecutive data transmissions, namely Interframe Space (IFS). The shortest one is SIFS (Short Interframe Space) used for special sequence of data exchange, e.g. ACK transmission to acknowledge a frame successful reception. The SIFS duration is specified by physical layer to give enough time for the system to switch from reception to transmission or inversely. The other intervals are given in duration increasing order: PIFS, used by stations with Point Coordination Function; DIFS, used by stations with Distributed Coordination Function; AIFS, used in Extended DCF (EDCF), and EIFS (Extended Interframe Space), used by stations after transmission error.

In order to provide stations with the equal access to the channel, it is necessary to determine the appropriate algorithm to choose the backoff time which is a number of basic time intervals called time-slots. To choose the random backoff, each station determines the Contention Window (CW) which is the range the backoff time is chosen from. The minimal CW is 31 time slots, and maximal one is 1023 time slots. The backoff is determined as:

$$Backoff = Random(CW) \cdot SlotTime,$$

where $Random(CW)$ is an integer uniformly chosen in the range $(0, CW - 1)$. At each time, the value CW depends on the number n of the attempts failed to send a packet and is given by

$$CW = CW_0 2^n,$$

where CW_0 is the minimal contention window, $0 \leq n \leq m$, and m is the maximal number of attempts allowed to send a packet. If the $m + 1$-th attempt is failed the packet is discarded.
When a station tries to get an access to the channel, after the DIFS expires the backoff starts to count down. If the channel is idle during DIFS and the backoff, the station immediately starts transmission as the backoff counter reaches 0.
After the successful packet transmission, the CW is determined anew. If another station transmits during the backoff time, the backoff counter is frozen until the channel becomes idle (transmission is finished and DIFS expired). With this procedure, it is easy so see that the more times the station freezes its backoff counter the grater the probability that the packet waiting for transmission will not collide.
The described algorithm to get an access to the wireless medium guarantees the equal access for all stations in the network. But under such procedure, the probability of collision (that an arbitrary packet collides upon its transmission) is still non-zero. The collision probability could be reduced by extension of the maximal CW. But it leads to the grater backoff value which decreases the channel throughput. Therefore, the DCF uses the following algorithm to minimize collisions. After each successful frame reception, the destination station sends the ACK (ACKnowledgement) after SIFS to acknowledge the transmission was successful. If collision happens upon transmission the sender station does not get an ACK, so it finds out the transmission failed. The sender waits for ACK during EIFS, and if the ACK is not received the sender increases its CW. Thus, if the CW is 31 slots for the first transmission attempt for the second one it is 63, 127 for the third, 255 for the fourth, 511 for the fifth, and 1023 for other attempts. It can be seen that the CW is dynamically increased with collision number increasing, which allows reducing both the delay and the collision probability.
Note that the sender station can not receive the ACK frame indicating the transmission was success due to collision or signal distortion. And both reasons are not distinguishable for the sender station.
As it is mentioned above, all stations are equal to get an access to the wireless medium due to the contention mechanism, and no station has a priority to transmit data. This restricts DCF to provide Quality of Service (QoS). In IEEE 802.11e protocol, QoS is enabled by Enhanced DCF (EDCF). The EDCF mechanism is similar to DCF, but the difference concerns the CW size and the backoff counter to provide the priority access for various applications to wireless medium. For EDCF the traffic is divided into categories (TC, Traffic Categories) which differ from each other by priority to get an access to the transmission medium. Meanwhile, the medium access mechanism is the same as for DCF contention based.
Before a station detecting the channel idle starts transmission, it waits for AIFS (Arbitration InterFrame Space) and then counts down the backoff counter. The backoff counter is uniformly chosen in the range $[1, CW(TC) + 1]$ where $CW(TC)$ is the contention window for the given TC. TC priority is provided by using the different values of minimal and maximum CW and AIFS. Thus, the minimal value of AIFS is always DIFS but it can be increased depending on the TC.
In case of collisions which happen when two or more backoff counter drop to zero simultaneously and AIFS intervals are the same, the CW is increased. For EDCF, the new CW size is

determined as $newCW(TC) = ((oldCW(TC) + 1)PF) - 1$, where PF is the constant CW scaling factor depending on the TC. Note that for DCF we have $PF = 2$.
Thus in EDCA, the TC is assigned a priority by variation of the parameters CWmin, CWmax, AIFS, PF. Each station of the wireless network can have up to 8 TC queues for transmission. But in this case, the TC backoff counters within the same station can drop to zero simultaneously which is called the virtual collision. To avoid virtual collisions, the station uses the special queue scheduler which provides the higher priority TC with the priority access to the transmission medium.
The considered mechanism of transmission medium multiple access control have the same bottleneck, so-called problem of hidden stations. The situation happens when two stations can not listen to each other directly due to natural barriers. Such stations are called hidden. To avoid the problem, the DCF and EDCF mechanisms have the optional technique known as Request-To-Send/Clear-To-Send (RTS/CTS).
Accordingly to RTS/CTS, before transmitting a packet, a station operating in RTS/CTS mode "reserves" the channel by sending a special Request-To-Send short frame. An RTS frame involves the information on the forthcoming transmission and destination station and is available to all stations in the network (except ones hidden from the sender station). It allows the other stations to postpone transmission for the declared transmission time, and during this time the channel is considered "virtually busy". The destination station acknowledges the receipt of an RTS frame by sending back a Clear-To-Send (CTS) frame, after which normal packet transmission and ACK response occurs. Since collision may occur only on the RTS frame, and it is detected by the lack of CTS response, the RTS/CTS mechanism allows to increase the system performance by reducing the duration of a collision when long messages are transmitted.
The DCF and EDCF are simple and reliable mechanisms of multiple access to transmission medium in IEEE 802.11 broadband wireless networks. But they have two shortcomings: the lack of both QoS support and solution of the hidden stations problem. Thereby, the IEEE 802.11 protocol was supplemented with the alternative methods to multiple access control.

2.3 Point coordination function

The DCF mechanism described above is basic for IEEE 802.11 protocols and can be used in both ad-hoc wireless networks and infrastructure networks (having Access Point, AP). But for the infrastructure networks, there are more appropriate methods to control the transmission medium multiple access, namely, Point Coordination Function (PCF) and Hybrid Coordination Function (HCF). Note that PCF and HCF mechanisms are optional and applied for networks with AP only.
In case of PCF, one of stations (access point) is central and called the Point Coordinator (PC). The PC controls is imposed a responsibility to control the multiple access on the base of the specified polling algorithm or information on station priority. Thus, the PC polls all stations in the network listed in its polling table and organizes data transmission between network stations. As opposed to DCF where each station decides for itself when to start transmission, the PCF implies that only PC decides what station can get an access to the channel. It is significant to note that this approach completely avoids the contention access to the channel unlike DCF, makes collisions impossible and provide the priority access for time-sensitive applications. Thus, PCF can be used to organize collision-free priority access to the transmission medium.

The Point Coordination Function does not contradict the Distributed Coordination Function and rather supplements it. In fact, the PCF networks can use both PCF and traditional DCF. During network operating, the time intervals for PCF and DCF alternate.
In order to provide alternating the PCF and DCF modes, the access point which realizes PCF has to have a priority access to transmission medium. It is possible if an access to the channel is contention one (as for DCF) but the time interval for the access point to wait the response is less then DIFS. In this case, when the access point tries to get an access to the channel it waits the end of transmission as other stations do and since it has the minimal inferframe space, it gets an access first. The interframe space for the access point is called PIFS (PCF Interframe Space), diven that $SIFS < PIFS < DIFS$.
The DCF and PCF mechanisms are combined in so called superframe which is the sum of PCF interval of the contention-free access called CFP (Contention-Free Period) and the succeeding DCF interval of CP (Contention Period). The CP length should be enough to enable the transmission of at least one frame by using PCF mechanism. It is necessary for the association procedure but it is out of our topic. The superframe starts with the beacon frame, and all stations having received it postpone their transmissions for the time determined by the CFP. The beacons contain the information on the CFP duration and allow synchronizing the operation of all stations.
Under PCF, the access point sends data packets (DATA) destined for stations (if available) and asks (polls) all stations about the frames waiting for transmission by sending them the service frame CF-POLL (invitation to transmit). The access point polls stations accordingly to its polling list (polling table). The methods to form and keep up the polling table are not specified in the standard thus the wireless network developers can freely decide on how to realize it.
A station can transmit packets to the channel only when it receives the CF-POLL. Having received the CF-POLL, the station sends the short frame containing both data (if available) and acknowledgement of CF-POLL reception after SIFS. If there are no data to transmit, the station answers with NULL frame containing the header only. If the access point gets the answer frame, it waits for SIFS and polls the next station. Otherwise if there is now answer during PIFS, the access point considers the station unaccessible and polls the next station.
In order to cut expenses, an access point can combine the CF-POLL with data transmission (the frame DATA+CF-POLL). Similarly, the stations are allowed to combine acknowledgement frame with data transmission (the frame DATA+CF-ACK). Under the PCF, there are four types of a frame:

- DATA, data frame;
- CF-ACK, acknowledgement frame;
- CF-POLL, poll frame;
- DATA+CF-ACK, combined frame of data and acknowledgement;
- DATA+CF-POLL, combined frame of data and poll;
- DATA+CF-ACK+CF-POLL, combined frame of data, acknowledgement and poll;
- CF-ACK+CF-POLL, combined frame of acknowledgement and poll.

Along with the extended distributed function EDCF in the IEEE 802.11e protocol, the Hybrid Coordination Function (HCF) is determined. Similarly as the EDCF is extension of the DCF, the HCF extends the PCF.

Due to the fact that the PCF and HCF realize centralized non-collision priority access to transmission medium, they completely solve the problem of hidden stations and provide QoS. But regardless of their advantages, the PCF and HCF are harder to be realized then the DCF. Besides, the point coordination functions imply relatively large number of service frames (CF-POLL, etc.) that essentially increases overhead expenses of data transmission in the wireless medium. To organize the wireless network with PCF or HCF, all stations support these regimes and one station serves as an access point while the DCF allows organizing the ad-hoc network without access point. The distributed coordination functions are reasonable to be used in the simplest wireless networks without hidden stations and delay-sensitive applications.

2.4 Problem of hidden stations in wireless metropolitan area networks

In IEEE 802.11 broadband wireless network, the basic structure unit is a radio-cell having star-shape structure: the center is a base station with omnidirectional antenna which antennae of all subscriber stations are focused on presenting the radio bridges between wireless network and local cable networks.

In typical conditions of the wireless metropolitan area network, the subscriber stations do not have a radio visibility for each other (they are hidden from each other) and have to communicate through the retransmitting base station disposed on the high altitude (tower buildings, TV tower, etc.) and providing an access to the outer network. Thus, the broadband wireless metropolitan area networks have the mesh structure: a mesh is a radio cell and subscriber stations are connected with the high-speed backborn network.

Below we consider the operation of a radio cell in the broadband wireless metropolitan area network.

A radio cell has the following features:

- all subscriber stations are hidden from each other and their antennae are focused on the base station, i.e. data transmission between subscriber stations is possible through the base station only;
- the distances between the base station and subscriber ones are large enough (several kilometers) and different.

Each of the IEEE 802.11 protocols supports its own set of transmission rates. For IEEE 802.11b, they are the channel rates 1, 2, 5.5 and 11 Mbit/sec, for IEEE 802.11a they are 6, 9, 12, 18, 24, 48 and 54 Mbit/sec. A rate is a type of modulation which modulate the carrier radio signal or its part in the system with several carriers. The "faster" modulations are less noise-resistant, and otherwise. For this reason, the IEEE 802.11 based devises use more noise-resistant modulations, or lower transmission rates in cases of the signal depression and large number of losses when transmitting frames. It is obvious that a frame of n bits is transmitted longer by rate 1 Mbit/sec (hence it reserves a channel for a longer time) than by rate 11 Mbit/sec. In case when a subscriber station is far off or is placed in a nasty place, the radio signal it receives from the base station is weak, and the subscriber station automatically decrease the transmission rate. It results in decreasing of the network throughput. In case of PCF, the situation gets worth as it is impossible to control the subscriber station behavior. Remind, that accordingly to the DCF and EDCF mechanisms, the subscriber station decides by itself when to transmit data. Besides, regardless of data transmission directions within a radio cell, they come through the base station, thus it is logical to impose the base station on the radio cell control responsibility. It easy to see that due to the above-mentioned features, it is not reasonable to use the distributed coordination functions in broadband wireless metropolitan area networks. Even though the equipment supporting the centralized control is more complicated for development and production hence it is more expensive, it much more efficiently uses two most valuable resources of the broadband wireless network, they are frequency and throughput. The centralized control in the MANs allows to avoid the problem of hidden stations, efficiently schedule an order of station access to the wireless channel, flexibly control the radio cell operation and change its parameters correspondingly to the current situation by adjusting only the base station. The adaptive centralized control is a prospective direction of developing the IEEE 802.11 broadband wireless networks.

2.5 Adaptive dynamical polling in wireless networks

Since both the Point Coordination Function and Hybrid Coordination Function are based on the centralized control (polling), the main attention should be focused on the method of the centralized control realization. The specific polling mechanism and its parameters are the main factors determining the efficiency of the broadband wireless MAN with centralized control.
The IEEE 802.11 standards allow the developers freely construct the methods of PCF and HCF realization. The standard does not specify: the method to poll the subscriber stations, method to construct and support the polling table, the policy to form queues of frames for transmission, and recommendations on the relation between DCF and PCF intervals in a superframe. Further, within the Section the main attention is given to development and modelling of the algorithms to poll subscriber stations, the schemes of adaptive dynamic polling (ADP).
For each subscriber station, the base station forms a queue of frames. Within a queue, the frames are ordered accordingly to the priority of traffic they belong to. Then the base station starts the polling cycle. As the cycle starts, it polls the first station in the polling list (polling table) sending frames from the corresponding queue to the station. Then the base station polls it sending the CF-POLL (or QoS CF-POLL) and receives the frames to be transmitted from the station (if available). As the base station finishes polling the subscriber one, it switches to polling the next station accordingly to the polling table. Note that PCF leads to the overhead expenses which can not be avoided during data exchange between the base station and the subscriber one, they are the time to switch to subscriber stations and service frames for polling them (CF-POLL, etc.).
The development the algorithm of adaptive dynamic polling lead to the following problems:

- choosing the method to poll the subscriber stations;
- choosing the policy to work with a subscriber station (receiving and transmission);
- development of methods to minimize the overhead expenses and;
- development of methods to optimize the system parameters.

Since the base station is substantially the central device in a radio cell of the broadband wireless MAN, it is reasonable to provide it with the complete functionality concerning the radio cell control, service of the queues, determination of the polling order and minimization of the overhead expenses. For the equipment developers, it means that the base station has to carry the majority of the network features. A subscriber station plays a passive role just answering the service frames from the base station.
Efficiency of the broadband wireless PCF network performance considerably depends on the method of polling mechanism realization. At the same time, the choosing of a cyclic polling type and a queue service discipline is determined by the method of the broadband wireless MAN (Metropolitan Area Network) application. Normally, there are two situations during the broadband wireless MAN operating. In the first one, uplink traffic from the base station to end

stations dominates over the downlink one. Such a traffic is typical while using a broadband wireless MAN cell as a "last mile" for Internet provider. In this case, each end station has its own segment of LAN (Local Area Network) that gets an access to Internet via the wireless network. The investigation of the real broadband wireless MAN (Vishnevsky, 2000) working as the "last mile" shows that traffic from the outer network to the local segment is much higher than the backward traffic.

In the second case, downlink traffic from an end station to the base one dominates over the uplink one. Such a traffic is typical while using the broadband wireless network as backbone network to transmit information from the objects "behind" the end stations to the outer network (usually, to an information data storage center). The situation takes place when the wireless network is used for the video monitoring systems, automatic process control systems, telemetering, systems of collection, storage and processing of data.

Further we say a cell works in the "last mile" mode if the downlink traffic is dominate and it works in the "data collection" mode if the uplink traffic is dominate.

The fundamental difference between two situations is that the base station (coordinator of the wireless network cell work) knows the parameters of the queue of frames to be transmitted to end stations in the case of dominating downlink traffic. Thus, the base station can choose the policy to work with an end station queue before it polls the station. In the second case, the base station knows nothing about the queues of frames to be transmitted when the uplink traffic is dominate. Hence, it needs to poll the end station first and then to decide on the policy to work with the queue.

When a cell works in the "data collection" mode we can neglect the data traffic from the base station to the end ones considering just uplink traffic from the end stations to the base one. In this case, the base station polls an end one first, that is connects to the end station and starts transmitting. Then, the base station makes an attempt to switch to the next end station accordingly to the polling table. Moreover, the base station does not know if the end station will respond to the poll and if it will have the frames for transmission. In order to cut down expenses of polling the empty end stations and stations that stopped working for some reason we propose not to poll those stations at the next polling cycle.

Thus the rule to poll the end stations could be as follows. The base station polls a subscriber one if it polled it in the previous cycle and the end station had frames for transmission or it was skipped. In the case the end station did not respond to the base one in the previous cycle or it was empty when being polled, it is skipped (not polled) by the base station in the current cycle. Since the base station can not estimate the number of frames in the queue to be transmitted in advance (before polling the end station) it is not reasonable to serve the queue until it is empty. Thus, we propose the discipline to serve the end station: the base station transmits only the frames which were present in the queue at the station polling epoch.

The adequate model to investigate the performance characteristics of the broadband wireless network working in the "data collection" mode described above is the polling system with adaptive polling mechanism and gated service analyzed in Section 3 where we presented the analytical and simulation results.

When a cell works in the "last mile" mode, the data traffic from the end station to the base one can be neglected. So we consider just downlink data traffic from the base station to the end one. In this case, the base station sends frames from the queue to the corresponding end station. Then, it waits for the successful data transmission acknowledgment from the subscriber station (during PIFS) and starts sending frames from the next queue. Time to switch between queues is random so it is impossible to say in advance if the data are transmitted successfully. If the queue of frames is short it is obvious that data transmission costs are high. So, it is better to serve the queue only if its length exceeds the given value called the threshold in order to cut down the expenses. To maximize the system throughput, the queues of each end station should be served until empty. But in the case when one or several queues are long enough, the frame mean waiting time in the queue is large that is unacceptable for some network applications. So, the most important system parameters are the mean queue length and the frame mean waiting time. These parameters can be estimated by means of stochastic model with exhaustive threshold service considered in Section 4.

3. Adaptive polling system

The polling systems are varieties of the queuing systems with multiple queues and one (or more) server(s) common to all queues that poll the queues and serve the queued customers. Classification of polling systems and methods to study them are presented in reviews (Levy & Sidi, 1990; Vishnevsky & Semenova, 2006) and books (Takagi, 1986; Borst, 1996; Vishnevsky & Semenova, 2006).

The adaptive polling mechanism assumes that the server polls not all queues in a cycle. The queues that were empty at the previous polling epoch are skipped (not visited) by the server and are visit in the next cycle only. Unfortunately, the exact analysis of adaptive polling systems is cumbersome, and in this Section, we provide an approximate analysis.

We consider the polling system with N queues (having unlimited waiting space) and a single server which is common for all queues. Each queue, say i, has a Poisson input of customers with parameter λ_i. Service times at queue i are independent and identically distributed with the distribution function $B_i(t)$ with the moments $b_i^{(r)} = \int_0^\infty t^r dB_i(t)$, $r \geq 1$, and Laplace-Stieltjes transform (LST) $\beta_i(s) = \int_0^\infty e^{-st} dB_i(t)$, $i = \overline{1,N}$.

The server visits queues in cyclic order from queue 1 to queue N accordingly to an adaptive scheme. A cycle is referred to as the time the server spends visiting (serving) queues 1 through N. With adaptive mechanism, the server skips (does not visit) queues which were visited in the previous cycle and were found empty at their polling moments. After being skipped, a queue is always visited in the next cycle.

If queue i is polled (or visited) by the server, the switchover time is incurred with distribution function $S_i(t)$ having the LST $\tilde{S}_i(t)$ and the moments $s_i^{(r)}$, $r \geq 1$, $r = \overline{1,N}$. A polling moment is referred to as a moment when the switchover time is finished and the server is ready to start working at a queue. The service discipline is gated, i. e. the server serves only those customers that presented at a queue at its polling instant. The customers arriving during the queue service time will be served in the next cycle.

If N queues were sequentially found empty at their polling moments, the server takes a vacation having the distribution function $F(t)$ with LST $\varphi(s)$, and moments $\varphi^{(k)}$, $k \geq 1$. After a vacation, server starts working at the queue that he stopped polling at and went for a vacation.

The approach we use to investigate the model with several queues is based on the decomposition of the polling system to separate queues, further analysis of a queue as a queueing system with server's vacations, and then, application of the obtained results to the system with several queues. The analysis aims at deriving the first and second moments of the mean waiting time in a separate queue and is partially based on the results obtained in (Sumita, 1988) for the queueing system with server's vacations which are discussed in Section 3.1.

3.1 Analysis of a single queue

First, consider a single queue with server's vacations. Within this subsection, we omit the lower index denoting the queue number. We assume that the vacation time depends on the fact whether the queue was empty at the moment the server finishes the previous vacation or it was not. If not, the vacation time has distribution function $H(t)$ with LST $h(s)$ and the moments $h^{(k)},\ k \geq 1$. If the queue is empty when server finishes a vacation, the next vacation period is distributed with the function $\tilde{H}(t)$ having the LST $\tilde{h}(s)$, and the moments $\tilde{h}^{(k)},\ k \geq 1$.
To investigate the queue, consider the Markov chain describing the queue state at the end of vacations. Let t_k be the kth epoch when vacation is finished, $k \geq 1$, and i_{t_k} be the number of customers at the queue at the epoch t_k. It is easy to see that the process $i_{t_k},\ k \geq 1$, presents the discrete-time Markov chain, and its one-step transition probabilities

$$p_{i,j} = P\{i_{t_{k+1}} = j | i_{t_k} = i\},\ i, j \geq 0,$$

have the form

$$p_{0,j} = \tilde{y}_j,\ p_{i,j} = \sum_{l=0}^{j} a_l^{(i)} y_{j-l},\ i > 0,\ j \geq 0,$$

where

$$a_l^{(i)} = \int_0^{\infty} \frac{(\lambda t)^l}{l!} e^{-\lambda t} dB^{(*i)}(t),\ y_l = \int_0^{\infty} \frac{(\lambda t)^l}{l!} e^{-\lambda t} dH(t),\ \tilde{y}_l = \int_0^{\infty} \frac{(\lambda t)^l}{l!} e^{-\lambda t} d\tilde{H}(t),\ l \geq 0,$$

and $B^{(*i)}(t)$ is the i-fold convolution of the distribution function $B(t)$.
It can be shown that under the condition $\rho = \lambda b^{(1)} < 1$ fulfilled, the stationary state probabilities exist

$$q^{(j)} = \lim_{k \to \infty} P\{i_{t_k} = j\},\ j \geq 0.$$

These probabilities satisfy the following set of the balance equations, $j \geq 0$,

$$q^{(j)} = q^{(0)} \int_0^{\infty} \frac{(\lambda t)^j}{j!} e^{-\lambda t} d\tilde{H}(t) + \sum_{i=1}^{\infty} \int_0^{\infty} \frac{(\lambda t)^l}{l!} e^{-\lambda t} dB^{(*i)}(t) \int_0^{\infty} \frac{(\lambda t)^{j-l}}{(j-l)!} e^{-\lambda t} dH(t). \quad (1)$$

By multiplying the equations (1) by the corresponding powers of z and summing them up, it readily follows that the probability generating function (PGF) $Q(z) = \sum_{j=0}^{\infty} q^{(j)} z^j$ of the stationary probabilities satisfies the functional equation

$$Q(z) = (Q(\beta(\lambda - \lambda z)) - q^{(0)}) h(\lambda - \lambda z) + q^{(0)} \tilde{h}(\lambda - \lambda z),\ |z| \leq 1. \quad (2)$$

The equations of type (2) have been solved in (Sumita, 1988) on the base of the method to solve the functional equations like (2) described in (Kuczma, 1968) and the corresponding result is included into (Takagi, 1991), pp. 223-225.
Below, we briefly describe the solution of (2). Consider the sequence of functions $\eta_j(z),\ j \geq 0$, which are defined recursively,

$$\eta_0(z) = z,\ \eta_{j+1}(z) = \beta(\lambda - \lambda \eta_j(z)),\ j \geq 0. \quad (3)$$

It was proven in (Sumita, 1988) that if $\rho = \lambda b^{(1)} < 1$, the sequence of functions $\eta_j(z),\ j \geq 0$, converges uniformly to 1 as $j \to \infty$ for all $z,\ 0 \leq z \leq 1$. By substitution $\eta_j(z)$ instead of z in (2), we get

$$Q(\eta_j(z)) = Q(\eta_{j+1}(z))h(\lambda - \lambda\eta_j(z)) + q^{(0)}(\tilde{h}(\lambda - \lambda\eta_j(z)) - h(\lambda - \lambda\eta_j(z))).$$

Continuing the substitution for $j = 0, 1, \ldots, n-1$, we have

$$Q(z) = Q(\eta_n(z))\prod_{j=0}^{n-1} h(\lambda - \lambda\eta_j(z)) + q^{(0)}\sum_{j=0}^{n-1}(\tilde{h}(\lambda - \lambda\eta_j(z)) - h(\lambda - \lambda\eta_j(z)))\prod_{k=0}^{j-1} h(\lambda - \lambda\eta_k(z)). \tag{4}$$

Here we assume that $\prod_{k=a}^{b} c_k = 1$ if $b < a$.
Now, let $n \to \infty$ in (4). Using the normalization condition $Q(1) = 1$, we obtain

$$Q(z) = \prod_{j=0}^{\infty} h(\lambda - \lambda\eta_j(z)) + q^{(0)}\sum_{j=0}^{\infty}(\tilde{h}(\lambda - \lambda\eta_j(z)) - h(\lambda - \lambda\eta_j(z)))\prod_{k=0}^{j-1} h(\lambda - \lambda\eta_k(z)). \tag{5}$$

Setting $z = 0$ at the latter equation, we get the relation for the probability $q^{(0)}$:

$$q^{(0)} = \prod_{j=0}^{\infty} h(\lambda - \lambda z^{(j)})\left[1 - \sum_{j=0}^{\infty}(\tilde{h}(\lambda - \lambda z^{(j)}) - h(\lambda - \lambda z^{(j)}))\prod_{k=0}^{j-1} h(\lambda - \lambda z^{(k)})\right]^{-1}, \tag{6}$$

where the quantities $z^{(j)} = \eta_j(0),\ j \geq 0$, are given by the recursion

$$z^{(0)} = 0,\ z^{(j+1)} = \beta(\lambda - \lambda z^{(j)}),\ j \geq 0. \tag{7}$$

Finally, the equations (1), (5)–(7) give the probabilities $q^{(j)},\ j \geq 0$.
To obtain the performance characteristics of the system with adaptive polling scheme, we need to derive formulas for the first three derivatives $Q'(1),\ Q''(1),\ Q'''(1)$ of the PGF $Q(z)$ at $z = 1$, which can be found by using (5). The formula (5) involves the LSTs $(\tilde{h}(\lambda - \lambda\eta_j(z))$ and $h(\lambda - \lambda\eta_j(z))$, where the functions $\eta_j(z),\ j \geq 0$, are given by recursion (3). Thus, in order to obtain the formulas for the derivatives $Q'(1),\ Q''(1),\ Q'''(1)$ of the PGF $Q(z)$, we have to find the explicit expressions for the derivatives of the functions $\eta_j(z),\ j \geq 0$, at $z = 1$:

$$\eta_j(1) = 1,\ \eta_j{}'(1) = \rho^j,\ j \geq 0,\ \eta_j{}''(1) = \lambda^2 b^{(2)}\rho^{j-1}\frac{1-\rho^j}{1-\rho},\ j \geq 0,$$

$$\eta_j{}'''(1) = \lambda^3 b^{(3)}\rho^{j-1}\frac{1-\rho^{2j}}{1-\rho^2} + 3\lambda^4(b^{(2)})^2\rho^{j-1}\frac{(1-\rho^{j-1})(1-\rho^j)}{(1-\rho)(1-\rho^2)},\ j \geq 0.$$

Using the formulas obtained above, we can calculate the derivatives of functions $h(\lambda - \lambda\eta_j(z)),\ j \geq 0$, at $z = 1$: $h(\lambda - \lambda\eta_j(z))|_{z=1} = 1$,

$$(h(\lambda - \lambda\eta_j(z)))'|_{z=1} = \lambda h^{(1)}\rho^j,\ (h(\lambda - \lambda\eta_j(z)))''|_{z=1} = \lambda^2 h^{(2)}\rho^{2j} + \lambda^3 h^{(1)} b^{(2)}\rho^{j-1}\frac{1-\rho^j}{1-\rho},$$

$$(h(\lambda - \lambda\eta_j(z)))'''|_{z=1} = \lambda^3 h^{(3)}\rho^{3j} + 3\lambda^4 h^{(2)} b^{(2)} \rho^{j-1}\frac{1-\rho^j}{1-\rho} +$$

$$+\lambda h^{(1)}\rho^{j-1}\left[\lambda^3 b^{(3)}\frac{1-\rho^{2j}}{1-\rho^2} + 3\lambda^4 (b^{(2)})^2 \frac{(1-\rho^{j-1})(1-\rho^j)}{(1-\rho)(1-\rho^2)}\right].$$

Finally, after cumbersome simplifications, the formulas for the derivatives $Q'(1)$, $Q''(1)$, $Q'''(1)$ of the PGF $Q(z)$ at $z = 1$ are obtained as

$$Q'(1) = \frac{\lambda h^{(1)}}{1-\rho} + q^{(0)}\frac{\lambda(\tilde{h}^{(1)} - h^{(1)})}{1-\rho}, \tag{8}$$

$$Q''(1) = \frac{\lambda^2 h^{(2)}}{1-\rho^2} + \frac{\lambda^3 h^{(1)} b^{(2)} + (\lambda h^{(1)})^2 2\rho}{(1-\rho)(1-\rho^2)} + q^{(0)}\left[\frac{\lambda^2(\tilde{h}^{(2)} - h^{(2)})}{1-\rho^2} + \right. \tag{9}$$

$$\left. +(\tilde{h}^{(1)} - h^{(1)})\frac{\lambda^3 b^{(2)} + 2\rho\lambda^2 h^{(1)}}{(1-\rho)(1-\rho^2)}\right],$$

$$Q'''(1) = \frac{\lambda^3 h^{(3)}}{1-\rho^3} + \frac{\lambda^4 h^{(1)} b^{(3)}}{(1-\rho)(1-\rho^3)} + \frac{3\rho\lambda^4 h^{(2)} b^{(2)} + 3\rho(1+2\rho)\lambda^3 h^{(1)} h^{(2)}}{(1-\rho^2)(1-\rho^3)} + \tag{10}$$

$$+\frac{3\rho\lambda^5 h^{(1)}(b^{(2)})^2 + 3\lambda^4(1+2\rho^2)(h^{(1)})^2 b^{(2)}}{(1-\rho)(1-\rho^2)(1-\rho^3)} + \frac{6\rho^3(\lambda h^{(1)})^3}{(1-\rho)(1-\rho^2)(1-\rho^3)} + q^{(0)}\left[\frac{\lambda^3(\tilde{h}^{(3)} - h^{(3)})}{1-\rho^3} + \right.$$

$$+\frac{\lambda^4(\tilde{h}^{(1)} - h^{(1)})b^{(3)}}{(1-\rho)(1-\rho^3)} + \frac{3\rho\lambda^4(\tilde{h}^{(2)} - h^{(2)})b^{(2)}}{(1-\rho^2)(1-\rho^3)} +$$

$$+\frac{3\rho^2\lambda^3(\tilde{h}^{(2)} - h^{(2)})h^{(1)}}{(1-\rho^2)(1-\rho^3)} + \frac{3\rho(1+\rho)\lambda^3(\tilde{h}^{(1)} - h^{(1)})h^{(2)}}{(1-\rho^2)(1-\rho^3)} +$$

$$\left. +\frac{3\rho\lambda^5(\tilde{h}^{(1)} - h^{(1)})(b^{(2)})^2 + 3\lambda^4(1+2\rho^2)((\tilde{h}^{(1)} - h^{(1)})h^{(1)} b^{(2)}}{(1-\rho)(1-\rho^2)(1-\rho^3)} + \frac{6\rho^3\lambda^3(h^{(1)})^2(\tilde{h}^{(1)} - h^{(1)})}{(1-\rho)(1-\rho^2)(1-\rho^3)}\right],$$

Let ζ be the duration of the queue service (between two successive server's vacations). Using the relations obtained above, we can get the moments $\hat{\psi}^{(r)}$, $r = 1,2,3$ of ζ. The number of customers served between two successive server's vacations is a random variable, say τ, which has the first three moments $L_1 = Q'(1)$, $L_2 = Q''(1) + Q'(1)$, $L_3 = Q'''(1) + 3L_2 - 2L_1$, and the service time of the kth customer in a service period is a random variable, say ξ_k, which has the moments $b^{(r)}$, $r = 1,2,3$. It is easy to see that $\zeta = \sum_{k=1}^{\tau} \xi_k$. The random variables ξ_k are mutually independent and independent of τ. Thus, ζ equals to the sum of a random number of the random variables. Using the technique of the conditional expectations, it can be shown that the moments $\hat{\psi}^{(r)}$, $r = 1,2,3$, of the random variable ζ are

$$\hat{\psi}^{(1)} = b^{(1)}L_1, \quad \hat{\psi}^{(2)} = b^{(2)}L_1 + (b^{(1)})^2(L_2 - L_1), \tag{11}$$

$$\hat{\psi}^{(3)} = L_1 b^{(3)} + 3b^{(1)}b^{(2)}(L_2 - L_1) + (b^{(1)})^3(L_3 - 3L_2 + 2L_1). \tag{12}$$

Denote by $\psi^{(r)}$, $r = 1,2,3$, the conditional moments of the distribution of ζ, duration of the queue service between two successive vacations, given that the queue is not empty. It is easy to see that the conditional moments $\psi^{(r)}$, $r = 1, 2, 3$, are expressed in the terms of the moments $\hat{\psi}^{(r)}$, $r = 1, 2, 3$, as follows:

$$\psi^{(r)} = \frac{\hat{\psi}^{(r)}}{1 - q^{(0)}}, \ r = 1, 2, 3.$$

Thus, the formulas (5)–(7) give the stationary distribution of $q^{(i)}$, $i \geq 0$, the number of customers in the queue at the epoch the server finishes a vacation, and the formulas (8)–(10) express the first three moments of that distribution.

Let $W(x), x \geq 0$, be the distribution of the waiting time in the queue, and $w(s) = \int_0^{\infty} e^{-sx} dW(w)$ be its LST. It is easy to see that

$$w(s) = w^{(0)}(s) + w^{(1)}(s) + w^{(2)}(s),$$

where $w^{(0)}(s)$ is the LST of the waiting time of an arbitrary customer arrived to the system when the server was busy, $w^{(1)}(s)$ is the LST of the waiting time of an arbitrary customer arrived to the system when the server was on an ordinary vacation (with distribution $H(t)$), $w^{(2)}(s)$ is the LST of the waiting time of an arbitrary customer arrived to the system when the server was on a special vacation (with distribution $\tilde{H}(t)$).

Using the probabilistic sense of LST, one can verify the following formulas are valid

$$w^{(0)}(s) = \tau^{-1} h(s) \frac{Q(\beta(\lambda(1 - \beta(s)))) - Q(\beta(s))}{s - \lambda(1 - \beta(s))},$$

$$w^{(1)}(s) = \tau^{-1}(Q(\beta(\lambda(1 - \beta(s)))) - q^{(0)}) \frac{h(\lambda(1 - \beta(s))) - h(s)}{s - \lambda(1 - \beta(s))}, \ w^{(2)}(s) =$$

$$= \tau^{-1} q^{(0)} \frac{\tilde{h}(\lambda(1 - \beta(s))) - \tilde{h}(s)}{s - \lambda(1 - \beta(s))}.$$

These formulas lead to the following result.

Theorem 1. *The LST $w(s)$ of the waiting time is given by*

$$w(s) = \frac{\tau^{-1}}{s - \lambda(1 - \beta(s))} (Q(\beta(s))(1 - h(s)) + q^{(0)}(h(s) - \tilde{h}(s))).$$

By differentiating $w(s)$ at $s = 0$, we obtain the formula for the mean waiting time

$$W^{(1)} = \frac{v^{(2)}}{2v^{(1)}} + \frac{\lambda b^{(2)} + 2\rho h^{(1)}}{2(1 - \rho)} \tag{13}$$

and for second moment $W^{(2)}$ of the waiting time

$$W^{(2)} = \frac{v^{(3)}}{3v^{(1)}} + \frac{\lambda b^{(3)} + 3\lambda b^{(2)} h^{(1)} + 3\rho h^{(2)}}{3(1 - \rho)} + W^{(1)} \left(\frac{\lambda b^{(2)}}{1 - \rho} + \frac{2\rho^2 h^{(1)}}{1 - \rho^2} \right) \tag{14}$$

with $v^{(k)} = (1 - q^{(0)}) h^{(k)} + q^{(0)} \tilde{h}^{(k)}$, $k = 1, 2, 3$.

3.2 Approximate analysis of the system with N queues

As it follows from (6)–(14), the key role in calculation of the performance characteristics of the system considered is played by the form of the functions $h(s)$ and $\tilde{h}(s)$, the LSTs of the distributions of vacation period after visiting an empty or non-empty queue, respectively. In order to apply the results obtained for the system with single queue to the polling system with N queues, we have to estimate those LSTs. It is easy to see that they depend on the time the server spends visiting other queues, and are unknown. Below, we elaborate a procedure of successive approximations to calculate the first and second moments of waiting time in each queue on the base of the analysis given in Section 3.1. Note that the unknown LSTs $h(s)$ and $\tilde{h}(s)$ of vacation duration are improved at each stage of the procedure. And below, we elaborate such a heuristic procedure.

In the procedure, we use the quantities $z_i^{(j)}$, $j \geq 0$, $q_i^{(0)}$, $\psi_i^{(r)}$, $r = 1, 2, 3$, and $W_i^{(r)}$, $r = 1, 2$, $i = \overline{1, N}$, given by formulas (6)–(14) where all LSTs and the corresponding moments are given the subscript i, the number of the queue.

Since a queue being polled and found empty will not be visited at the next cycle and will be polled again two cycles later, it seems reasonable to assume that the time when the server is away from the queue is equal to the double intervisit time as if the queue was visited each cycle (after being polled and found non-empty). Besides, we need to take into account the fact that, if all queues are found empty, they are all visited in the cycle following the server's vacation. Thus, we assume that $\tilde{h}_i(w) = \tilde{\chi}_i(w)\tilde{S}_i(w)$, where $\tilde{\chi}_i(w) = (\chi_i(w))^2 + \bar{q}_i(\varphi(w + \lambda_i(1 - \beta_i(w)))r_i(w) - \chi_i(w))$, the functions $\chi_i(s)$ are given by formula (17),

$$\bar{q}_i = \prod_{j=1, j\neq i}^{N} q_j^{(0)}, \quad r_i(w) = \prod_{j=1, j\neq i}^{N} (q_j^{(0)} + (1 - q_j^{(0)})\psi_j(w))\tilde{S}_j(w).$$

It follows that the moments of the distribution functions $\tilde{H}(t)$ and $H(t)$ of a vacation period in the system with single queue considered in Section 3.1 are defined as

$$\tilde{h}_i^{(1)} = \tilde{\chi}_i^{(1)} + s_i, \quad \tilde{h}_i^{(2)} = \tilde{\chi}_i^{(2)} + s_i^{(2)} + 2\tilde{\chi}_i^{(1)} s_i, \quad \tilde{h}_i^{(3)} = \tilde{\chi}_i^{(3)} + s_i^{(3)} + 3\tilde{\chi}_i^{(2)} s_i + 3\tilde{\chi}_i^{(1)} s_i^{(2)},$$

where $\tilde{\chi}_i^{(l)}$, $i = \overline{1, 3}$, are calculated as

$$\tilde{\chi}_i^{(1)} = 2\chi_i^{(1)} + \bar{q}_i(\hat{\varphi}^{(1)} + r_i^{(1)} - \chi_i^{(1)}),\ \tilde{\chi}_i^{(2)} = 2\chi_i^{(2)} + 2(\chi_i^{(1)})^2 + \bar{q}_i(\hat{\varphi}^{(2)} + 2\hat{\varphi}^{(1)} r_i^{(1)} + r_i^{(2)} - \chi_i^{(2)}),$$

$$\tilde{\chi}_i^{(3)} = 2\chi_i^{(3)} + 6\chi_i^{(1)}\chi_i^{(2)} + \bar{q}_i(\hat{\varphi}^{(3)} + 3\hat{\varphi}^{(2)} r_i^{(1)} + 3\hat{\varphi}^{(1)} r_i^{(2)} + r_i^{(3)} - \chi_i^{(3)}),$$

$\varphi^{(r)}$, $r = \overline{1, 3}$, are given by

$$\hat{\varphi}^{(1)} = \varphi^{(1)}(1 + \rho_i),\ \hat{\varphi}^{(2)} = \varphi^{(2)}(1 + \rho_i)^2 + \varphi^{(1)}\lambda_i b_i^{(2)},$$

$$\hat{\varphi}^{(3)} = \varphi^{(3)}(1 + \rho_i)^3 + 3\varphi^{(2)}(1 + \rho_i)\lambda_i b_i^{(2)} + \varphi^{(1)}\lambda_i b_i^{(3)},$$

and the moments $r_i^{(l)}$ are defined as

$$r_i^{(1)} = \sum_{j=1,\ j\neq i}^{N} (q_j^{(0)} s_j^{(1)} + (1 - q_j^{(0)}) a_j^{(1)}),$$

$$r_i^{(2)} = \sum_{j=1,\ j\neq i}^{N} (q_j^{(0)} s_j^{(2)} + (1-q_j^{(0)}) a_j^{(2)} +$$

$$+ (q_j^{(0)} s_j^{(1)} + (1-q_j^{(0)}) a_j^{(1)}) \sum_{k=1,\ k\neq i, k\neq j}^{N} (q_k^{(0)} s_k^{(1)} + (1-q_k^{(0)}) a_k^{(1)})),$$

$$r_i^{(3)} = \sum_{j=1,\ j\neq i}^{N} (q_j^{(0)} s_j^{(3)} + (1-q_j^{(0)}) a_j^{(3)} + 2(q_j^{(0)} s_j^{(2)} + (1-q_j^{(0)}) a_j^{(2)}) \sum_{k=1,\ k\neq i, k\neq j}^{N} (q_k^{(0)} s_k^{(1)} +$$

$$+ (1-q_k^{(0)}) a_k^{(1)}) + (q_j^{(0)} s_j^{(1)} + (1-q_j^{(0)}) a_j^{(1)}) \sum_{k=1,\ k\neq i, k\neq j}^{N} (q_k^{(0)} s_k^{(2)} + (1-q_k^{(0)}) a_k^{(2)} +$$

$$+ (q_k^{(0)} s_k^{(1)} + (1-q_k^{(0)}) a_k^{(1)}) \sum_{m=1,\ m\neq i, m\neq j, m\neq k}^{N} (q_m^{(0)} s_m^{(1)} + (1-q_m^{(0)}) a_m^{(1)}))).$$

Further, we discuss the problem of determining the LST $h(s)$.
On the first step of the procedure, assume that each queue is visited (polled) in each cycle and the server serves only one customer in a queue per a visit. From this assumption it follows that the initial form of the LST $h_i(s)$ of the vacation (intervisit time for queue i) could be

$$h_i(s) = \frac{\sigma_i}{s + \sigma_i}$$

with

$$(\sigma_i)^{-1} = \sum_{j=1,\ j\neq i}^{N} (b_j^{(1)} + s_j^{(1)}),\ i = \overline{1, N}.$$

Then, the moments of the vacation period (intervisit time for queue i) are given by equations

$$h_i^{(1)} = \sigma_i^{-1},\ h_i^{(2)} = 2\sigma_i^{-2},\ h_i^{(3)} = 6\sigma_i^{-3}.$$

Using LSTs $h_i(s)$, $i = \overline{1, N}$, and the moments $h_i^{(r)}$, $r = \overline{1, 3}$, we calculate the values of $z_i^{(j)}$, $j \geq 0$, $q_i^{(0)}$, $\psi_i^{(r)}$, $r = 1, 2, 3$, $W_i^{(r)}$, $r = 1, 2$, $i = \overline{1, N}$, by the formulas (6)–(14) for each queue i, $i = \overline{1, N}$. Note that we can use the Lyapunov inequality to check the correctness of numerical calculation of the moments. For instance, the values $\psi_i^{(r)}$, $r = 1, 2, 3$, have to satisfy the inequalities

$$\psi_i^{(2)} \geq (\psi_i^{(1)})^2,\ \psi_i^{(3)} \geq (\psi_i^{(1)})^3,\ \psi_i^{(3)} \geq (\psi_i^{(2)})^{\frac{3}{2}}.$$

Having analyzed the moments $\psi_i^{(r)}$, $r = 1, 2, 3$, of queue i service period, we choose the form of the LST $\psi_i(s)$ as follows.

- If the value $c_\psi = \frac{\psi_i^{(2)}}{(\psi_i^{(1)})^2}$ is approximately equal to 1, we set $\psi_i(s) = \frac{1}{1 + s\psi_i^{(1)}}$, i.e. the vacation (intervisit time) is exponentially distributed.

- If $c_\psi \approx \frac{1}{k}$, where k is some positive integer, we set

$$\psi_i(s) = \left(\frac{1}{1 + s\frac{\psi_i^{(1)}}{k}} \right)^k,$$

i.e. the vacation has Erlang-k distribution.

- If $c_\psi > 1$, we suppose that a vacation has hyper-exponential distribution, i.e.

$$\psi_i(s) = p_i \frac{\mu_i^{(1)}}{\mu_i^{(1)} + s} + (1 - p_i)\frac{\mu_i^{(2)}}{\mu_i^{(2)} + s}. \tag{15}$$

The parameters p_i, $\mu_i^{(1)}$, $\mu_i^{(2)}$ are calculated through values $\psi_i^{(r)}$, $r = 1,2,3$, in the following way. First, calculate the values

$$v_l = \frac{\psi_i^{(l)}}{l!}, \quad l = 1,2,3, \quad f_1 = \frac{v_3 - v_1 v_2}{v_2 - (v_1)^2}, \quad f_2 = \frac{v_1 v_3 - (v_2)^2}{v_2 - (v_1)^2},$$

$$\mu_i^{(1)} = \frac{2}{f_1 + \sqrt{f_1^2 - 4f_2}}, \quad \mu_i^{(2)} = \frac{2}{f_1 - \sqrt{f_1^2 - 4f_2}}, \quad p_i = \frac{\mu_i^{(1)}(\mu_i^{(2)} v_1 - 1)}{\mu_i^{(2)} - \mu_i^{(1)}}.$$

If the inequalities $v_2 > v_1^2$, $v_3 > v_1 v_2$, $v_1 v_3 > v_2^2$ and $f_1^2 > 4f_2$, are fulfilled then we define the function $\psi_i(s)$ from (15). Otherwise, the values $\mu_i^{(1)}$, $\mu_i^{(2)}$ and p_i are chosen to satisfy the relations

$$p_i \neq \frac{2(\psi_i^{(1)})^2}{\psi_i^{(2)}}, \quad 0 \le p_i \le 1, \quad \mu_i^{(2)} = \frac{2\psi_i^{(1)}(1-p_i) - \sqrt{2(1-p_i)p_i(\psi_i^{(2)} - 2(\psi_i^{(1)})^2)}}{2(\psi_i^{(1)})^2 - \psi_i^{(2)} p_i},$$

$$\psi_i^{(1)}\mu_i^{(2)} - 1 + p_i > 0, \quad \mu_i^{(1)} = \frac{p_i \mu_i^{(2)}}{\psi_i^{(1)}\mu_i^{(2)} - (1-p_i)}.$$

Note that the value p_i should be chosen to minimize $\left| \frac{6p_i}{(\mu_i^{(1)})^3} + \frac{6(1-p_i)}{(\mu_i^{(2)})^3} - \psi_i^{(3)} \right|$ for the better approximation (Kazimirsky, 2002).

- If $c_\psi < 1$ but c_ψ does not equal to $\frac{1}{k}$ approximately, the vacation period distribution can be approximated by the phase distribution.
- If $c_\psi \approx 0$, we use the following form:

$$\psi_i(s) = e^{-\psi_i^{(1)} s}.$$

Now, suppose the LSTs $\psi_i(s)$, $i = \overline{1,N}$, are known. Note that queues may have different forms of the LST of intervisit times (server's vacations).
Then, we make a simplifying assumption that the probability that queue i is found empty at an arbitrary polling epoch is independent of the state of the other queues. Note that this assumption may not be valid at all (e.g. if the other queues are empty, queue i gets more time for its customers to be served, hence the probability that the queue is empty is greater then in the case when all other queues are not empty). But this assumption makes it possible to investigate the model with adaptive polling analytically. Thus, we can rewrite the form of the LST $h_i(s)$ of vacation duration for queue i (its intervisit time) can be rewritten as

$$h_i(w) = \chi_i(w)\tilde{S}_i(w), \tag{16}$$

with

$$\chi_i(w) = \prod_{j=1,\ j\neq i}^{N} (q_j^{(0)} + (1 - q_j^{(0)})\psi_j(w)\tilde{S}_j(w)). \tag{17}$$

The derivation of (16)–(17) is based on the probabilistic interpretation of LST as the probability that a catastrophe from a Poisson input with parameter w does not occur during the period considered. A catastrophe does not occur during a vacation time for the queueing system with vacations corresponding to queue i if it does not occur during switchover time for this queue (with probability $\tilde{S}_i(w)$) and during the switchover and service periods for the rest of queues in the current cycle (with probability $\chi_i(w)$). The first term in (17) implies that a catastrophe does not occur during the switchover time to queue j and the following service period with probability 1, if the queue is not visited by the server, and with probability $\psi_j(w)\tilde{S}_j(w)$ if it was.
Formulas (16) and (17) result in the relations for the moments of the intervisit time for queue i:

$$h_i^{(1)} = \chi_i^{(1)} + s_i^{(1)}, \quad h_i^{(2)} = \chi_i^{(2)} + s_i^{(2)} + 2\chi_i^{(1)}s_i^{(1)}, \quad h_i^{(3)} = \chi_i^{(3)} + s_i^{(3)} + 3\chi_i^{(2)}s_i^{(1)} + 3\chi_i^{(1)}s_i^{(2)}, \tag{18}$$

where

$$\chi_i^{(1)} = \sum_{j=1,\ j\neq i}^{N} (1 - q_j^{(0)})a_j^{(1)}, \quad \chi_i^{(2)} = \sum_{j=1,\ j\neq i}^{N} (1 - q_j^{(0)})a_j^{(2)} + \tag{19}$$
$$+ \sum_{j=1,\ j\neq i}^{N} (1 - q_j^{(0)})a_j^{(1)} \sum_{k=1,\ k\neq i, k\neq j}^{N} (1 - q_k^{(0)})a_k^{(1)},$$

$$\chi_i^{(3)} = \sum_{j=1,\ j\neq i}^{N} (1 - q_j^{(0)})a_j^{(3)} + 2\sum_{j=1,\ j\neq i}^{N} (1 - q_j^{(0)})a_j^{(2)} \sum_{k=1,\ k\neq i, k\neq j}^{N} (1 - q_k^{(0)})a_k^{(1)} +$$
$$+ \sum_{j=1,\ j\neq i}^{N} (1 - q_j^{(0)})a_j^{(1)} \sum_{k=1,\ k\neq i, k\neq j}^{N} (1 - q_k^{(0)})a_k^{(1)} \sum_{m=1,\ m\neq i, m\neq j, m\neq k}^{N} (1 - q_m^{(0)})a_m^{(1)}, \tag{20}$$

where, for $m = \overline{1,N}$,

$$a_m^{(1)} = s_m^{(1)} + \psi_m^{(1)}, \quad a_m^{(2)} = s_m^{(2)} + \psi_m^{(2)} + 2s_m^{(1)}\psi_m^{(1)}, \quad a_m^{(3)} = s_m^{(3)} + \psi_m^{(3)} + 3s_m^{(2)}\psi_m^{(1)} + 3s_m^{(1)}\psi_m^{(2)}.$$

Then using formulas (16)–(20) for the LSTs and the moments of the vacation duration (of intervisit time for a queue), we recalculate the values $q_i^{(0)}$, $\psi_i^{(r)}$, $r = 1,2,3$, $W_i^{(r)}$, $r = 1,2$, $i = \overline{1,N}$, by formulas (6)–(14) for all i, $i = \overline{1,N}$.
The iterative procedure described above should be repeated until the values $q_i^{(0)}$, $\psi_i^{(r)}$, $r = 1,2,3$, $W_i^{(r)}$, $r = 1,2$, $i = \overline{1,N}$, calculated at the succeeding steps coincide with the necessary accuracy. Thus, we get the moments, $W_i^{(r)}$, $r = 1,2$, $i = \overline{1,N}$, of the waiting time in the queues.

3.3 Numerical results

Here we give numerical examples to illustrate how the algorithm works for polling systems with various numbers of queues and traffic intensity comparing to the simulation results obtained using the general-purpose simulation system GPSS World (Schriber, 1974). The object of modeling was represented by a regional broadband wireless network consisting of several devices operating with one base station. The rates of packet arrivals to the devices and the rate of processing them are different. The devices are polled cyclically. Packet servicing is gated, that is, only those packets are transmitted which were at the queue at the polling moment. The input flows are assumed to be of the Poisson nature, and the times of packet servicing and polling initialization are assumed to be exponentially distributed.
We assume for simulation that the system is in the stationary mode when at duplication of the number of the packets passing through the system none of the comparison parameters changes more than by 0.5%. In the experiments, more than three million of packets passed through the system.
Average packet service time, switchover time, etc. in the numerical examples are taken from the real IEEE 802.11a broadband wireless network under PCF mode with realistic packet sizes and load levels. Thus, the obtained results correspond to such networks and mean waiting times satisfy the real systems requirements.
Case N = 2. First, consider the symmetric system of two queues with the mean service times $b_1^{(1)} = b_2^{(1)} = 0.311$, the mean switchover times $s_1^{(1)} = s_2^{(1)} = 0.091$ and the mean time of server's vacation $\varphi^{(1)} = 0.005$. The mean waiting time calculated by the Algorithm (column "A"). Simulation results (column "S") and relative error of comparison (column "Δ") are shown in Table 1. The first column describes the customers input intensities and the corresponding traffic intensities. Two last lines of the Table present the results obtained for different mean server's vacation times $\varphi^{(1)}$ given that $\lambda_1 = \lambda_2 = 0.5$.

	A	S	Δ, %
$\lambda_1 = \lambda_2 = 0.321, \rho = 0.2$	0.289	0.268	7.8
$\lambda_1 = \lambda_2 = 0.5, \rho = 0.311$	0.392	0.358	9.5
$\lambda_1 = \lambda_2 = 0.803, \rho = 0.5$	0.659	0.601	9.7
$\lambda_1 = \lambda_2 = 1.28, \rho = 0.8$	1.73	1.93	10.4
$\varphi^{(1)} = 0.05$	0.392	0.358	9.5
$\varphi^{(1)} = 0.1$	0.417	0.384	8.6

Table 1. System with two queues

Case N = 3. Now, consider the case of three queues with symmetric service $b_i^{(1)} = 0.044$, $s_i^{(1)} = 0.1$, $i = \overline{1,3}$, $\varphi^{(1)} = 0.1$. The mean waiting times $W_i^{(1)}$, $i = \overline{1,3}$ obtained by using the algorithm and simulation are presented in Table 2 for various customer input intensities. The last two lines contain results for fully symmetric system (all λ_i, $i = \overline{1,3}$ are the same).

Case N = 5. And, finally, consider the case of five queues with $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 0.5$, $\lambda_4 = 6$, $\lambda_5 = 0.5$, $b_i^{(1)} = 0.05$, $s_i^{(1)} = 0.05$, $i = \overline{1,5}$, $\varphi^{(1)} = 0.05$. We vary the input intensity by multiplying all λ_i by α which takes values 0.285, 0.714, 1, and 1.143. Thus, the traffic intensity, ρ, varies from 0.2 to 0.8. The results are given in Table 3. The last four lines present results for the fully symmetric system with $\lambda_i = 2$ multiplied by the same values of α.

	A	S	Δ, %		A	S	Δ, %
$\lambda_1 = 2.5$	0.342	0.365	6.3	$\lambda_1 = 4.375$	0.658	0.698	5.7
$\lambda_2 = 6$	0.335	0.361	7.2	$\lambda_2 = 10.5$	0.781	0.834	3.0
$\lambda_3 = 0.5$	0.410	0.440	6.8	$\lambda_3 = 0.875$	0.778	0.805	3.4
Symmetric system							
$\lambda_i = 3$, $i = \overline{1,3}$	0.387	0.382	1.3	$\lambda_i = 5.25$, $i = \overline{1,3}$	0.702	0.771	8.9

Table 2. System with three queues

		A	S	Δ, %			A	S	Δ, %
$\alpha = 0.285$, $\rho = 0.2$	$W_1^{(1)}$	0.203	0.220	7.7	$\alpha = 1$, $\rho = 0.7$	$W_1^{(1)}$	0.714	0.672	6.2
	$W_2^{(1)}$	0.199	0.215	7.4		$W_2^{(1)}$	0.661	0.618	7.0
	$W_3^{(1)}$	0.205	0.222	7.7		$W_3^{(1)}$	0.776	0.738	5.1
	$W_4^{(1)}$	0.197	0.203	3.0		$W_4^{(1)}$	0.705	0.679	3.8
	$W_5^{(1)}$	0.205	0.224	8.5		$W_5^{(1)}$	0.776	0.745	4.2
$\alpha = 0.714$, $\rho = 0.5$	$W_1^{(1)}$	1.036	0.974	6.4	$\alpha = 1.143$, $\rho = 0.8$	$W_1^{(1)}$	0.374	0.398	6.0
	$W_2^{(1)}$	0.353	0.370	4.6		$W_2^{(1)}$	0.967	0.934	3.5
	$W_3^{(1)}$	0.393	0.419	6.2		$W_3^{(1)}$	1.153	1.080	6.8
	$W_4^{(1)}$	0.340	0.355	4.2		$W_4^{(1)}$	1.152	1.120	2.9
	$W_5^{(1)}$	0.393	0.422	6.9		$W_5^{(1)}$	1.153	1.090	5.8
Symmetric system									
$\rho = 0.2$	$W_i^{(1)}$	0.207	0.216	4.2	$\rho = 0.7$	$W_i^{(1)}$	0.759	0.686	10.6
$\rho = 0.5$	$W_i^{(1)}$	0.455	0.391	16.4	$\rho = 0.8$	$W_i^{(1)}$	1.003	1.040	3.6

Table 3. System with five queues

The results for non-symmetric service ($b_1^{(1)} = 0.07$, $b_2^{(1)} = 0.015$, $b_3^{(1)} = 0.1$, $b_4^{(1)} = 0.025$, $b_5^{(1)} = 0.4$) are given in Table 4.

Below, we discuss the results in brief. For the symmetric systems with $N = 2$ and $N = 3$, the relative error of comparison grows as the traffic intensity ρ grows. But the situation is different in case $N = 5$: the approximate and simulation results coincide up to 5% for low and high traffic intensity, but for $\rho = 0.5$ and $\rho = 0.7$, the error becomes unacceptable (grater than 10%). For the system with five queues, the dependence of the relative error on the total traffic intensity, or the traffic intensity to a queue, is not well-understood. In case of non-symmetric system with five queues (Tables 3 and 4), the results can not be well explained. In case of non-symmetric input of customers (Table 3), the coincidence gets better as ρ increases, but when we make the service non-symmetric (Table 4), it holds only for queues with relatively high traffic intensities, namely, queues 4 and 5, as long as the results for the rest of queues behave the different way (the relative error grows as the values of ρ_i, $i = \overline{1,3}$, grow). Note that in Table 3, the results for queues 3 and 5 have to be the same as the queues are identical but the simulation results differ up to 1%, which is the simulation error.

		A	S	Δ, %			A	S	Δ, %
$\alpha = 0.4,$ $\rho = 0.2$	$W_1^{(1)}$	0.251	0.250	0.4	$\alpha = 1,$ $\rho = 0.5$	$W_1^{(1)}$	0.570	0.506	12.7
	$W_2^{(1)}$	0.248	0.244	1.6		$W_2^{(1)}$	0.535	0.475	12.7
	$W_3^{(1)}$	0.251	0.254	1.2		$W_3^{(1)}$	0.592	0.548	7.9
	$W_4^{(1)}$	0.244	0.228	7.0		$W_4^{(1)}$	0.516	0.455	13.4
	$W_5^{(1)}$	0.223	0.254	12.2		$W_5^{(1)}$	0.538	0.559	3.8
$\alpha = 0.6,$ $\rho = 0.3$	$W_1^{(1)}$	0.318	0.314	1.3	$\alpha = 1.4,$ $\rho = 0.7$	$W_1^{(1)}$	1.016	0.902	12.6
	$W_2^{(1)}$	0.311	0.302	3.0		$W_2^{(1)}$	0.901	0.831	8.4
	$W_3^{(1)}$	0.322	0.325	0.9		$W_3^{(1)}$	1.095	0.994	10.2
	$W_4^{(1)}$	0.305	0.281	8.5		$W_4^{(1)}$	0.938	0.896	4.7
	$W_5^{(1)}$	0.281	0.325	13.5		$W_5^{(1)}$	1.082	1.080	0.2

Table 4. System with five queues and non-symmetric service

To complete the numerical analysis, we compare two polling schemes in the system working in "data collection" mode described in Section 2.5. The comparison is based on the radio cell model with parameters $N = 4$, input intensities $\lambda_1 = \lambda_2 = 1500$, $\lambda_3 = \lambda_4 = \lambda$ take values from 1 to 200 with step 10, mean service times are $b_1^{(1)} = b_2^{(1)} = b_4^{(1)} = 1/4500$, $b_3^{(1)} = 1/3000$ and the mean switch-over times are $s_1^{(1)} = s_2^{(1)} = s_3^{(1)} = s_4^{(1)} = 1/1500$. The obtained results are shown on Fig. 1. The figure shows that adaptive dynamical polling can make significant profit when some stations have light traffic, since the waiting times are the same for classical cyclic polling system and one with adaptive polling in case of heavy traffic.

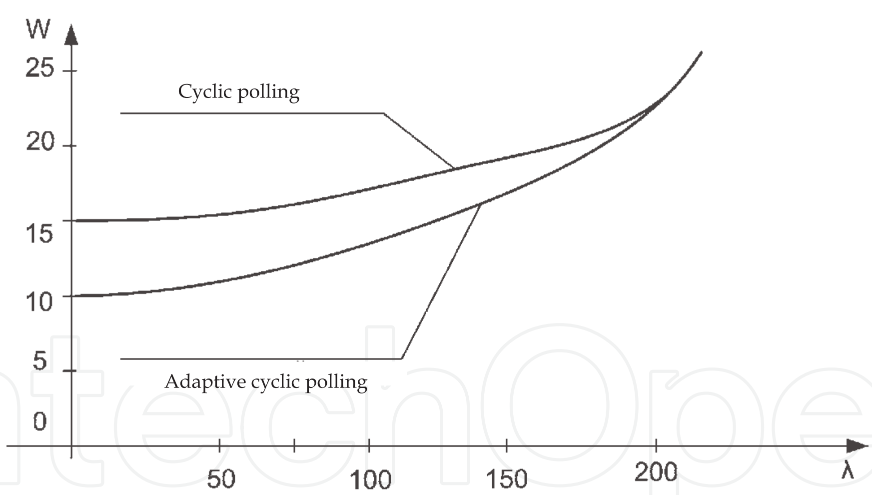

Fig. 1. The dependence of the mean waiting time on λ under the adaptive cyclic polling

4. System with exhaustive threshold service

In this Section, we present the polling system modelling a radio-cell working in the "last mile" mode as described in Section 2.5. We consider the polling system described in Section 3, but we assume that the service at a queue is exhaustive (the queue is served until empty).
The server visit a queue only if the queue length exceeds the given threshold (k_i for queue i), $k_i \geq 0$, $i = \overline{1, N}$. Before starting service at the queue i, the server needs a random time exponentially

distributed with the parameter s_i to prepare for work. This time can be considered the time to switch to queue i which is incurred only if the queue is served. The queue also has a finite waiting space h_i ($h_i \geq k_i$).

The service time in queue i is exponentially distributed with parameter μ_i, $i = \overline{1, N}$. The queue is served until it becomes empty, then the server moves to the nearest queue which has the sufficient number of customers (having reached its threshold). If the numbers of customers in all queues are insufficient to get service (less than k_i for queue i), the server stops polling until the number of customers in any queue reaches the threshold.

4.1 Stationary distribution of the system states and the performance characteristics

A system state at an arbitrary time t in a steady-state is presented by a random process

$$\xi(t) = (m(t), i(t), \mathbf{n}(t)), \quad t \geq 0,$$

where $m(t) = 0$, if at time t the server is idle, $m(t) = 1$, if at time t the server switches to a queue, $m(t) = 2$, if at time t the server serves the customers, $t \geq 0$; $i(t)$ is the number of queue which server is attended at time t, $i(t) = 0$, if the server is idle; $\mathbf{n}(t)$ is a vector

$$\mathbf{n}(t) = (n_1(t), n_2(t), \ldots, n_N(t)),$$

$n_j(t)$ is the number of customers at queue j at time t, $j = \overline{1, N}$.

The stochastic process $\xi(t)$, $t \geq 0$, is Markovian. Introduce the stationary state probabilities of the process $\xi(t)$, $t \geq 0$ for $\mathbf{r} = (r_1, r_2, \ldots, r_N)$, $i = \overline{1, N}$,

$$a(\mathbf{r}) = \lim_{t\to\infty} P\{m(t) = 0, i(t) = 0, \mathbf{n}(t) = \mathbf{r}\}, 0 \leq r_m < k_m, m = \overline{1, N},$$

$$p_i(\mathbf{r}) = \lim_{t\to\infty} P\{m(t) = 1, i(t) = i, \mathbf{n}(t) = \mathbf{r}\}, r_m = \overline{0, h_m}, m = \overline{1, N}, m \neq i, r_i = \overline{k_i, h_i},$$

$$q_i(\mathbf{r}) = \lim_{t\to\infty} P\{m(t) = 2, i(t) = i, \mathbf{n}(t) = \mathbf{r}\}, \overline{0, h_m}, m = \overline{1, N}, m \neq i, r_i\overline{1, h_i}.$$

The balance equations for the stationary probabilities are

$$\lambda a(\mathbf{r}) = \sum_{j=1}^{N} \lambda_j a(\mathbf{r} - \mathbf{e}_j) I_{\{r_j > 0\}} + \sum_{j=1}^{N} \mu_j q_j(\mathbf{r} + \mathbf{e}_j) I_{\{r_j = 0\}}, \ r_1 < k_1, \ldots, r_N < k_N, \tag{21}$$

$$\left(\sum_{m=1}^{N} \lambda_m I_{\{r_m < h_m\}} + \mu_i\right) q_i(\mathbf{r}) = \sum_{j=1}^{N} \lambda_j q_i(\mathbf{r} - \mathbf{e}_j) I_{\{r_j > \delta_{ij}\}} + \mu_i q_i(\mathbf{r} + \mathbf{e}_i) I_{\{r_i < h_i\}} + s_i p_i(\mathbf{r}) I_{\{r_i \geq k_i\}}, \tag{22}$$

$$\begin{aligned}\left(\sum_{m=1}^{N} \lambda_m I_{\{r_m < h_m\}} + s_i\right) p_i(\mathbf{r}) = \sum_{j=1}^{N} \lambda_j p_i(\mathbf{r} - \mathbf{e}_j) I_{\{r_j > k_i \delta_{ij}\}} + \mu_{i-1} q_{i-1}(\mathbf{r} + \mathbf{e}_{i-1}) I_{\{r_{i-1} = 0\}} + \\ + \lambda_i a(r_1, \ldots, r_{i-1}, k_i - 1, r_{i+1}, \ldots, r_N) I_{\{r_1 < k_1, \ldots, r_{i-1} < k_{i-1}, r_i = k_i, r_{i+1} < k_{i+1}, \ldots, r_N < k_N\}} + \\ + \sum_{j=1}^{i-2} \mu_j q_j(\mathbf{r} + \mathbf{e}_j) I_{\{r_j = 0, r_{j+1} < k_{j+1}, \ldots, r_{i-1} < k_{i-1}\}} + \\ + \sum_{j=i+1}^{N} \mu_j q_j(\mathbf{r} + \mathbf{e}_j) I_{\{r_j = 0, r_{j+1} < k_{j+1}, \ldots, r_N < k_N, r_1 < k_1, \ldots, r_{i-1} < k_{i-1}\}}, \\ 0 \leq r_m \leq h_m, \ m = \overline{1, N}, \ m \neq i, \ k_i \leq r_i \leq h_i, \quad i = \overline{1, N}.\end{aligned} \tag{23}$$

By replacing one of the equations of system (21)–(23) by normalization condition

$$\sum_{\mathbf{r}\in\Lambda} a(\mathbf{r}) + \sum_{i=1}^{N}\sum_{\mathbf{r}\in\Pi_i} q_i(\mathbf{r}) + \sum_{i=1}^{N}\sum_{\mathbf{r}\in\chi_i} p_i(\mathbf{r}) = 1,$$

with $\Lambda = \{(r_1,\ldots,r_N) : r_m < k_m,\ m = \overline{1,N}\}$, $\Pi_i = \{(r_1,\ldots,r_N) : 0 < r_i \le h_i,\ 0 \le r_m \le h_m,\ m = \overline{1,N}, m \ne i\}$, $\chi_i = \{(r_1,\ldots,r_N) : k_i \le r_i \le h_i,\ 0 \le r_m \le h_m,\ m = \overline{1,N}, m \ne i\}$, $\quad i = \overline{1,N}$, we get a system of equations for $\sum_{i=1}^{N}(2h_i - k_i + 1)\prod_{j=1,\,j\ne i}^{N}(h_j+1) + \prod_{j=1}^{N} k_j$ unknowns.
Having calculated the stationary state distribution, we can readily derive the following performance characteristics:

1. Mean length of queue j at time when queue i is served (excluding a customer being served): $L_j^i = \sum\limits_{\mathbf{r}\in\Pi_i}(r_j - \delta_{ij})q_i(\mathbf{r}), \quad i,j = \overline{1,N}$;
2. Mean length of queue j at time when the server switches to queue i: $S_j^i = \sum\limits_{\mathbf{r}\in\chi_i} r_j p_i(\mathbf{r})$, $i,j = \overline{1,N}$;
3. Mean length of queue j at time when the server is idle: $U_j = \sum_{\mathbf{r}\in\Lambda} r_j a(\mathbf{r}), \quad j = \overline{1,N}$;
4. Mean length of queue j at arbitrary time: $L_j = \sum_{i=1}^{N}(L_j^i + S_j^i) + U_j, \quad j = \overline{1,N}$;
5. The mean fraction of time the server is idle: $\bar{a} = \sum_{\mathbf{r}\in\Lambda} a(\mathbf{r})$.

Since the waiting space in the system is limited, some arriving customers can be lost. The probability P_{lost}^j that an arbitrary customer arriving to queue j is lost equals to the probability that an arbitrary time all h_j waiting places are occupied,

$$P_{lost}^j = \sum_{i=1\, i\ne j}^{N}\sum_{\mathbf{r}\in\Pi_i} q_i(\mathbf{r}) I_{\{r_j = h_j\}} + \sum_{i=1}^{N}\sum_{\mathbf{r}\in\chi_i} p_i(\mathbf{r}) I_{\{r_j = h_j\}}, \quad j = \overline{1,N}.$$

The mean waiting time in queue j can be obtained by Little's law $W_j = L_i/\lambda_i$.
Consider the model of the broadband wireless network radio cell working in the "last mile" mode with adaptive polling mechanism. The model parameters are the following: $N = 4$, $\lambda_1 = \lambda_2 = 1500$, $\lambda_3 = \lambda_4 = \lambda$ take the values 30, 60, 100. The service intensities $\mu_1 = \mu_3 = \mu_4 = 4500$, $\mu_2 = 3000$, the intensities of switching between queues $s_1 = s_2 = s_3 = s_4 = 1500$. The Fig. 2 shows the dependence of the frame mean waiting time on the threshold value $k_i = k$ ($i = \overline{1,4}$) assumed to be the same for all the queues. We see that the optimal value k varies from curve to curve. The less the queues are loaded the grater the k minimizing the mean waiting time. If the system has queues with low traffic it is reasonable to increase their service thresholds. As a result, the frame mean waiting time in queues with low traffic increases but it decreases for queues with high traffic so it can reduce the weighted sum of the mean waiting times in the system. The optimal threshold choosing can be a troublesome problem in practise since the threshold value depends on the relation between numbers of stations with light traffic and heavy traffic, station parameters, etc.

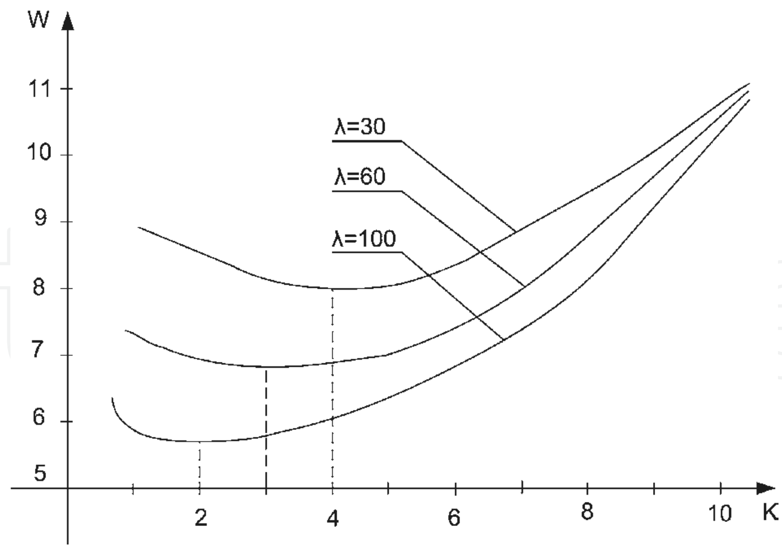

Fig. 2. The dependence of the mean waiting time on the threshold

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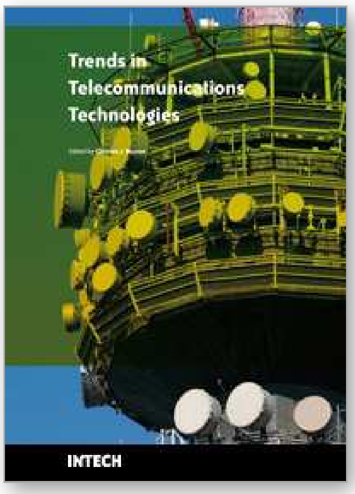

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The main focus of the book is the advances in telecommunications modeling, policy, and technology. In particular, several chapters of the book deal with low-level network layers and present issues in optical communication technology and optical networks, including the deployment of optical hardware devices and the design of optical network architecture. Wireless networking is also covered, with a focus on WiFi and WiMAX technologies. The book also contains chapters that deal with transport issues, and namely protocols and policies for efficient and guaranteed transmission characteristics while transferring demanding data applications such as video. Finally, the book includes chapters that focus on the delivery of applications through common telecommunication channels such as the earth atmosphere. This book is useful for researchers working in the telecommunications field, in order to read a compact gathering of some of the latest efforts in related areas. It is also useful for educators that wish to get an up-to-date glimpse of telecommunications research and present it in an easily understandable and concise way. It is finally suitable for the engineers and other interested people that would benefit from an overview of ideas, experiments, algorithms and techniques that are presented throughout the book.