A New Trend in Mitigating the Peak Power Problem in OFDM System

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1. Introduction

The Orthogonal Frequency Division Multiplexing (OFDM) is introduced as a wonder cure against everything that counteracts the high data rate wireless transmission (Andrews et al., 2007). Unfortunately, the combination of different signals with different phase and frequency give a large dynamic range that is used to be characterized by a high Peak to Average power Ratio (PAR), which results in severe clipping effects and nonlinear distortion if the composite time signal is amplified by a power amplifier, which have nonlinear transfer function. This degrades the performance of an OFDM system. A measure of the degradation can be very helpful in evaluating the performance of a given system, and in designing a signaling set that avoids degradation. The high PAR sets strict requirements for the linearity of the PA. In order to limit the adjacent channel leakage, it is desirable for the PA to operate in its linear region. High linearity requirement for the PA leads to low power efficiency. This poor efficiency causes high power consumption, which leads to warming in physical devices. This is a problem especially in a base station where the transmitted power is usually high. (Gregorio & Laakso, 2005)
The high peak to average power ratio (PAR) levels of OFDM signals attracts the attention of many researchers during the past decade. Several options appear in the literature related with OFDM systems and nonlinearities. Existing approaches that attack the PAR issue are abundant, but no general framework or quantitative comparisons among them exist to date. Actually the peak power problem in OFDM signals has been limited to the PAR reduction only, and the PAR reduction is considered as the issue regardless of the effect of this reduction on the system performance.
In this chapter, a new trend in mitigating the peak power problem in OFDM systems is proposed, based on modeling the effect of power amplifier nonlinearities on OFDM systems. By analyzing the obtained model, We showed that the distortion due to Power amplifier effects, either clipping or nonlinear effects, is highly related to the dynamic range itself rather than the clipping level or the saturation level of the nonlinear amplifier which is used to be an indication of the effect of the PAR on the OFDM system degradation, and thus we propose two criteria to reduce the dynamic range of the OFDM, firstly, through the generation of an OFDM signal with inherently small dynamic range regardless of the PAR level, through examining the effect of modulation choice on OFDM system, and looking for

modulation formats which minimize peak power and retain high spectral efficiency we propose the use of N-MSK modulation instead of Qudrature QAM modulation and Secondly, the use of Walsh Hadamard Transform (WHT) as an intelligent factor to reduce the dynamic range of the OFDM signal without the risk of amplifying the noise when restoring the signal to its original level. Computer simulations of the OFDM system using Matlab are completely matched with the deduced model in terms of OFDM signal quality metrics such as BER, ACPR and EVM. Also simulation results show that even the reduction of PAR using the two proposed criteria's is not significant, the reduction in the distortion due to HPA with very small Input Back Off (IBO) values is truly delightful.

2. PAR in OFDM systems

OFDM signals have a higher Peak-to-Average Ratio (PAR)—often called a Peak-to- Average Power Ratio (PAPR) than single-carrier signals do. The reason is that in the time domain, a multicarrier signal is the sum of many narrowband signals. At some time instances, this sum is large and at other times is small; the peak power is broken into a sum (in terms of decibels) of average power and a peak-to-average power ratio, not to mention that traditional modulation and coding schemes have been designed from the standpoint of minimizing average power (Miller & O'Dea, 1998), which means that the peak value of the signal is substantially larger than the average value. This high PAR is one of the most important implementation challenges that face OFDM, because it reduces the efficiency and hence increases the cost of the RF power amplifier, which is one of the most expensive components in the radio (Hanzo et al., 2003). An alternative measure of the envelope variation of a signal is the Crest Factor (CF), which is defined as the maximum signal value divided by the RMS signal value. For an unmodulated carrier, the Crest factor is 3 dB. This 3-dB difference between the PAR ratio and Crest factor also holds for other signals, provided that the center frequency is large in comparison with the signal bandwidth.
In this section, we quantify the PAR problem, and explain its severity in OFDM systems. Let *A= (Ao, A1, …, AN-1)* be a modulated data sequence of length N during the time interval [0, T], where *Ai* is a symbol from a signal constellation and T is the OFDM symbol duration. Then the complex envelope of the base-band OFDM signal for N carriers is given by:

$$s(t) = \sum_{n=0}^{N-1} A_n e^{jw_o nt} \tag{1}$$

Where, $w_o 2 = \pi / T$ and j=$\sqrt{-1}$. In practical systems, a guard interval (cyclic prefix) is inserted by the transmitter in order to remove inter-symbol interference (ISI), and Inter-channel interference (ICI) in the multipath environment. However, it can be ignored since it does not affect the PAR. The PAR of the transmit signal *s*(*t*), defined above in (1), is the ratio of the maximum instantaneous power and the average power, given by (Tellado, 1999):

$$PAR(A) = \max_{0 \le t < T} |s(t)|^2 \; / \; E\left\{ |s(t)|^2 \right\} \tag{2}$$

Where, *E{.}* denotes the expectation operator. Usually, the continuous time PAR of *s(t)* is approximated using the discrete time PAR, which is obtained from the samples of the OFDM signal. While the discrete-time PAR is several decibels less than that of the un-coded case. This shows that both caution and care must be exercised when the discrete-time PAR is used as a measure of PAR-reduction. It is shown that; with an over-sampling of L=4, the difference between the continuous-time and discrete-time PAR is negligible.
Recentely, (Litsyn & Yudin, 2005) studied some mathematical relationships between the PAR in continuous and sampled time points, they gave an upper bound and a lower bound for the ratio of the two PAR values, and a probably tighter upper bound for the continuous PAR, given the sampled PAR and its derivative against time.
The issue of over-sampling was also considered. The mathematics may be interesting, and the results are helpful in estimating the continuous PAR. However, the work assumes that the sub-carrier modulations are constant-modulus.

2.1 The PAR Problem

When transmitted through a nonlinear device, such as a high-power amplifier (HPA) or a digital to analog converter (DAC) a high peak signal, generates out-of-band energy (spectral regrowth) and in-band distortion (constellation tilting and scattering). These degradations may affect the system performance severely.

2.1.1 Sensitivity to Nonlinear Amplification

In general, nonlinear amplifier clips the amplitude of input signal. The sudden changes of the input amplitude generate higher spectral components in the power spectrum of the signal, so it results in a spectrum spreading. The spectrum spreading by nonlinear amplification is a cause of Adjacent Channel Interference (ACI) (Hara & Prasad, 2003).
The nonlinear behavior of an HPA can be characterized by amplitude modulation/ amplitude modulation (AM/AM) and amplitude modulation/phase modulation (AM/PM) responses. A typical AM/AM response for an HPA is shown in Figure 1, with the associated input and output back-off regions (IBO and OBO, respectively).
Figure 1 A typical power amplifier response.
Operation in the linear region is required in order to avoid distortion, so the peak value must be constrained to be in this region, which means that on average, the power amplifier is under-utilized by a back-off amount. To avoid such undesirable nonlinear effects, a waveform with high peak power must be transmitted in the linear region of the HPA by decreasing the average power of the input signal. This is called (input) *backoff* (IBO) and results in a proportional output backoff (OBO). High backoff reduces the power efficiency

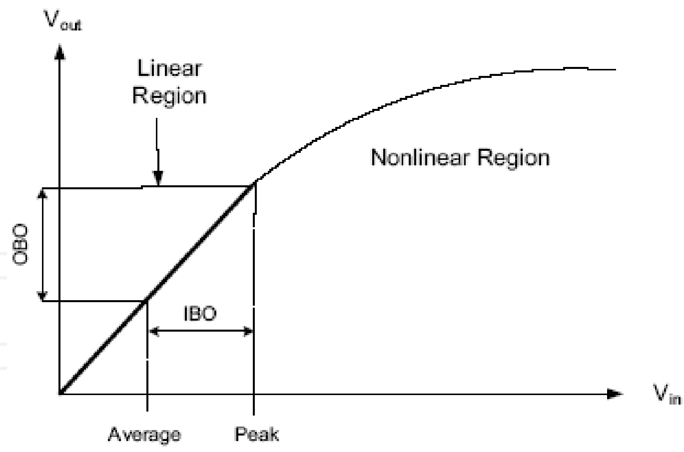

Fig. 1. A typical power amplifier response.

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$$IBO = 10\log_{10}\left(P_{in\ sat} \,/\, \overline{P_{in}}\right) \tag{3}$$

Where *Pin sat* is the saturation power, above which is the nonlinear region, and P_{in} is the average input power. The amount of backoff is usually greater than or equal to the PAR of the signal. The power efficiency of an HPA can be increased by reducing the PAR of the transmitted signal. For example, the efficiency of a class A amplifier is halved when the input PAR is doubled or the operating point (average power) is halved (Narayanaswami, 2001). Clearly, it would be desirable to have the average and peak values are as close together as possible in order to maximize the efficiency of the power amplifier. in P

Let us consider a case where an OFDM signal is amplified by a nonlinear device. The OFDM signal has non constant envelope; in other words, the waveform is like that of a narrowband Gaussian noise because of the central limiting theorem. Therefore, even if we try to reduce the spectrum spreading with a large input back-off, we cannot perfectly eliminate sudden inputs of the larger amplitudes. Furthermore, the out-band radiation generated from a sub-carrier also becomes "inter-sub-carrier interference" for the neighboring subcarriers. Therefore, the severe inter-sub-carrier interference drastically deteriorates the BER performance.

The nonlinearity severely influences the spectral characteristics of an OFDM signal. As can be seen from Figure 2, the OFDM signal corrupted by the amplifier corresponding to NLA for an IBO of 3 dB, 9 dB and 15 dB. As expected, there is a severe out-of-band radiation, and a very high IBO is necessary to reduce this radiation.

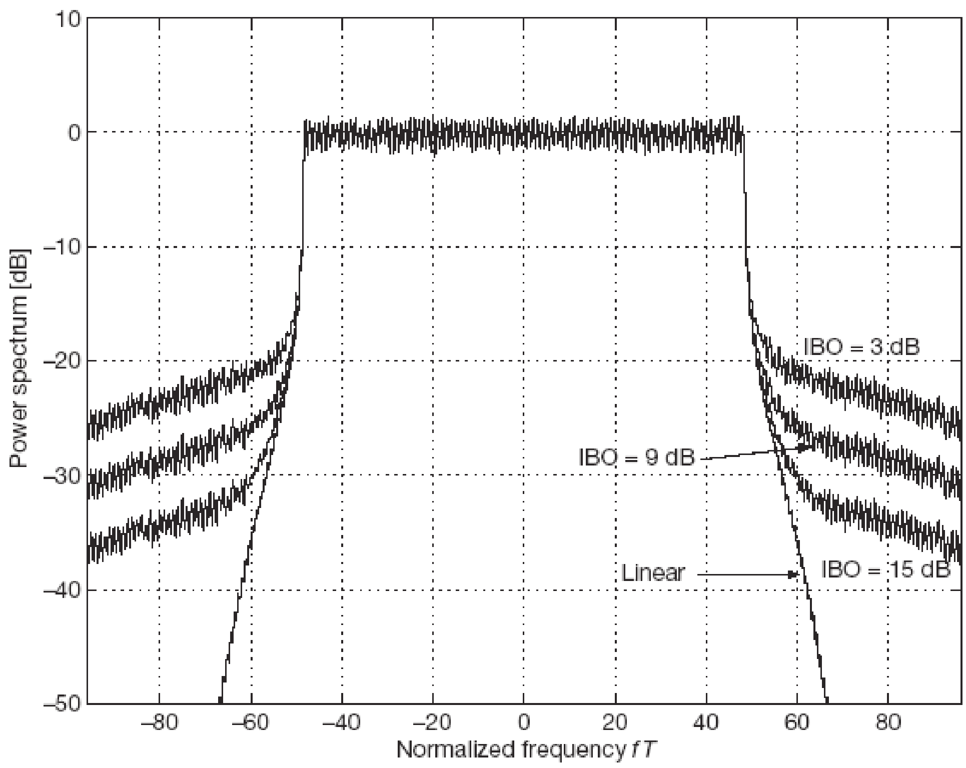

Fig. 2. Spectrum of an OFDM signal with a nonlinear amplifier. (Schulze & Luders, 2005)

Inside the main lobe, the useful signal is corrupted by the mixing products between all subcarriers. Simulations of the bit error rate would be necessary to evaluate the performance degradations for a given OFDM system and a given amplifier for the concrete modulation and coding scheme. For a given modulation scheme, the disturbances caused by the nonlinearities can be visualized by the constellation diagram in the signal space.

2.1.2 Sensitivity of (A/D and D/A) Resolutions

In addition to the large burden placed on the HPA, a high PAR requires high resolution for both the transmitter's DAC and the receiver's ADC, since the dynamic range of the signal is proportional to the PAR. High-resolution D/A and A/D conversion places an additional complexity, cost, and power burden on the system.

Assume that the digital signal processors at the transmitter and receiver have the same *q* bit-resolution. If we can deal with OFDM signal as a narrow band Gaussian noise with an average of zero and power of σn2 , then 99.7% of amplitude values ranges in [-3σs, 3σs] and 99.994% of amplitude values ranges in [-4σs, 4σs] here we call σs effective amplitude. Different resolutions have different optimum values in the quantization ranges to minimize the BER (Hara & Prasad, 2003).

3. Peak to Average Power Ratio Reduction Techniques

Simply dimensioning the system components to be able to cope with the worst-case signal peaks is practically impossible. That is why solutions have been proposed over the years, to counteract the PAR problem. This section reviews the existing techniques used for PAR reduction. All these techniques are categorized among three groups, namely: Coding, clip effect transformation and probabilistic techniques. A comparative study on these techniques has been done. All existing solutions involve some form of compromise. The most obvious one is the trade-off between bandwidth and computational complexity.

Practical systems do not use the full signal amplitude range, from a systematic approach; a general framework can be postulated. In this framework, the dynamic range of the system is set to Δ, which is smaller than the maximum signal level. When Δ is normalized by dividing it by the square root of the average signal power σ_x^2 *, it is refered to as* $\delta(=\Delta/\sigma_x)$. The dynamic range of the D/A and A/D only needs to be equal to Δ now. Also the power amplifier has to deal with a smaller input signal range.

When a signal is larger than this threshold Δ, it is clipped, which refers to hard amplitude limiting (Mestdagh, & Spruyt, 1996). However, the conversion from the digital to the analog domain causes peak regrowth and therefore some limiting could also be needed in the analog part of the system. Another and probably better option to avoid peak regrowth is to consider an over-sampled digital signal in the first place (Bahai et al, 2002). In any case, clipping causes in-band signal degradation, called clipping noise, which is approximately white in the signal band. For an over-sampled signal or analog clipping, also out-of-band radiation is generated. Appropriate filtering can reduce this effect (Gross & Veeneman, 1994). The amount of clipping noise depends on the probability that the signal is clipped, which is the probability that the signal amplitude is above Δ. Since each time sample obtained via an IFFT can be approximated as a large sum of independent contributions (for N > 64), the probability density function of the signal amplitude is approximately Gaussian.

The probability P(δ) that at least one of the N elements of x is clipped is thus given by $P(\delta)=1-(1-e^{-\delta^2})^N$, in the case of no over-sampling is used (Muller& Huber, 1997). As mentioned above, this equation neglects D/A regrowth. The probability that the maximum peak of the OFDM signal is above δ, can be approximated by (Ochiai & Imai, 2001):

$$P(\delta) = 1 - e^{-\sqrt{\frac{\pi}{3}}N.\delta.e^{-\delta^2}} \tag{4}$$

Setting the value of δ at such a level that the clipping noise is negligible (e.g. 50 dB below the signal level is obtained when δ = 4) is not optimal. Lowering Δ for a constant number of bits reduces the quantization noise quadratically. The clipping noise however increases, as the clipping probability is larger. Trading off one noise source versus the other minimizes the total noise and results in an increased overall Signal-to-Noise Ratio (SNR). Alternatively, for a constant SNR, the number of bits in the D/A and A/D can be decreased, lowering the implementation cost. To recapitulate: this framework imposes a limited signal range and clips peaks accordingly, governed by the total noise tradeoff. Existing PAR reduction techniques fit into this framework, as they all improve this noise tradeoff in some way.

Block coding schemes aim at completely eliminating the clipping noise by allowing only symbols that have a peak amplitude smaller than Δ, at the cost of a lower data rate. Another class of techniques, clip effect transformation, tries not to eliminate clipping, but to reduce the effect it has on the system. Finally, a third class reduces the clipping noise directly by lowering the probability of clipping. This class is therefore called probabilistic, in contrast to the deterministic behavior of block coding.

4. Modeling the effect of HPA on the OFDM signal

The OFDM baseband signals can be modeled by complex Gaussian processes with Rayleigh envelope distribution and uniform phase distribution, if the number of carriers is sufficiently large. The degradation of instantaneous nonlinear amplifiers and the signal-to-distortion ratio can be derived and expressed in an easy way. As a consequence, the output spectrum and the bit-error rate (BER) performance of OFDM systems are predictable both for uncompensated amplitude modulation/amplitude modulation (AM/AM) and amplitude modulation/pulse modulation (AM/PM) distortions. The aim of this work is to obtain the analytical expressions for the total degradation due to a nonlinear device and for the BER performance.
Analog-to-digital (A/D) converters, mixers, and power amplifiers are usually the major sources of nonlinear distortions due to the limited range that they allow for signal dynamics. It is possible to distinguish between two different classes of nonlinear distortions: the first, hereafter named Cartesian, acts separately on the baseband components of the complex signal, while the second acts on the envelope of the complex signal. A/D distortions, called Cartesian clipping, belong to the first class, while AM/AM and AM/PM (amplitude and phase distortion which depends on the amplitude of the input) introduced by power amplifiers belong to the second class.
The Bussgang theorem for real functions (Giunta, G. et.al, 1997) gives the separateness of a nonlinear output as the sum of a useful attenuated input replica and an uncorrelated nonlinear distortion, expressed by (Banelli & Cacopardi, 2000):

$$\mathbf{S_d}\ \mathbf{(t)} = \mathbf{S_u}\ \mathbf{(t)} + \mathbf{n_d}\ \mathbf{(t)} = \alpha \cdot \mathbf{s}\ \mathbf{(t)} + \mathbf{n_d}\ \mathbf{(t)} \tag{5}$$

In this analysis, we will focus on nonlinear distortions introduced by power amplifiers as it is considered as the main source of degradation.

4.1. OFDM Signal Statistical Properties

It is well known that according to the central limit theory that, the real and imaginary parts of the OFDM signal completely agree with the normal distribution and consequently its absolute agrees with the Rayleigh distribution with Probability density function expressed by:

$$P(x) = \frac{x}{s} e^{\frac{x^2}{2s^2}}, \ x \in \left[0, \infty\right] \tag{6}$$

Where s is a parameter, and Mean $\mu = s\sqrt{\pi / 2}$, and variance $\sigma^2 = \frac{4-\pi}{2} s^2$.

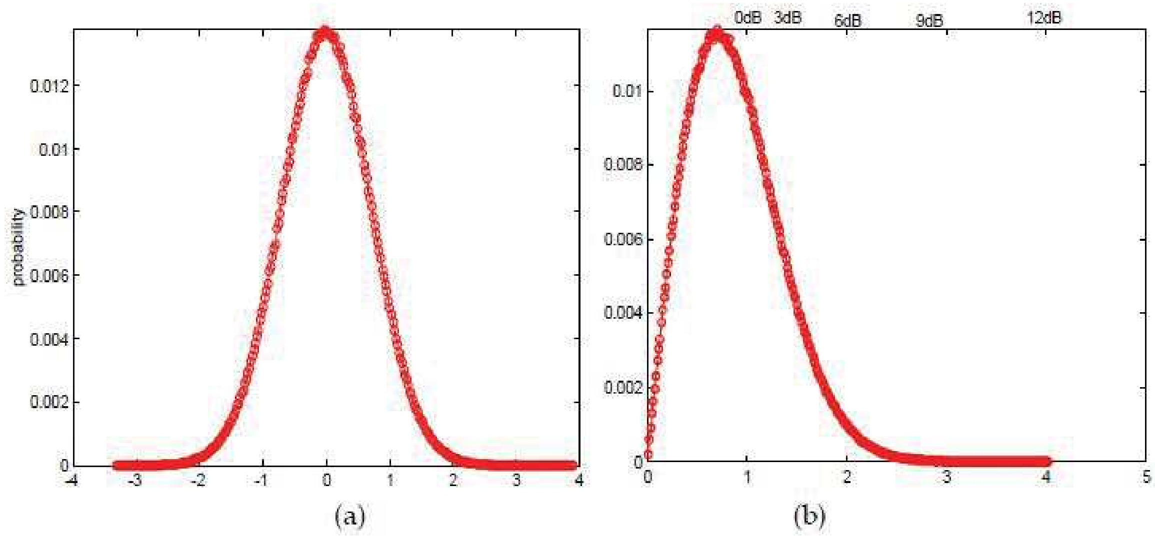

Fig. 3. Histogram of an OFDM signal (a) I component (b) Amplitude

Figure 3 explicitly shows that the measured amplitude histogram of the in-phase component/Quadrature component and the signal absolute for a 256-subcarrier OFDM signal. It is clear that the distribution in figure 3-) obeys a Gaussian distribution while that in figure 3-b obeys a Rayleigh distribution.

4.2. Power Amplifier Model

Consider an input signal in polar coordinates as (Gregorio & Laakso, 2005)

$$x = \rho e^{j\varphi} \tag{7}$$

The output of the power amplifier can be written as

$$g(x) = M(\rho)e^{j(\varphi+P(\rho))} \tag{8}$$

Where M(ρ) represents the AM/AM conversion and P(ρ) the AM/PM conversion characteristics of the power amplifier.
Several models have been developed for nonlinear power amplifiers, the most commonly used ones are as follows:

4.2.1 Limiter Transfer characteristics

A Limiter (clipping) amplifier is expressed as (Behravan & Eriksson, 2002):

$$M(\rho) = \begin{cases} \rho & |\rho| < A \\ A & |\rho| \geq A \end{cases}, \tag{9}$$

Where A is the clipping level. This model does not consider AM/PM conversion.

4.2.2 Solid-State Power Amplifier (SSPA)

The conversion characteristics of solid-state power amplifier are modeled by Rapp's SSPA with characteristic (Behravan & Eriksson, 2002):

$$V_{out} = \frac{V_{in}}{(1+(|V_{in}|/V_{sat})^{2p})^{\frac{1}{2p}}} \tag{10}$$

Where V_{out} and V_{in} are complex i/p & o/p, v_{sat} is the output at the saturation point, and P is "knee factor" that controls the smoothness of the transition from the linear region to the saturation region of characteristic curve (a typical value of P is 1).

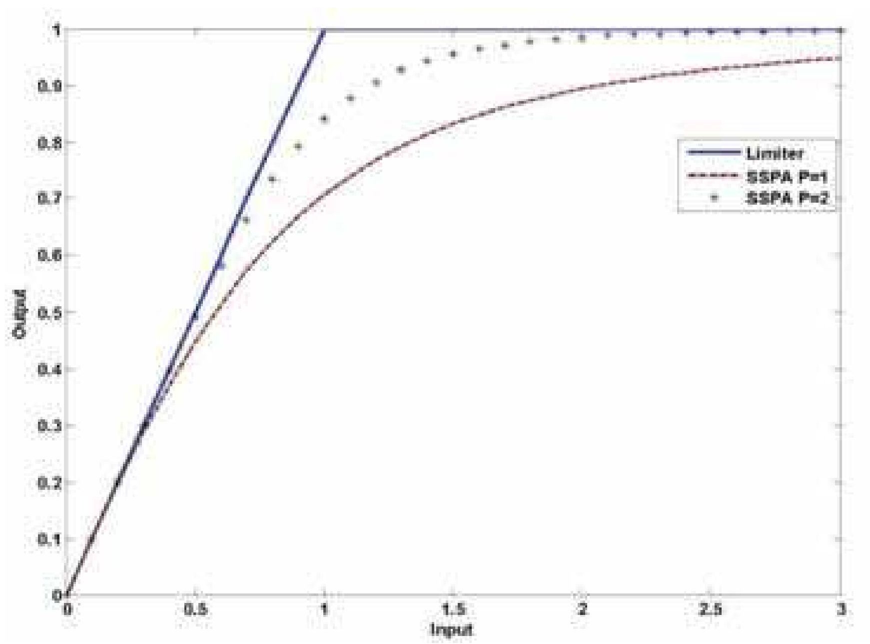

Fig. 4. AM/AM conversion of HPA

Figure 4 shows the AM/AM conversion of the two described models with ***A=1***. It is clear from the figure; as the value of knee factor increases the SSPA model approaches the Limiter Model.

4.3 Nonlinearity Distortion Analysis

Here we will analyze the effect of nonlinear amplifier on the OFDM signal, first: we will consider the NLA as a limiter that is expressed by:

$$g(x) = \begin{cases} x & x \le a \\ a & x > a \end{cases}.$$

And thus the distortion due to the NLA as a limiter can be represented by an extra Gaussian noise with variance $\sigma^2_{\lim iter}$ where

$$\sigma_{\lim iter} = \int_a^\infty (x-a)^2 P(x)dx = \int_a^\infty (x-a)^2 \frac{x}{s} e^{-\frac{x^2}{2s^2}} dx \ .$$

$$\therefore \sigma_{\lim iter} = \frac{1}{s}\left(e^{-\frac{x^2}{2s^2}}(-x^2s^2 - (a^2+2s^2)s^2 + 2axs^2) - a\sqrt{2\pi}s^3 erf(\frac{x}{2s}) \right)_a^\infty$$

$$= \frac{1}{s}\left(0 - a\sqrt{2\pi}s^3) - \frac{1}{s}(e^{-\frac{a^2}{2s^2}}(-a^2s^2 - (a^2+2s^2)s^2 + 2a^2s^2) - a\sqrt{2\pi}s^3 erf(\frac{a}{2s}) \right) .$$

This leads to

$$\sigma_{\lim iter} = a\sqrt{2\pi}s^2(erf(\frac{a}{\sqrt{2}s}) - 1) + 2s^3 e^{-\frac{a^2}{2s^2}} \ .$$

And finally

$$\sigma_{\lim iter\_} = 2s^3 e^{-\frac{a^2}{2s^2}} + a\sqrt{2\pi}s^2 erfc(\frac{a}{\sqrt{2}s}) \ , \quad (5)$$

If we consider the solid state power amplifier model given by Rapp:

$$g(x) = \frac{x}{(1+(x/V_{sat})^{2p})^{\frac{1}{2p}}} \ ,$$

And let p=1 and $V_{sat} = a$

The NLA distortion can be represented by

$$\sigma_{SSPA} = \int_0^\infty (x - g(x))^2 P(x)dx$$

$$= \int_0^\infty (x - \frac{x}{\sqrt{1+(x/V_{sat})^2}})^2 \frac{x}{s} e^{\frac{x^2}{2s^2}} dx \ .$$

$$\sigma_{SSPA} = \frac{1}{2s}\left(-2a^2s^2 + 4s^4 + a^4 e^{\frac{a^2}{2s^2}} Ei\left[-\frac{a^2}{2s^2}\right] - 2\sqrt{a^2} e^{-\frac{a^2}{2s^2}} \sqrt{2\pi}s(a^2-s^2) erf\left[\frac{a}{\sqrt{2}s}\right] + 2\sqrt{a^2} e^{-\frac{a2}{2s^2}} \sqrt{2\pi}s(a^2-s^2) \right)$$

Where Ei(x) is the Exponential integral, is defined as

$$E_i(x) = \int_{-\infty}^x \frac{e^t}{t} dt$$

$$\therefore \sigma_{SSPA} = \frac{1}{2s}\left(2ae^{-\frac{a^2}{2s^2}} \sqrt{2\pi}s(a^2-s^2)(1 - erf\left[\frac{a}{\sqrt{2}s}\right]) + a^4 e^{2\frac{a^2}{2s^2}} Ei\left[-\frac{a^2}{2s^2}\right] - 2a^2s^2 + 4s^4 \right)$$

And finally

$$\sigma_{SSPA} = \left(ae^{-\frac{a^2}{2s^2}} \sqrt{2\pi}(a^2 - s^2) erfc\left[\frac{a}{\sqrt{2}s}\right] + \frac{a^4}{2s} e^{\frac{a^2}{2s^2}} Ei\left[-\frac{a^2}{2s^2}\right] - a^2 s + 2s^3 \right), \tag{6}$$

When plotting the deduced distortion models in eq. (11, 12) versus the distribution parameter (s) with saturation level a= 2 ($A_{IP}3$ = 20 dB) we notice as shown in figure 5 below that the distortion due to the SSPA non-linearity is much more larger than that of its limiting effect, also it is obvious that the distortion is highly sensitive to any variation of the parameter (s) as the slopes of the curves show.

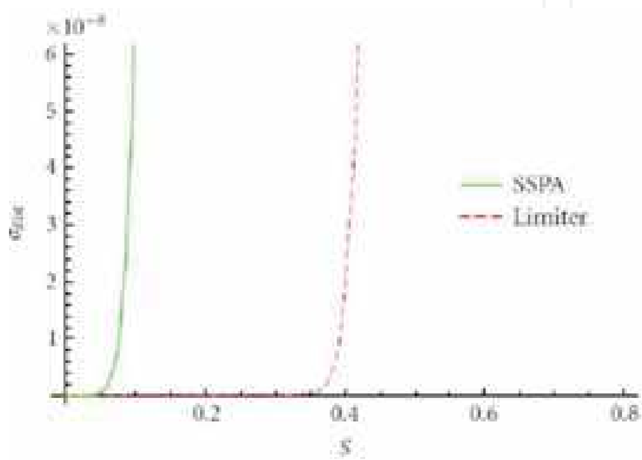

Fig. 5. NLA Distortion Versus *s*, with saturation level a= 2 (A_{IP3} = 20dB)

On the other hand, when plotting distortion model versus the saturation level with parameter (s=0.08) as depicted in figure 6, it is shown that the distortion decays as the value of saturation level increases.

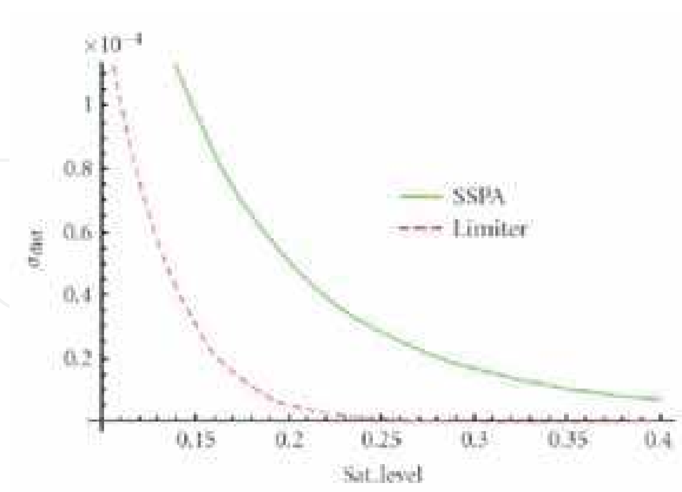

Fig. 6. NLA Distortion Versus saturation level **a,** with **s=** 0.08

From figures 5, 6 it is clear that the distortion due to these effects is highly related to (s) the distribution parameter, which controls the dynamic range itself, rather than the clipping level or the saturation level of the nonlinear amplifier.

An OFDM system is simulated using 512 carriers with cyclic prefix length equal to 4. Each carrier is modulated using 16-QAM constellation. AWGN noise is included. BER simulations compared with theoretical results considering the power amplifier distortion models deduced above in equations (11), (12) are shown in Figure 7.

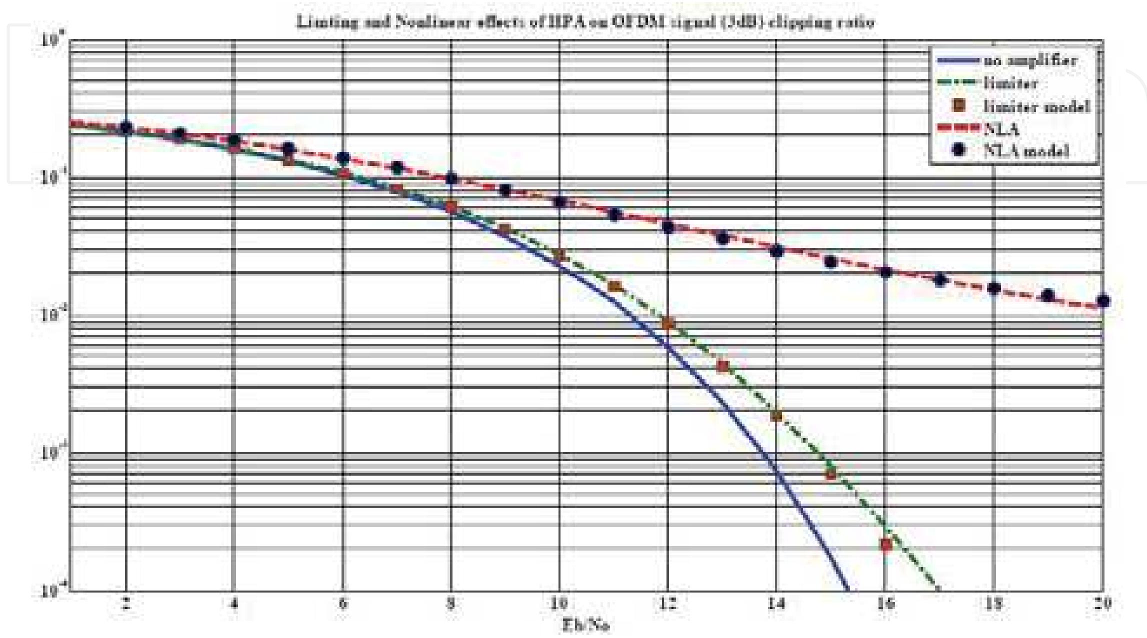

Fig. 7. BER of OFDM system with HPA

From figure 7, it is possible to see that it was predicted in the previous analysis. The effect of nonlinear power amplifier is illustrated, where a limiter amplifier is included in the simulations with IBO of 10 *dB*. The harmful effect of the nonlinearity can also be clearly seemed in this figure. And finally the figure shows that the computer simulations of BER are completely matched with the deduced models both the limiting and the nonlinearity effect. Table (1) shows The ACPR value for both limiting and non-linear effects with different limiting values relative to the maximum absolute value of the OFDM composite time signal **Ymax.**

Sat.Level/	ACPR (dB)		EVM	
Ymax	Limiting	SSPA	Limiting	SSPA
1	-44.7094	-28.7268	1.3e-016	0.0929
2	-44.7094	-31.0497	1.3e-016	0.0273
10	-44.7094	-32.8791	1.3e-016	0.0012
0.75	-30.4841	-27.2594	0.0126	0.1448
0.5	-23.4736	-25.0624	0.0969	0.2474
0.25	-14.5232	-22.3549	0.3972	0.4741

Table 1. ACPR & EVM for different limiting values

It is clear that as the limiting value decreases the ACPR increases. It can also be noted that the effect of nonlinearity on ACPR value is negligible as compared to that of limiting as the clip level varies; this is due to the fact that the spectral leakage that causes the ACPR to increase is mainly due to the clipping that can be viewed as windowing the spectrum by rectangular window. Table (1) shows also The EVM value for both limiting and non-linear effects with different limiting values, it is clear that as the limiting value decreases the EVM increases. And as the EVM is a measure of the total distortion it is highly affected by the nonlinearity rather than the limiting effect.

5. OFDM Using Different Modulators

In this section we consider the choice of low crest factor modulation techniques to be used in the OFDM system. We have compared among several families of modulation techniques, compare among these modulations techniques in OFDM for the spectral efficiency, BER, and PAR reduction, which reduce the effect of nonlinear amplifier. We propose to use a modulation technique with high spectral efficiency and less sensitive to nonlinear channel effects, which is the Minimum Shift Keying (MSK). For the sake of preserving the spectral efficiency we extend out proposal to the family of Multi-Amplitude
Minimum Shift Keying (N-MSK) to be used with the OFDM system.
When designing a constellation diagram for a modulation technique, some considerations must be given to (Hanzo et.al, 2000):
The minimum Euclidean distance amongst phasors, which is characteristics of the noise immunity of the scheme
The minimum phase rotation amongst constellation points, determining the phase jitter immunity
The peak to average phasor power, which is a measure of robustness against non-linear distortion introduced by power amplifiers.
The Bandwidth efficiency can be increased either by increasing the Number of signal phase levels, or by increasing the Number of signal amplitude levels,
Increasing the signal amplitude levels has the drawback that the signal envelope is not constant and therefore non-linear amplification may cause spreading of the signal spectrum and increase in BER.
Increasing the Number of phase levels will highly increase the BER.
We have simulated an OFDM system shown in figure 8 with 256 carriers and oversampling factor of 2, with different modulation techniques, namely, M-PSK , M- QAM (with M=4, 8 and 16) and N-MSK (with N=1,2,4).

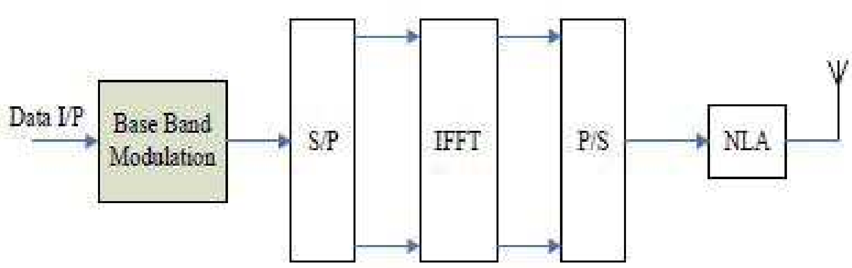

Fig. 8. Block diagram of an OFDM system

When analyzing the statistical properties of the OFDM composite time signal we deduced the following: As shown in figure 9; the probability density function of the absolute OFDM signal agrees with the Rayleigh distribution for all used modulation techniques.

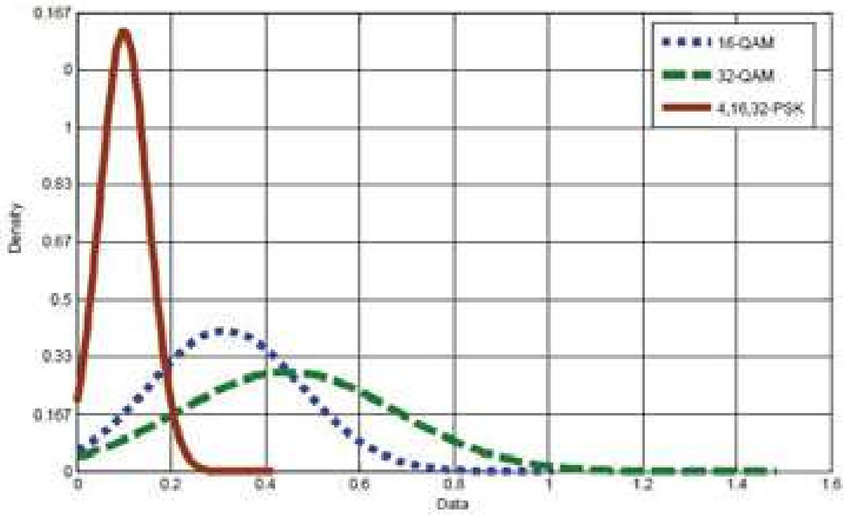

Fig. 9. PDF of OFDM with different modulation

It is clear that as M-increases in M-QAM the dynamic range increases and is not changed in M-PSK cases.

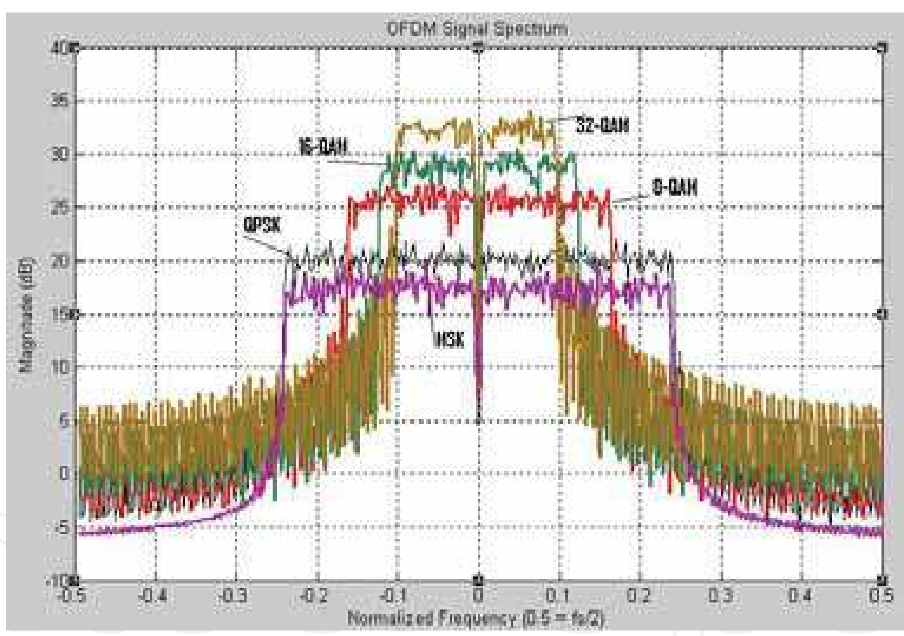

Fig. 10. OFDM Spectrum with different modulation

As shown in figure 10 in agreement to figure 9, We have noted that for M-PSK, as M increases no effect has been occurred to the dynamic range and the Magnitude remains nearly the same, while for M-QAM, as M increases the dynamic range increase and so the Magnitude, but both of them agree in the spectral efficiency increase as M increase. While in the case of MSK the dynamic range is less than that of QPSK. It is also noticeable from figure 10 that the Bandwidth of MSK is nearly the same as that of QPSK, but with lower power.
Table 2 Shows PAR, standard deviation 8 and the dynamic range of the OFDM signal with the above mentioned modulation techniques

Modulation	PAR (dB)	STD (8)	Absolute signal range
MSK	8.3902	0.0427	0.0642
QPSK	8.7001	0.0562	0.0912= MSK + 3 dB
8-QAM	9.0063	0.0965	0.1470= QPSK +4 dB
16-QAM	9.2989	0.1247	0.2213= 8QAM +3.5 dB
32-QAM	10.082	0.1777	0.2780= 16QAM +2 dB

Table 2. PAR and STD of OFDM signal

It is again in agreement with the above results. It is clear that although the PAR reduction due to the use of MSK instead of QPSK is slightly small, the true gain is the reduction in the dynamic range by **3 dB,** which enables us to use a low linearity and high efficiency power amplifiers. In addition a new indicator arises in the table which is the standard deviation 8, it is obvious from the table that MSK has the lowest 8 while for M-QAM, as M increase 8 increases also, since we can deal with OFDM signal as a narrow band Gaussian noise with a mean of zero and variance of 8^2 , then 68% of amplitude values ranges in [-8,8] and 99.994% of amplitude values ranges in [-48, 48], this can be a good indicator for clipping efficiency. Regarding to the BER performance of the used modulation techniques, it is clear from figure 11 that, for M-PSK and M-QAM, the BER increases ad M increase while for the same M the BER of M-PSK is larger than that of M-QAM for the same Eb/No.

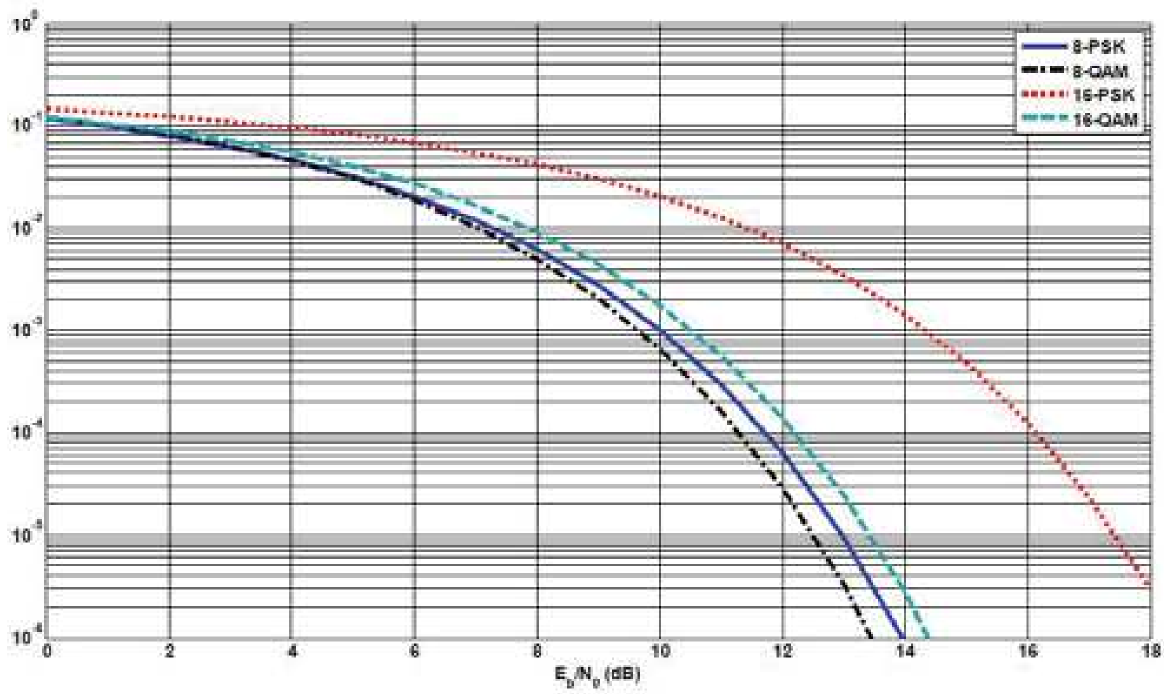

Fig. 11. BER of OFDM using 8, 16 -PSK&QAM

Also we have applied the amplifier (using the SSPA model discussed earlier) on the OFDM signal using M-QAM with M=4, 8 and 16. Using IBO of 5 dB as in figure 12-a and IBO of 10 dB as in figure 12-b, we noticed that as M increases the distortion due to NLA increases and so the BER as shown in figure 12-a, b.

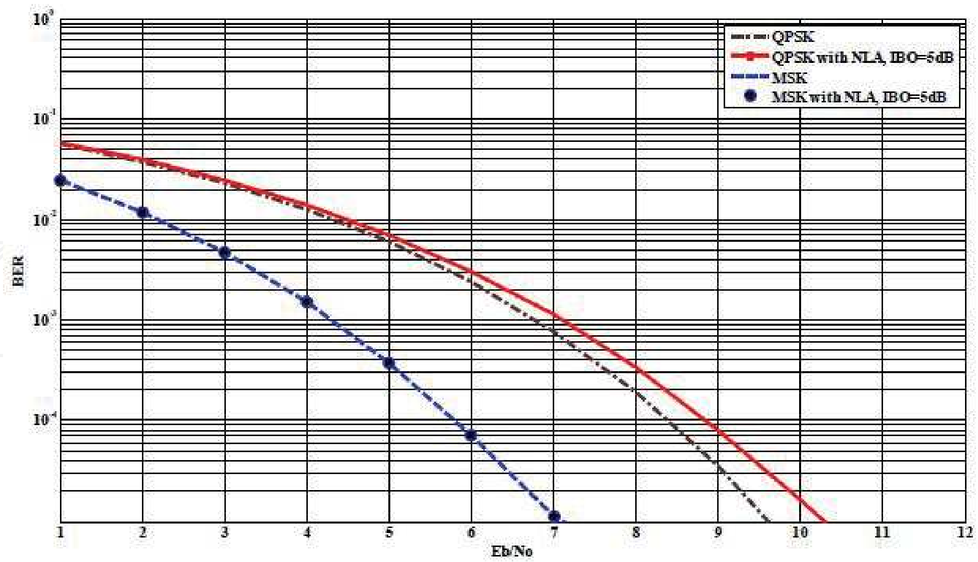

Fig. 12 -a effect of NLA on MSK & QPSK

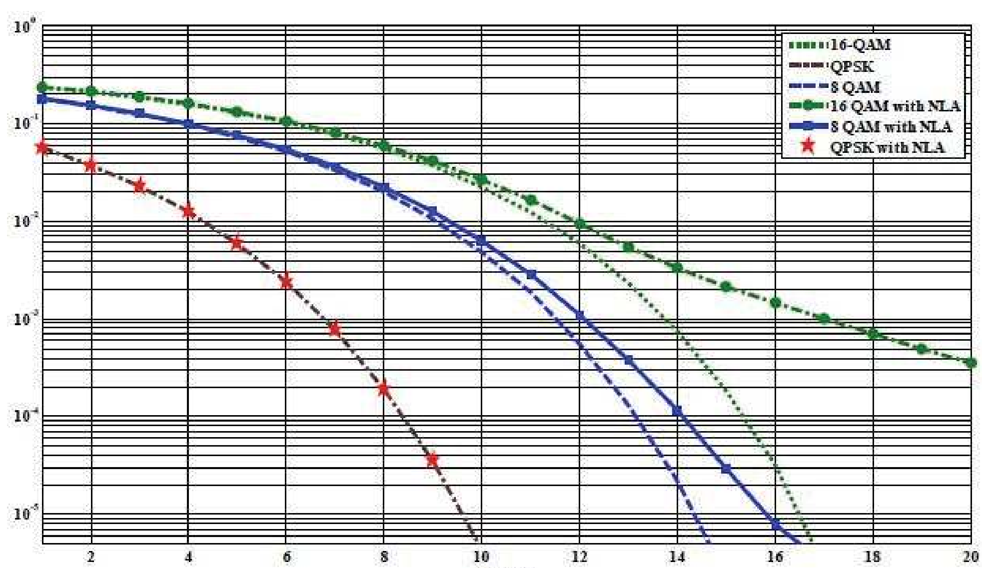

Fig. 12-b effect of NLA on M-QAM

As regarding to our previous results, it can be noticed that, the MSK modulation gives us the lowest PAR when used in OFDM besides its main advantage, that it ignores any fading introduced amplitude fluctuation present in the received signal, and hence facilitates the utilization of power efficient class C amplifier (Javornik et.al, 2001).

On the other hand, the MSK has a spectral efficiency that is nearly equal to that of QPSK, which is much less than those of higher order M-ary modulation techniques; and thus it is better to check the performance of N-MSK modulation techniques with OFDM systems in the presence of high power amplifiers nonlinearities.

***N-MSK signal description**

Minimum shift keying (MSK) is a special type of continuous phase-frequency shift keying (CPFSK) expressed as:[92]

$$\mathbf{s(t)=Acos(w_c t+j(t)),\ wc=(wo+w1)/2} \tag{7}$$

$\phi(\mathrm{t}) = +\Delta \mathrm{wt} / 2 + \phi(0)$,

Where Δw is the separation between *wo, wl* used to represent 0, 1

$\phi(0)$ is the initial phase, $\Delta \mathrm{w} = \pi / \mathrm{Tb}$

Consequently the modulation index h=$\Delta w/w_c$ =*0.5*
That corresponds to the minimum frequency spacing that allows two FSK signals to be coherently orthogonal, MSK, conventionally, is a special case of offset Qudrature Phase-Shift Keying (OQPSK) with sinusoidal symbol weighting. That is, two sinusoidally-shaped bit streams, one having a bit-period time-offset relative to the other, are employed to modulate orthogonal components of a carrier as [92]:

$$s(t) = I\cos(\frac{\pi t}{2T})\cos(2\pi f_c t) + Q\sin(\frac{\pi t}{2T})\sin(2\pi f_c t) \tag{8}$$

It also can be expressed in the trigonometric form as:

$$s(t) = \cos(2\pi f_c t + b_k(t)\frac{\pi t}{2T}) \quad \mathrm{s(t) = cos(2nf_c t + b_k\ (t)jT\ )} \tag{9}$$

Where b_k is -1 if I&Q are the same and +1 otherwise.
The Multi-amplitude minimum shift keying (N-MSK) is obtained by superposition of N MSK signals with different amplitudes. It is a bandwidth efficient modulation scheme that has the continuous phase property of MSK and provides higher spectral efficiency by using multilevel modulation. However, for N-MSK there is no requirement for the signals to be orthogonal, in fact, they are co-phased. Furthermore, the amplitudes of the constituent signals are not equal and this prevents the phase trajectory from passing through the origin (Figure 13). For example, a two level N-MSK signal can be expressed as:

$$s(t) = 2A\cos(2\pi f_c t + \phi_2(t,I)) + A\cos(2\pi f_c t + \phi_1(t,J)) \tag{10}$$

Where

$$\phi_2(t,I) = \frac{\pi}{2}\sum_{k=-\infty}^{n-1} I_k + \frac{\pi}{2T_s} I_n(t-nT) \qquad nT \le t \le (n+1)T$$

$$\phi_2(t,J) = \frac{\pi}{2}\sum_{k=-\infty}^{n-1} J_k + \frac{\pi}{2T_s} J_n(t-nT) \qquad nT \le t \le (n+1)T$$

Figure 13-a shows the constellation diagram of 2-MSK signal with Constituent signals have equal amplitude, while Figure 13-b shows the constellation of 2-MSK signal with Constituent signals have unequal amplitudes

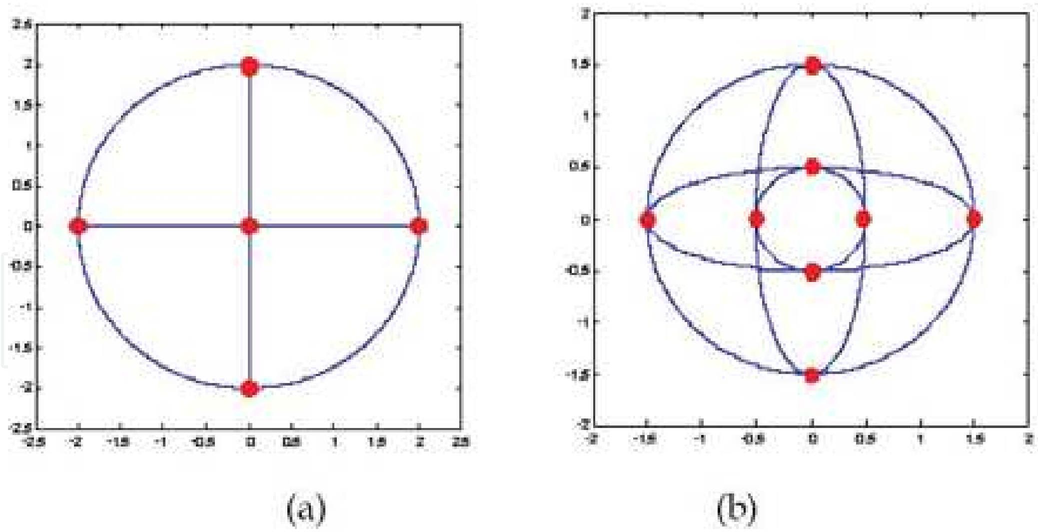

Fig. 13. Signal constellation for N-MSK, with Constituent signals (a) have equal amplitude (b) have unequal amplitudes

It is noticed for the case when constituent signals have unequal amplitudes; the 2-MSK signal never reaches the zero signal energy, which improves the signal performance in nonlinear channels.
Since the N-MSK signal can be considered as a sum of two MSK signals its spectral efficiency is twice that of MSK. However, it loses, slightly, the constant envelope property due to multilevel modulation.
When testing the above simulated OFDM system with N-MSK (N=2, 4) we found the following:
When analyzing the statistical properties of the OFDM composite time signals when using 2-MSK, with the two cases of equality between the two constituent signals: As shown in Figure 14;

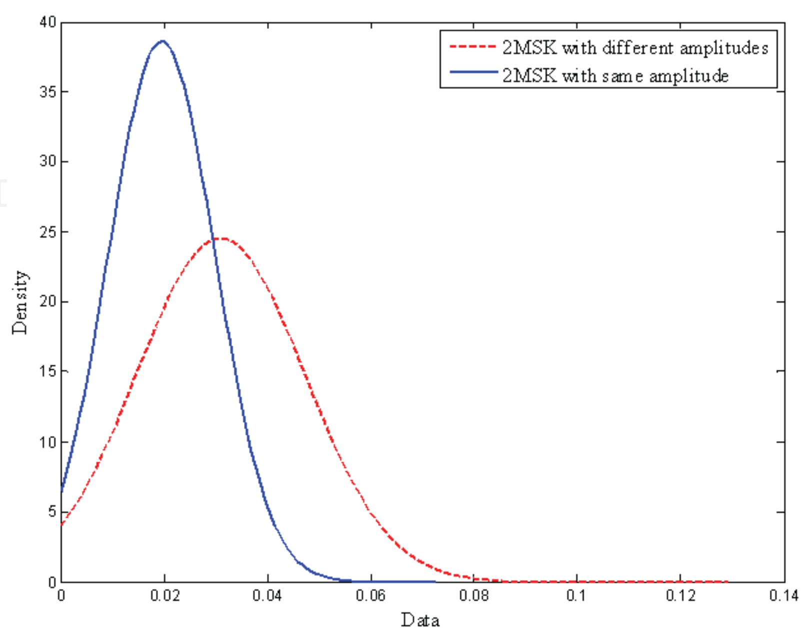

Fig. 14. PDF of OFDM with 2-MSK

The probability density function of the absolute OFDM signal agrees with the Rayleigh distribution in the two cases. And the dynamic range is larger in the case of different amplitudes.
Table (3) shows PAR of OFDM signal with the above mentioned modulation techniques

Modulation	Range		PAR (dB)	
MSK	0.0422		8.5752	
2-MSK	Same amplitude	Different amplitudes	Same amplitude	Different amplitudes
	0.0444	0.0735	8.9145	9.3439
4-MSK	Same amplitude	Different amplitudes	Same amplitude	Different amplitudes
	0.0429	0.1158	8.1945	8.2528
QPSK	0.0692		9.0054	
16-QAM	0.1365		9.9054	

Table 3. PAR of OFDM signal

It is clear from the table that when using N-MSK where N=2 or 4 , with the same amplitude of the two or four constituent signals respectively, the PAR change with respect to MSK is negligible; and there will be no change in the dynamic range that they are nearly the same, but when using the constituent signals with different amplitudes the increase in the dynamic range will be somewhat larger (4.37 dB and 8.72 dB for N=2 and 4 respectively) but it is of course much less than the increase in the dynamic range when using QPSK or 16-QAM, 4.29 dB and 10 dB respectively with respect to MSK. It is also noticeable from figure 15 that the Bandwidth of MSK is nearly the same as that of QPSK, while BW of 2-MSK is nearly the same as 16-QAM. Both MSK and 2-MSK have lower amplitudes than QPSK and 16-QAM respectively.

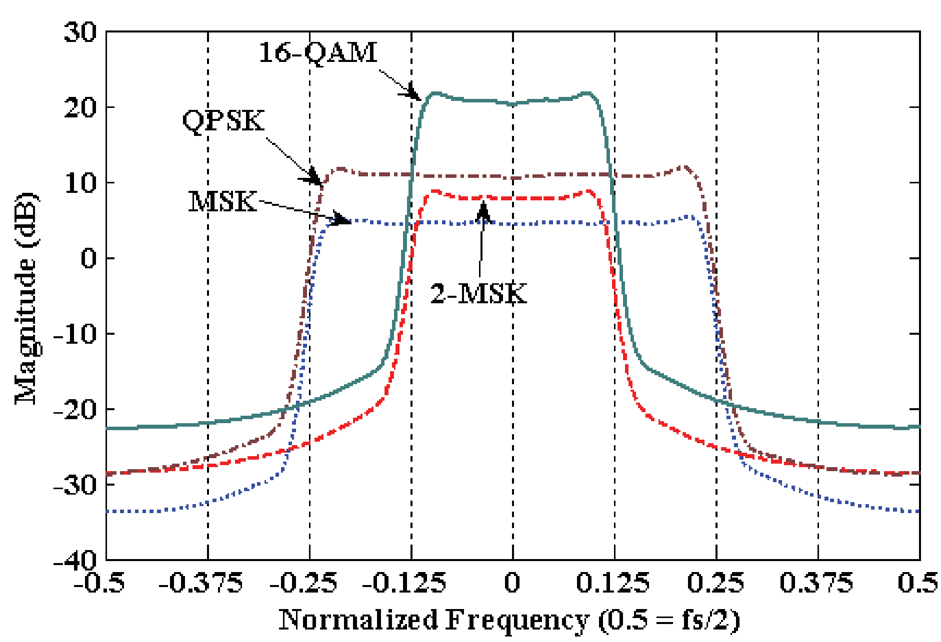

Fig. 15. OFDM Spectrum with N-MSK

Regarding to the BER performance of the used modulation techniques, it is clear from figure 16 that, the BER of 2-MSK is larger than that of MSK for the same Eb/No. it is also noticed that the BER of N-MSK is less than that of M-QAM with the same spectral efficiency.

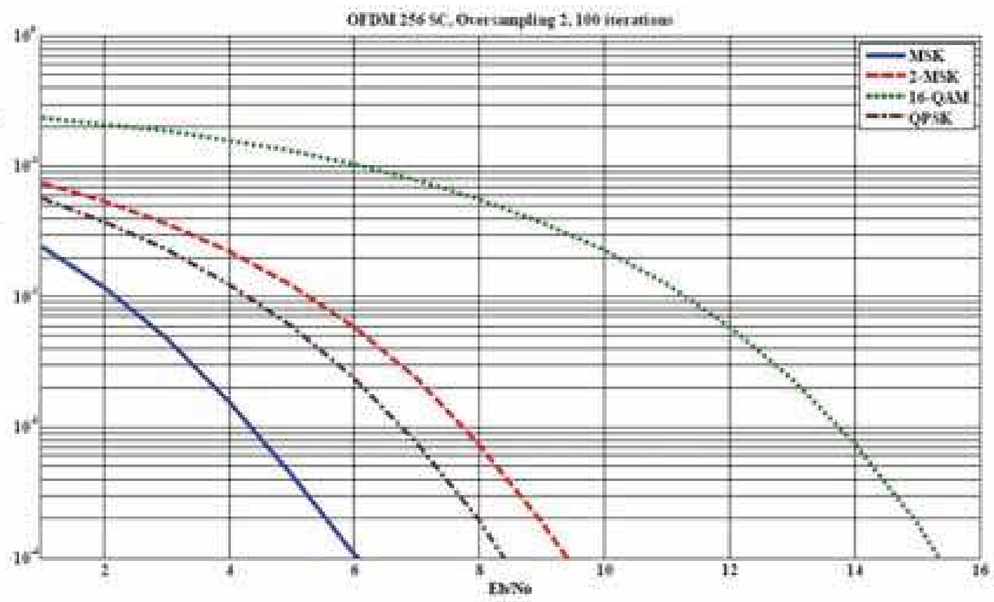

Fig. 16. BER of OFDM with N-MSK& M-QAM

The impact of the nonlinear power amplifier on the spectrum of the OFDM signal is shown in Figure 17. The spectrum of both 16-QAM and 2-MSK with and without the non-linear amplifier are shown. A clear spreading and in-band distortion are observed when passing the OFDM signal with 16-QAM through the SSPA nonlinearity. On the other hand, the effect of nonlinearity is shown to be negligible on the OFDM signal spectrum with 2-MSK.

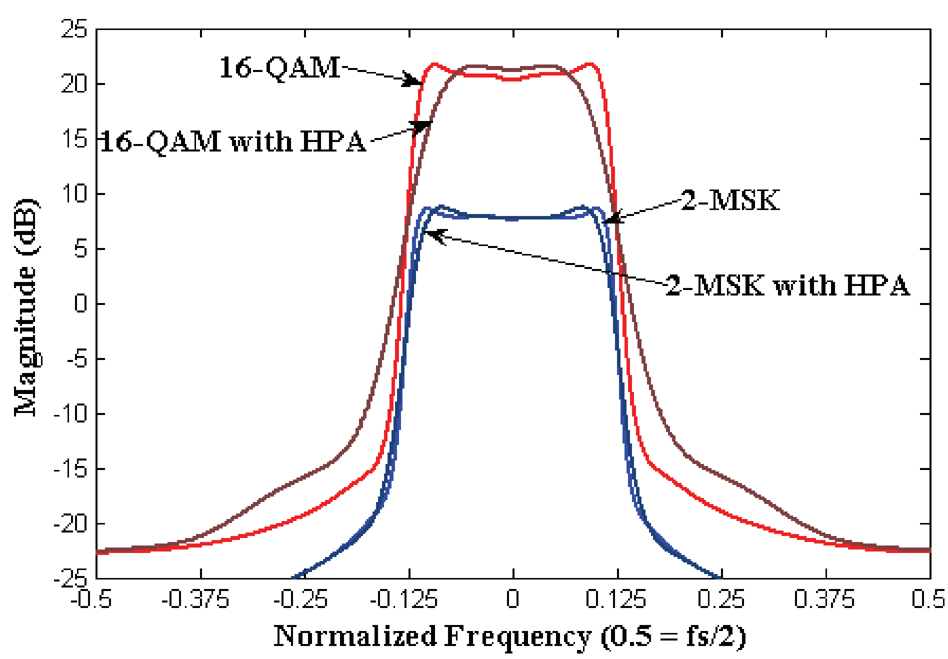

Fig. 17. OFDM Spectrum with HP A

Regarding the BER performance of the OFDM system using these modulation techniques, figure 18 shows the effect of SSPA non-linearity on the BER curves when using 2-MSK and 16-QAM. It can be noticed that, there is no effect of the HPA in the case of MSK, and a very small effect occurs in the 2-MSK case it can be said to be negligible as compared to the large degradation in the BER performance when using 16-QAM.

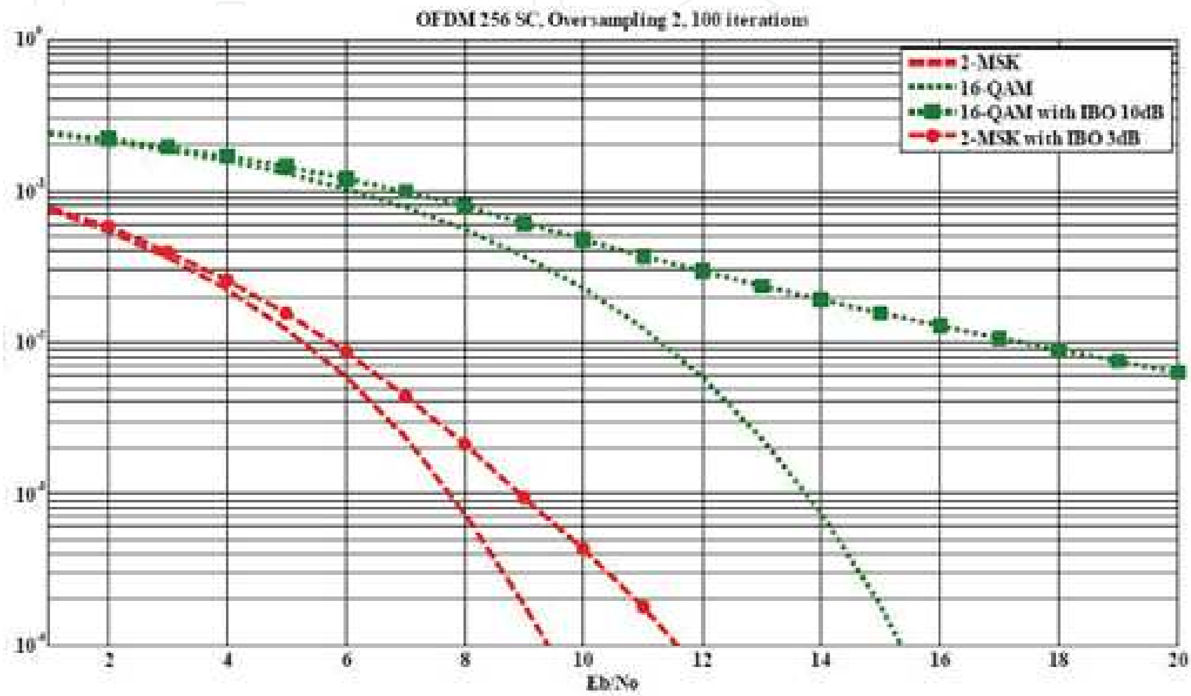

Fig. 18. OFDM BER with HPA

Provided that a reduction of 7 dB in the IBO for the amplifier used in the N-MSK cases.

A differential 2MSK receiver, based on regeneration of the MSK signal component with larger amplitude in the receiver, was used (Javornik & Kandus, 2002). As shown in the block diagram in figure 19. For the case of 4-MSK we have used a nested 2MSK receiver of the above structure. It is clear that, as N increases the complexity of the receiver dramatically increases.

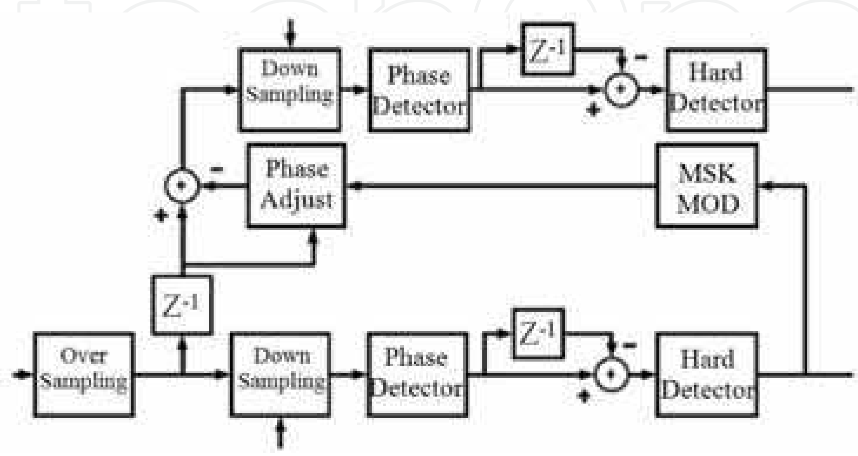

Fig. 19. Block diagram of A differential 2MSK receiver [93]

6. PAR reduction in OFDM using Walsh Hadamard Transform (WHT)

We propose the use of Walsh Hadamard Transform (WHT) as an intelligent scaling factor to reduce the dynamic range of the OFDM signal without the risk of amplifying the noise when restoring the signal to its original level. This technique offers an excellent solution to all of peak power problems in OFDM systems and without any loss in terms of spectral efficiency and without any side information being transmitted, and can be applied with low computational complexity.

6.1 Walsh Hadamard Transform

The Walsh-Hadamard Transform (WHT) is perhaps the best known of the non sinusoidal orthogonal transforms (Furis et.al, 2005). The Walsh-Hadamard transform of a signal x, of size N= 2n, is the matrix-vector product WHTN x, where

$$WHT_N = \overset{n}{\underset{i=1}{\otimes}} DFT_2 = DFT_2 \ \otimes \ldots \otimes \ \mathrm{DFT}_2 \ , \tag{11}$$

$DFT_2 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is the 2-point DFT matrix, and ® denotes the tensor or Kronecker product. The tensor product of two matrices is obtained by replacing each entry of the first matrix by that element multiplied by the second matrix. Thus, for example,

$$WHT_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \otimes \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

The WHT has gained prominence in digital signal processing applications, since it can be computed using additions and subtractions only consequently, its hardware implementation is simpler.

6.2 Fast Walsh-Hadamard Transform (FWHT)

As the FFT is an algorithm to compute the DFT efficiently, similarly the FWHT is an algorithm to compute the WHT efficiently. The FWHT can be expressed as: (Furis et.al, 2005)

$$W(u) = \frac{1}{N}\sum_{x=0}^{N-1} f(x)g(x,u), \tag{12}$$

Where $g(x,u) = [\prod_{i=0}^{n-1}(-1)^{b_i(x)b_{n-1-i}(u)}]$.

The FWHT can be derived using matrix factoring or matrix partitioning techniques. The signal flow graph for a 4 point FWHT is shown in Figure 20.

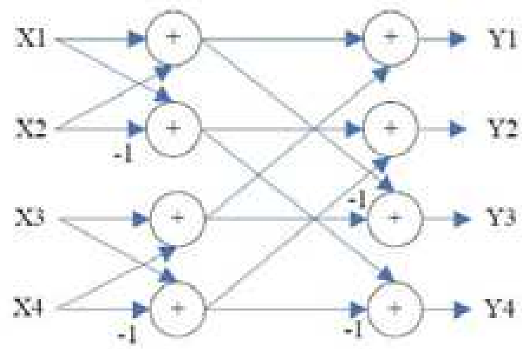

Fig. 20. four point FWHT

In a digital system the 1/4 multiplier can be simply implemented in two arithmetic shifts. The number of additions and subtractions needed to compute the four WHT coefficients is 4 x $\log_2 4 = 8$.

6.3 OFDM system using FWHT

The use of Hadamard transform with OFDM for the purpose of PAR reduction is proposed in (Park et.al, 2000), as a "Hadamard Transform-based OFDM". Through the construction of an OFDM system with Discrete Hadamard Transform (DHT) instead of the DFT. This act as a cubic constellation shaping that leads to a great PAR reduction. Also the Hadamard Transform is used for the optimum phase sequences for the Partial Transmit Sequence technique (Cimini & Sollenberger, 2000).

We have examined the use of FWHT with OFDM in a different way; with different lengths.

The idea: since the peak value of the OFDM symbol results at the instant of coherent addition of sub-carriers, it is possible to reduce this peak by modifying the sub-carriers initial phases so as to avoid this condition [96]. This can be done via successive phase shifts, we have tried when summing up a number of orthogonal sinusoidal waveforms with successive phase shifts with different phase shift step in each time we try and we have got the optimum case at **phase shift step =** n which is equivalent to modifying the OFDM symbol representation to be as follows:

$$y_n = \sum_{k=0}^{N-1} (-1)^k X_k e^{j2\pi \frac{nk}{N}} \tag{13}$$

The above equation can be approximately implemented as shown below in figure 21 in the proposed block diagram of OFDM system.

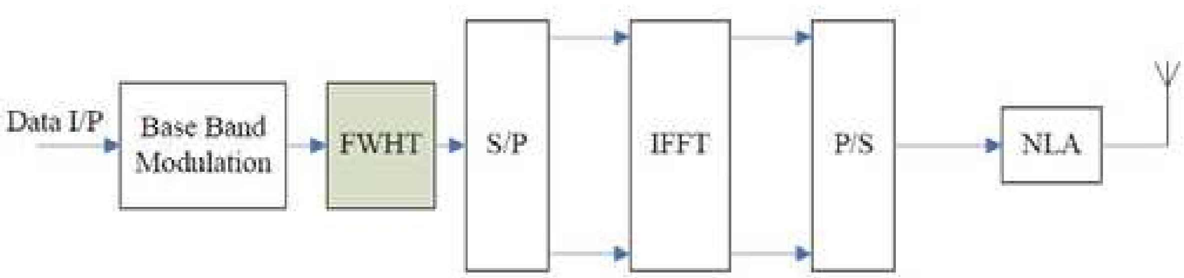

Fig. 21. OFDM system block diagram

In the simulations, 16- QAM modulation scheme is selected in OFDM with 512 sub-carriers and an oversampling factor of 2, i.e., IFFT length = 1024.
Figure 22 depicts the constellation diagram of an OFDM mapped signal with and without FWHT

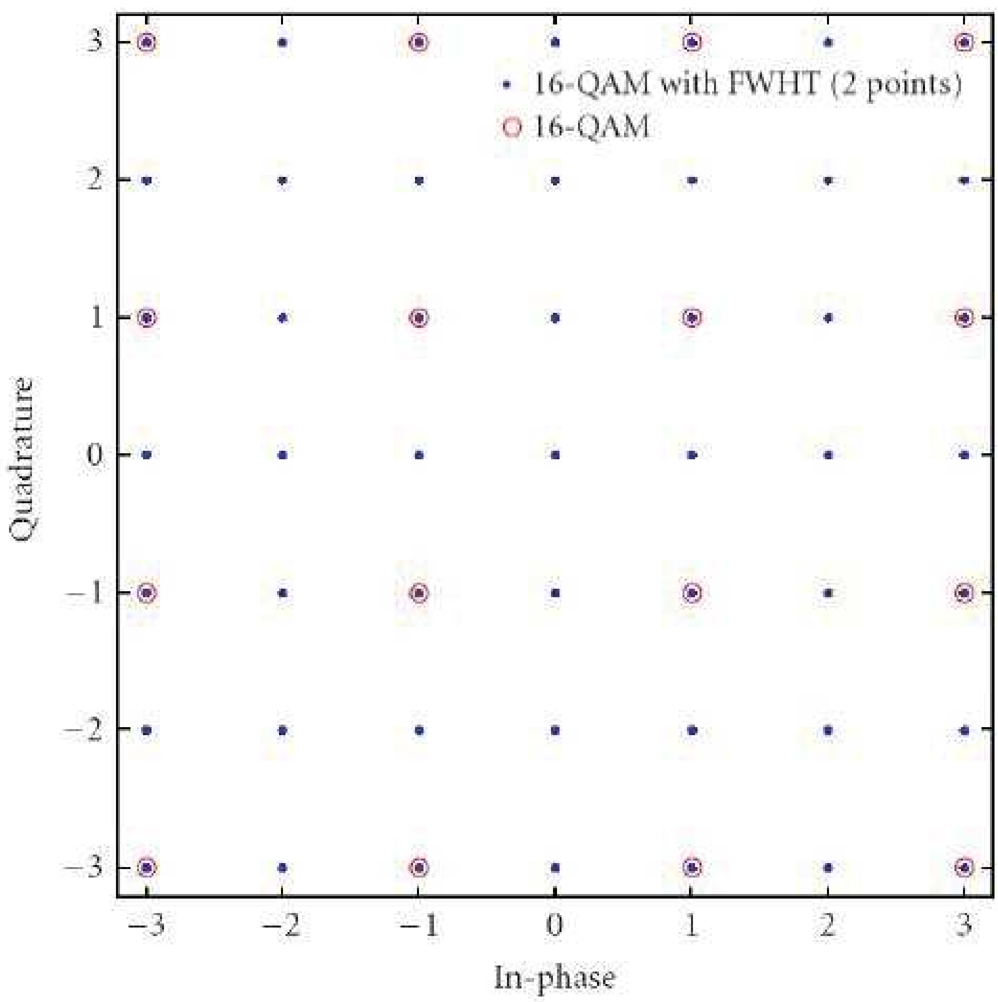

Fig. 22. constellation diagram of OFDM signal

It is clear that the FWHT can be viewed as an Active Constellation Extension (ACE) method, one of the latest PAR reduction, which alter or introduce new signal constellations to combat large signal peaks, but our concern here is its effect on the dynamic range of the OFDM signal. Simulations shows that when using the OFDM shown in Figure 21 with 16-QAM without FWHT the PAR is found to be 10.8 dB, while when using the FWHT (2 and 4 points) the PAR equals 10.5 and 10.15 dB respectively, provided that the dynamic range is reduced by 3 and 6 dB in the same cases respectively.
Regarding the statistical properties of PAR, figure 23 shows the power survivor (equivalent to the traditionally used CCDF) instead of the PAR survivor. It is noticed that when using FWHT with size $\mathbf{N=2^n}$ the total power of the OFDM system is reduced by **3n dB,** which is a very good chance to perform an intelligent scaling that avoids the risk of amplifying the noise when restoring the signal to its original level at the receiver.

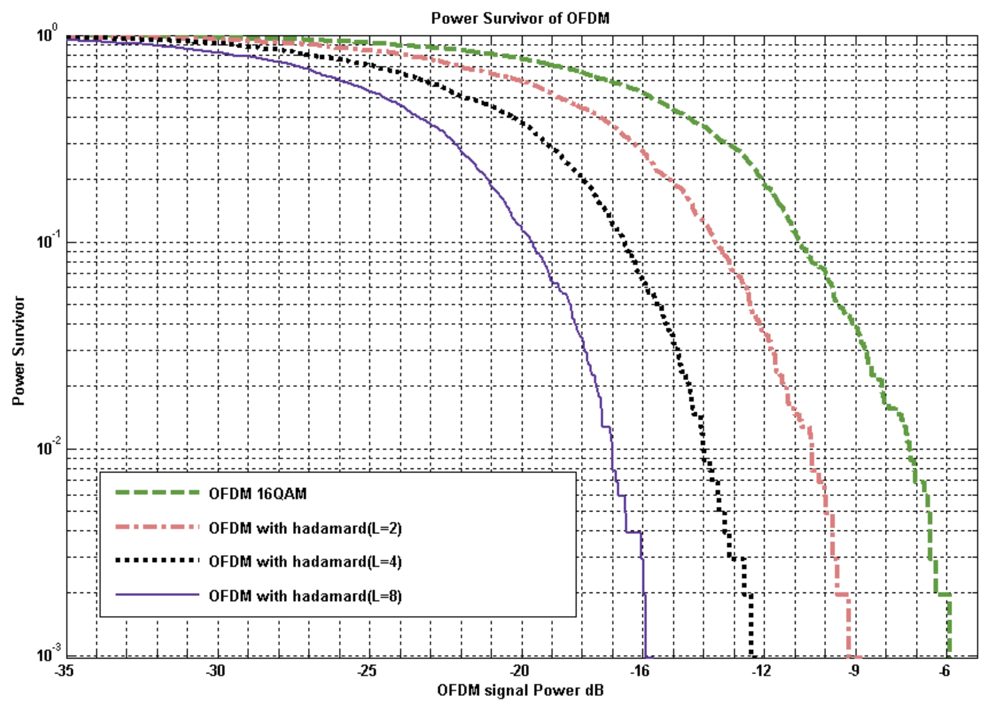

Fig. 23. power survivor of OFDM with WHT

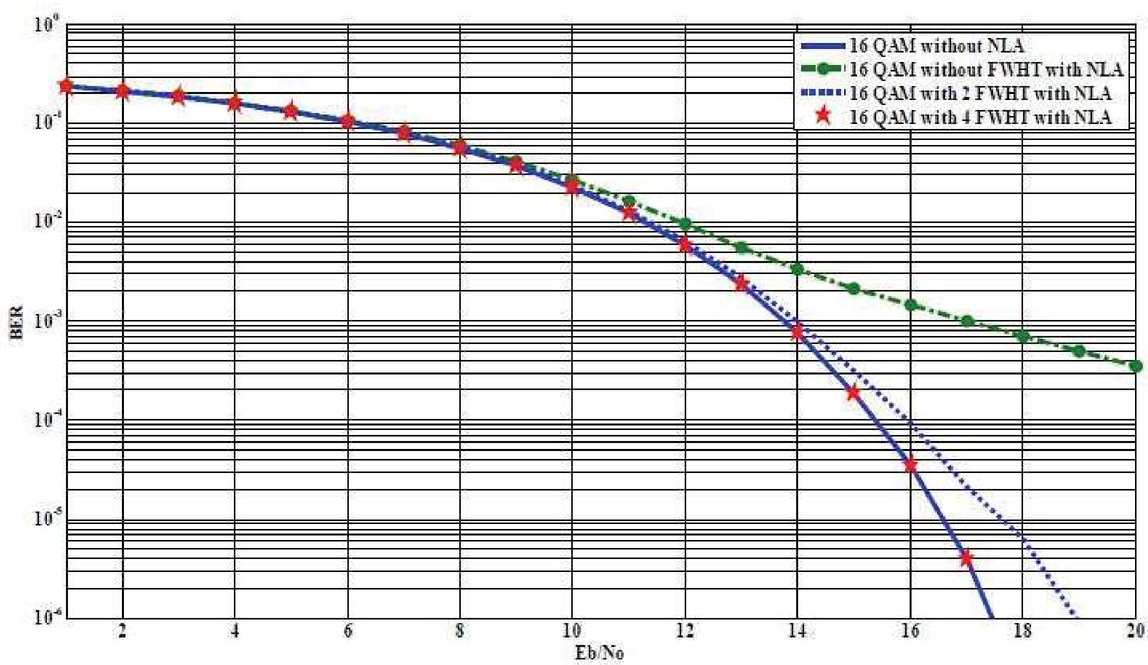

Fig. 24. effect of NLA on BER with/without FWHT

The BER performance as a function of the signal to noise ratio (SNR) Eb/No in OFDM systems with and without FWHT in the presence of NLA with vsat=0.1 is depicted in figure 24. It can be noticed that the use of FWHT (4 points) greatly avoids the distortion due to NLA (Raap's SSPA model) as compared to the 16-QAM case, despite of the negligible reduction in the PAR. It is clear that the real reason in this good performance is due to the

reduction in the dynamic range of about 6 dB, again in a complete agreement with the above results.
The BER performance as a function of the signal to noise ratio (SNR) Eb/No in OFDM systems with HPA with and without FWHT are plotted in Figure 25

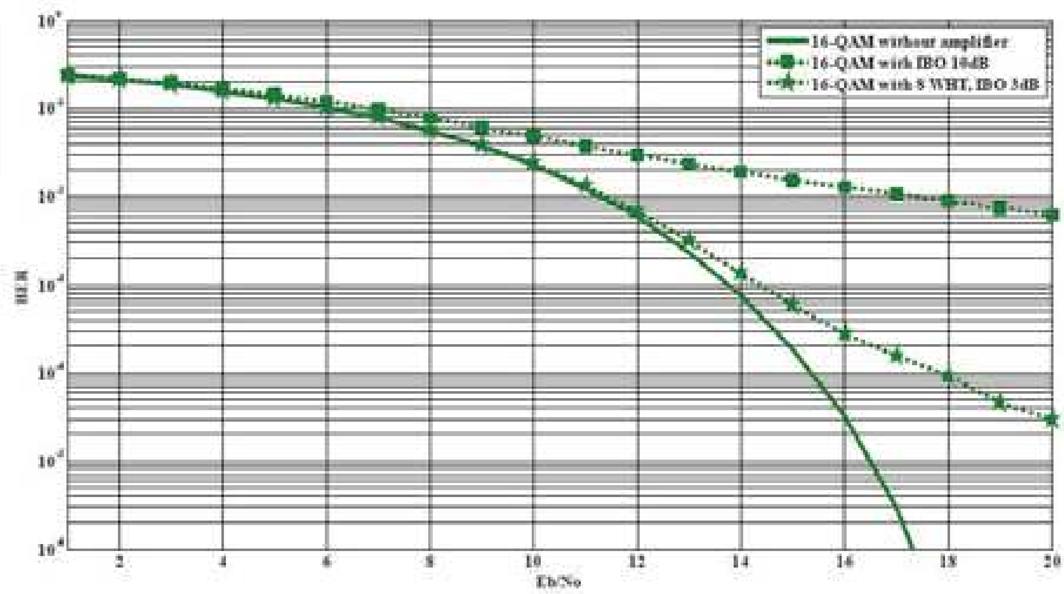

Fig. 25. effect of NLA on BER performance With different IBO

Figure 25 shows that the use of FWHT enables the use of an amplifier with IBO of 3dB with better performance than the case when using an amplifier with IBO of 10dB without using FWHT. In a complete agreement with our conclusions from the models depicted in equations (11) and (12).

7. Conclusions

In this chapter, the effects of nonlinearities in the power amplifier over OFDM systems were modeled, analyzed and simulated. We can conclude that:
-The distortion due to high power amplifier, either limiting or nonlinearity effects, is highly related to the distribution parameter (s), that controls the dynamic range itself, rather than the clipping level or the saturation level of the amplifier.
-Also it is noticed that the effect of nonlinearity on ACPR value is negligible as compared to that of limiting as the clip level varies; this is due to the fact that the spectral leakage that causes the ACPR to increase is mainly due to the clipping that can be viewed as windowing the spectrum by rectangular window.
-The EVM is a measure of the total distortion it is highly affected by the nonlinearity rather than the limiting effect. And generally as the limiting value decreases the EVM increases.
-Although the PAR reduction due to the use of MSK instead of QPSK is slightly small, the true gain is the reduction in the dynamic range by 3 dB, which enables us to use a low linearity and high efficiency power amplifiers like class B or C.
-The Multi-amplitude minimum shift keying (N-MSK) is a bandwidth efficient modulation scheme that has the continuous phase property of MSK and provides higher spectral

efficiency by using multi-level modulation that boost the spectral efficiency and maintaining PAR in an acceptable range.
-For N-MSK there is no requirement for the signals to be orthogonal, in fact, they are co-phased. Furthermore, the amplitudes of the constituent signals are not equal and this prevents the phase trajectory from passing through the origin.
-Although the N-MSK loses the constant envelope property, the effect of SSPA non-linearity is negligible as compared to the M-QAM with the same spectral efficiency that is because N-MSK signal never reaches the zero signal energy.
-For N-MSK as N increases the complexity of the receiver dramatically increases.
-When using N-MSK with the same amplitude of the constituent signals, the PAR change with respect to MSK is negligible; and there will be no change in the dynamic range that they are nearly the same, but when using the constituent signals with different amplitudes the increase in the dynamic range will be somewhat larger.
-Maintaining the dynamic range at a very low level as compared to M-QAM is a very important property of the N-MSK that facilitates the use of nonlinear amplifiers with negligible effect.
-Walsh Hadamard Transform is used with OFDM systems as an intelligent scaling factor to reduce the dynamic range of the OFDM signal without the risk of amplifying the noise when restoring the signal to its original level. This technique offers an excellent solution to all of peak power problems in OFDM systems and without any loss in terms of spectral efficiency and without any side information being transmitted, and can be applied with low computational complexity.
-The use of WHT with OFDM system enables the use of the high efficiency Class C amplifier without affecting the BER performance.

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The techniques of computer modelling and simulation are increasingly important in many fields of science since they allow quantitative examination and evaluation of the most complex hypothesis. Furthermore, by taking advantage of the enormous amount of computational resources available on modern computers scientists are able to suggest scenarios and results that are more significant than ever. This book brings together recent work describing novel and advanced modelling and analysis techniques applied to many different research areas.