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Adaptive Robust Estimator of a Location Parameter For Some Symmetric Distributions

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1. Introduction

This chapter discusses the properties of the trimmed means as an adaptive robust estimator of a location paramer. In addition, it gives a thorough treatment of the measure of tail length R in adaptive robust estimation. Specifically, it provides the mathematical details in attempting to explain the properties of the trimmed mean, the derivations of the asymptotic property of the trimmed mean and the ratio R of the tail lengths as well as estimate of the variance of the trimmed mean, the results on the efficiency of the trimmed mean versus the untrimmed mean and the independence of the ratio R and the trimmed mean, and the demonstration on the the performance of the adaptive trimmed means for estimating a location parameter empirically.

The least squares method of estimation has been practiced for a long time, and is still used frequently. It is closely tied to the normal distribution. Moreover, it has been generally realized that outliers in the data, which do not appear to come from the normal distribution but may have arisen from distribution with long(heavy) tails like the Double Exponential, Mixture Normal, Weibull Distribution, Student's T and Cauchy Distributions or possibly from gross errors, have unusually large influence on the least squares estimates. Robust methods of estimation have been developed to reduce the influence of outliers in the data, especially on the estimates.

Statistics which are represented as a linear combination of order statistics, called L-estimates make a proper class of robust estimates for estimating a location parameter. The trimmed mean is a special class of L-estimates. Denote the trimmed mean by $T_{\alpha,\beta}$, where α *and* $1-\beta$ are the trimming proportions of the left side and right side of the symmetric distributions , respectively.

A number of nice properties of $T_{\alpha,\beta}$, have been cited in the textbook Staudte and Sheather (1990). We shall refer to the textbook by S.S. for short in the following. The properties are given as follows: (1) $T_{\alpha,\beta}$ is robust to outliers up to 100 α % on the left side and 100 $(1-\beta)\%$ on the right side, (ii) the asymptotic efficiency relative to the untrimmed mean is ≥ $(1-\alpha-\beta)^2$ and, (iii). It is simple to compute and its standard error may be estimated from the $(\alpha + \beta)-$ winsorized sample. Moreover, $T_{\alpha,\beta}$, is consistent and asymptotically normal when the underlying distribution F is continuous at the unique quantiles $\xi_\alpha = F^{-1}(x)$ and $\xi_{1-\beta} = F^{-1}(1-\beta)$, see Stigler (1969).

Generally, a larger proportion of data points should be trimmed when F has a longer tail and a smaller proportion, otherwise. Therefore, the choice of the trimming proportions would depend upon a priori knowledge of the tail-length of F. This information may not be available. Therefore, there is a need to determine the trimming proportions from the sample itself. The trimmed mean is called adaptive when the trimming proportions are determined from the data.

Hogg(1967) considered the kurtosis as a measure of the tail-length, and use the sample kurtosis to determine the trimming proportions. His estimate of the center of a symmetric distribution is given by a combination of several trimmed means, associated with different trimming proportions. Subsequently, Hogg (1974) proposed a choice of the trimming proportions, based on the ratio R of two L-estimates. In a recent paper, Alam and Mitra (1996), have reviewed Hogg's method and presented certain theoretical and empirical results on the application of R as a measure of tail-length, to determine the trimming proportions of an adaptive trimmed mean. In this chapter the robustness and asymptotic properties of the estimates $T_{\alpha,\beta}$, and R are derived. Moreover, Proportions of R as a suitable measure of the tail-length of F have been investigated.

2. Properties of, $T_{\alpha,\beta}$

Let F_n denote the empirical distribution of F, derived from a sample of n observations from F. We restrict consideration to estimators T_n of the form $T_n = T(F_n)$, where T maps a class of distributions, including F and F_n, into the real line. The mapping T is called a descriptive measure, and T_n is an estimator of T. The stability (robustness) of T_n may be derived from the smoothness of T in the sense of continuity with respect to appropriate topologies, in a neighborhood of F. It is assumed here that the probability that F_n lies inside the neighborhood is quite high.

Consider the three criteria of robustness of the descriptive measure T(F), based on the theory of robustness developed by Huber (1964) and Hampel (1968). They are called breakdown point, influence function and qualitative robustness. The robustness of the estimator T_n is assessed by these properties. Consider an, $\varepsilon-$ neighborhood of F, consisting of all distributions G such that

$$\sup_y \left|F(y) - G(y)\right| < \varepsilon \tag{2.1}$$

The breakdown point is the value of ε, as ε increases from 0 to 1, for which T(G) is arbitrarily far from T(F). The breakdown point is a quantitative measure of robustness, indicating the maximum proportion of outliers which the induced estimator T_n can accept.

On the other hand, with regard to the second criterion , consider a mixture distribution,

$$F_{x,\varepsilon} = (1-\varepsilon)F + \varepsilon\Delta_x \tag{2.2}$$

which yields an observation from F with probability 1 - ε and an observation x with probability ε .
The limiting value of

$$(T(F_{x,\varepsilon}) - T(F)) / \varepsilon \tag{2.3}$$

as $\varepsilon \to 0$ is defined as the influence function of T(F) at the point x. Continuity and finiteness of the influence function implies robustness of T(F). The influence function is a directional derivative of T at F. When a continuous linear functional exists which provides a good approximation to T(G) – T(F) for all G near F, the kernel of the derivative agrees with the influence function. That is, a series expansion of T(G) for G is a neighborhood of F gives

$$TG) + T(F) + \int \vartheta_{(T,F)}(x)\, d(G(x) - F(x)) + R \tag{2.4}$$

where $\vartheta_{T,F}(x)$ denotes the influence function, and R is a remainder term.

Qualitative robustness is associated with a strong form of continuity of the functional T. It implies that a slight change in F should result in a small change in T(F_n), uniformly in n. Note that qualitative robustness is closely related to but not identical with a nonzero breakdown. This can be illustrated by the following three examples such as: (1) The Arithmetic mean is nowhere qualitatively robust and nowhere continuous at ε = 0. This shows that ordinary continuity of T_n (x_1, x_2,…x_n) as a function of the observations x_1, x_2,…x_n is not suitable for a robust concept.; (2) Median is qualitatively robust and continuous at F if F^{-1} (1/2) contains only one point, but not that it always satisfies ε = 1/2. ; and (3) The α – trimmed mean 0 < α < ½ is qualitatively robust and continuous at all distributions , with ε = α . This simple behavior is quite popular because it appeals to the intuition that is, one removes the both the (α n) smallest and (α n) largest observations and calculates the mean of the remaining ones.
Consider now the robustness of $T_{\alpha,\beta}$ with respect to the given measures of robustness.
Denote the descriptive measure of the trimmed mean by $T_{\alpha,\beta}$ **(F)** and the sample estimate $T_{\alpha,\beta}$ **(F_n)** by $T_{\alpha,\beta}$ suppressing n for convenience. F can be assumed to be a continuous distribution with a location parameter θ which we want to estimate. If $\beta = (1-\alpha)$ then $T_{\alpha,\beta}$ (F) is a measure of location of F by a definition of location measure (see S.S. 4.3.1). By an outlier in a location parameter context we mean, without being very specific, an observation which is considerably larger in absolute value than the bulk of the sample values. Various specific definitions of an outlier can be given. For example, an observation may be designated as an outlier if it is more than two or three times the interquartile range from the median.
Following S.S. (3.2.1) the breakdown point ε^* of T(F) is defined as the minimum proportion ε of outlier contamination at x for which $T(F_{x,\varepsilon})$ is unbounded in x, where $F_{x,\varepsilon}$ is given by (2.2). The finite sample breakdown point ε^* is the smallest proportion of the n observations

in the sample which can render the estimator out of bound. It is easily seen that the breakdown point ε^* for the trimmed mean is equal to $\min(\hat{\alpha}, 1-\beta)$.

The influence function of $T_{\alpha,\beta}$ is derived as follows. Let ξ_α and ξ_β denote the $\hat{\alpha}$ *and* β quantiles of F. It is assumed that the two quantiles are uniquely determined. The descriptive measure $T_{\alpha,\beta}(F)$ is given by

$$T_{\alpha,\beta} = \frac{1}{\beta-\alpha}\int_{\xi_\alpha}^{\xi_\beta} y\, dF(y) \tag{2.5}$$

Substituting the contamination model $F_{x,\varepsilon}$ for F in (2.4) and putting $\xi_{\alpha,\varepsilon} = F_{x,\varepsilon}^{-1}(\alpha)$ *and* $\xi_\beta = F_{x,\varepsilon}^{-1}(\beta)$, it follows that $T_{\alpha,\beta} = \frac{1}{\beta-\alpha}\int_{\xi_{\alpha,\varepsilon}}^{\xi_{\beta,\varepsilon}} y\, dF(y) + \frac{\varepsilon}{\beta-\alpha}\int_{\xi_{\alpha,\varepsilon}}^{\xi_{\beta,\varepsilon}} y\, d(\nabla_x)$.

$$T_{\alpha,\beta} = \frac{1}{1-\alpha-\beta}\int_{\alpha,\varepsilon}^{1-\beta,\varepsilon} y dF(y) + \frac{\varepsilon}{1-\beta}\int_{\alpha,\varepsilon}^{1-\beta,\varepsilon} y d(\nabla_x - F)$$

Differentiating with respect to ε and letting $\varepsilon \downarrow 0$ after simplification the influence function of $T_{\alpha,\beta}$ can be obtained as

$$(\beta-\alpha)\varsigma_{T_{\alpha,\beta},F}(x) = \begin{cases} \xi_\alpha - W_{\alpha,\beta}, & x < \xi_\alpha \\ x - W_{\alpha,\beta}, & \xi_\alpha \le x \le \xi_\beta \\ \xi_\beta - W_{\alpha,\beta}, & x > \xi_\beta \end{cases} \tag{2.6}$$

where

$$W_{\alpha,\beta} = (\beta-\alpha)T_{\alpha,\beta}(F) + \alpha\xi_\alpha + (1-\beta)\xi_\beta$$

denotes the $(\beta-\alpha)-$ Winsorized mean. Note that E $\varsigma_{T\alpha,\beta,\ F}(x) = 0$

For comparison with the mean and median we have that the influence function of the mean = μ(F) is given by

$$\varsigma_{\mu,F}(x) = x - \mu(F)$$

and that the influence function of the ε th quantile function T(F) = $F^{-1}(\varepsilon)$ is given by

$$\varsigma_{\varepsilon,F}(x) = \begin{cases} (\varepsilon-1)/f(\xi_\varepsilon), & x < \xi_\varepsilon \\ 0, & x = \xi_\varepsilon \\ \varepsilon/f(\xi_\varepsilon), & x > \xi_\varepsilon \end{cases} \tag{2.7}$$

where f(x) is the density function of F. Note the discontinuity of $\varsigma_{\varepsilon,F}$ *at* ξ_ε .

2.1 Asymptotic Property

The asymptotic property of the trimmed mean can be derived from the relation (2.4). Putting F_n for G in (2.4) , the resulting equation is

$$
\begin{aligned}
T_{\alpha,\beta} &= T_{\alpha,\beta}(F) + \int \varsigma_{T_{\alpha,\beta},F}(x)\,d(F_n - F(x) + R_n \\
&= T_{\alpha,\beta}(F) + \int \varsigma_{T_{\alpha,\beta},F}\, d\,F_n(x) + R_n \\
&= T_{\alpha,\beta}(F) + 1/n \sum_{i=1}^{n} \varsigma_{T_{\alpha,\beta}} F(x_i) + R_n
\end{aligned}
\tag{2.8}
$$

where $\sqrt{n}\,R_n \xrightarrow{P} 0,$ as $n \to \infty$ and x_i denote the sample values. In fact, $R_n = O_p(1/n)$ under reasonable conditions. Therefore

$$
\sqrt{n}\,(T_{\alpha,\beta} - T_{\alpha,\beta}(F) \approx n^{1/2} \sum_{i=1}^{n} \varsigma_{T_{\alpha,\beta}}, F(x_i)
\tag{2.9}
$$

An application of the central limit theorem shows that

$$
\sqrt{n}\,(T_{\alpha,\beta} - T_{\alpha,\beta}(F)) \xrightarrow{d} N(0, V(F))
\tag{2.10}
$$

where

$$
\begin{aligned}
V(F) &= \operatorname{var} \varsigma_{T_{\alpha,\beta}}, F(x) \\
&= E(\varsigma_{T_{\alpha,\beta}}, F(x))^2 .
\end{aligned}
\tag{2.11}
$$

Here x denotes a random observation from the distribution F. From (2.6) it can be shown that

$$
(\beta - \alpha)^2 V(F) = \alpha(\xi_\alpha - W_{\alpha,\beta})^2 + (1-\beta)(\xi_\beta - W_{\alpha,\beta})^2 + \int_{\xi_\alpha}^{\xi_\beta} (x - w_{\alpha,\beta})^2 dF(x).
\tag{2.12}
$$

In comparison, the asymptotic variance of the ε th quantile is given by

$$
\operatorname{var}(\varsigma_{\varepsilon,F}(x)) = \varepsilon(1-\varepsilon)/(f(\xi_\varepsilon))^2.
\tag{2.13}
$$

2.2 Relative Efficiency

Since the trimmed mean is a special member of the class of L-estimator, it is interesting to compare its asymptotic variance with the asymptotic variance of any other member of the class. Such as the untrimmed mean. The descriptive measure of an L-estimate, measuring location, is given by

$$T(F) = \int_0^1 F^{-1}(t)\, dk(t) \tag{2.14}$$

where k is a probability distribution on (0,1).

The trimmed mean $T_{\alpha,1-\alpha}(F)$ is obtained by taking k uniform on $(\alpha, 1-\alpha)$. Let T_1 and T_2 be two L-estimators determined by k_1 and k_2, and let f_1 and f_2 denote the densities of K_1 and K_2, respectively. Suppose that

$$0 \le (f_2(t), f_1(t)) \le A \tag{2.15}$$

where $A > 1$. Bickel and Lehmann (1976) have shown that (2.15) implies $V(T_2(F)) \le AV(T_1(F))$ Here $V(T_1, F)$ and $V(T_2, F)$ denote the asymptotic variances of T_1 and T_2, respectively. It follows from the above result that the asymptotic relative efficiency of $T_{\alpha,1-\alpha}$ relative to the untrimmed mean is bounded below by (β-α) or $(1-\alpha)^2$.

2.3 Estimate of Variance of T

Let $r = [n\alpha]$ *and* $s = [n(1-\beta)]$ where [x] denotes the integer part of x, and let $X_{(I)}$ denote the I^{th} order statistic from the sample. The winsorized sample is given by $y_1, \ldots\ldots, y_n$, where

$$y_i = \begin{cases} x(r+1), & r \ge i \\ x_{(i)} & r < i \le s \\ x_{(s)} & s < i \end{cases}$$

the winsorized sample variance, denoted by S_w^2, is given by

$$S_w^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2$$

where $\overline{y}$ denotes the winsorized sample mean. From (2.6) and the asymptotic relation (2.9) we derive an estimate of the variance of $T_{\alpha,\beta}$, given by

$$\hat{v}ar\, T_{\alpha,\beta} = S_w^2 / (n(\beta - \alpha)^2).$$

It is called the influence function estimate of the variance. It is also proportional to a Jackknife estimate of the variance (see Efron 1982), Sec. 3.3).

3. Tail-length

A sample from a distribution with longer tail is likely to contain a larger number of outliers compared to a sample from a distribution with shorter tail. Therefore, for estimating a location parameter, using a trimmed mean, the sample should be trimmed more if the underlying distribution has longer tail, for the sake of robustness. In order to determine the trimming proportions a measure of the tail-length is needed which can be estimated with some degree of precision. A measure of tail-length which has been referred to in the introduction is given, as follows.

It is assumed throughout that the distribution F is symmetric about a location parameter θ and that the trimmed mean is symmetrized, that is, the trimming proportions are equal ($\beta = 1 - \alpha$). In this case the descriptive measure of the trimmed mean is denoted by

$$T_\alpha(F) = \frac{1}{1-2\alpha}\int_{\xi_\alpha}^{\xi_{1-\alpha}} x \, dF(x)$$

and the sample estimate of $T_\alpha(F)$ is given by

$$T_\alpha(F_n) = \frac{1}{n-2m}\sum_{i=(m+1)}^{n-m} X_{(i)}$$

where $m = [n\alpha]$ and $x_{(i)}$ denotes the ith ordered value in the sample.

Let $0 < \gamma < \delta < \alpha < 1/2$. The tail-length of F is given by

$$R_{\gamma,\delta}(F) = (T_{1-\gamma,1}(F) - T_{0,\gamma}(f) / (T_{1-\delta,1-\gamma}(F) - T_{\gamma,\delta}(F)). \tag{3.1}$$

It can be noticed that the numerator and denominator of the right side of (3.1) are each invariant with respect to translation. Therefore, $R_{\gamma,\delta}(F)$ is invariant with respect to both translation and scale transformation. Clearly, $R_{\gamma,\delta}(F) \geq 1$. Moreover, it is also clear that a longer value of the tail-length will induce a larger value of $R_{\gamma,\delta}$. Therefore, $R_{\gamma,\delta}(F)$ is a suitable measure of the tail-length of F. The sample estimate of the tail-length is given by

$$R_{\gamma,\delta}(F_n) = (T_{1-\gamma,1}(F_n) - T_{0,\gamma}(F_n)) / (T_{1-\delta,1-\gamma}(F_n) - T_{\gamma,\delta}(F_n)). \tag{3.2}$$
$$= A_n/B_n, \text{ say.}$$

The asymptotic property of $R_{\gamma,\delta}(F_n)$ is derived from an application of the asymptotic relation (2.9). Using (2.9) and the symmetry of F, a simple algebraic computation shows that the trimmed mean $T_\alpha(F_n)$ and A_n and B_n are pair-wise uncorrelated when n is large. Moreover they are jointly normally distributed, asymptotically. Thus, the Theorem 3.1 can be stated as:

Theorem 3.1

If F is symmetrically distributed then $R_{\gamma,\delta}(F_n)$ and $T_\alpha(F_n)$ are asymptotically independent, as $n \to \infty$ Moreover, $R_{\gamma,\delta}(F_n)$ is asymptotically distributed as a ratio of two independent normal random variables, given by A_n/B_n.

The statistical independence of the trimmed mean $T_\alpha(F_n)$ and the tail-length $R_{\gamma,\delta}(F_n)$, given by Theorem 3.1 , is a useful result. If the trimming proportions α of the trimmed mean is based on the tail-length $R_{\gamma,\delta}(F_n)$ the result implies that the asymptotic distribution of the adaptive trimmed mean is the same as if α was fixed a priori. Therefore, in testing a hypothesis or constructing a confidence interval, the nominal level of significance of the confidence level remains unchanged, at least for large values of n.

In order to see how good is $R_{\gamma,\delta}(F_n)$ as a measure of tail-length, Table 1 below gives its values for certain proportions γ *and* δ for a number of symmetric distributions with varying tail-length. In particular, consider the student 's t-distribution with υ degrees of freedom. Here the tail-length increases as υ decreases. For υ =1 the distribution is Cauchy which has a very long tail. On the other hand, as $\nu \to \infty$. The distribution approaches the normal distribution whose tail-length is of typically normal size, so to speak. It is also shown in the table the asymptotic variance of the symmetrized trimmed mean, T_α for certain values of α .

In (3), φ denotes the standard normal density function. The distribution (2) is a modified form of the standard Weibull distribution.

Distribution	Density Function	Asymptotic Variance
(1) Double exponential	$\frac{1}{2} e^{-\lvert x \rvert}$	2
(2) Weibull (λ =1/2)	$(2\Gamma(1+\frac{1}{\lambda})^{-1} e^{-\lvert x \rvert^{\lambda}}$	$\Gamma(3/\lambda)/\Gamma(1/\lambda)$
(3) Normal Mixture ($\varepsilon = 0.2$)	$(1-\varepsilon)\varphi(x) + \varepsilon\varphi(\frac{x}{3})$	1 + 8 ε
(4). Student's t_ν	$c_\upsilon (1 + x^2/\upsilon)^{-(\upsilon+1)/2}$ where $c_\upsilon = \Gamma\left(\frac{\upsilon+1}{2}\right)(\sqrt{\upsilon\pi}\,\Gamma\left(\frac{\upsilon}{2}\right))^{-1}$	$\nu/(\nu-2), \quad \nu > 2$

Table 1. Symmetric Distributions with the Density functions and the asymptotic variances

The following figures give the graphs of the density functions of the Double Exponential,. Weibull, Student's T and normal mixture distribution as displayed below:

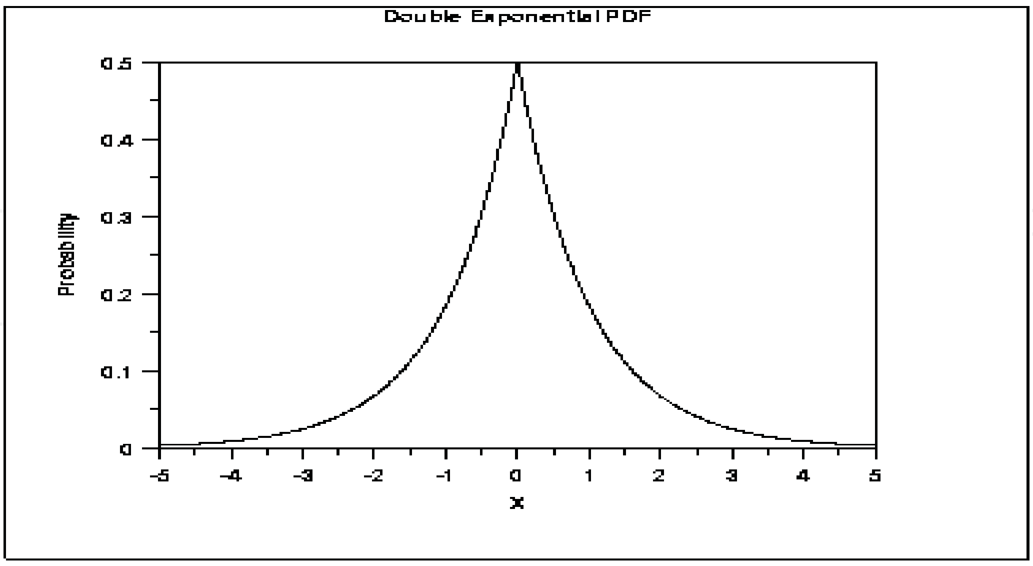

Fig. 1. The graph of the Double Exponential Density function

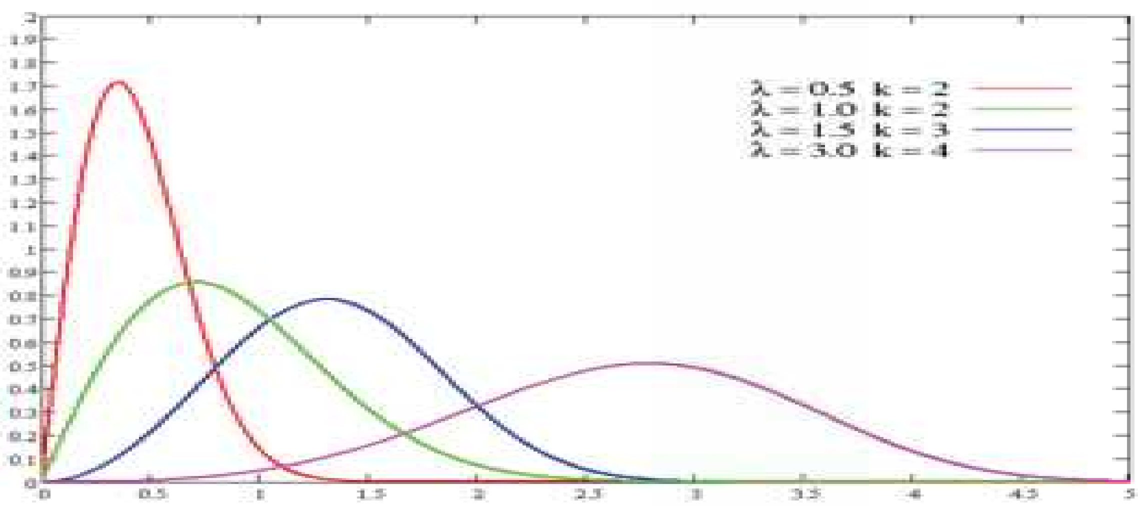

Fig. 2. The graphs of the density functions of the Weibull distribution at various parameters

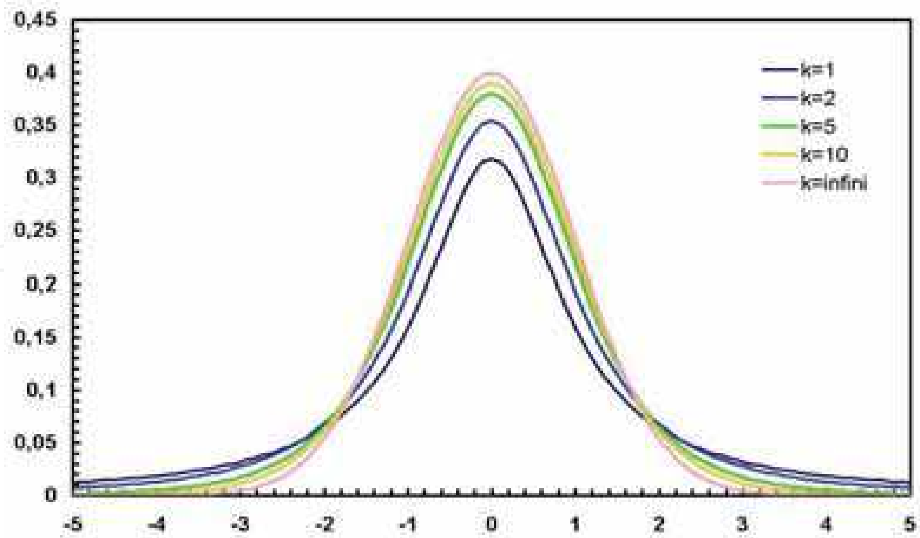

Fig. 3. The graphs of the density functions of the Student's T distributions at various parameters

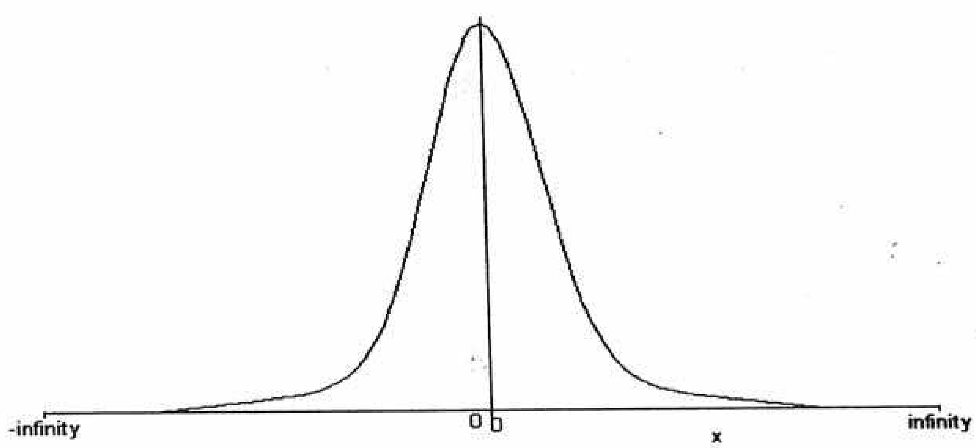

Fig.4. Graph of Mixture of Normal : .8N(0,1) + .2N0,9)

$R_{\gamma,\delta}$ $V_\alpha(T)$

(γ,δ)	(.10,.20)	(.10,.25)	(.15,.30)	(.20,.30)	α: 0	.1	.15	.20	.25
Double Exponential	2.10	2.37	2.65	2.70	2.00	1.85	1.61	1.45	1.32
Weibull(λ=1/2)	3.06	3.58	4.17	4.28	120	45.41	34.73	22.97	20.64
Normal Mixture (ε =.2)	2.27	2.52	2.66	2.62	2.60	2.45	2.44	2.60	2.98
Student's t_v									
υ=2.0	2.76	3.12	3.54	3.45	∞	2.51	2.19	1.73	1.90
3.0	2.28	2.53	2.76	2.73	3.00	2.04	1.83	1.72	1.66
4.0	2.05	2.28	2.44	2.50	2.00	1.81	1.61	1.60	1.54
5.0	1.95	2.17	2.34	2.34	1.67	1.73	1.76	1.48	1.70
6.0	1.71	2.08	2.30	2.32	1.50	1.75	1.60	1.50	0.98
∞	1.67	1.84	1.91	2.04	1.00	1.36	1.31	1.30	1.26

Table 2. Tail-Length $R_{\gamma,\delta}(F)$ and Asymptotic Variance $V_\alpha(T)$

Values of α	A. Select	A′. Select
	x = $R_{.1,.2}(F)$ if	y = $R_{.15,.3}(F)$ if
.05	x < 1.5	y < 2.0
.10	1.5 ≤ x < 2.0	2.0 ≤ y < 2.5
.15	2.0 ≤ x < 2.5	2.5 ≤ y < 3.0
.20	2.5 ≤ x < 3.0	3.0 ≤ y < 3.0
.30	3.0 ≤ x	3.5 ≤ y

Table 3. Selection Rule for the values of Tail lengths and α

In Table 3, the selection rule is presented given the tail lengths which can be summarized as follow:

Selection rule says that

- if the tail lengths are x = $R_{.1,.2}(F)$ < 1.5 and y = $R_{.15,.3}(F)$ < 2.0 then use α = .05.
- if the tail lengths are 1.5 ≤ x < 2.0 and / or 2.0 ≤ y < 2.5 then use α = .10.
- if the tail lengths are 2.0 ≤ x < 2.5 and/ or 2.5 ≤ y < 3.0 then use α = .15.
- if the tail lengths are 2.5 ≤ x < 3.0 and / or 3.0 ≤ y < 3.0 then use α = .20.
- if the tail lengths are 3.0 ≤ x and / or 3.5 ≤ y then use α = .30.

4. Empirical Results

From each of the distributions listed in Table 1, random samples of size n = 40 observations were generated. From these sample values, the adaptive Trimmed means T_α were computed according to the rule given in Table 3. Likewise, the trimmed mean T_α for two fixed values of α = .05 and .10 were also computed. Iterating the procedures m = 100 times, m values of

T_A , $T_{A'}$, $T_{.05}$ and $T_{.10}$ were obtained together with their respective standard errors denoted by $S(T_A)$, $S(T_{A'})$, $S(T_{.05})$ and $S(T_{.10})$. These values can be found in Table 4 below:

Distribution	$S(T_A)$	$S(T_{A'})$	$S(T_{.05})$	$S(T_{.10})$
Double Exponential	.185	.185	.203	.193
Weibull(λ=1/2)	.560	.553	.759	.632
Normal Mixture (ε =.2)	.358	.341	.465	.411
Student's t_v				
υ=2.0	.177	.177	.266	.224
3.0	.156	.151	.218	.187
4.0	.167	.163	.220	.200
5.0	.149	.143	.193	.173
6.0	.154	.145	.200	.174
∞	.430	.417	.507	.471

Table 4. Standard Errors of T_A , $T_{A'}$, $T_{.05}$ and $T_{.10}$

A comparison between the standard deviations and the adaptive trimmed means T_A and $T_{A'}$ and the nominal trimmed means $T_{.05}$ and $T_{.10}$ whose trimming proportions have been fixed **a' priori** would indicate the relative efficiency of the adaptive procedure. It is seen from Table 4 that the sample estimates of the standard deviations of the adaptive trimmed means are always smaller than those of the nominal trimmed means. These results substantiate the findings given in the paper by Alam and Mitra (1996).

5. Findings in the Study

Considering the measures of robustness such as breakdown point, influence function and a qualitative robustness, the following results are revealed.
5.1 The trimmed mean $T_{\alpha,\beta}$ has a breakdown point of min (α,1-β). This implies that at this proportion of n observations in the sample the estimator can be out of bound.
5.2 The asymptotic property of the trimmed mean derived is that as the number of n observations approach infinity, its distribution is normal with mean 0 and variance $V_{\alpha,\beta}$ defined as expectation of the square influence function of the trimmed mean. This variance is a function of the quantiles and winsorized means.
5.3 The tail length of F given by the ratio $R_{\alpha,\beta}$ (F) is invariant with respect to translation and scale transformation and its value is greater than or equal to 1.
5.4 If F is symmetrically distributed then the ratio R and the trimmed mean T are asymptotically independent, as n goes to infinity. This is a very important result since this implies that the asymptotic distribution of the adaptive trimmed mean is the same as if α was fixed a priori.
5.5 The ratio R is asymptotically distributed as a ratio of two independent random normal variables.
5.6 It was shown empirically that the adaptive trimmed means are relatively more efficient than the nominal trimmed means fixed at 5% and 10%.

6. Conclusion

6.1 The ratio $R_{\alpha,\beta}$ (F) is a good measure of tail length. The larger is its value the more outliers are there in the distribution of data. Its goodness can be attributed to the asymptotic independence of the ratio R to the trimmed means $T_{\alpha,\beta}$.
6.2 The empirical results showed that relatively the adaptive trimmed means are efficient than the nominal trimmed means where the trimming proportions were fixed a priori. Hence, in the absence of **a ' priori** trimming proportions, the choice of the appropriate trimming proportions using the proposed **ad hoc rule** may be used with advantage**.**

7. Future Direction

7.1 The refinement and applicability of the selection rule on the trimming proportions corresponding to the tail lengths $R_{\gamma,\delta}(F)$ specifically for some mentioned symmetric distributions like the double exponential, mixtures of normal, Weibull, and the Student's T distributions are recommended.
7.2 Investigation on the applicability of the selection rule with some adjustments on the degree of skewness for assymetrical distribution are hereby recommended. The extension of the procedure can be explored for quantile regression analysis on the choice of the τ th quantile values.
7.3 The Hodges-Lehmann estimator is the median of the set of all averages of pairs of observations. It combines the robustness property of the mean with the efficiency of the averaging process. Its asymptotic variance is given by

$$V_{HL}(F) = \frac{1}{12}\left(\int f^2(y)\,dy\right)^{-2} \tag{7.1}$$

for symmetric F, where f denotes the density function. It would be interesting to compare the asymptotic variance of the trimmed mean with 7.1 for a future study.

8. References

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