Performance of Wireless Communication Systems with MRC over Nakagami–*m* Fading Channels

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1. Introduction

The Nakagami–*m* distribution (*m*–distribution) (Nakagami, 1960) received considerable attention due to its greater flexibility as compared to Rayleigh, log-normal or Rician fading distribution (Al–hussaini & Al–bassiouni, 1985; Aalo, 1995; Annamalai et al., 1999; Zhang, 1999; Alouini et al., 2001). The distribution also includes Rayleigh and one-sided Gaussian distributions as special cases. It can also accommodate fading conditions that are widely more or less severe than that of the Rayleigh fading. Nakagami–*m* fading is, therefore, often encountered in practical applications such as mobile communications.

This chapter discusses the performance analysis of wireless communication systems where the receiver is equipped with maximal–ratio–combining (MRC), for performance improvement, in the Nakagami-*m* fading environment. In MRC systems, the combined signal–to–noise ratio (SNR) at the output of the combiner is a scaled sum of squares of the individual channel magnitudes of all diversity branches. Over Nakagami-*m* fading channels, the combined output SNR of the MRC combiner is a sum of, normally, correlated Gamma random variables (r.v.'s). Therefore, performance analysis of this diversity–combining receiver requires knowledge of the probability density function (PDF) or the moment generating function (MGF) of the combined SNR. The PDF of the sum of Gamma r.v.'s has also long been of interest in mathematics (Krishnaiah & Rao, 1961; Kotz & Adams, 1964; Moschopoulos, 1985) and many other engineering applications.

The current research progress in this area is as follows. The characteristic function (CF) of the sum of identically distributed, correlated Gamma r.v.'s is derived in (Krishnaiah & Rao, 1961) and (Kotz & Adams, 1964). Then, the PDF of the sum of statistically independent Gamma r.v.'s with non–identical parameters is derived in (Moschopoulos, 1985). The results derived in (Krishnaiah & Rao, 1961; Kotz & Adams, 1964; Moschopoulos, 1985) are used for performance analysis of various wireless communication systems in (Al–hussaini & Al–bassiouni, 1985; Aalo, 1995; Annamalai et al., 1999; Zhang, 1999; Alouini et al., 2001) and references therein. In (Win et al., 2000), the CF of a sum of arbitrarily correlated Gamma r.v.'s with non–identical but integer fading orders is derived by using a so-called virtual branch technique. This technique is also used in (Ghareeb & Abu-Surra, 2005) to derive the

CF of the sum of arbitrarily correlated Gamma r.v.'s. In (Alouini et al., 2001), using the results derived in (Moschopoulos, 1985), the PDF of the sum of arbitrarily correlated, non–identically distributed Gamma r.v.'s but with identical fading orders (both integer as well as non–integer) is derived. Performance of an MRC receiver for binary signals over Nakagami–m fading with arbitrarily correlated branches is analyzed in (Lombardo et al., 1999) for the case of identical fading orders m's (both integer as well as non–integer). The distribution of multivariate Nakagami–m r.v.'s is recently derived in (Karagiannidis et al., 2003a) also for the case of identical fading orders. The joint PDF of Nakagami-m r.v.'s with identical fading orders using Green's matrix approximation is derived in (Karagiannidis et al., 2003b). A generic joint CF of the sum of arbitrarily correlated Gamma r.v.'s with non–identical and non-integer fading orders is recently derived in (Zhang, 2003).

For a large number of diversity branches the virtual branch technique proposed in (Win et al., 2000) has a high computational complexity since the eigenvalue decomposition (EVD) is performed over a large matrix. Although the joint CF derived in (Zhang, 2003) is very general, it does not offer an immediate simple form of the PDF and therefore analyzing some performance measures can be complicated. In this chapter, we provide some improvements over the existing results derived in (Win et al., 2000) and (Zhang, 2003). Firstly, we transform the correlated branches into multiple uncorrelated virtual branches so that the EVDs are performed over several small matrices instead of a single large matrix. Secondly, we derive the exact PDF of the sum of arbitrarily correlated Gamma r.v.'s, with non-identical and half-of-integer fading orders, in the form of a single Gamma series, which greatly simplifies the analysis of many different performance measures and systems that are more complicated to analyze by the CF– or MGF–based methods. Note that parts of this chapter are also published in (Tran & Sesay, 2007).

The chapter is organized as follows. Section 2 describes the communication signal model. We derive the MGF and PDF of the sum of Gamma r.v.'s in Section 3. In Section 4, we address the application of the derived results to performance analysis of wireless communication systems with MRC or space-time block coded (Su & Xia, 2003) receivers. Numerical results and discussions are presented in Section 5 followed by the conclusion in Section 6.

The following notations are used throughout this chapter: $E\{x\}$ denotes the statistical average of random variable x; lowercase, bold typeface letters, e.g. $\mathbf{x}$, represent vectors; uppercase, bold typeface letters, e.g. $\mathbf{X}$, represent matrices; $\triangleq$ denotes the definition; $\mathbf{I}_m$ denotes an $m \times m$ identity matrix; $\mathbf{x}^T$ and $\mathbf{X}^T$ denote the transpose of vector $\mathbf{x}$ and matrix $\mathbf{X}$, respectively; $\|\mathbf{x}\|^2 \triangleq \mathbf{x}^T\mathbf{x}$; $\lceil x \rceil$ denotes the smallest integer greater than or equal x; $P(\cdot|x)$ denotes the statistical conditional function given random variable x; $j \triangleq \sqrt{-1}$ denotes the complex imaginary unit; $|x|^2 \triangleq xx^*$, where x^* denotes the complex conjugate of x; $Q(x)$ denotes the Q-function, defined as $Q(x) \triangleq (1/\sqrt{2\pi})\int_x^\infty \exp(-z^2/2)dz$, and $\text{erfc}(x)$ denotes the complementary error function, defined as $\text{erfc}(x) \triangleq (2/\sqrt{\pi})\int_x^\infty \exp(-z^2)dz$; $x = f^{-1}(y)$ denotes the inverse function of function $y = f(x)$.

2. Communication Signal Model

Consider a wireless communication system equipped with one transmit antenna and L receive antennas and assume perfect channel estimation is attained at the receiver. The low–pass equivalent received signal at the kth receive antenna at time instant t is expressed by

$$r_k(t) = \alpha_k(t) e^{j\varphi_k(t)} s(t) + w_k(t), \tag{1}$$

where $\alpha_k(t)$ is an amplitude of the channel from the transmit antenna to the kth receive antenna. In (1), $\alpha_k(t)$ is an m-Nakagami distributed random variable (Nakagami, 1960), $\varphi_k(t)$ is a random signal phase uniformly distributed on $[0, 2\pi)$, $s(t)$ is the transmitted signal that belongs to a signal constellation Ξ with an averaged symbol energy of $\overline{E}_s \triangleq E\{|s(t)|^2\}$, and $w_k(t)$ is an additive white Gaussian noise (AWGN) sample with zero mean and variance σ_w^2. The overall instantaneous combined SNR at the output of the MRC receiver is then given by

$$\eta(t) \triangleq \frac{\overline{E}_s}{\sigma_w^2} \sum_{k=1}^{L} \alpha_k^2(t) = \frac{\overline{E}_s}{\sigma_w^2} \gamma(t), \tag{2}$$

where $\gamma(t) \triangleq \sum_{k=1}^{L} x_k(t)$ with $x_k(t)$ being defined as $x_k(t) \triangleq \alpha_k^2(t)$. From now on the time index t is dropped for brevity. Since α_k is an m-Nakagami distributed random variable, the marginal PDF of x_k is a Gamma distribution given by (Proakis, 2001)

$$p_{X_k}(x_k) = \frac{x_k^{m_k - 1}}{\Gamma(m_k)} \left(\frac{m_k}{\Omega_k} \right)^{m_k} \exp\left(-\frac{m_k x_k}{\Omega_k} \right), \tag{3}$$

where

$$\Omega_k = E\{x_k\} \text{ and } m_k = \Omega_k^2 / E\{(x_k - \Omega_k)^2\} \geq 1/2. \tag{4}$$

In (4), the Ω_k's and m_k's are referred to as fading parameters in which the m_k's are referred to as *fading orders*, and $\Gamma(\cdot)$ is the Gamma function (Gradshteyn & Ryzhik, 2000). Finding the PDF or MFG of $\gamma \triangleq \gamma(t)$, which is referred to as the received SNR coefficient, is essential to the performance analysis of diversity combining or space-time block coded receivers of wireless communication systems which is addressed in this chapter.

3. Derivation of the Exact MGF and PDF of γ

3.1 Moment Generating Function

In this section, we derive the MGF of γ for the case $m_k = n_k / 2$ with n_k being an integer and $n_k \geq 1$. First, without loss of generality, assume that the x_k's are indexed in increasing fading orders m_k's, i.e., $m_1 \leq m_2 \leq \cdots \leq m_L$. Let $\mathbf{z}_k$ denote a $2m_k \times 1$ vector defined as $\mathbf{z}_k \triangleq [z_{k,1}, z_{k,2}, \ldots, z_{k,2m_k}]^T$, $k = 1, \ldots, L$, where the $z_{k,i}$'s are independently and identically distributed zero–mean real Gaussian random variables with variances of $E\{z_{k,i}^2\} = \Omega_k / 2m_k$. The random variables x_k's, $1 \le k \le L$, are then constructed by $x_k \triangleq \sum_{i=1}^{2m_k} z_{k,i}^2 = \|\mathbf{z}_k\|^2$. Therefore, the received SNR coefficient γ is expressed by $\gamma \triangleq \sum_{k=1}^{L} \|\mathbf{z}_k\|^2$. Following (Win et al., 2000), the elements of the vectors $\mathbf{z}_k$'s, $k = 1,\ldots,L$, are constructed such that their correlation coefficients are given by

$$E\{z_{k,i} z_{l,w}\} = \begin{cases} \Omega_k / 2m_k, & \text{if } i = w \text{ and } k = l \\ \rho_{k,l}\sqrt{\Omega_k \Omega_l / 4m_k m_l}, & \text{if } k \ne l \text{ but } i = w \text{ and } 1 \le i,w \le 2\min\{m_k, m_l\} \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

and $0 \le \rho_{k,l} \le 1$. Here, $\rho_{k,l}$ is the normalized correlation coefficient between $z_{k,i}$ and $z_{l,w}$. The correlation coefficient between two branches, x_k and x_l, is related to $\rho_{k,l}$ though

$$\begin{aligned} \rho_{x_k x_l} &\triangleq \frac{E\{(x_k - \Omega_k)(x_l - \Omega_l)\}}{\sqrt{\operatorname{var}(x_k)\operatorname{var}(x_l)}} \\ &= \rho_{k,l}^2 \sqrt{\frac{\min(m_k, m_l)}{\max(m_k, m_l)}}. \end{aligned} \tag{6}$$

Further analysis is complicated by the fact that $\rho_{k,l} \ne 0$ even for some $l \ne k$. However, we observe from (5) that the correlation coefficient $\rho_{k,l} = 0$ for both $l = k$ and $l \ne k$ as long as $w \ne i$. We exploit this fact to rearrange the r.v.'s in the received SNR coefficient γ as follows. Let $\mathbf{v}_w$ denote an $L_w \times 1$ ($1 \le L_w \le L$ with $L_1 = L$) vector, which is defined as $\mathbf{v}_w \triangleq [z_{L-L_w+1,w}, z_{L-L_w+2,w}, \ldots, z_{L,w}]^T$ for $w = 1, 2, \ldots, 2m_L$, where the vector length L_w depends on the fading order m_w. The indexing is selected such that if $L - L_w + 1 > 2m_w$ then $z_{w,i} = 0$ and is removed from the vector $\mathbf{v}_w$. Also, let γ_w's denote *new* r.v.'s defined by $\gamma_w \triangleq \sum_{i=L-L_w+1}^{L} z_{i,w}^2 = \|\mathbf{v}_w\|^2$ for $w = 1, 2, \ldots, 2m_L$. Therefore, the random variables γ_w's are formed by summing all the wth elements of the random variables x_k's, $k = 1,2,\ldots,L$. From (5), we note that the r.v.'s $z_{k,i}$ and $z_{l,w}$ are uncorrelated if $i \ne w$. Furthermore, since the r.v.'s $z_{k,i}$ and $z_{l,w}$ are Gaussian by definition, they are also statistically independent if $i \ne w$. Consequently, the newly formed r.v.'s γ_w's are also statistically independent. From the definitions of the vectors $\mathbf{z}_k$ and $\mathbf{v}_w$, we have

$$\gamma = \sum_{w=1}^{2m_L} \|\mathbf{v}_w\|^2 = \sum_{w=1}^{2m_L} \gamma_w. \tag{7}$$

In the sum of γ, we have grouped the wth, $w = 1, 2, \ldots, 2m_L$, elements of x_k, $k = 1,2,\ldots,L$, together so that different groups in the sum are statistically independent. Therefore, such a rearrangement of the elements of the Gamma random variables in the sum of γ actually transforms L correlated branches into $2m_L$ independent branches. The wth new independent branch is a sum of L_w correlated Gamma variables with a *common* fading order of 0.5. Let $\Phi_{\gamma_w}(s)$ denote the MGF of γ_w. Since the r.v.'s γ_w's are statistically independent, we have

$$\Phi_\gamma(s) = \prod_{w=1}^{2m_L} \Phi_{\gamma_w}(s). \tag{8}$$

Let $\mathbf{R}_{V,w}$ denote the correlation matrix of vector $\mathbf{v}_w$, where the $(k,l)^{\text{th}}$ element of $\mathbf{R}_{V,w}$ can be shown to be (Win et al., 2000)

$$\begin{aligned}\mathbf{R}_{V,w}(k,l) &\triangleq \frac{E\left\{\left(z_{kk,w}^2 - \frac{\Omega_{kk}}{2m_{kk}}\right)\left(z_{ll,w}^2 - \frac{\Omega_{ll}}{2m_{ll}}\right)\right\}}{\sqrt{\text{var}(z_{kk,w}^2)\,\text{var}(z_{ll,w}^2)}} \\ &= \rho_{kk,ll}^2,\end{aligned} \tag{9}$$

where $kk \triangleq L - L_w + k$, $ll \triangleq L - L_w + l$ for $k,\ l = 1,\ 2, \ldots,\ L_w$, and $\rho_{k,k}^2 = 1$. Since $\mathbf{R}_{V,1}$ is an $L \times L$ matrix, from the construction given in (5) and the definition of vector $\mathbf{v}_w$, we have

$$\mathbf{R}_{V,w} = \mathbf{R}_{V,1}(L - L_w + 1 : L, L - L_w + 1 : L),\ w = 2,3,\ldots,2m_L. \tag{10}$$

The Matlab notation $\mathbf{R}_{V,1}(k:l,m:n)$, denotes a sub matrix of the matrix $\mathbf{R}_{V,1}$ whose rows and columns are, respectively, the kth through lth rows and the mth through nth columns of the matrix $\mathbf{R}_{V,1}$. Let $\mathbf{\Theta}_w$ be an $L_w \times L_w$ positive definite matrix (i.e., its eigenvalues are positive) defined by

$$\mathbf{\Theta}_w = \text{diag}\left(\frac{\Omega_{L-L_w+1}}{m_{L-L_w+1}}, \frac{\Omega_{L-L_w+2}}{m_{L-L_w+2}}, \ldots, \frac{\Omega_L}{m_L}\right)\sqrt{\mathbf{R}_{V,w}}, \tag{11}$$

where the square root operation in (11) implies taking the square root of each and every element of the matrix $\mathbf{R}_{V,w}$. The joint characteristic function (CF) of vector the $\mathbf{v}_w$ is given by (Krishnaiah & Rao, 1961; Kotz & Adams, 1964; Lombardo et al., 1999)

$$\begin{aligned}\psi_{\mathbf{v}_w}(t_1, \ldots, t_{L_w}) &= E\left\{\exp\left(j\textstyle\sum_{i=1}^{L_w} z_{L-L_w+i}^2 t_i\right)\right\} \\ &= \left[\det\left(\mathbf{I}_{Lw} - j\mathbf{T}_w\mathbf{\Theta}_w\right)\right]^{-1/2},\end{aligned} \tag{12}$$

where $\mathbf{T}_w \triangleq \text{diag}(t_1, \ldots, t_{L_w})$. Let $\{\lambda_{w,i} > 0\}_{i=1}^{L_w}$ denote the set of eigenvalues of the matrix $\mathbf{\Theta}_w$. Using (12), the CF of the r.v. γ_w is given by (Krishnaiah & Rao, 1961; Kotz & Adams, 1964)

$$\psi_{\gamma_w}(t) = \prod_{i=1}^{L_w}\left(1 - jt\lambda_{w,i}\right)^{-1/2}. \tag{13}$$

Therefore, the MGF of γ_w is given by

$$\Phi_{\gamma_w}(s) = \prod_{i=1}^{L_w} \left(1 - s\lambda_{w,i}\right)^{-1/2}. \tag{14}$$

Substituting (14) into (8) gives

$$\Phi_{\gamma}(s) = \prod_{w=1}^{2m_L} \prod_{i=1}^{L_w} \left(1 - s\lambda_{w,i}\right)^{-1/2}. \tag{15}$$

Remark: By working with the vectors $\mathbf{v}_w$'s instead of the vectors $\mathbf{z}_k$'s, we only deal with the sum of correlated Gamma random variables with an *identical fading order* of 0.5, which greatly simplifies the analysis. In the proposed method, the most computationally complex EVD's are performed over $L \times L$ matrices; all other EVD's are performed over $L_w \times L_w$ matrices with $L_w < L$. Therefore, the improvement over (Win et al., 2000) is that the EVD proposed in (Win et al., 2000) is performed over a $D_T \times D_T$ matrix, where $D_T \triangleq \sum_{k=1}^{L} 2m_k$. When the number of branches, L, is large and the m_k's are greater than 1, we have $D_T \gg L$ and thus the EVD in (Win et al., 2000) is much more computationally complex than the EVD's proposed in this chapter.

3.2 Probability Density Function

The PDF of γ, which is denoted by $p_{\gamma}(\gamma)$, is also desired for the cases where performance is harder to analyze by the MGF– or CF–based methods. The PDF of the sum of statistically independent, non–identically distributed Gamma random variables is derived in (Moschopoulos, 1985). This PDF is in the form of a single Gamma series and is thus desired by the performance analysis. Here, we apply the MGF derived in (15) for the case of *non–identical*, *non–integer* m_k's and *non–identical* Ω_k's to Eq. (2.9) in (Moschopoulos, 1985) to obtain the PDF of γ. First, let us re-write (15) as

$$\begin{aligned} \Phi_{\gamma}(s) &= \prod_{w=1}^{2m_L} \prod_{i=1}^{L_w} \left(1 - s\lambda_{w,i}\right)^{-1/2} \\ &= \prod_{i=1}^{M} \left(1 - s\lambda_i\right)^{-1/2}, \end{aligned} \tag{16}$$

where $M \triangleq \sum_{w=1}^{2m_L} L_w$. The eigenvalues λ_i's in (16) are defined as

$$\lambda_i \triangleq \lambda_{w,i-L^{(w)}}, \quad \text{for} \quad L^{(w)} + 1 \le i \le L^{(w)} + L_w, \; w = 1, 2, \ldots, 2m_L, \tag{17}$$

where $L^{(w)} \triangleq \sum_{k=1}^{w-1} L_k$ with $L^{(1)} = 0$. The MGF given in (16) is now in a form that can readily be applied to Eq. (2.9) in (Moschopoulos, 1985) to obtain the exact PDF of γ, which is given by

$$p_\gamma(\gamma) = c\sum_{k=0}^{\infty}\frac{\delta_k \gamma^{A_k-1}e^{-\gamma/\lambda_1}}{\Gamma(A_k)\lambda_1^{A_k}}, \quad \gamma > 0 \\ = 0, \ \text{otherwise}, \tag{18}$$

where $A_k \triangleq k + M/2$, $c \triangleq \prod_{i=1}^{M}(\lambda_1/\lambda_i)^{1/2}$ and δ_k's are recursively computed by

$$\delta_{k+1} = \frac{1}{2(k+1)}\sum_{w=1}^{k+1}\delta_{k+1-w}\sum_{i=1}^{M}\left[1-(\lambda_1/\lambda_i)\right]^w, \quad k = 0,\ 1,\ 2,\ldots \tag{19}$$

with $\delta_0 = 1$. In (18), we assume, without any loss of generality, that $\lambda_1 = \min_i\{\lambda_i\}$. If $\lambda_1 \neq \min_i\{\lambda_i\}$, we can simply find the minimum eigenvalue and put it at the first position. A low-complexity computation of the parameters δ_{k+1} is discussed in Appendix I.

For practical numerical evaluations, it is desired to obtain a truncated version of (18) and the associated truncation error. The PDF in (18) can be truncated to give

$$p_\gamma(\gamma, K) = c\sum_{k=0}^{K}\frac{\delta_k \gamma^{A_k-1}e^{-\gamma/\lambda_1}}{\Gamma(A_k)\lambda_1^{A_k}}, \ \gamma > 0 \tag{20}$$

and 0 elsewhere. By applying the upper bound given by Eq. (2.12) in (Moschopoulos, 1985) for $p_\gamma(\gamma)$, the associated truncation error produced by (20) is upper–bounded by

$$e_K(\gamma) \triangleq p_\gamma(\gamma) - p_\gamma(\gamma, K) \\ \leq \frac{c\gamma^{\frac{M}{2}-1}e^{-\gamma(1-g)/\lambda_1}}{\lambda_1^{M/2}\Gamma\left(\frac{M}{2}\right)} - p_\gamma(\gamma, K), \tag{21}$$

where $g \triangleq \max_{2\leq i\leq M}\left[1-(\lambda_1/\lambda_i)\right]$. From (21), we can choose K such that the error is in the desired regime.

Remark: *The upper bound of the PDF given by Eq. (2.12) in (Moschopoulos, 1985) is attained when* $\lambda_1 = \cdots = \lambda_M$ *and is tight when* $\chi \triangleq \max_k\{\lambda_k\}/\min_k\{\lambda_k\} \approx 1$. *However, when* $\chi \gg 1$, *this upper bound becomes extremely loose and thus* (21) *cannot be used to determine* K *for a good truncation.*

In the case when χ is large, we propose that K be determined as follows. For a specific K, using Eq. (3.462–9) in (Gradshteyn & Ryzhik, 2000) the error of the area under the PDF due to truncation is given by

$$E_{er}(K) \triangleq \int_0^\infty p_\gamma(\gamma)d\gamma - \int_0^\infty p_\gamma(\gamma, K)d\gamma \\ = 1 - \int_0^\infty p_\gamma(\gamma, K)d\gamma \\ = 1 - c\sum_{k=0}^{K}\delta_k. \tag{22}$$

It is pointed out in (Moschopoulos, 1985) that the interchange of the integration and summation in (22) is justified due to the uniform convergence of $p_\gamma(\gamma)$. For a pre–

determined threshold of error $E_{er}(K) \le \varepsilon$, we can easily choose K from (22) such that this condition is satisfied. The advantage of this method compared to the bounding method derived in (Moschopoulos, 1985), given in (21), is that K can be determined for an arbitrary χ.
Note that the PDF derived in (18) is an extension of the PDF given by Eq. (5) derived in (Alouini et al., 2001) to the case of *non–identical* fading orders m_k's. The MGF and PDF of γ derived in (15) and (20), respectively, can be used for general performance analysis in wireless communication systems such as (a) outage probability, (b) bit error probability and (c) Shannon capacity analysis as shown in (Alouini et al., 2001).

4. Application to Performance Analysis of MRC Systems

4.1 Preliminaries

Bit error probability (BEP) analysis can be performed using either the PDF or MGF of the received SNR coefficient at the output of the MRC combiner.

Method 1**: **BEP analysis using PDF of the received SNR coefficient. It is shown in (Lu et al., 1999) that the conditional BEP, given the received SNR coefficient γ, of an $\bar{M}$-PSK or $\bar{M}$-QAM modulated system, is a function of

$$P_b(\xi \mid \gamma) \triangleq \operatorname{erfc}\left(\sqrt{\xi\gamma}\right), \tag{23}$$

where $\operatorname{erfc}(x)$ is the complementary error function (Gradshteyn & Ryzhik, 2000), ξ is a deterministic variable determined from the unfaded received SNR $\bar{E}_s / \sigma_w^2$ at each branch and the digital modulation scheme used, which will be discussed in more details. In (23), $P_b(\xi \mid \gamma)$ is considered as elementary conditional BEP based upon which the overall conditional BEP is calculated. First, using the integral representation of $\operatorname{erfc}(\sqrt{\xi\gamma})$ given by Eq. (7.4.11) in (Abramowitz & Stegun, 1972) and a change of variable, we arrive at

$$P_b(\xi \mid \gamma) = \tfrac{2}{\pi}\int_0^{\infty}\left(z^2+1\right)^{-1}\exp\left[-\xi\gamma(z^2+1)\right]dz. \tag{24}$$

Then, using the PDF $p_\gamma(\gamma)$ of the received SNR coefficient γ, the statistical average of the elementary BEP of the receiver is then computed by

$$\begin{aligned} P_b(\xi) &\triangleq E\left\{P_b(\xi \mid \gamma)\right\} \\ &= \tfrac{2}{\pi}\int_0^{\infty}\int_0^{\infty}\left(z^2+1\right)^{-1}\exp\left[-\xi\gamma(z^2+1)\right]p_\gamma(\gamma)dz\,d\gamma. \end{aligned} \tag{25}$$

Method 2: BEP analysis using MGF of the received SNR coefficient. The elementary BEP in (25) can also be manipulated as

$$P_b(\xi) = \frac{2}{\pi}\int_0^{\infty}\left(\frac{1}{z^2+1}\right)E\left\{\exp[-\xi\gamma(z^2+1)]\right\} dz$$
$$= \frac{2}{\pi}\int_0^{\infty}\left(\frac{1}{z^2+1}\right)\Phi_\gamma\left(-\xi\left(z^2+1\right)\right) dz. \quad (26)$$

If a closed-form solution to (26) is not available, we can resort to numerical analysis with a high degree of accuracy using the well-known Gaussian-Chebyshev Quadrature (GCQ) given in (Abramowitz & Stegun, 1972). First, apply the substitution $y = (1-z^2)/(1+z^2)$ to (26) and then use an *N*-point integral GCQ, we arrive at

$$P_b(\xi) = \frac{1}{N}\sum_{n=1}^{N}\Phi_\gamma\left(-\frac{2\xi}{\cos(\theta_n)+1}\right) + R_N(\xi)$$
$$= \frac{1}{N}\sum_{n=1}^{N}\prod_{i=1}^{M}\left(1+\frac{2\xi\lambda_i}{\cos(\theta_n)+1}\right)^{-1/2} + R_N(\xi), \quad (27)$$

where $\theta_n \triangleq [(2n-1)\pi/2N]$ and $R_N(\xi)$ is the remainder given in (Abramowitz & Stegun, 1972). It is shown in (Annamalai et al., 1999; Zhang, 1999) and references therein that using (27) is very accurate even with only a small *N*.

4.2 Bit Error Probability Analysis

Method 1: BEP analysis using the PDF $p_\gamma(\gamma)$. Application of the PDF of the received SNR coefficient at the output of the MRC combiner $p_\gamma(\gamma)$, given in (18), to (25) gives the elementary BEP of the receiver as

$$P_b(\xi) = \frac{2c}{\pi}\sum_{k=0}^{\infty}\Delta_k\int_0^{\infty}\int_0^{\infty}(z^2+1)^{-1}\gamma^{A_k-1}\exp\left\{-\gamma\left[\xi(z^2+1)+1/\lambda_1\right]\right\}d\gamma dz, \quad (28)$$

where $\Delta_k \triangleq \left\{\delta_k/\left[\Gamma(A_k)\lambda_1^{A_k}\right]\right\}$. Like (22), the interchange of the integration and summation in (28) is justified due to the uniform convergence of $p_\gamma(\gamma)$ as shown in (Moschopoulos, 1985). Then, by using Eq. (3.462-.9) and Eq. (3.259-.3) in (Abramowitz & Stegun, 1972), we have

$$P_b(\xi) = c\sum_{k=0}^{\infty} f(k,\xi)B\left(\frac{1}{2},A_k+\frac{1}{2}\right){}_2F_1\left(A_k,\frac{1}{2};A_k+1,\frac{1}{1+\lambda_1\xi}\right), \quad (29)$$

where $B(\cdot,\cdot)$ is the beta function, ${}_2F_1(\cdot,\cdot\,;\,\cdot,\cdot)$ is the Gauss hypergeometric function given in (Gradshteyn & Ryzhik, 2000), and $f(k,\xi) \triangleq \delta_k/[\pi(1+\xi\lambda_1)^{A_k}]$. Since $1 > 1/(1+\lambda_1\xi)$, the Gauss hypergeometric function ${}_2F_1(\cdot,\cdot\,;\,\cdot,\cdot)$ in (29) converges. We then invoke the results derived in (Lu et al., 1999) and use the relationship $Q(x) = 0.5\text{erfc}(x/\sqrt{2})$ to obtain, after straightforward manipulations, the conditional BEP given the received SNR coefficient γ of a coherent $\bar{M}$-PSK modulated system with MRC receiver as

$$
\begin{aligned}
P_b^{PSK}(\cdot\,|\,\gamma) &\cong a_P \sum_{k=1}^{K_P} \operatorname{erfc}\left(\sqrt{\xi_k \gamma}\right) \\
&= a_P \sum_{k=1}^{K_P} P_b(\xi_k\,|\,\gamma),
\end{aligned}
\tag{30}
$$

where $\bar{M}$ is the size of the signal constellation, $a_P \triangleq 1/\max(b,2)$ with $b \triangleq \log_2 \bar{M}$ is the number of information bits per modulated symbol, deterministic variable ξ_k, as mentioned in (23), is defined by the branch unfaded received SNR $\bar{E}_s/\sigma_w^2$ and the modulation scheme as $\xi_k \triangleq \left\{\bar{E}_s \sin^2[(2k-1)\pi/\bar{M}]/\sigma_w^2\right\}$, and $K_P \triangleq \left\lceil \max(\bar{M}/4,1) \right\rceil$. The statistical average of (30) is finally computed by

$$
\begin{aligned}
\bar{P}_b^{PSK} &\triangleq E\left\{P_b^{PSK}(\cdot\,|\,\gamma)\right\} \\
&\cong E\left\{a_P c \sum_{k=1}^{K_P} P_b(\xi_k\,|\,\gamma)\right\} \\
&= a_P c \sum_{k=1}^{K_P} E\left\{P_b(\xi_k\,|\,\gamma)\right\} \\
&= a_P c \sum_{k=1}^{K_P} P_b(\xi_k),
\end{aligned}
\tag{31}
$$

where $P_b(\xi_k\,|\,\gamma)$ and $P_b(\xi_k)$ are defined in (24) and (25), respectively. Note that from (31) we can also easily obtain the BEP of a coherent FSK modulated system by setting $\bar{M}=2$ and replacing ξ_k with $\xi_k/2$. Similarly, invoking the results derived in (Lu et al., 1999) gives the approximation of conditional BEP, given the received SNR coefficient γ, of a coherent, square $\bar{M}$-QAM modulated system with MRC receiver as

$$
\begin{aligned}
P_b^{QAM}(\cdot\,|\,\gamma) &\cong a_Q \sum_{k=1}^{K_Q} \operatorname{erfc}\left(\sqrt{\zeta_k \gamma}\right) \\
&= a_Q \sum_{k=1}^{K_Q} P_b(\zeta_k\,|\,\gamma),
\end{aligned}
\tag{32}
$$

where $a_Q \triangleq [2(\sqrt{\bar{M}}-1)/(b\sqrt{\bar{M}})]$, $K_Q \triangleq \left\lceil \sqrt{\bar{M}}/2 \right\rceil$ and $\zeta_k \triangleq \{3\bar{E}_s(2k-1)^2/[2\sigma_w^2(\bar{M}-1)]\}$. Therefore, the statistical average BEP of a square $\bar{M}$-QAM modulated system is approximated by

$$\begin{aligned}\overline{P}_b^{QAM} &\triangleq E\left\{P_b^{QAM}(\cdot\,|\,\gamma)\right\}\\ &\cong E\left\{a_Q c\sum_{k=1}^{K_Q} P_b(\zeta_k\,|\,\gamma)\right\}\\ &= a_Q c\sum_{k=1}^{K_Q} P_b(\zeta_k),\end{aligned} \tag{33}$$

Where, again, $P_b(\zeta_k\,|\,\gamma)$ and $P_b(\zeta_k)$ are defined in (24) and (25), respectively.

Method 2: BEP analysis using the MGF $\Phi_\gamma(s)$. Consider now the option of using the MGF $\Phi_\gamma(s)$ to analyze the BEP of the MRC receiver. Substituting (16) into (27) gives the elementary BEP as

$$P_b(\xi) = \frac{1}{N}\sum_{n=1}^{N}\prod_{i=1}^{M}\left(1+\frac{2\xi\lambda_i}{\cos(\theta_n)+1}\right)^{-1/2} + R_N(\xi). \tag{34}$$

Therefore, for an $\bar{M}$-PSK modulated system with MRC receiver, the overall BEP is approximated by

$$\overline{P}_b^{PSK} \cong \frac{a_P}{N}\sum_{k=1}^{K_P}\sum_{n=1}^{N}\prod_{m=1}^{M}\left(1+\frac{2\xi_k\lambda_m}{\cos(\theta_n)+1}\right)^{-1/2} + R_P, \tag{35}$$

where λ_m's are defined in (17) and $R_P \triangleq a_P\sum_{k=1}^{K_P} R_N(\xi_k)$ is the overall remainder of the N-point GCQ operation. Similarly, for a square $\bar{M}$-QAM modulated system with MRC receiver, the overall BEP is approximated by

$$\overline{P}_b^{QAM} \cong \frac{a_Q}{N}\sum_{k=1}^{K_Q}\sum_{n=1}^{N}\prod_{m=1}^{M}\left(1+\frac{2\zeta_k\lambda_m}{\cos(\theta_n)+1}\right)^{-1/2} + R_Q, \tag{36}$$

where $R_Q \triangleq a_Q\sum_{k=1}^{K_Q} R_N(\zeta_k)$ is the overall remainder of the N-point GCQ operation.

4.3 Outage Probability Analysis

Outage probability is also a useful performance measure of the receiver. The outage probability P_{out} is defined as the probability that the BEP P_b exceeds a BEP threshold $\overline{P}_0$ or equivalently the probability that the overall instantaneous SNR η falls below a predetermined SNR threshold, say $\overline{\eta}_0$, that is

$$\begin{aligned}P_{\text{out}} &\triangleq \Pr\left\{P_b \ge \overline{P}_0\right\} = \Pr\{\eta \le \overline{\eta}_0\}\\ &= \Pr\left\{\gamma \le \overline{\eta}_0\sigma_w^2/\overline{E}_s\right\},\end{aligned} \tag{37}$$

since $\eta \triangleq \gamma\overline{E}_s/\sigma_w^2$ as defined in (2), where $\overline{\eta}_0 \triangleq f^{-1}(\overline{P}_0)$. Therefore, we have

$$P_{out} = \int_0^{\overline{\eta}} p_\gamma(\gamma) d\gamma, \tag{38}$$

where $\overline{\eta} \triangleq \overline{\eta}_0 \sigma_w^2 / \overline{E}_s$. Substituting (18) into (38) and using Eq. (3.381-1) in (Gradshteyn & Ryzhik, 2000) gives

$$\begin{aligned} P_{out} &= c\sum_{k=0}^{\infty} \frac{\delta_k}{\Gamma(A_k)} \left[\Gamma(A_k) - \Gamma(A_k, \overline{\eta}/\lambda_1)\right] \\ &= 1 - c\sum_{k=0}^{\infty} \frac{\delta_k \Gamma(A_k, \overline{\eta}/\lambda_1)}{\Gamma(A_k)}, \end{aligned} \tag{39}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function (Gradshteyn & Ryzhik, 2000). A truncation version of (39) can be obtained by using (20) for practical computation purposes. Note that (39) is similar to the result derived in (Alouini et al., 2001), but for the case of *non-identical* and *non-integral* fading orders m_k's. A number of other performance measures as discussed in (Alouini et al., 2001) can also be easily analyzed using the PDF and MGF derived in this Chapter.

4.4 Application to Performance Analysis of Space-Time Block Coded Systems

In this section, we apply the derived results to analyze the performance of orthogonal space-time block coded (OSTBC) (Su & Xia, 2003) system over quasi-static frequency-flat Nakagami-*m* fading channels. Consider an OSTBC system equipped with N_t transmit-antennas and N_r receive-antennas with perfect channel estimation at the receiver. At the receiver, after a simple linear combining operation, the combined received signal, denoted by r_k, for the *k*th code symbol, denoted by s_k, is equivalent to

$$r_k = \left(\sum_{i=1}^{N_t} \sum_{l=1}^{N_t} |\alpha_{il}|^2 \right) s_k + \sum_{i=1}^{N_t} \sum_{l=1}^{N_r} \alpha_{il} w_{il}, \tag{40}$$

where α_{il}'s are channel fading gains from the *i*th transmit-antenna to the *l*th receive-antenna, which are Nakagami-*m* distributed random variables with fading parameters m_{il}'s and Ω_{il}'s, and w_{il}'s are AWGN samples with a zero-mean and a variance σ_w^2. The fading orders m_{il}'s are integers or half of integers. The SNR of r_k, given the channel fading gains α_{il}'s, is then computed by

$$\eta_{r,k} = \frac{\overline{E}_s}{N_t \sigma_w^2} \sum_{i=1}^{N_t} \sum_{l=1}^{N_r} |\alpha_{il}|^2. \tag{41}$$

The overall received SNR $\eta_{r,k}$ in (41) is a sum of correlated Gamma random variables, which is in the form of the received SNR given in (2). Therefore, the application of the results derived in this chapter to performance analysis of OSTBC systems over quasi-static frequency-flat Nakagami-*m* fading channels is immediate.

5. Numerical Results and Discussions

5.1 Exponentially Correlated Branches

It is shown in (Lee, 1993) and (Zhang, 1999) that it is reasonable to place two adjacent antennas such that their correlation is from 0.6 to 0.7. Therefore, we choose the power–correlation between two adjacent antennas to be 0.6 and the correlations amongst the antennas follow the exponential rule, i.e., $\rho_{x_k x_l} = 0.6^{|k-l|}$, for this example. An example for an arbitrary case is shown in the next subsection. For this example, we consider an MRC receiver with $L = 4$ diversity branches and branch power correlation matrix $\mathbf{R_{X1}}$ in (42). Let $\mathbf{m}_k = [m_{k,1}, m_{k,2}, m_{k,3}, m_{k,4}]^T$ and $\mathbf{\Omega}_k = [\Omega_{k,1}, \Omega_{k,2}, \Omega_{k,3}, \Omega_{k,4}]^T$ denote the fading parameter vectors. We consider three cases with the following fading parameters:

Case 1: $\mathbf{m}_1 = [0.5,\ 0.5,\ 0.5,\ 0.5]^T$ and $\mathbf{\Omega}_1 = [0.85,\ 1.21,\ 0.92,\ 1.12]^T$,

Case 2: $\mathbf{m}_2 = [0.5,\ 1.0,\ 1.5,\ 2]^T$ and $\mathbf{\Omega}_2 = [1.15,\ 1,\ 0.92,\ 1.2]^T$, and

Case 3: $\mathbf{m}_3 = [1.5,\ 2.5,\ 3,\ 3.5]^T$ and $\mathbf{\Omega}_3 = [1.35,\ 1,\ 0.95,\ 1.15]^T$.

$$\mathbf{R_{X1}} = \begin{pmatrix} 1 & 0.6 & 0.36 & 0.216 \\ 0.6 & 1 & 0.6 & 0.36 \\ 0.36 & 0.6 & 1 & 0.6 \\ 0.216 & 0.36 & 0.6 & 1 \end{pmatrix}. \tag{42}$$

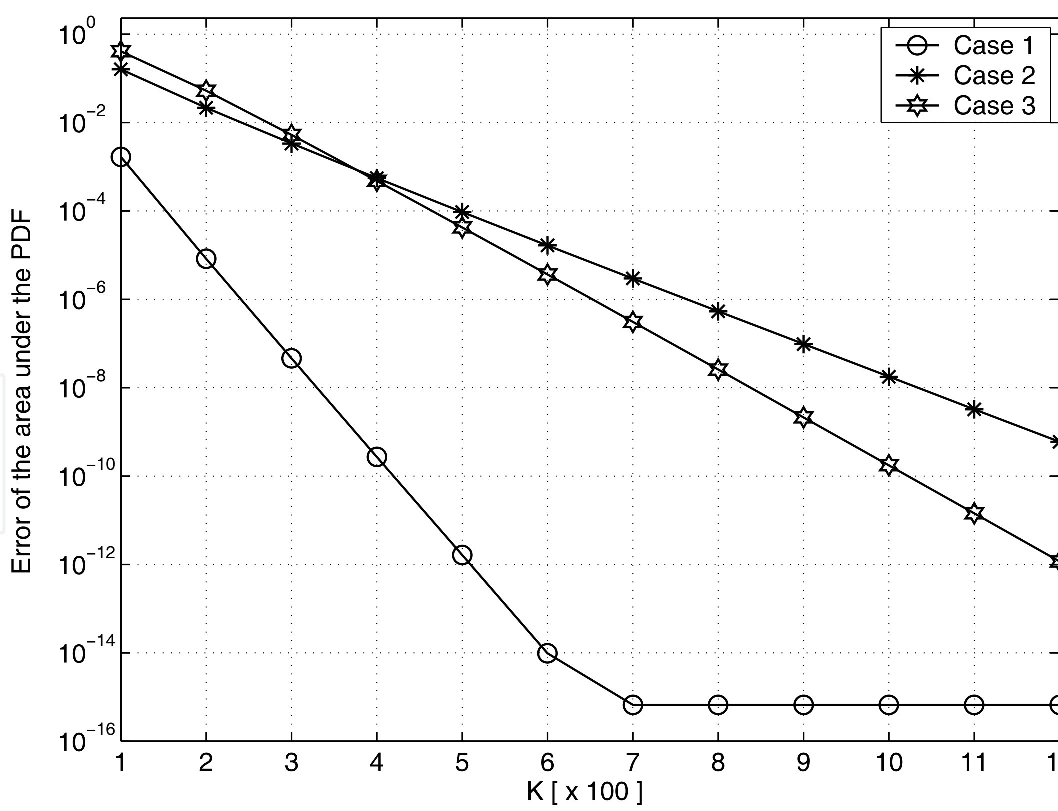

Fig. 1. The error of the areas under the PDFs, $E_{er}(K)$, due to truncation to $K+1$ terms with the branch power-correlation matrix $\mathbf{R_{X1}}$.

Note that these three cases do not necessarily reflect any particular practical system parameters. They, rather, demonstrate the generality of the derived results. These three cases cover a very wide range of fading conditions, from worse to better than the Rayleigh fading cases. They also represent a wide range of the variations of fading severities and gain imbalances among diversity branches. Using (6) and (9), the correlation matrix $\mathbf{R}_{V,1}$ relates to the branch power-correlation matrix $\mathbf{R}_{\mathbf{X}1}$ through (43).

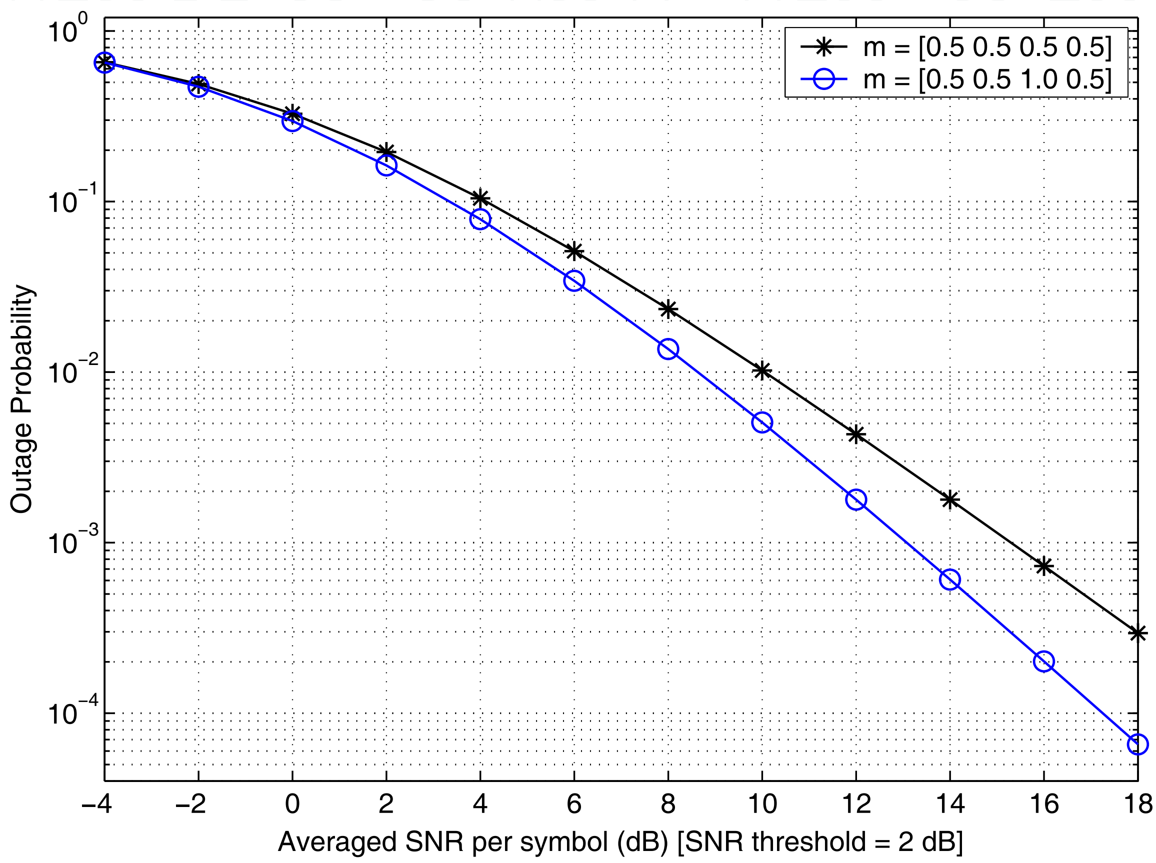

Fig. 2. Outage probability comparisons for different fading orders with the branch power-correlation matrix $\mathbf{R}_{\mathbf{X1}}$.

$$\mathbf{R}_{V,1}(k,l) = \mathbf{R}_{\mathbf{X1}}(k,l)\sqrt{\frac{\max(m_k, m_l)}{\min(m_k, m_l)}}. \tag{43}$$

The next step is to determine the truncation parameter K for each case. The truncation errors of the area under the PDFs computed by (22) for different values of K for the above cases are given in Figure 1. It is noted from Figure 1 that the convergence rate of the PDF depends heavily on the fading parameters (i.e., m_k's and Ω_k's). It is also evident from Figure 1 that using (22) we can easily choose a good truncation of the PDF for practical evaluation.

For a better view of the convergence of the infinite series in (18), we first compute the PDF in (18) with $K = 100{,}000$ and then compute the PDF's for smaller values of K and compare those PDF's with that of the case $K = 100{,}000$. We use sample mean-squared error (MSE) as the measure of the difference between different PDF's. The results are shown in Table 1. As observed from Table 1, the difference in MSE sense between the PDF computed for $K = 100{,}000$ and that computed for $K = 100$ is extremely small ($10^{-26.2482}$) and the convergence is fast. Note, however, that the convergence rate depends on the system parameters as evident from Figure 1.

K	MSE
1	$10^{-2.2726}$
10	$10^{-3.1351}$
20	$10^{-3.8496}$
30	$10^{-4.5195}$
50	$10^{-5.9959}$
70	$10^{-10.9634}$
100	$10^{-26.2482}$

Table 1. MSE's between the PDF's computed for different K's and the PDF computed for $K = 100{,}000$ with the branch power-correlation matrix $\mathbf{R}_{\mathbf{X1}}$.

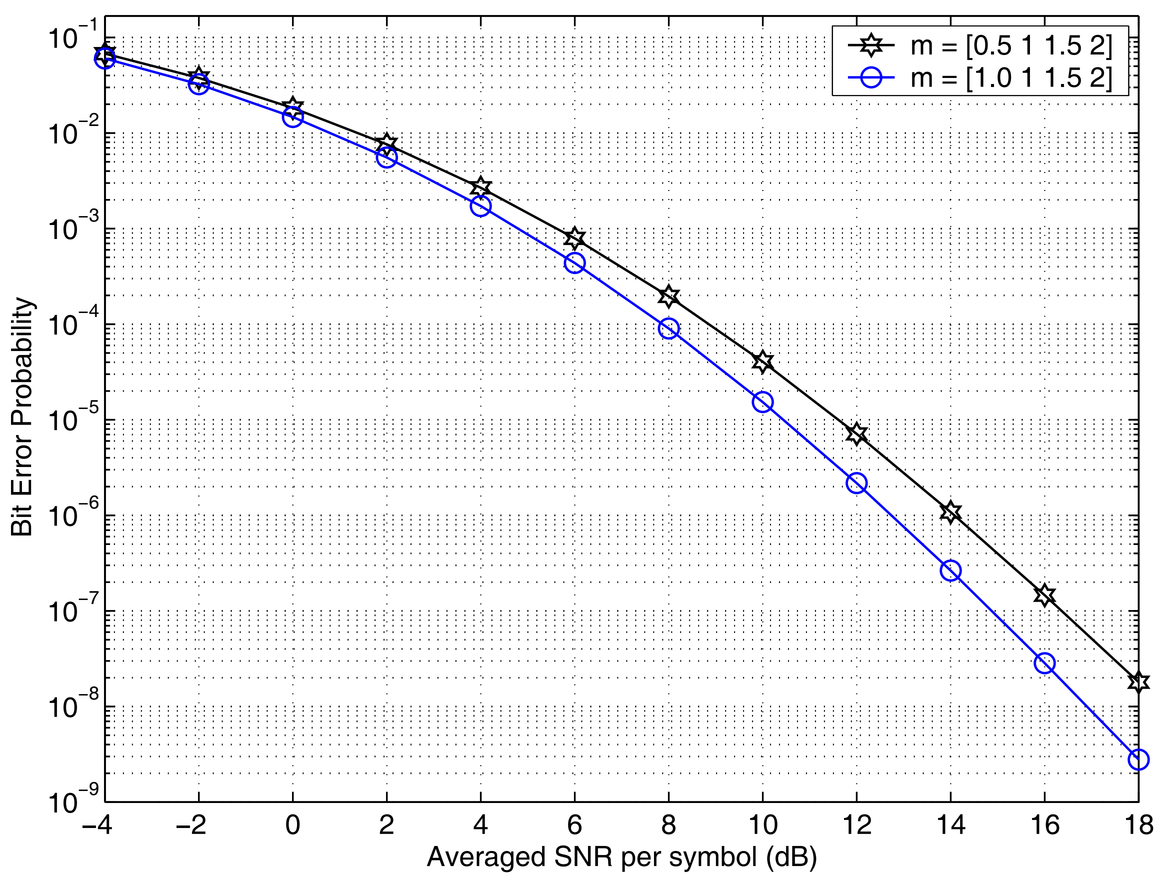

Fig. 3. Bit error probability comparisons for different fading orders with BPSK modulation, and branch power and branch correlation matrix of the Case 2 with the branch power-correlation matrix $\mathbf{R}_{\mathbf{X1}}$.

Practicality of the derived results: The question that remains to be answered is whether the performances of MRC wireless systems over Nakagami-m fading channels would vary significantly when the fading orders are changing to the nearest integers from half of integers.

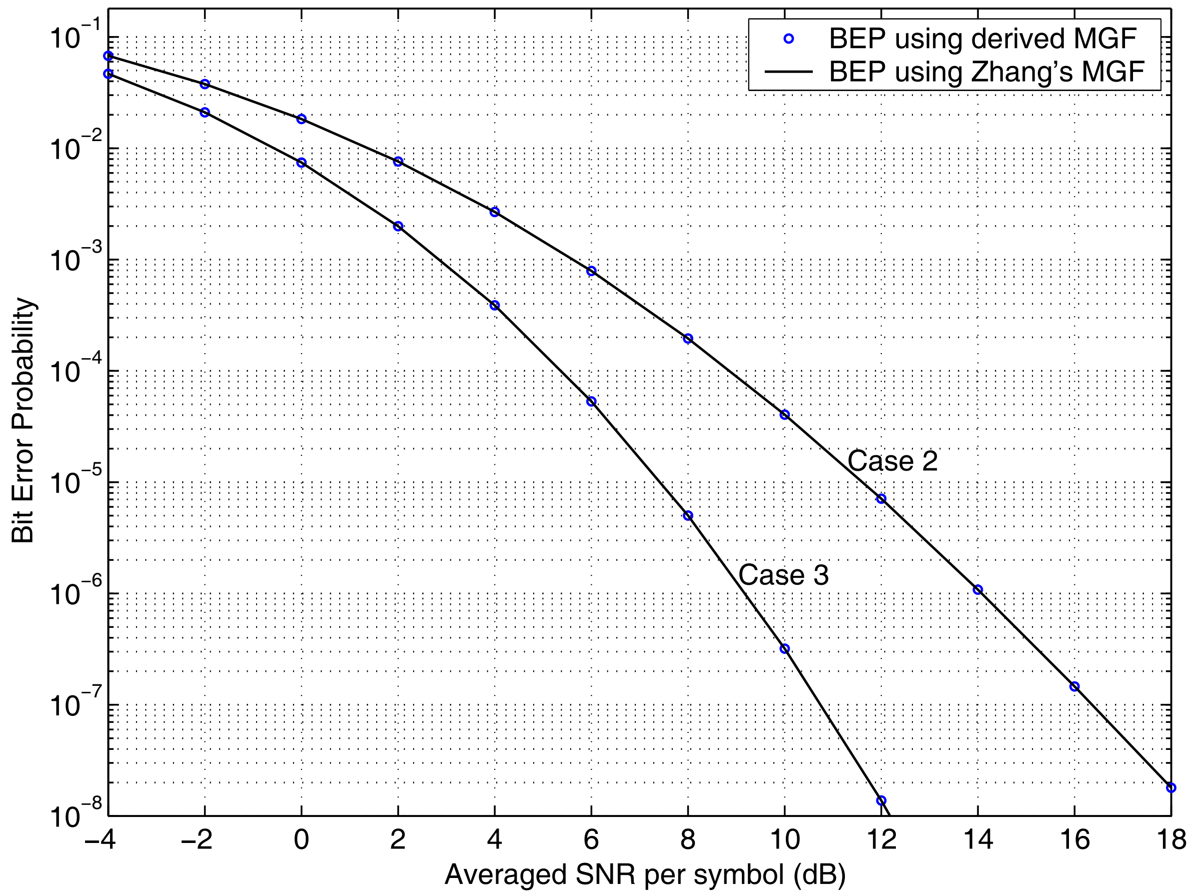

Fig. 4. Bit error probabilities of an MRC receiver derived in (35) using the MGF derived in (15) and the MGF given in (Zhang, 2003) for the Cases 2 and 3 with $L=4$ branches and the branch power-correlation matrix $\mathbf{R_{X1}}$ for BPSK modulation.

We investigate this question through numerical results. Figure 2 presents the outage probabilities for different values of fading orders. The results presented in Figure 2 are for a SNR threshold $\overline{\eta}=2\text{dB}$, the branch power vector $\mathbf{\Omega}_1$ and branch correlation matrix given in (42). As seen in Figure 2, among the four fading orders only one is rounded from 0.5 to 1 but the difference in outage probability is already significant, especially at high SNR's. The same is observed for BEP performance given in Figure 3. These observations are also reported in (Ghareeb & Abu-Surra, 2005). Clearly, the derived results in this chapter for the case of half-of- integer fading orders m's are practically useful.

Comparison to previous results: Figure 4 shows the BEP's of an MRC receiver with $L=4$ branches with the fading orders, branch powers and correlation matrix specified in Case 2 and Case 3 above for BPSK modulation. As seen in Figure 4, the BEP's derived in (35), using the MGF given in (15), very well match with those derived using Zhang's MGF in (Zhang, 2003). Note, however, that the MGF in (15) is derived by a totally different approach from that derived in (Zhang, 2003). Therefore, the approach presented in this chapter provides an alternative to, as well as a verification of, Zhang's approach (Zhang, 2003) for integer and half-of- integer fading orders.

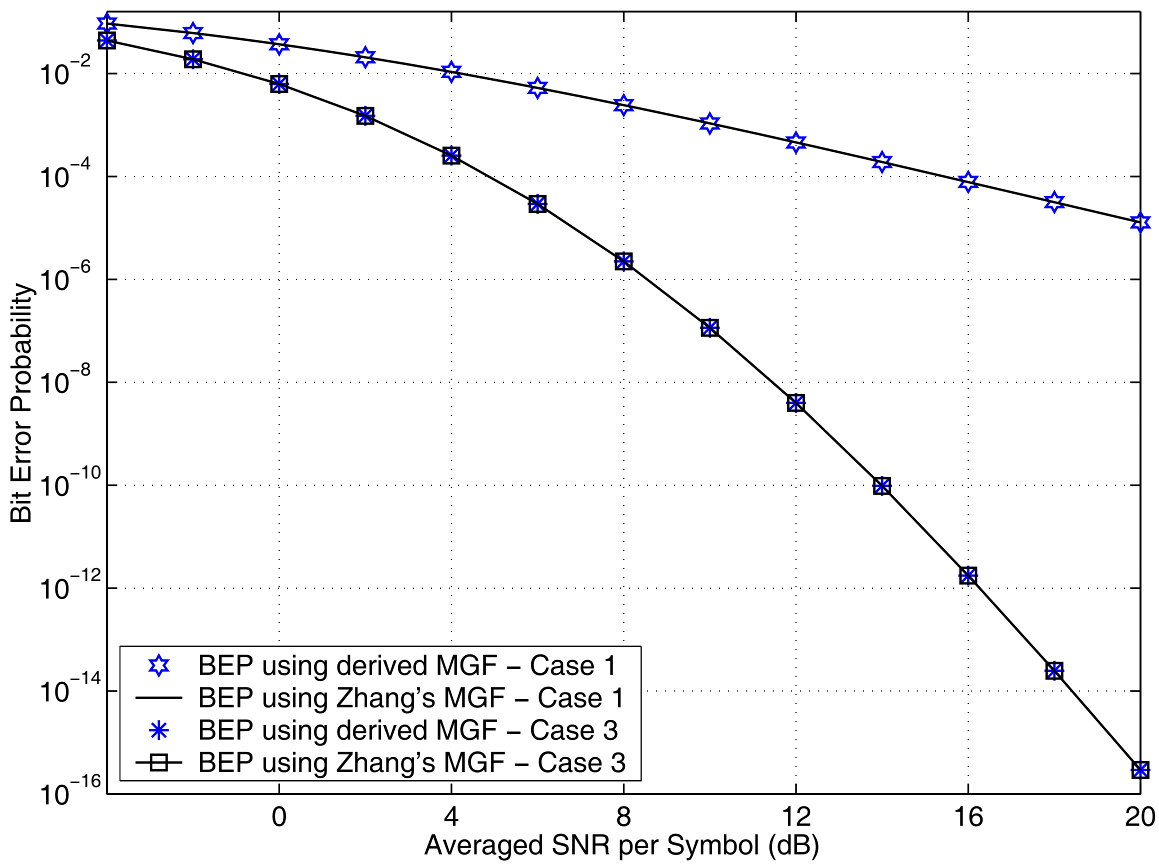

Fig. 5. Bit error probabilities of an MRC receiver derived in (35) using the MGF derived in (15) and the MGF given in (Zhang, 2003) for the Cases 1 and 3 with $L=4$ branches and the branch power-correlation matrix $\mathbf{R}_{X2}$ for BPSK modulation.

5.2 Arbitrarily Correlated Branches

Consider now the case $L=4$ diversity branches with an arbitrary branch power-correlation matrix $\mathbf{R}_{X2}$ given by

$$\mathbf{R}_{X2}=\begin{pmatrix}1 & 0.5 & 0.2 & 0.05\\ 0.5 & 1 & 0.5 & 0.2\\ 0.2 & 0.5 & 1 & 0.5\\ 0.05 & 0.2 & 0.5 & 1\end{pmatrix}. \quad (44)$$

For the branch power-correlation matrix in (44), the farther the antennas are away from one another the smaller the branch correlation is. This corresponds to an arbitrary case as opposed to the previous example. In this example, we still consider the same fading parameters as that considered in Subsection 5.1 for brevity. Figure 5 shows the BEP's of an MRC receiver having four diversity branches with the fading orders, branch powers and correlation matrix specified in Case 1 and Case 3 in Subsection 5.1 also for BPSK modulation. It is seen in Figure 5 that the BEP's derived in (35), using the MGF given in (15), also very well match with those derived from Zhang's MGF in (Zhang, 2003). The results in Figure 5 demonstrate the generality of the derived results.

6. Conclusion

Most of the existing works dealing with multivariate Gamma r.v.'s in the literature are for the cases of *identical* and *integer* fading order. In this chapter, we have derived the MGF and PDF of the sum of *arbitrarily correlated* Gamma r.v.'s with *non–integer* and *non–identical* fading parameters. The new results remove most of the restrictions imposed on existing results. We also derived a simple method to determine a good truncation of the PDF and the associated truncation error for any fading parameters including the cases where the upper bound given in (Moschopoulos, 1985) is too loose and cannot be used. The derived PDF and MGF can be used to evaluate a number of performance measures of diversity combining receivers in wireless communications over Nakagami–*m* fading channels. The derived MGF offers a low complexity for the analysis and the PDF is in the form of a single Gamma series, which greatly simplifies the analyses of a number of performance measures in an MRC wireless system.

Appendix I: Low–Complexity Computation of $\delta_1, \ldots, \delta_K$

First, we recognize that in (19), the common terms between δ_k and δ_{k+1} are

$$X(w) = \sum_{i=1}^{M} \left[1 - (\lambda_1 / \lambda_i)\right]^{w}, \ \ w = 1,\ 2, \ldots,\ k. \tag{45}$$

Therefore, when computing δ_{k+1}, we can use $X(w)$, $w = 1,\ 2, \ldots,\ k$, that we already computed for δ_k and need to compute only one more term, that is $X(k+1)$. Suppose that we need to truncate the PDF $p_\gamma(\gamma)$ at $K+1$ terms in (20). We then need to compute $\delta_1,\ \delta_2, \ldots,\ \delta_K$ (since $\delta_0 = 1$) according to (19). However, from (45) we note that $X(w)$, $w = 1,\ 2, \ldots,\ K$, used to compute δ_K can be used to compute $\delta_1,\ \delta_2, \ldots,\ \delta_{K-1}$ as well. Therefore, if we use (19), we have to repeat many computations, which are already done. With this in mind, we propose a very simple algorithm to compute $\delta_1,\ \delta_2, \ldots, \delta_K$ as follows. First, for $w = 1,\ 2, \ldots,\ K$, compute and store $X(w)$. Then, for $k = 0,\ 1, \ldots,\ K-1$, compute

$$\delta_{k+1} = \frac{1}{2(k+1)} \sum_{w=1}^{k+1} X(w) \delta_{k+1-w}. \tag{46}$$

Using (46) significantly reduces the computational complexity of (19), especially for a large K.

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Mobile and Wireless Communications have been one of the major revolutions of the late twentieth century. We are witnessing a very fast growth in these technologies where mobile and wireless communications have become so ubiquitous in our society and indispensable for our daily lives. The relentless demand for higher data rates with better quality of services to comply with state-of-the art applications has revolutionized the wireless communication field and led to the emergence of new technologies such as Bluetooth, WiFi, Wimax, Ultra wideband, OFDMA. Moreover, the market tendency confirms that this revolution is not ready to stop in the foreseen future. Mobile and wireless communications applications cover diverse areas including entertainment, industrialist, biomedical, medicine, safety and security, and others, which definitely are improving our daily life. Wireless communication network is a multidisciplinary field addressing different aspects raging from theoretical analysis, system architecture design, and hardware and software implementations. While different new applications are requiring higher data rates and better quality of service and prolonging the mobile battery life, new development and advanced research studies and systems and circuits designs are necessary to keep pace with the market requirements. This book covers the most advanced research and development topics in mobile and wireless communication networks. It is divided into two parts with a total of thirty-four stand-alone chapters covering various areas of wireless communications of special topics including: physical layer and network layer, access methods and scheduling, techniques and technologies, antenna and amplifier design, integrated circuit design, applications and systems. These chapters present advanced novel and cutting-edge results and development related to wireless communication offering the readers the opportunity to enrich their knowledge in specific topics as well as to explore the whole field of rapidly emerging mobile and wireless networks. We hope that this book will be useful for students, researchers and practitioners in their research studies.

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