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# Triangle Formation of Multi-Agent Systems with Leader-Following on Fuzzy Control

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## 1. Introduction

In recent years, there have been an increasing number of achievements dealing with the control of agent formations (Olfati-Saber, 2006; Tanner & Jadbabaie, 2007; Fax & Murray, 2004; Jadbabaie et al., 2003; Lin et al., 2004; Olfati-Saber & Murray, 2004). The formation control of multiple moving agents has emerged as a topic of widespread interest due to its broad range of applications in military missions, environmental surveying, and space missions.

Multiple mobile autonomous agents coupled with each other through interactions can generate certain ordered behaviors, such as aggregation, cohesion, alignment, rotation and synchronization (Fax & Murray, 2004; Jadbabaie et al., 2003; Lin et al., 2004; Olfati-Saber & Murray, 2004). Among the typical approaches to formation control, distributed control strategies have aroused researchers' outstanding attention because there is no centralized supervisor and only a little sense of communication information is needed. Recently, there has been a tendency to deal with the formation control as a consensus problem. In the multi-agent systems, consensus means to reach an agreement by means of an interaction rule that specifies the information exchange between an agent and its neighbors (Fax & Murray, 2004; Jadbabaie et al., 2003; Lin et al., 2004; Olfati-Saber & Murray, 2004). Fax & Murray (Fax & Murray, 2004) analyzed the stability of the formation control with first-order consensus protocols based on the graph Laplacian. Jadbabaie et al. (Jadbabaie et al., 2003) demonstrated the results of the alignment problem which is concerned with reaching an agreement without computing any objective functions. Lin et al. (Lin et al., 2004) studied three formation strategies for groups of mobile autonomous agents with local communication topology. Moreover, by using a Lyapunov-based approach, Olfati-Saber & Murray (Olfati-Saber & Murray, 2004) solved the consensus problems in networks of agents with directed interconnection graphs and time delays.

The key idea involved in nonlinear formation control is to preserve the inter-agent distances from decaying to zero during the motion for the purpose of collision avoidance. In the formation control, tracking control problems of multi-agent systems are studied increasingly, and many results have been obtained with nearest neighbor-based rule (Hong et al., 2006; Anderson et al., 2007; Chen & Tian, 2009). Hong et al. (Hong et al., 2006)

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considered the tracking control of mobile agents' consensus with unmeasurable velocity or acceleration information for an active leader. Anderson et al. (Anderson et al., 2007) solved the formation control problem for the system of three-coleader agents described by first-order integrators, that is, each agent should retain a distance from other agents. Promoted by this work, Chen & Tian (Chen & Tian, 2009)) tackled three-coleader formation control problem with second-order integrators of the agent dynamics by applying a backstepping method.

In this paper, like many predecessors, we study the formation control problem of mobile agent systems to maintain the desired velocity and the inter-agent distances. We consider a simple formation with just heterogeneous three-agent system, one agent moving forward as a leader and others following the leader, that is, agent 0 (leader agent) moves freely, and the following agent 1 should maintain a distance from agent 0, agent 2 should maintain a distance from agent 0, and agent 1 and agent 2 should maintain a distance from each other. By applying fuzzy logical controller (FLC), the formation of three agents is achieved. Best to our knowledge, there are not studies for the formation multi-agent system by applying FLC. This chapter is organized as follows: Section 2 gives the system model and the problem statement. The formation control based on fuzzy logical control is presented in Section 3. In Section 4, many computer simulations are applied to verify the formation control, and the robustness of the system is discussed by adding noise and bias. Conclusions are provided in Section 5.

## 2. Problem description

In this paper, we study a formation control problem of the system with three heterogeneous agents, one agent moving as a leader and the others following the leader (Fig. 1). Based on the distance preserved between each other, the followers will be moving in the direction of the leader.

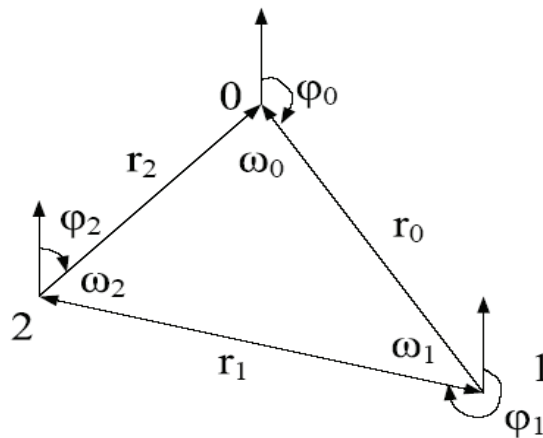


Fig. 1. Formation with three heterogeneous agents.

Notation is defined in reference to Fig. 1. Agent 0 is the leader with the dynamics:

$$\begin{aligned} \dot{z}_0 &= v_0, \\ \dot{v}_0 &= f(t, z_0, v_0). \end{aligned} \quad (1)$$

where  $z_0$  is the position of agent 0 and  $v_0$  is its velocity,  $f(.,.,.)$  is piecewise continuous in  $t$  and locally Lipschitz in  $z_0$  and  $v_0$ . Other agents are the followers with the variable position:

$$z_i = (x_i, y_i), i = 1, 2 \quad (2)$$

and the velocity  $v_i, i = 1, 2$ .

Fig. 1 illustrates this directed formation control system:  $\varphi_i$  is the angle between north and the direction of agent  $i$ , as seen from agent  $i + 1$  (agent 0 being identified with agent 3, here and subsequently);  $r_i$  is the current distance between agent  $i$  and agent  $i + 1$ ;  $d_i$  is the distance that ought to be maintained between agent  $i$  and  $i + 1$ ;  $\omega_i$  is the internal angle of the triangle formed by the three agents, at agent  $i$ , for  $i = 0, 1, 2$ .

In order to control the formation of three agents, we shall make two standing assumptions. First, in the triangle, it is assumed that for  $i \neq j \neq k$ , the triangle inequality  $d_i + d_j > d_k$  holds; thus the steady state to which the formation is supposed to tend is well-defined as a triangle. Second, it is assumed that following agent  $i$  just knows the positions and velocities about itself and its neighbors.

In Fig. 1, the summation of the internal angle of the triangle  $\omega_0 + \omega_1 + \omega_2 = \pi$  holds, and

$$\begin{aligned} \varphi_1 - \varphi_0 &= -(\pi + \omega_1), \\ \varphi_0 - \varphi_2 &= \pi - \omega_0, \\ \varphi_2 - \varphi_1 &= \pi - \omega_2. \end{aligned} \quad (3)$$

We assume the control law of agent  $i$  can only use its local information. The formation control problem is to design the directed control law by using local information, such that three agents achieve the nominated formation, i.e.

$$\lim_{t \rightarrow \infty} r_i = d_i, i = 0, 1, 2$$

with the common desired velocity,

$$\lim_{t \rightarrow \infty} v_i = v_0, i = 1, 2.$$

Manuscript must contain clear answers to following questions: What is the problem / What has been done by other researchers and where you can contribute / What have you done / Which method or tools you used / What are your results / What is new and good, what is not good / Future research

### 3. Fuzzy logical controller for the formation of multi-agent systems

In this section, we present a Fuzzy logical control law for the formation of heterogeneous three-agent system. Following the moving disciplinarian of the system, the agent should satisfy, for  $i = 1, 2$

$$\dot{z}_i = v_i.$$

Let

$$\dot{v}_i = u_i,$$

where  $u_i$  is the control input to adjust the velocity with the moving.

With the movement of the leader agent, the follower agent can calculate the distances between its neighbors. If the distance is larger than the maintained value, its velocity will be increased, otherwise, it will be decreased. Based on this moving mechanism, we construct a fuzzy logical controller to control the agent's dynamic track. This controller has two input variables and one output variable, which the distance and its difference variable are used as input parameters and the intensity of the velocity is determined by the output from the fuzzy controller.

In order to ensure the smooth of the moving track, it is asked to adjust the velocity slowly. If the regulation is too strong, the moving locus will be concussed greatly. By the upwards principle, we will build the following fuzzy logical controller.

Let the distance of two neighbors  $r_i(t)$ , the quantity estimate is  $E_i = \frac{r_i(t) - d_i}{d_i} \times 6$ , then the subjection function can be calculate by the value of  $E_i$  (Fig. 1). Let the difference variable of

the distance  $\Delta r_i(t) = r_i(t) - r_i(t-1)$ , the quantity estimate is  $\Delta E_i = \frac{\Delta r_i(t)}{2d_i} \times 6$ , then the

subjection function can be calculate by the value of  $\Delta E_i$  (Fig. 2). By fuzzy logical rules (Table 1), FLC determines the size of the output  $c$ , where ZO PS PM PB PVB NM and NB is Zero positive-small positive-middle positive-big positive-very-big negative-middle negative-big, respectively.

The output of the FLC determines the size of the repulsion/attraction. When the output is positive, it denotes that attraction is required, and the velocity of agent will be increased. When the output is negative, repulsion is asked, and the velocity of agent will be decreased.

By the quantity estimate, the fact value corresponding with the output  $c$  is  $\frac{c}{6} \times M$  with sufficient large number  $M$ .

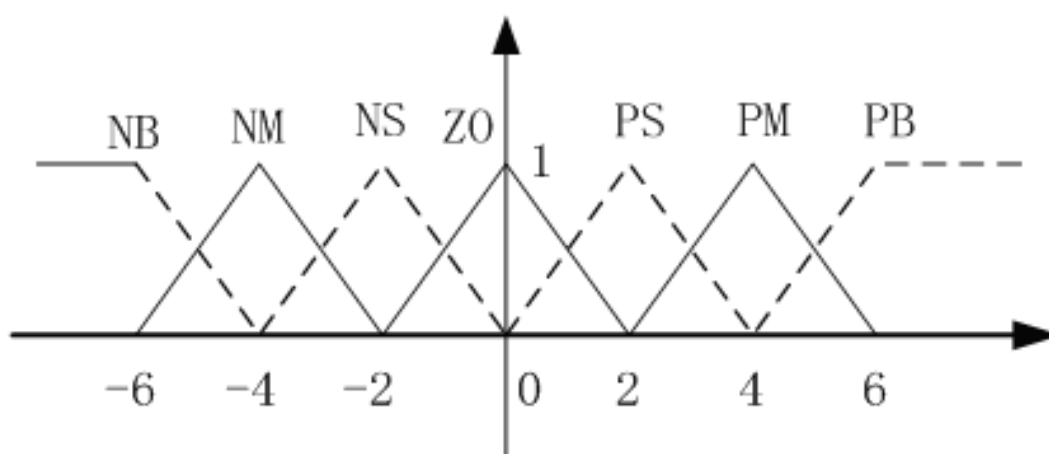


Fig. 2. The subjection function of the pair distance  $E_i$  and  $\Delta E_i$

$E_i \setminus \Delta E_i$	PB	PM	PS	ZO	NS	NM	NB
PB	PB	PB	PB	PB	PM	PS	ZO
PM	PB	PB	PB	PM	PS	ZO	NS
PS	PB	PB	PM	PS	ZO	NS	NM
ZO	PB	PM	PS	ZO	NS	NM	NB
NS	PM	PS	ZO	NS	NM	NB	NB
NM	PS	ZO	NS	NM	NB	NB	NB
NB	ZO	NS	NM	NB	NB	NB	NB

Table 1. Fuzzy rules

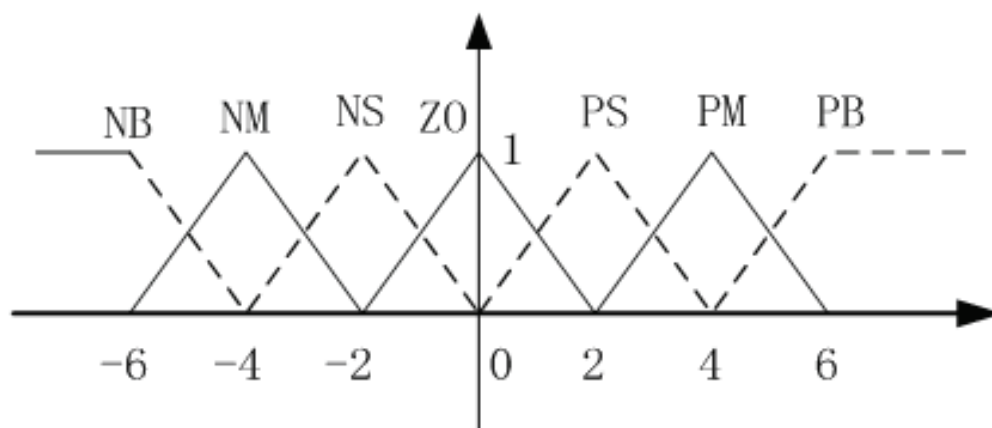


Fig. 3. The subjection function of the output  $c$

Suppose there are  $N$  “if-then” rules,

$$L^{(i)} : \text{If } x_1 \text{ is } F_1^i, \text{ and } x_2 \text{ is } F_2^i, \dots, \text{ and } x_n \text{ is } F_n^i \text{ then } y^i \text{ is } Y^i$$

for  $i = 1, \dots, N$ . Based on the constructing process, we apply the product discursion, single value fuzziness, center average defuzziness and trigonal subjection function, to establish the FLC

$$y(x) = \frac{\sum_{i=1}^N \prod_{k=1}^n \mu_{F_k^i}(x_k) y^i}{\sum_{i=1}^N \prod_{k=1}^n \mu_{F_k^i}(x_k)} \tag{4}$$

In order to comprehend the process of FLC easily, we illuminate the use of FLC by an example. Suppose the parameters  $d1=3cm, d2=4cm, d3=5cm$ , the distance between agent  $i$

and agent  $j$  is  $r_2 = 4.5\text{cm}$ , the diversification of the distance  $\Delta r_i = -2\text{cm}$ . Applying the fuzzy process,  $x_1=0.75$ ,  $x_2= -1.5$ , and the following fuzzy rules obtain from the table 1:

*If  $x_1$  is ZO with grade 0.635 and  $x_2$  is ZO with grade 0.25, then  $c$  is ZO*  
*If  $x_1$  is ZO with grade 0.635 and  $x_2$  is NS with grade 0.75, then  $c$  is NS*  
*If  $x_1$  is PS with grade 0.375 and  $x_2$  is ZO with grade 0.25, then  $c$  is PS*  
*If  $x_1$  is PS with grade 0.375 and  $x_2$  is NS with grade 0.75, then  $c$  is ZO*

Then the every rule output  $c$  is 0 , -2 , 2 , 0 by Fig.2, respectively. From Eq.(4), the last output of the FLC

$$c = \frac{0.635 * 0.25 * 0 + 0.635 * 0.75 * (-2) + 0.375 * 0.25 * 2 + 0.375 * 0.75 * 0}{0.635 * 0.25 + 0.635 * 0.75 + 0.375 * 0.25 + 0.375 * 0.75}$$

$$= -0.765$$

Corresponding the function value of the basis fields

$$\frac{c}{6} \times M = -0.1275M .$$

We can apply this output to control the moving of multi-agents by adjusting the parameter  $M$ .

#### 4. Simulations analysis

In this section, we apply the simulations to analyze the results of the formation control of heterogeneous three-agent system. Suppose the maintained distances  $d_0 = 3$ ,  $d_1 = 4$ ,  $d_2 = 5$ , the dynamical velocity of the leader (agent 0)

$$v_0 = (0.1t^2, \sin 0.2t)^T .$$

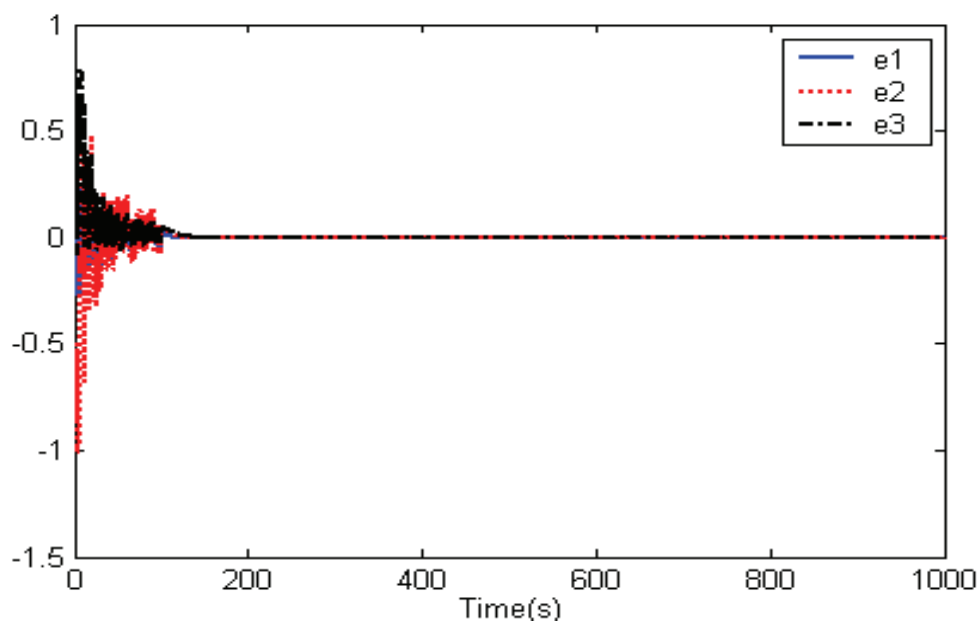


Fig. 4. Plot of errors between current distances and expected values

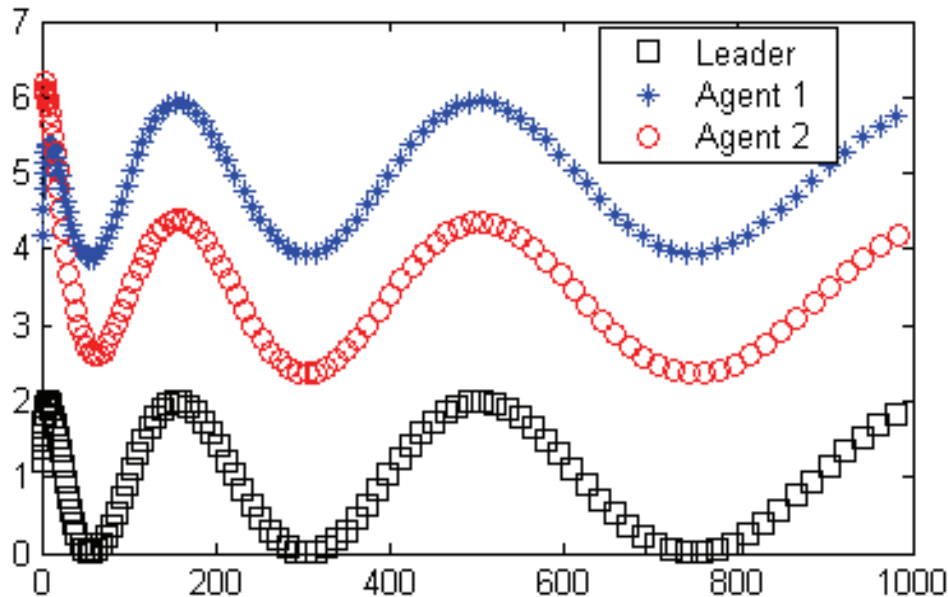


Fig. 5. Plot of agents' tracks.

Let initialized positions get randomly by computer. The errors ( $e_i = r_i - d_i$ ) between the current distance  $r_i$  and the expected value  $d_i$  are shown in Fig. 4, and the information states of the three agents in Fig 5, respectively. These figures illuminate that the distances among the three agents reach the expected values, and the synchronizations of the moving tracks have been achieved.

## 5. Conclusion

This paper studies a directed formation control problem of heterogeneous multi-agent systems. The system is composed of a leader agent and two following agents with each one required maintaining a nominated distance from its neighbors. The follower is allowed to determine its movement strategy by using local knowledge of the direction of its neighbors and the current and desired distance from its neighbors. Based on the moving mechanism, a Fuzzy logical controller for multi-agent systems with leader-following is presented, which can not only accomplish the desired triangle formation but also ensure that the followers' speeds converge to the leader's velocity without collision during the motion. Simulation results are provided to illustrate the effectiveness of the control law. The proposed Fuzzy logical controller is interesting for the design of optimization algorithms that can ensure the triangle formation that multi-agent systems are required maintaining a nominated distance.

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While several books are available today that address the mathematical and philosophical foundations of fuzzy logic, none, unfortunately, provides the practicing knowledge engineer, system analyst, and project manager with specific, practical information about fuzzy system modeling. Those few books that include applications and case studies concentrate almost exclusively on engineering problems: pendulum balancing, truck backeruppers, cement kilns, antilock braking systems, image pattern recognition, and digital signal processing. Yet the application of fuzzy logic to engineering problems represents only a fraction of its real potential. As a method of encoding and using human knowledge in a form that is very close to the way experts think about difficult, complex problems, fuzzy systems provide the facilities necessary to break through the computational bottlenecks associated with traditional decision support and expert systems. Additionally, fuzzy systems provide a rich and robust method of building systems that include multiple conflicting, cooperating, and collaborating experts (a capability that generally eludes not only symbolic expert system users but analysts who have turned to such related technologies as neural networks and genetic algorithms). Yet the application of fuzzy logic in the areas of decision support, medical systems, database analysis and mining has been largely ignored by both the commercial vendors of decision support products and the knowledge engineers who use them.

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