Sensitivity of Safe Game Ship Control on Base Information from ARPA Radar

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1. Introduction

The problem of non-collision strategies in the steering at sea appeared in the Isaacs works (Isaacs, 1965) called "the father of the differential games" and was developed by many authors both within the context of the game theory (Engwerda, 2005; Nowak & Szajowski, 2005), and also in the control under uncertainty conditions (Nisan et al., 2007). The definition of the problem of avoiding a collision seems to be quite obvious, however, apart from the issue of the uncertainty of information which may be a result of external factors (weather conditions, sea state), incomplete knowledge about other ships and imprecise nature of the recommendations concerning the right of way contained in International Regulations for Preventing Collision at Sea (COLREG) (Cockcroft & Lameijer, 2006). The problem of determining safe strategies is still an urgent issue as a result of an ever increasing traffic of ships on particular water areas. It is also important due to the increasing requirements as to the safety of shipping and environmental protection, from one side, and to the improving opportunities to use computer supporting the navigator duties (Bist, 2000; Gluver & Olsen, 1998). In order to ensure safe navigation the ships are obliged to observe legal requirements contained in the COLREG Rules. However, these Rules refer exclusively to two ships under good visibility conditions, in case of restricted visibility the Rules provide only recommendations of general nature and they are unable to consider all necessary conditions of the real process. Therefore the real process of the ships passing exercises occurs under the conditions of indefiniteness and conflict accompanied by an imprecise co-operation among the ships in the light of the legal regulations. A necessity to consider simultaneously the strategies of the encountered ships and the dynamic properties of the ships as control objects is a good reason for the application of the differential game model - often called the dynamic game (Osborne, 2004; Straffin, 2001).

2. Safe ship control

2.1 Integrated of navigation

The control of the ship's movement may be treated as a multilevel problem shown on Figure 1, which results from the division of entire ship control system, into clearly determined subsystems which are ascribed appropriate layers of control (Lisowski, 2007a), (Fig. 1).
This is connected both with a large number of dimensions of the steering vector and of the status of the process, its random, fuzzy and decision making characteristics - which are

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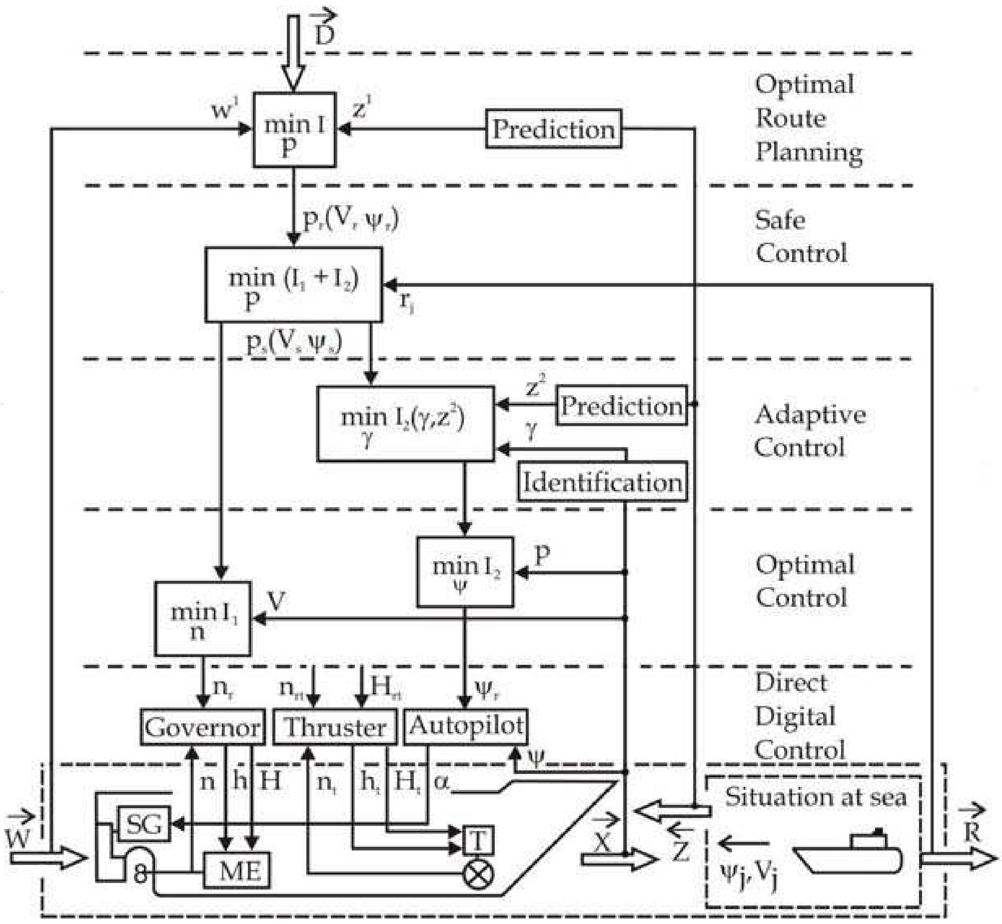


Fig. 1. Multilevel ship movement steering system

affected by strong interference generated by the current, wind and the sea wave motion on the one hand, and a complex nature of the equations describing the ship's dynamics with non-linear and non-stationary characteristics. The determination of the global control of the steering systems has in practice become too costly and ineffective (Lisowski 2002).

The integral part of the entire system is the process of the ship's movement control, which may be described with appropriate differential equations of the kinematics and dynamics of a ship being an object of the control under a variety of the ship's operational conditions such as:

- stabilisation of the course or trajectory,
- adjustment of the ship's speed,
- precise steering at small speeds in port with thrusters or adjustable-pitch propeller,
- stabilisation of the ship's rolling,
- commanding the towing group,
- dynamic stabilisation of the drilling ship's or the tanker's position.

The functional draft of the system corresponds to a certain actual arrangement of the equipment. The increasing demands with regard to the safety of navigation are forcing the ship's operators to install the systems of integrated navigation on board their ships. By improving the ship's control these systems increase the safety of navigation of a ship - which is a very expensive object of the value, including the cargo, and the effectiveness of the carriage goods by sea (Cymbal et al., 2007; Lisowski, 2005a, 2007b).

2.2 ARPA anti-collision radar system of acquisition and tracking

The challenge in research for effective methods to prevent ship collisions has become important with the increasing size, speed and number of ships participating in sea carriage. An obvious contribution in increasing safety of shipping has been firstly the application of radars and then the development of ARPA (Automatic Radar Plotting Aids) anti-collision system (Bole et al., 2006; Cahill, 2002), (Fig. 2).

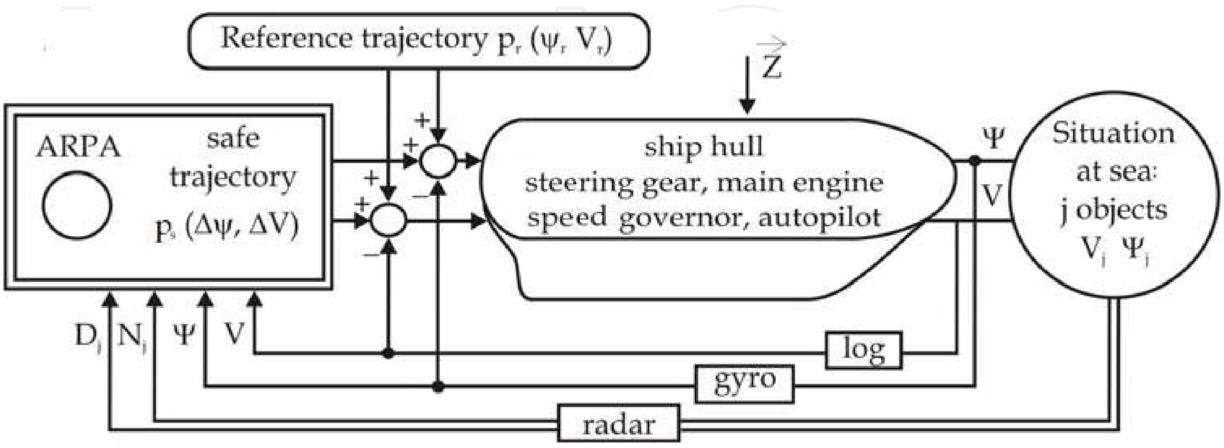


Fig. 2. The structure of safe ship control system

The ARPA system enables to track automatically at least 20 encountered j objects as is shown on Figure 3, determination of their movement parameters (speed V_j, course ψ_j) and elements of approach to the own ship ($D^j_{min} = DCPA_j$ - Distance of the Closest Point of Approach, $T^j_{min} = TCPA_j$ - Time to the Closest Point of Approach) and also the assessment of the collision risk r_j (Lisowski, 2001, 2008a).

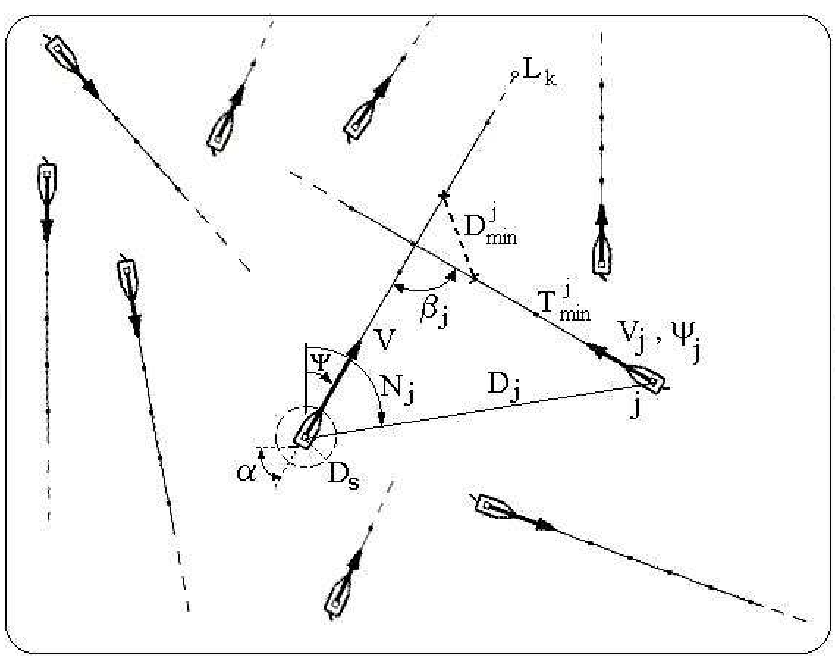


Fig. 3. Navigational situation representing the passing of the own ship with the j-th object

The risk value is possible to define by referring the current situation of approach, described by parameters D^j_{min} and T^j_{min}, to the assumed evaluation of the situation as safe, determined by a safe distance of approach D_s and a safe time T_s – which are necessary to execute a collision avoiding manoeuvre with consideration of distance D_j to j-th met object - shown on Figure 4 (Lisowski, 2005b, 2008c):

$$r_j = \left[k_1 \left(\frac{D^j_{min}}{D_s} \right)^2 + k_2 \left(\frac{T^j_{min}}{T_s} \right)^2 + \left(\frac{D_j}{D_s} \right)^2 \right]^{-\frac{1}{2}} \tag{1}$$

The weight coefficients k_1 and k_2 are depended on the state visibility at sea, dynamic length L_d and dynamic beam B_d of the ship, kind of water region and in practice are equal:

$$0 \le [k_1(L_d, B_d),\ k_2(L_d, B_d)] \le 1 \tag{2}$$

$$L_d = 1.1\,(1 + 0.345\ V^{1.6}) \tag{3}$$

$$B_d = 1.1\,(B + 0.767\ LV^{0.4}) \tag{4}$$

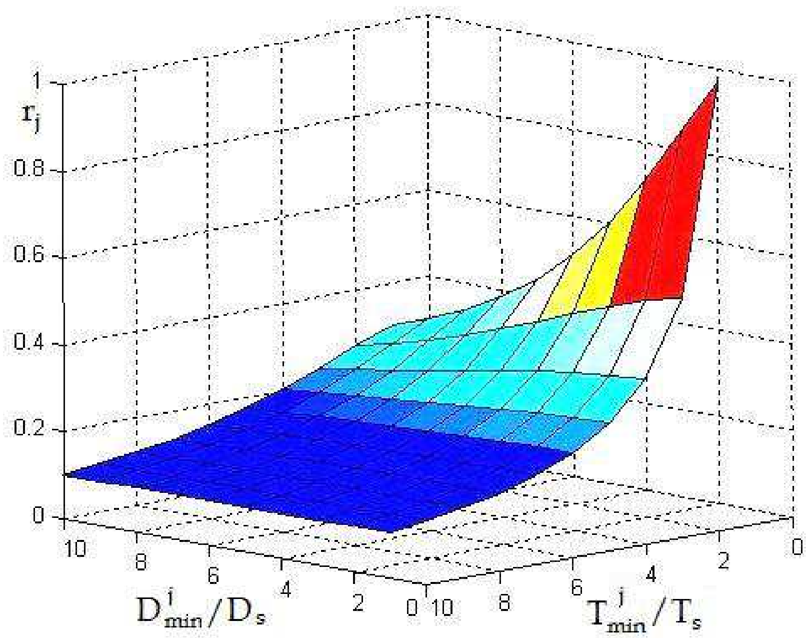


Fig. 4. The ship's collision risk space in a function of relative distance and time of approaching the j-th object

2.3 ARPA anti-collision radar system of manoeuvre simulation

The functional scope of a standard ARPA system ends with the simulation of the manoeuvre altering the course $\pm\Delta\psi$ or the ship's speed $\pm\Delta V$ selected by the navigator as is shown on Figure 5 (Pasmurow & Zimoviev, 2005).

2.4 Computer support of navigator manoeuvring decision

The problem of selecting such a manoeuvre is very difficult as the process of control is very complex since it is dynamic, non-linear, multi-dimensional, non-stationary and game making in its nature.

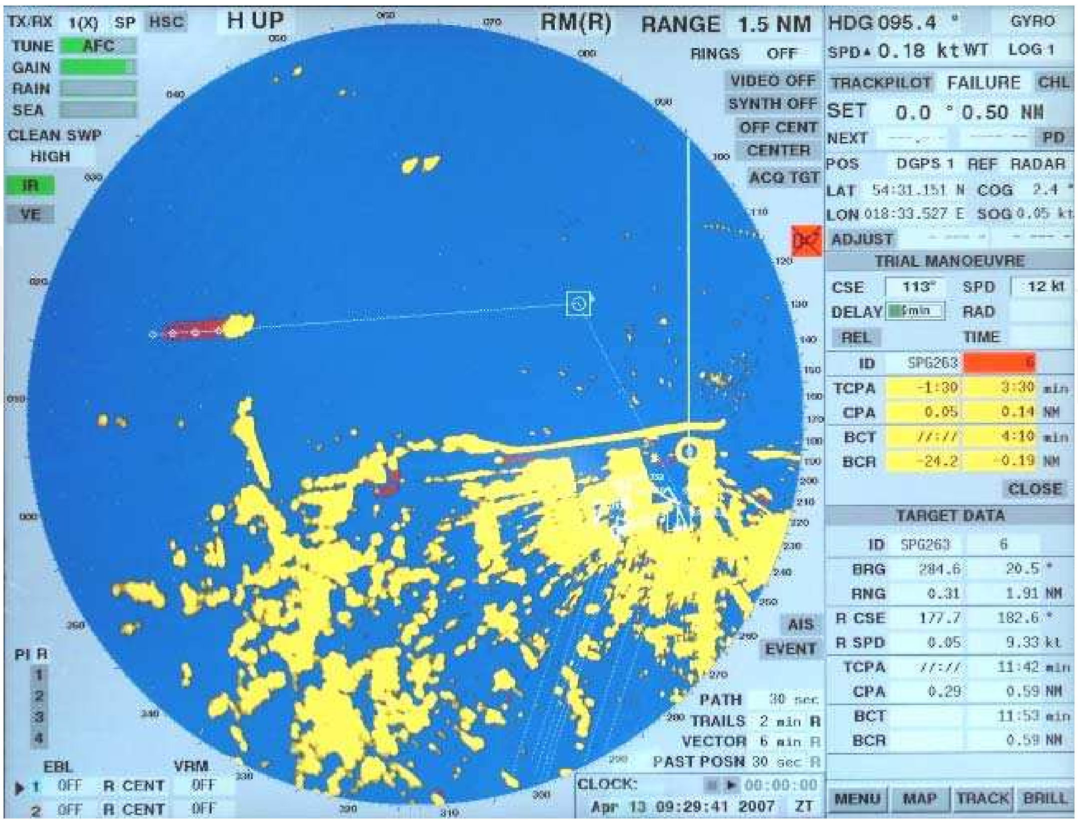


Fig. 5. The screen of SAM Electronics ARPA on the sailing vessel s/v DAR MLODZIEZY (ENAMOR Gdynia, Poland)

In practice, methods of selecting a manoeuvre assume a form of appropriate steering algorithms supporting navigator decision in a collision situation. Algorithms are programmed into the memory of a Programmable Logic Controller PLC (Fig. 6). This generates an option within the ARPA anti-collision system or a training simulator (Lisowski, 2008a).

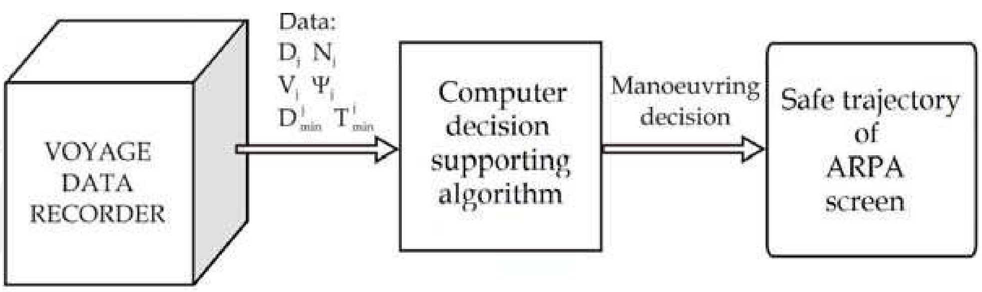


Fig. 6. The system structure of computer support of navigator decision in collision situation

3. Game control in marine navigation

3.1 Processes of game ship control

The classical issues of the theory of the decision process in marine navigation include the safe steering of a ship (Baba & Jain 2001; Levine, 1996).

Assuming that the dynamic movement of the ships in time occurs under the influence of the appropriate sets of control:

$x_{j,1} = D_j$ – distance to j-th ship,

$x_{j,2} = N_j$ – bearing of the j-th ship,

$x_{j,3} = \psi_j$ – course of the j-th ship,

$x_{j,4} = \dot{\psi}_j$ – angular turning speed of the j-th ship,

$x_{j,5} = V_j$ – speed of the j-th ship,

where: $\vartheta_0 = 6$, $\vartheta_j = 4$.

While the control values are represented by:

$u_{0,1} = \alpha_r$ – rudder angle of the own ship,

$u_{0,2} = n_r$ – rotational speed of the own ship screw propeller,

$u_{0,3} = H_r$ – pitch of the adjustable propeller of the own ship,

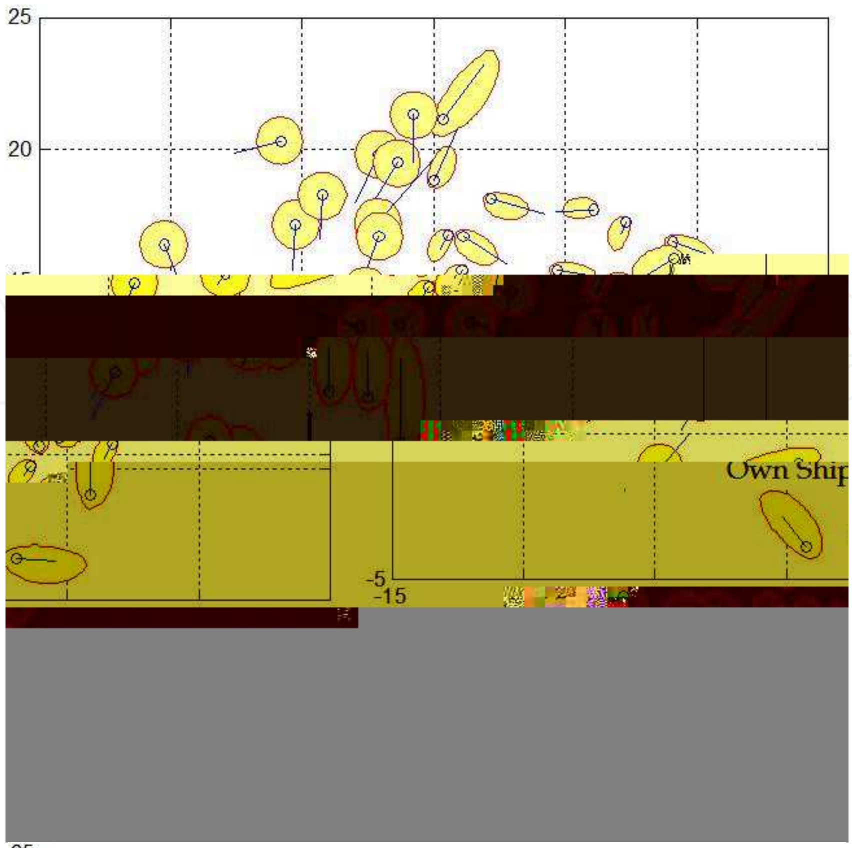
$u_{j,1} = \alpha_{r,j}$ – rudder angle of the j-th ship,

$u_{j,2} = n_{r,j}$ – rotational speed of the j-th ship screw propeller,

where: $\nu_0 = 3$, $\nu_j = 2$.

Values of coefficients of the process state equations (8) for the 12 000 DWT container ship are given in Table 1.

Coefficient	Measure	Value
a_1	m^{-1}	$-4.143 \cdot 10^{-2}$
a_2	m^{-2}	$1.858 \cdot 10^{-4}$
a_3	m^{-1}	$-6.934 \cdot 10^{-3}$
a_4	m^{-1}	$-3.177 \cdot 10^{-2}$
a_5	–	-4.435
a_6	–	-0.895
a_7	m^{-1}	$-9.284 \cdot 10^{-4}$
a_8	–	$1.357 \cdot 10^{-3}$
a_9	–	0.624
a_{10}	s^{-1}	-0.200
a_{11+j}	s^{-1}	$-5 \cdot 10^{-2}$
a_{12+j}	s^{-1}	$-4 \cdot 10^{-3}$
b_1	m^{-2}	$1.134 \cdot 10^{-2}$
b_2	m^{-1}	$-1.554 \cdot 10^{-3}$
b_3	s^{-1}	0.200
b_4	s^{-1}	0.100



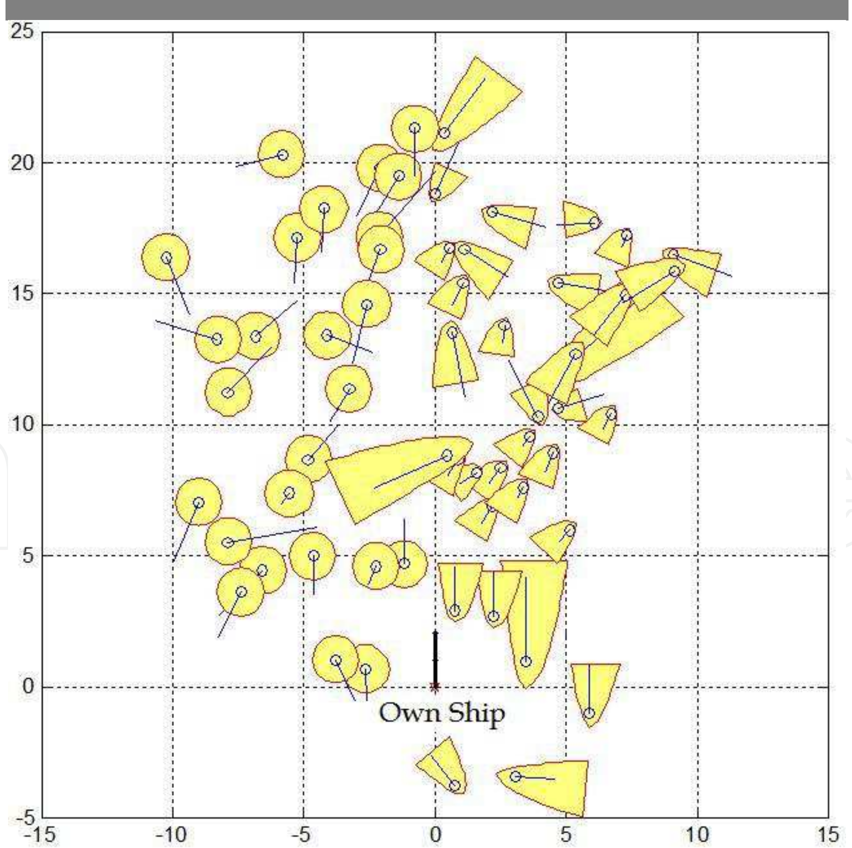


Fig. 8. The shapes of the neural domains in the situation of 60 encountered ships in English Channel

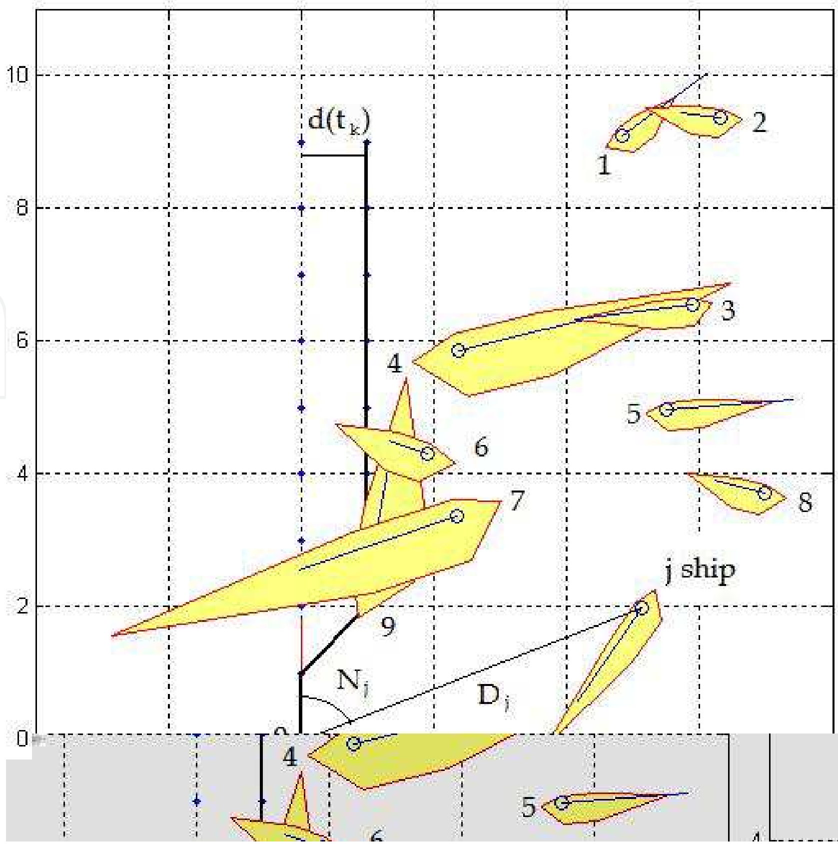


Fig. 9. Navigational situation representing the passing of the own ship with the j-th encountered ship

The application of reductions in the description of the own ship dynamics and the dynamic of the j-th encountered ship and their movement kinematics lead to approximated models: multi-stage positional game, multi-step matrix game, fuzzy matrix game, fuzzy dynamic programming, dynamic programming with neural state constraints, linear programming (LaValle, 2006; Lisowski, 2004).

4. Algorithms of safe game ship control

4.1 Multi-stage positional game trajectory POSTRAJ

The general model of dynamic game is simplified to the multi-stage positional game of j participants not co-operating among them, (Fig. 10).

State variables and control values are represented by:

$$\left.\begin{array}{r} x_{0,1} = X_0,\ x_{0,2} = Y_0,\ x_{j,1} = X_j,\ x_{j,2} = Y_j \\ u_{0,1} = \psi,\ \ u_{0,2} = V,\ \ u_{j,1} = \psi_j,\ \ u_{j,2} = V_j \\ j = 1, 2, \ldots, m \end{array}\right\} \quad (11)$$

The essence of the positional game is to subordinate the strategies of the own ship to the current positions $p(t_k)$ of the encountered objects at the current step *k*. In this way the process model takes into consideration any possible alterations of the course and speed of the encountered objects while steering is in progress. The current state of the process is determined by the co-ordinates of the own ship's position and the positions of the encountered objects:

$$\left.\begin{array}{l} x_0 = (X_0,\ Y_0),\ x_j = (X_j,\ Y_j) \\ j = 1, 2, ..., m \end{array}\right\} \tag{12}$$

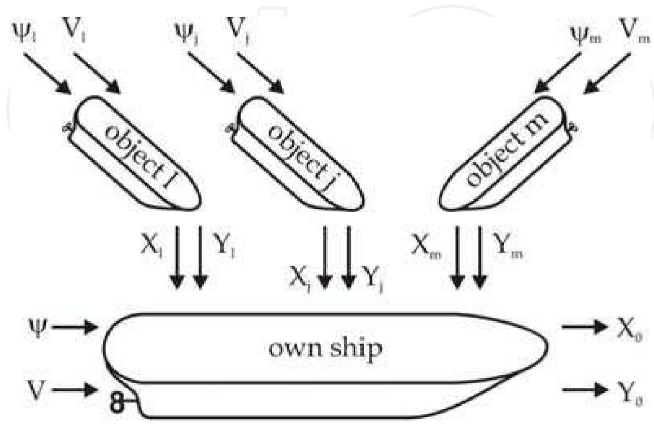


Fig. 10. Block diagram of the positional game model

The system generates its steering at the moment t_k on the basis of data received from the ARPA anti-collision system pertaining to the positions of the encountered objects:

$$p(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \quad j = 1, 2,..., m \quad k = 1, 2,..., K \tag{13}$$

It is assumed, according to the general concept of a multi-stage positional game, that at each discrete moment of time t_k the own ship knows the positions of the objects.
The constraints for the state co-ordinates:

$$\left\{ x_0(t), x_j(t) \right\} \in P \tag{14}$$

are navigational constraints, while steering constraints:

$$u_0 \in U_0,\ u_j \in U_j \quad j = 1, 2,..., m \tag{15}$$

take into consideration: the ships' movement kinematics, recommendations of the COLREG Rules and the condition to maintain a safe passing distance as per relationship (6).
The closed sets U_0^j and U_j^0, defined as the sets of acceptable strategies of the participants to the game towards one another:

$$\{U_0^j[p(t)],\ U_j^0[p(t)]\} \tag{16}$$

are dependent, which means that the choice of steering u_j by the j-th object changes the sets of acceptable strategies of other objects.
A set U_o^j of acceptable strategies of the own ship when passing the j-th encountered object at a distance D_s - while observing the condition of the course and speed stability of the own ship and that of the encountered object at step k is static and comprised within a half-circle of a radius V_r (Fig. 11).

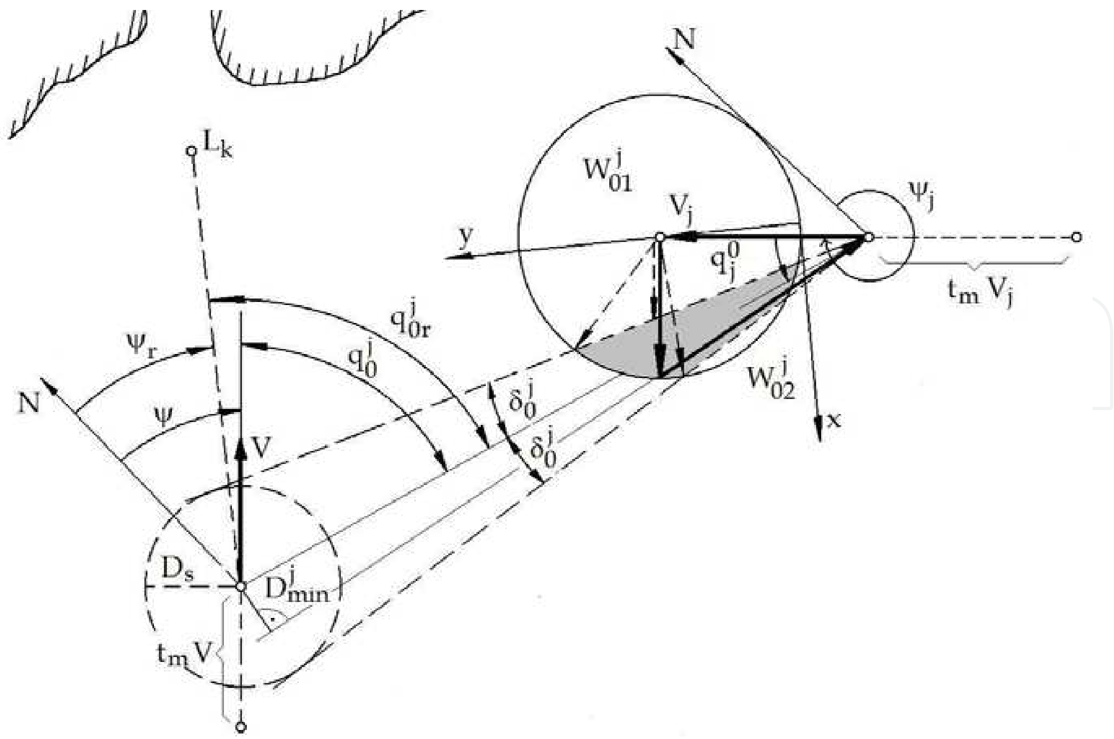


Fig. 11. Determination of the acceptable areas of the own ship strategies $U_0^j = W_{01}^j \cup W_{02}^j$

Area U_0^j is determined by an inequality (Fig. 12):

$$a_0^j \; u_0^x + b_0^j \; u_0^y \le c_0^j \tag{17}$$

$$\left(u_0^x\right)^2 + \left(u_0^y\right)^2 \le V_r^2 \tag{18}$$

where:

$$\left.\begin{array}{l}
\vec{V}_r = \vec{u}_0(u_0^x, u_0^y) \\
a_0^j = -\chi_0^j \cos(q_{0r}^j + \chi_0^j \, \delta_0^j) \\
b_0^j = \chi_0^j \sin(q_{0r}^j + \chi_0^j \, \delta_0^j) \\
c_0^j = -\chi_0^j \begin{bmatrix} V_j \sin(q_j^0 + \chi_0^j \, \delta_0^j) + \\ V_r \cos(q_{0r}^j + \chi_0^j \, \delta_0^j) \end{bmatrix} \\
\chi_0^j = \begin{cases} 1 \;\; \text{dla} \;\; W_{01}^j \;\; \text{(Starboard side)} \\ -1 \;\; \text{dla} \;\; W_{02}^j \;\; \text{(Port side)} \end{cases}
\end{array}\right\} \tag{19}$$

The value χ_0^j is determined by using an appropriate logical function Z_j characterising any particular recommendation referring to the right of way contained in COLREG Rules.
The form of function Z_j depends of the interpretation of the above recommendations for the purpose to use them in the steering algorithm, when:

$$Z_j = \begin{cases} 1 \;\; \text{then} \;\; \chi_0^j = \;\; 1 \\ 0 \;\; \text{then} \;\; \chi_0^j = -1 \end{cases} \tag{20}$$

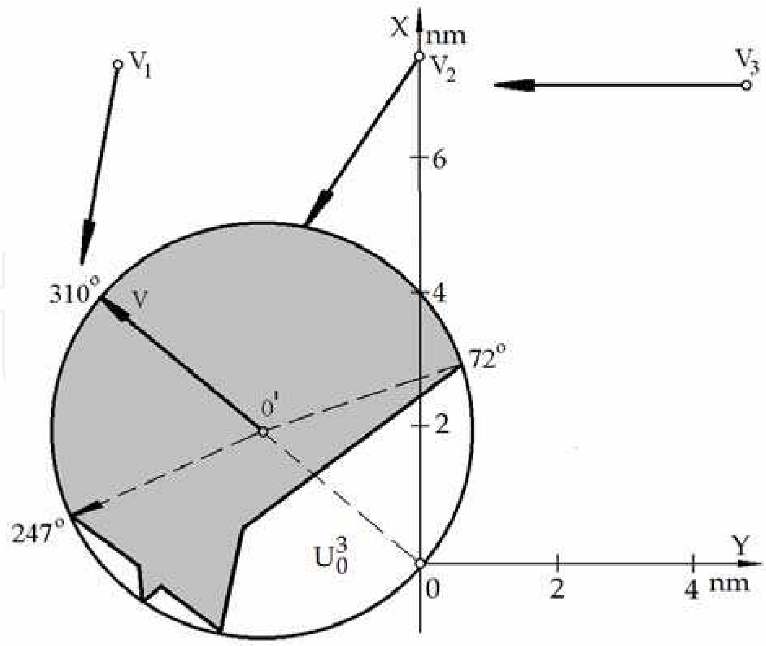


Fig. 12. Example of summary set U_0^3 of acceptable manoeuvres for three encountered ships

Interpretation of the COLREG Rules in the form of appropriate manoeuvring diagrams developed by A.G. Corbet, S.H. Hollingdale, E.S. Calvert and K.D. Jones enables to formulate a certain logical function Z_j as a semantic interpretation of legal regulations for manoeuvring.
Each particular type of the situation involving the approach of the ships is assigned the logical variable value equal to one or zero:
A – encounter of the ship from bow or from any other direction,
B – approaching or moving away of the ship,
C – passing the ship astern or ahead,
D – approaching of the ship from the bow or from the stern,
E – approaching of the ship from the starboard or port side.
By minimizing logical function Z_j by using a method of the Karnaugh's Tables the following is obtained:

$$Z_j = A \cup \overline{A}(\overline{B}\,\overline{C} \cup \overline{D}\,\overline{E}) \tag{21}$$

The resultant area of acceptable manoeuvres for m objects:

$$U_0 = \bigcap_{j=1}^{m} U_0^j \qquad j = 1, 2, \ldots, m \tag{22}$$

is determined by an arrangement of inequalities (17) and (18).
A set for acceptable strategies U_j^0 of the encountered j-th object relative to the own ship is determined by analogy:

$$a_j^0\, u_j^x + b_j^0\, u_j^y \le c_j^0 \tag{23}$$

$$\left(u_j^x\right)^2 + \left(u_j^y\right)^2 \le V_j^2 \tag{24}$$

where:

$$\left.\begin{aligned} \vec{V}_j &= \vec{u}_j(u_j^x, u_j^y) \\ a_j^0 &= -\chi_j^0 \cos(q_j^0 + \chi_j^0 \delta_j^0) \\ b_j^0 &= \chi_j^0 \sin(q_j^0 + \chi_j^0 \delta_j^0) \\ c_j^0 &= -\chi_j^0 V_0 \sin(q_0^j + \chi_j^0 \delta_j^0) \end{aligned}\right\} \tag{25}$$

The sing χ_j^0 is determined analogically to χ_0^j.

Taking into consideration of navigational constraints – shoal and shore line, presents additional constraints of the set of acceptable strategies:

$$a_0^{l,l-1} u_0^x + b_0^{l,l-1} u_0^y \le c_0^{l,l-1} \tag{26}$$

where: l – the closest point of intersection for the straight lines approximating the shore line (Cichuta & Dalecki, 2000).

The optimal steering of the own ship $u_0^*(t)$, equivalent for the current position p(t) to the optimal positional steering $u_0^*(p)$, is determined in the following way:

- sets of acceptable strategies $U_j^0[p(t_k)]$ are determined for the encountered objects relative to the own ship and initial sets $U_0^{jw}[p(t_k)]$ of acceptable strategies of the own ship relative to each one of the encountered objects,
- a pair of vectors u_j^m and u_0^j relative to each j-th object is determined and then the optimal positional strategy for the own ship $u_0^*(p)$ from the condition:

$$I^* = \left\{ \min_{u_0 \in U_0 = \bigcap_{j=1}^{m} U_0^j} \left\{ \max_{u_j^m \in U_j} \min_{u_0^j \in U_0^j(u_j)} S_0[x_0(t_k), L_k] \right\} = S_0^*(x_0, L_k) \quad U_0^j \subset U_0^{jw} \quad j = 1, 2, ..., m \right\} \tag{27}$$

where:

$$S_0[x_0(t), L_k] = \int_{t_0}^{t_{L_k}} u_0(t)\,dt \tag{28}$$

refers to the continuous function of the own ship's steering goal which characterises the ship's distance at the moment t_0 to the closest point of turn L_k on the assumed voyage route (Fig. 3).

In practice, the realization of the optimal trajectory of the own ship is achieved by determining the ship's course and speed, which would ensure the smallest loss of way for a safe passing of the encountered objects, at a distance which is not smaller than the assumed value D_s, always with respect to the ship's dynamics in the form of the advance time to the manoeuvre t_m, with element $t_m^{\Delta\psi}$ during course manoeuvre $\Delta\psi$ or element $t_m^{\Delta V}$ during speed manoeuvre ΔV (Fig. 13).

The dynamic features of the ship during the course alteration by an angle $\Delta\psi$ is described in a simplified manner with the use of transfer function:

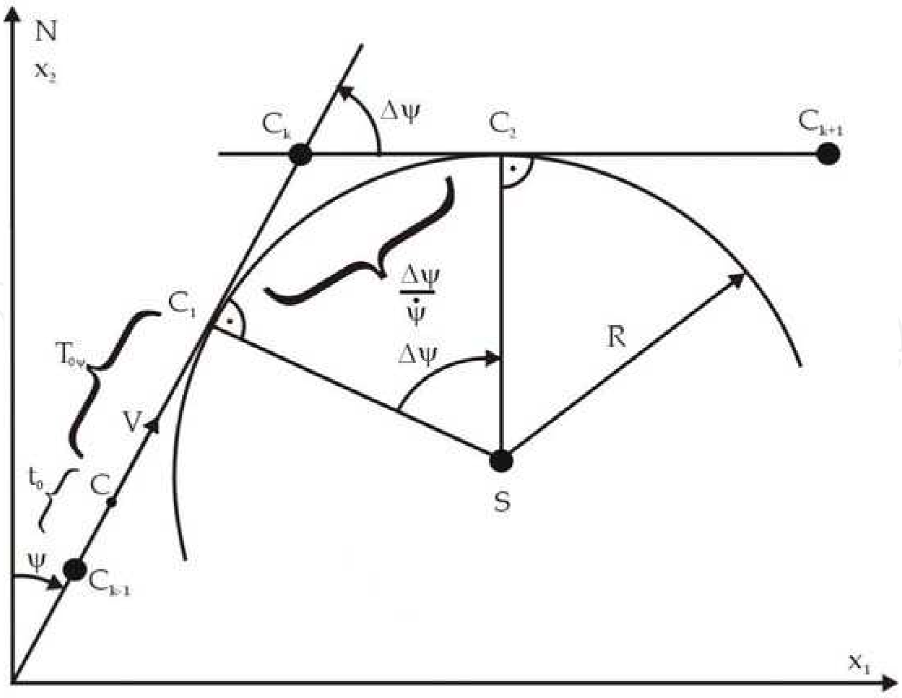


Fig. 13. Ship's motion during Δψ course changing

$$G_1(s) = \frac{\Delta\psi(s)}{\alpha(s)} = \frac{k_\psi(\alpha)}{s(1+T_\psi s)} \cong \frac{k_\psi(\alpha)\cdot e^{-T_{o\psi}s}}{s} \tag{29}$$

where:
T_ψ - manoeuvre delay time which is approximately equal to the time constant of the ship as a course control object,
$k_\psi(\alpha)$ - gain coefficient the value of which results from the non-linear static characteristics of the rudder steering.
The course manoeuvre delay time:

$$t_m^{\Delta\psi} \cong T_{o\psi} + \frac{\Delta\psi}{\dot{\psi}} \tag{30}$$

Differential equation of the second order describing the ship's behaviour during the change of the speed by ΔV is approximated with the use of the first order inertia with a delay:

$$G_2(s) = \frac{\Delta V(s)}{\Delta n(s)} = \frac{k_V e^{-T_{oV}s}}{1+T_V s} \tag{31}$$

where:
T_{oV} - time of delay equal approximately to the time constant for the propulsion system: main engine - propeller shaft – screw propeller,
T_V - the time constant of the ship's hull and the mass of the accompanying water.
The speed manoeuvre delay time is as follows:

$$t_m^{\Delta V} \cong T_{oV} + 3\,T_V \tag{32}$$

The smallest loss of way is achieved for the maximum projection of the speed vector maximum of the own ship on the direction of the assumed course ψ_r. The optimal steering of the own ship is calculated at each discrete stage of the ship's movement by applying Simplex method for solving the linear programming task.
At each one stage t_k of the measured position $p(t_k)$ optimal steering problem is solved according to the game control principle (27) (Fig. 14).
By using function *lp – linear programming* from Optimization Toolbox of the MATLAB software POSTRAJ algorithm was developed to determine a safe game trajectory of a ship in a collision situation (Łebkowski, 2001).

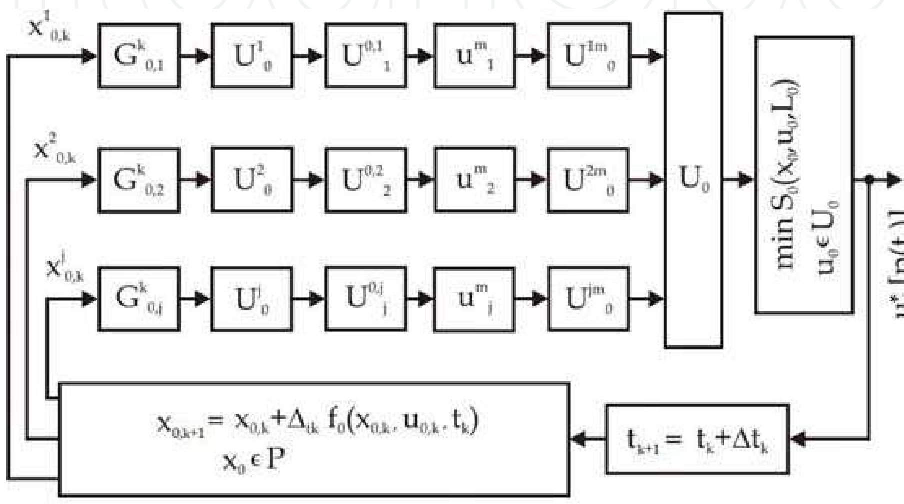


Fig. 14. Block diagram of the positional pattern for positional game steering: $G^k_{0,j}$ - a set of parameters of the own ship approach relative to j-th object taken from ARPA radar system

4.2 Multi-step matrix game trajectory RISKTRAJ

When leaving aside the ship's dynamics equations the general model of a dynamic game for the process of preventing collisions is reduced to the matrix game of j participants non-co-operating among them (Fig. 15).

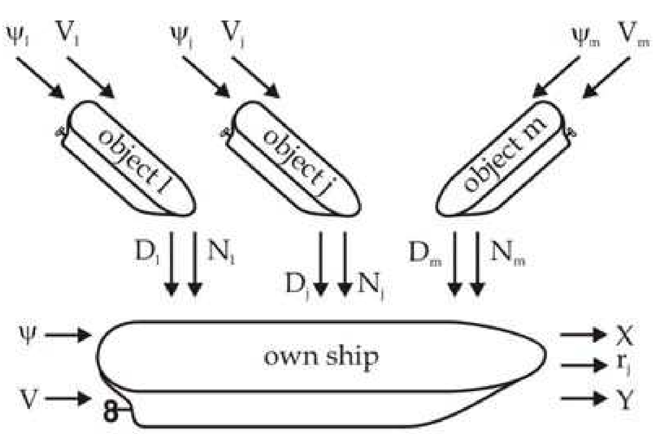


Fig. 15. Block diagram of a matrix game model

The state and steering variables are represented by the following values:

$$x_{j,1} = D_j,\ x_{j,2} = N_j,\ u_{0,1} = \psi,\ u_{0,2} = V,\ u_{j,1} = \psi_j,\ u_{j,2} = V_j \qquad j = 1, 2, ..., m \tag{33}$$

The game matrix $\mathbf{R}[r_j(\upsilon_j, \upsilon_0)]$ includes the values of the collision risk r_j determined on the basis of data obtained from the ARPA anti-collision system for the acceptable strategies υ_0 of the own ship and acceptable strategies υ_j of any particular number of *j* encountered objects. The risk value is defined by equation (1). In a matrix game player I - own ship has a possibility to use υ_0 pure various strategies, and player II - encountered ships has υ_j various pure strategies:

$$R = [r_j(\upsilon_j, \upsilon_0)] = \begin{vmatrix} r_{11} & r_{12} & \cdots & r_{1,\upsilon_0-1} & r_{1\upsilon_n} \\ r_{21} & r_{22} & \cdots & r_{2,\upsilon_0-1} & r_{2\upsilon_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\upsilon_1 1} & r_{\upsilon_1 2} & \cdots & r_{\upsilon_1,\upsilon_0-1} & r_{\upsilon_1\upsilon_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\upsilon_j 1} & r_{\upsilon_j 2} & \cdots & r_{\upsilon_j,\upsilon_0-1} & r_{\upsilon_j\upsilon_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\upsilon_m 1} & r_{\upsilon_m 2} & \cdots & r_{\upsilon_m,\upsilon_0-1} & r_{\upsilon_m\upsilon_n} \end{vmatrix} \tag{34}$$

The constraints for the choice of a strategy (υ_0, υ_j) result from the recommendations of the way priority at sea (Radzik, 2000). Constraints are limiting the selection of a strategy result from COLREG Rules. As most frequently the game does not have a saddle point, therefore the balance state is not guaranteed. In order to solve this problem we may use a dual linear programming.

In a dual problem player I aims to minimize the risk of collision, while player II aims to maximize the collision risk. The components of the mixed strategy express the distribution of the probability of using by the players their pure strategies. As a result for the goal control function in the form:

$$\left(I_0^{(j)}\right)^* = \min_{\upsilon_0} \max_{\upsilon_j} r_j \tag{35}$$

probability matrix **P** of applying each one of the particular pure strategies is obtained:

$$P = [p_j(\upsilon_j, \upsilon_0)] = \begin{vmatrix} p_{11} & p_{12} & \cdots & p_{1,\upsilon_0\;1} & p_{1\upsilon_n} \\ p_{21} & p_{22} & \cdots & p_{2,\upsilon_0\;1} & p_{2\upsilon_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{\upsilon_1 1} & p_{\upsilon_1 2} & \cdots & p_{\upsilon_1,\upsilon_0\;1} & p_{\upsilon_1\upsilon_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{\upsilon_j 1} & p_{\upsilon_j 2} & \cdots & p_{\upsilon_j,\upsilon_0\;1} & p_{\upsilon_j\upsilon_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ p_{\upsilon_m 1} & p_{\upsilon_m 2} & \cdots & p_{\upsilon_m,\upsilon_0\;1} & p_{\upsilon_m\upsilon_n} \end{vmatrix} \tag{36}$$

The solution for the control problem is the strategy representing the highest probability:

$$\left(u_0^{(\upsilon_0)}\right)^* = u_o^{(\upsilon_0)} \left\{ \left[p_j\left(\upsilon_j, \upsilon_o\right)\right]_{max} \right\} \tag{37}$$

The safe trajectory of the own ship is treated as a sequence of successive changes in time of her course and speed. A safe passing distance is determined for the prevailing visibility conditions at sea D_s, advance time to the manoeuvre t_m described by equations (30) or (32) and the duration of one stage of the trajectory Δt_k as a calculation step. At each one step the most dangerous object relative to the value of the collision risk r_j is determined. Then, on the basis of semantic interpretation of the COLREG Rules, the direction of the own ship's turn relative to the most dangerous object is selected.

A collision risk matrix **R** is determined for the acceptable strategies of the own ship υ_0 and that for the j-th encountered object υ_j. By applying a principle of the dual linear programming for solving matrix games the optimal course of the own ship and that of the j-th object is obtained at a level of the smallest deviations from their initial values.

Figure 16 shows an example of possible strategies of the own ship and those of the encountered object while, Figure 17 presents the hyper surface of the collision risk for these values of the strategy.

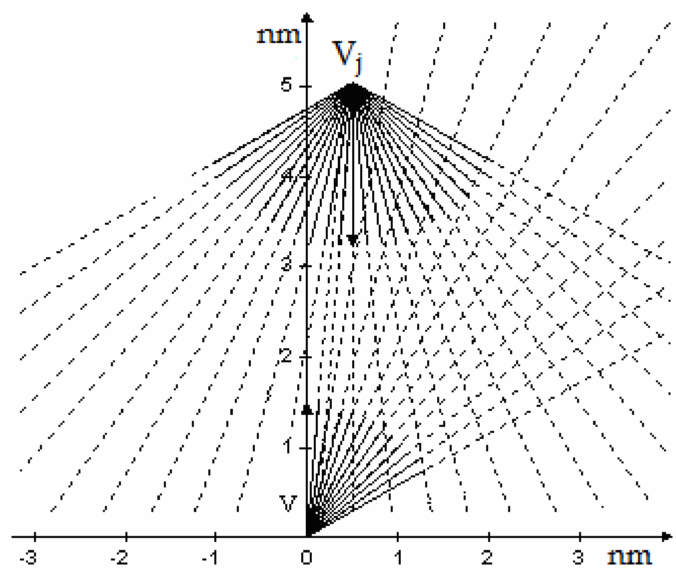


Fig. 16. Possible mutual strategies of the own ship and those of the encountered ship

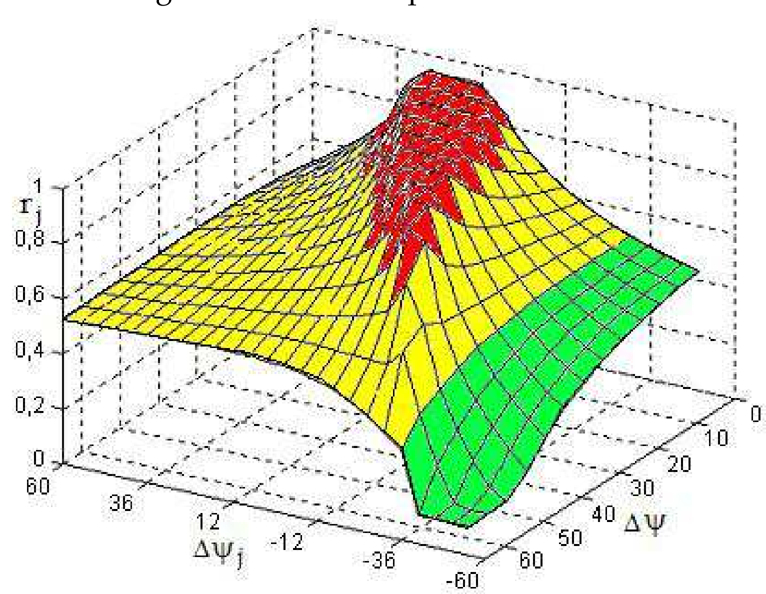


Fig. 17. Dependence of the collision risk on the course strategies of the own ship and those of the encountered ship

If, at a given step, there is no solution at the own ship's speed V, then the calculations are repeated for a speed decreased by 25%, until the game has been solved. The calculations are repeated step by step until the moment when all elements of the matrix **R** are equal to zero and the own ship, after having passed encountered objects, returns to her initial course and speed.
By using function *lp – linear programming* from Optimization Toolbox of the MATLAB software RISKTRAJ algorithm was developed to determine a safe game trajectory of a ship in a collision situation (Cichuta & Dalecki, 2000).

5. Sensitivity of game ship control

5.1 Definition of sensitivity

The investigation of sensitivity of game control fetch for sensitivity analysis of the game final payment (10) measured with the relative final deviation of $d(t_k)=d_k$ safe game trajectory from the reference trajectory, as sensitivity of the quality first-order (Wierzbicki, 1977). Taking into consideration the practical application of the game control algorithm for the own ship in a collision situation it is recommended to perform the analysis of sensitivity of a safe control with regard to the accuracy degree of the information received from the anti-collision ARPA radar system on the current approach situation, from one side and also with regard to the changes in kinematical and dynamic parameters of the control process.
Admissible average errors, that can be contributed by sensors of anti-collision system can have following values for:

- radar,
  - bearing: ±0,22°,
  - form of cluster: ±0,05°,
  - form of impulse: ±20 m,
  - margin of antenna drive: ±0,5°,
  - sampling of bearing: ±0,01°,
  - sampling of distance: ±0,01 nm,
- gyrocompas: ±0,5°,
- log: ±0,5 kn,
- GPS: ±15 m.

The sum of all errors, influent on picturing of the navigational situation, cannot exceed for absolute values ±5% or for angular values ±3°.

5.2 Sensitivity of control to inaccuracy of information from ARPA radar

Let $X_{0,j}$ represent such a set of state process control information on the navigational situation that:

$$X_{0,j} = \{V, \psi, V_j, \psi_j, D_j, N_j\} \tag{38}$$

Let then $X_{0,j}^{ARPA}$ represent a set of information from ARPA anti-collision system impaired by measurement and processing errors:

$$X_{0,j}^{ARPA} = \{V \pm \delta V, \psi \pm \delta\psi, V_j \pm \delta V_j, \psi_j \pm \delta\psi_j, D_j \pm \delta D_j, N_j \pm \delta N_j\} \tag{39}$$

Relative measure of sensitivity of the final payment in the game s_{inf} as a final deviation of the ship's safe trajectory d_k from the reference trajectory will be:

$$s_{inf} = (X_{0,j}^{ARPA}, X_{0,j}) = \frac{d_k^{ARPA}(X_{o,j}^{ARPA})}{d_k(X_{0,j})}$$
$$s_{inf} = \{s_{inf}^{V}, s_{inf}^{\psi}, s_{inf}^{V_j}, s_{inf}^{\psi_j}, s_{inf}^{D_j}, s_{inf}^{N_j}\} \qquad (40)$$

5.3 Sensitivity of control to process control parameters alterations

Let X_{param} represents a set of parameters of the state process control:

$$X_{param} = \{t_m, D_s, \Delta t_k, \Delta V\} \qquad (41)$$

Let then X'_{param} represents a set of information saddled errors of measurement and processing parameters:

$$X'_{param} = \{t_m \pm \delta t_m, D_s \pm \delta D_s, \Delta t_k \pm \delta \Delta t_k, \Delta V \pm \delta \Delta V\} \qquad (42)$$

Relative measure of sensitivity of the final payment in the game as a final deflection of the ship's safe trajectory d_k from the assumed trajectory will be:

$$s_{dyn} = (X'_{param}, X_{param}) = \frac{d'_k(X'_{param})}{d_k(X_{param})}$$
$$s_{dyn} = \{s_{dyn}^{t_m}, s_{dyn}^{D_s}, s_{dyn}^{\Delta t_k}, s_{dyn}^{\Delta V}\} \qquad (43)$$

where:
t_m - advance time of the manoeuvre with respect to the dynamic properties of the own ship,
Δt_k - duration of one stage of the ship's trajectory,
D_s – safe distance,
ΔV - reduction of the own ship's speed for a deflection from the course greater than 30^o.

5.4 Determination of safe game trajectories

Computer simulation of POSTRAJ and RISKTRAJ algorithms, as a computer software supporting the navigator decision, were carried out on an example of a real navigational situation of passing j=16 encountered ships. The situation was registered in Kattegat Strait on board r/v HORYZONT II, a research and training vessel of the Gdynia Maritime University, on the radar screen of the ARPA anti-collision system Raytheon.
The POSGAME algorithm represents the ship game trajectories determined according to the control index in the form (27) (Fig. 18).
The RISKTRAJ algorithm was developed for strategies: $\upsilon_0 = 13$ and $\upsilon_j = 25$ (Fig. 19).

5.5 Characteristics of control sensitivity in real navigational situation at sea

Figure 20 represents sensitivity characteristics which were obtained through a computer simulation of the game control POSTRAJ and RISKTRAJ algorithms in the Matlab/Simulink software for the alterations of the values $X_{0,j}$ and X_{param} within ±5% or ±3°.

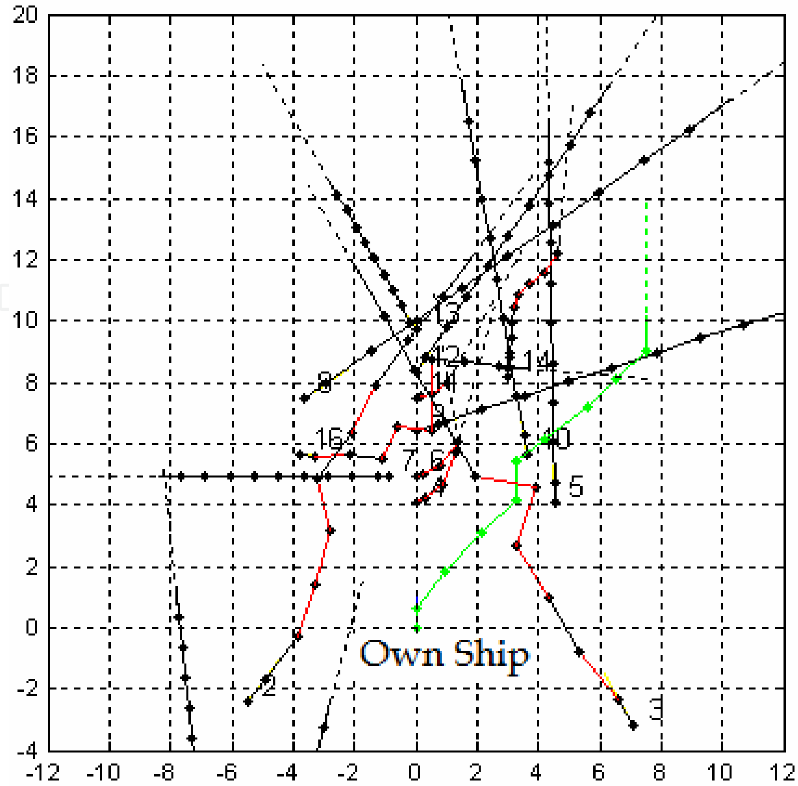


Fig. 18. The own ship game trajectory for the POSTRAJ, in good visibility, D_s=0,5 nm, $r(t_k)$=0, $d(t_k)$=7,72 nm, in situation of passing j=16 encountered ships

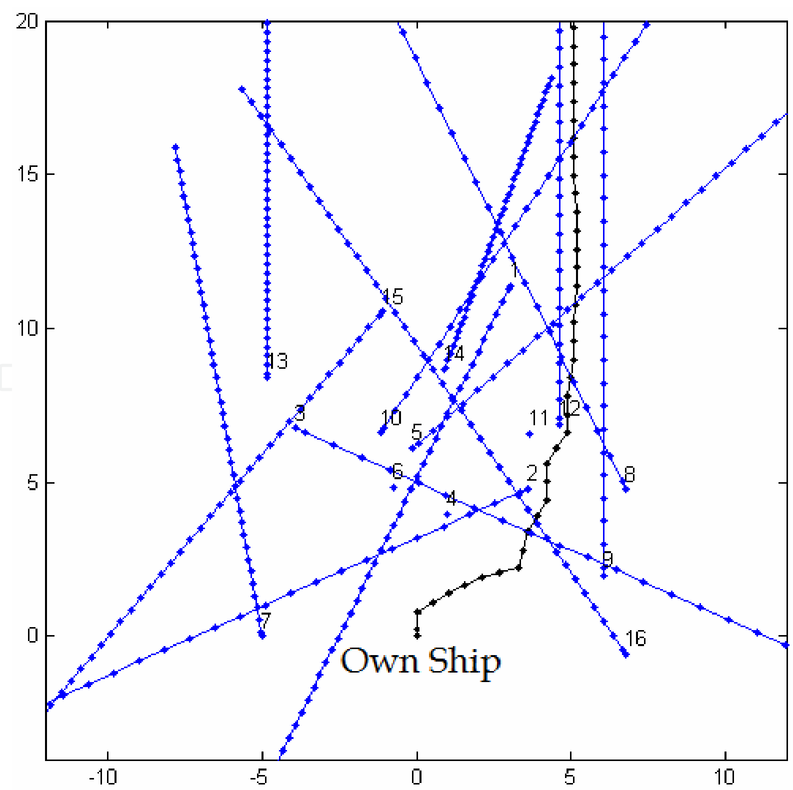


Fig. 19. The own ship game trajectory for the RISKTRAJ, in good visibility, D_s=0,5 nm, $r(t_k)$=0, $d(t_k)$=6,31 nm, in situation of passing j=16 encountered ships

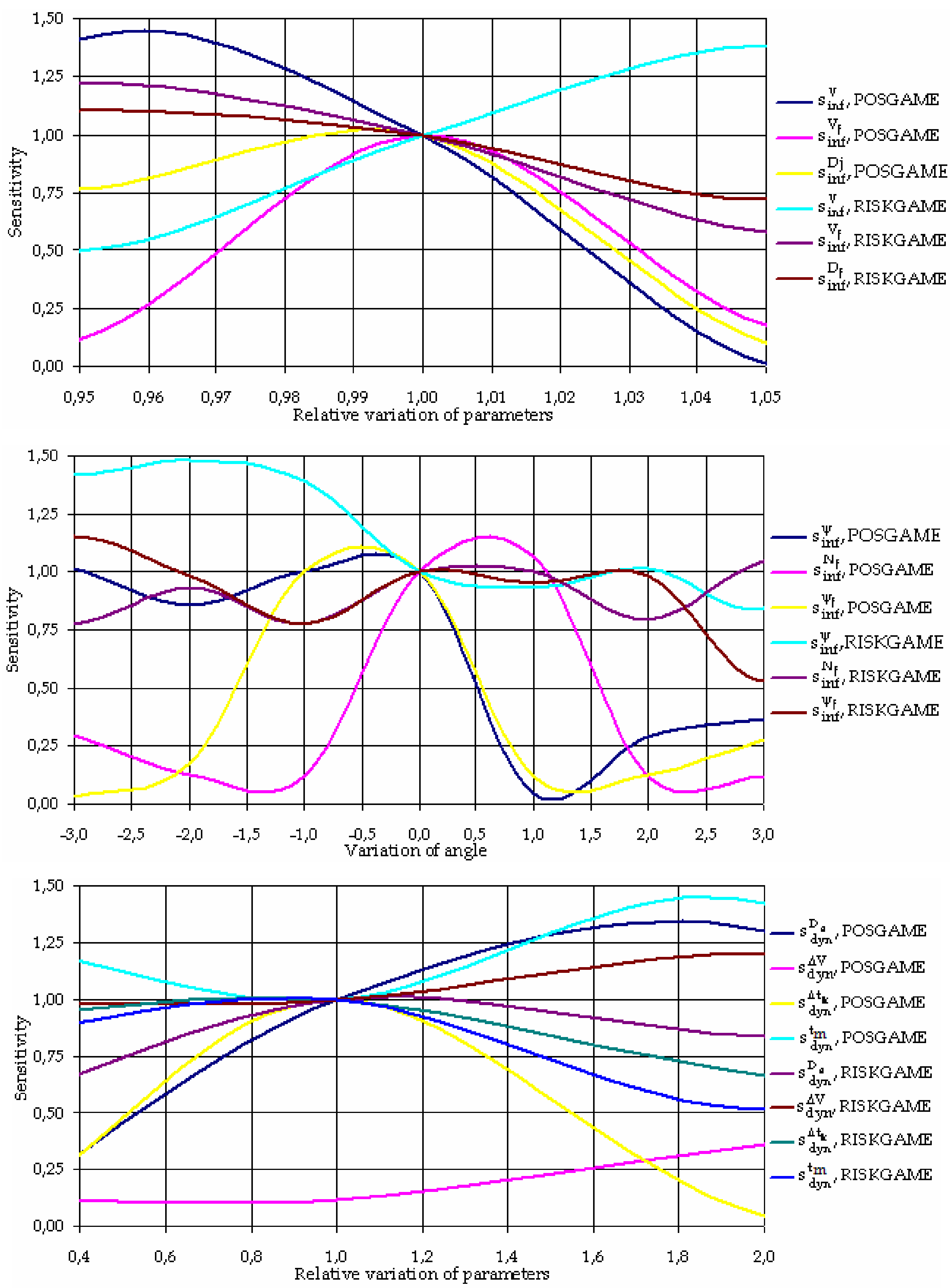


Fig. 20. Sensitivity characteristics of safe game ship control according to POSGAME and RISKTRAJ algorithms on an example of a navigational situation in the Kattegat Strait

6. Conclusion

The application of simplified model of the dynamic game of the process to the synthesis of the optimal control allows the determination of the own ship safe trajectory in situations of passing a greater number of the encountered ships as a certain sequence of the course and speed manoeuvres. The developed RISKTRAJ algorithm takes also into consideration the Rules of the COLREG Rules and the advance time of the manoeuvre approximating the ship's dynamic properties and evaluates the final deviation of the real trajectory from the reference value.

The sensitivity of the final game payment:

- is least relative to the sampling period of the trajectory and advance time manoeuvre,
- most is relative to changes of the own and met ships speed and course,
- it grows with the degree of playing character of the control process and with the quantity of admissible strategies.

The considered control algorithm is, in a certain sense, formal model of the thinking process of a navigator conducting a ship and making manoeuvring decisions. Therefore they may be applied in the construction of both appropriate training simulators at the maritime training centre and also for various options of the basic module of the ARPA anti-collision radar system.

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