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Controller Synthesis for a Class of State-Dependent Switched Nonlinear Systems

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1. Introduction

In the last decade, switched systems have gained much attention since a large class of practical systems can be modeled as switched systems (see e.g., (Dayawansa & Martin, 1999)) and there exist systems that cannot be asymptotically stabilized by a single continuous feedback control law (see e.g., (Brockett, 1983)). For stability analysis of switched systems, many interesting results have been proposed in the literature, see e.g., (Narendra & Balakrishnan, 1994; Johansson & Rantzer, 1998; Branicky, 1998; Ye et al., 1998; Dayawansa & Martin, 1999; Liberzon et al., 1999; Skafidas et al., 1999; Mancilla-Aguilar, 2000; and Chatterjee & Liberzon, 2007). Another topic is the derivation of stabilizing switching rules for switched systems, see et al., (Peleties & DeCarlo, 1991; Liberzon & Morse, 1999; and Pettersson, 2003). For feedback controller synthesis of switched control systems (with input signals), most of the proposed results consider the linear subsystems case, see e.g., (Daafouz et al., 2002; Sun & Ge, 2003; Petterson, 2004; Hespanha & Morse, 2004; and Seatzu et al., 2006). Only a few results have been proposed for feedback controller synthesis of switched nonlinear control systems, see e.g., (Sun & Zhao, 2001; El-Farra et al., 2005; Wu, 2008; and Wu, 2009). In (El-Farra et al., 2005), an integrated synthesis of feedback controllers together with switching laws has been proposed. In (Sun & Zhao, 2001), a *common control Lyapunov function* (CCLF) approach has been introduced also for constructing feedback control laws together with switching signals for switched nonlinear control systems. The concept of CCLF is motivated by the *control Lyapunov function* approach (see, e.g., (Artstein, 1983) and (Sontag, 1983 and 1989)) for designing stabilizing feedback laws for (non-switched) nonlinear systems. In (Wu, 2008), for switched nonlinear control systems under arbitrarily switching, conditions for the existence of CCLFs has been derived and a globally uniformly asymptotically stabilizing feedback law has been proposed. However, no systematical approaches have been provided for constructing CCLFs. Moreover, the obtained feedback law is complicated. In (Wu, 2009), for switched nonlinear control systems, which arbitrarily switching between a set of subsystems in strict feedback form, the backstepping approach (see e.g., (Krstic et al., 1995) and (Sepulchre et al., 1997)) has been employed to construct CCLFs, and a simpler stabilizing feedback law has been proposed.

However, till now, few results have been reported in the literature about stabilizing feedback controllers design for state-dependent switched nonlinear control systems. The

purpose of this chapter is to give a constructive approach for this problem. The state space is partitioned, by a set of switching surfaces, into several operation regions. In each of these regions, a nonlinear dynamical system (in feedback linearizable form) is given. Whenever the state trajectory passes a switching surface, a new dynamical model dominates the system's behavior. That is, the switching signal is state-dependent and predetermined. For the stability analysis of state-dependent switched systems, a *common Lyapunov function* for all subsystems is easier to develop but too conservative, see e.g., (Liberzon, 2003). It is known that the *multiple Lyapunov function* approach is a less conservative method. Based on the concepts of *multiple Lyapunov functions* and *control Lyapunov functions*, this chapter introduces a *switched control Lyapunov function* (SCLF) approach for designing stabilizing feedback controllers for state-dependent switched nonlinear control systems. It should be emphasized that the derivation of CCLFs or SCLFs for switched nonlinear control systems is an open problem unless the systems are in some particular form. Therefore, in this chapter we restrict our attention to switched nonlinear systems in *feedback linearizable* form for the reason that, in this case, SCLFs can be chosen as piecewise quadratic form and can be obtained by solving bilinear matrix inequalities (BMIs) with equality constraints. Although the considered systems are in feedback linearizable form, we do not use the feedback linearization technique in the design procedure. We show that the considered stabilization problem for switched nonlinear control systems can be solved by directly solving a matrix problem. We will show that an explicit stabilizing switched feedback law, based on the Sontag's formula (see (Sontag, 1989)), can be easily derived once a SCLF has been obtained.

NOTATIONS: That $A \setminus B$ is the set of all elements which belong to set A but not belong to set B; $A \cup B$ is the union of sets A and B; $A \cap B$ is the intersection of sets A and B; clA is the closure of set A; $\overline{A}$ is the boundary of set A; $IntA$ is the interior of set A (i.e., $IntA = clA \setminus \overline{A}$); $P > 0$ ($P < 0$) means that the matrix P is positive (negative) definite; $P \geq 0$ ($P \leq 0$) means that the matrix P is positive (negative) semidefinite; ϕ denotes an empty set; $\forall$ means "for all".

2. Problem Formulation and Preliminaries

The intension of this section is to present some preliminaries and to explicitly formulate the problem to be solved.

2.1 Switched nonlinear control systems
In this chapter we are focused on switched nonlinear systems with input signals:

$$\dot{x} = A_{\sigma(x)}x + C_{\sigma(x)}f_{\sigma(x)}(x) + B_{\sigma(x)}g_{\sigma(x)}(x)u , \ \sigma(x) \in \{1,...,q\} \tag{1}$$

where $x \in R^n$ is the system state, $u \in R^m$ is the control input, $A_i \in R^{n \times n}$, $C_i \in R^{n \times s}$, $B_i \in R^{n \times m}$, $i=1,...,q$, are constant matrices, $\sigma : R^n \mapsto \{1,...,q\}$ is a predetermined state-dependent switching signal, and $f_i(x) \in R^s$ and $g_i(x) \in R^{m \times m}$, $i=1,...,q$, are locally Lipchitz functions. Suppose that $f_i(0) = 0$, $g_i(x)$ is nonsingular for all $x \in \Omega_i$. Suppose also that

$$N(C_i^T) \subset N(B_i^T) \text{ for each } i \in \{1,...,q\} . \tag{2}$$

Define the index set $I_S = \{1,...,q\}$. Associated with the considered switched control system (1), a family of subsystems is defined:

$$\dot{x} = A_i x + C_i f_i(x) + B_i g_i(x)u, \ i \in I_S. \tag{3}$$

By (2), it can be seen that the subsystems in (3) are feedback linearizable (see Khalil, 1996).

2.2 State space partition

Specially in this chapter, the state space is partitioned into q regions Ω_i, $i=1,...,q$, given by quadratic forms:

$$\Omega_i \equiv \left\{ x \in R^n \left| x^T Q_i x \geq 0 \right. \right\}, \ i=1,...,q, \tag{4}$$

for some symmetric matrices $Q_i \in R^{n \times n}$, $i=1,...,q$. Let $Int\Omega_i \cap Int\Omega_j = \phi$ if $i \neq j$, and $\Omega_1 \cup \cdots \cup \Omega_q = R^n$. That is, the overlap between two adjacent regions is the boundary between these two regions. The i-th subsystem of (3) can be active only in part of the state space, specified by region Ω_i. Define the adjacent index set

$$I_A = \left\{ \{i,j\} \in I_S \times I_S \left| \Omega_i \cap \Omega_j \setminus \{0\} \neq \phi \right. \right\}. \tag{5}$$

That is, if $\{i,j\} \in I_A$, then Ω_i and Ω_j are adjacent regions and thus a switching region S_{ij} is defined:

$$S_{ij} \equiv \left\{ x \in R^n \left| x^T(Q_i - Q_j)x = 0 \right. \right\} \tag{6}$$

In fact, $S_{ij} = \Omega_i \cap \Omega_j$. Switches of the i-th subsystem into the j-th subsystem (or, switches of the j-th subsystem into the i-th subsystem) can occur only in the region S_{ij}. Note that in (Pettersson, 2004), for switched linear systems, the partition of state space is determined by the designer. That is, Q_i, $i=1,...,q$, are parameters to be determined. But in this chapter, we consider the case that they are predetermined.

2.3 Switching rule

In this chapter, we consider the case that the switching signal is state-dependent and is given by:

$$\sigma(x(t)) = i, \text{ if } x(t) \in Int\Omega_i, \text{ or } x(t) \in \overline{\Omega}_i \text{ and } \sigma(x(t^-)) = i. \tag{7}$$

By (7), the switching signal $\sigma(x)$ changes it value only if the state trajectory leaves one of the regions Ω_i, $i=1,...,q$. It holds constant value if the state trajectory keeps within a particular region (including its boundary).

2.4 Problem formulation

The goal of this chapter is to construct a state feedback law

$$u = h_{\sigma(x)}(x) \tag{8}$$

to globally asymptotically stabilize the switched control system (1). That is, we want to find a feedback law (8) such that the closed-loop system

$$\dot{x} = A_{\sigma(x)}x + C_{\sigma(x)}f_{\sigma(x)}(x) + B_{\sigma(x)}g_{\sigma(x)}(x)h_{\sigma(x)}(x) \tag{9}$$

becomes globally asymptotically stable under the switching rule (7).

2.5 Multiple Lyapunov functions

As stated in (Liberzon, 2003), *common Lyapunov function* approach will be too conservative for stability analysis of state-dependent switched systems. The *multiple Lyapunov function* approach will be less conservative. Here we briefly review the concept of *multiple Lyapunov functions*.

To analyze the stability of state-dependent switched system (9), for each $i \in I_S$, we first find a Lyapunov-like function $V_i(x)$, which vanishes at the origin and is positive for all $x \in \Omega_i \setminus \{0\}$, for the i-th subsystem of (9). System (9) is stable if, for each $i \in I_S$, the values of $V_i(x)$ at every switching instants, when we enter (switch into) the i-th subsystem, form a monotonically decreasing sequence.

However, using the *multiple Lyapunov function* approach in practically analyzing stability is difficult since, for verifying the monotonically decreasing property, one must have some information about the solutions of the switched systems (Liberzon, 2003). That is, one needs to know the values of suitable Lyapunov-like functions at switching times, which in general requires the knowledge of the state at these times. This is to be contrasted with the classical Lyapunov stability results, which do not require the knowledge of solutions, see (Liberzon, 2003). To simply the analysis procedure, an additional assumption that the *multiple Lyapunov function* is continuous on the boundaries between regions (i.e., switching surfaces) can be introduced. This assumption is conservative but leads to a simpler condition for verifying stability.

2.6 Switched control Lyapunov functions

Multiple Lyapunov function approach can be used to determine the stability of switched systems without input signals. However, it cannot tell us how to find a stabilizing feedback law (8) for the switched control system (1). Here we introduce the *switched control Lyapunov function* (SCLF) for feedback controller synthesis.

Definition 1: A switched function $V(x) \equiv V_{\sigma(x)}(x)$, which is differentiable in $Int\Omega_i$, for all $i \in I_S$, and continuous on S_{ij}, for all $\{i, j\} \in I_A$, is a SCLF of (1) if,

$$V(0) = 0 \tag{10}$$

$$V(x) > 0, \ \forall x \in R^n \setminus \{0\} \tag{11}$$

$$V(x) \to \infty, \text{ as } \|x\| \to \infty \tag{12}$$

and, along each possible nonzero solution of (1), a control signal u exists such that $V(x)$ monotonically decreases. ∎

Similar to the statement about *multiple Lyapunov function* in the previous subsection, for simplifying the design procedure, we make the assumption that the SCLF is continuous everywhere. By Definition 1, if we can find a SCLF for the switched system (1), then for each solution of (1) we can derive a control signal u such that SCLF monotonically decreases. The next problems are how to derive SCLFs for (1) and how to develop stabilizing feedback controllers by the obtained SCLF.

3. Stabilizing Controller Synthesis

In this section we propose the main results, a sufficient condition for the existence of SCLFs for (1) and a stabilizing feedback law derived by the obtained SCLF.
We first define the *regional control Lyapunov functions* (*RCLFs*) for the subsystems in (3), which will be used to construct SCLFs for (1).
Definition 2: A differentiable function $V_i(x)$ is a Ω_i-RCLF for the *i*-th subsystem of (3) if,

$$V_i(0) = 0 \tag{13}$$

$$V_i(x) > 0, \ \forall x \in \Omega_i \setminus \{0\} \tag{14}$$

$$V_i(x) \to \infty, \text{ as } x \in \Omega_i \text{ and } \|x\| \to \infty \tag{15}$$

and

$$\inf_{u \in R^m} \frac{\partial V_i(x)}{\partial x} \left(A_i x + C_i f_i(x) + B_i g_i(x) u \right) < 0, \ \forall x \in \Omega_i \setminus \{0\} \tag{16}$$

∎

Define

$$a_i(x) \equiv \frac{\partial V_i(x)}{\partial x} \left(A_i x + C_i f_i(x) \right) \tag{17}$$

and

$$b_i(x) \equiv \frac{\partial V_i(x)}{\partial x} B_i g_i(x). \tag{18}$$

By Definition 2, a differentiable function $V_i(x)$, satisfying (13)-(15), is a Ω_i-RCLF for the *i*-th subsystem if

$$\forall x \in \Omega_i \setminus \{0\}, \ b_i(x) = 0 \Rightarrow a_i(x) < 0. \tag{19}$$

Notice that if we can find a Ω_i-RCLF, $V_i(x)$, for the *i*-th subsystem, then for all $x \in \Omega_i \setminus \{0\}$ we can derive an u such that $a_i(x) + b_i(x) u < 0$.
Now we recall the S-procedure and the Finsler's lemma which will be used latter for deriving conditions for the existence of RCLFs.

Lemma 1 (S-procedure) (Boyd et al., 1994)*:* Let $P \in R^{n \times n}$ and $Q \in R^{n \times n}$ be symmetry. Then

$$x^T P x > 0 \text{ for all } x \neq 0 \text{ satisfies } x^T Q x \geq 0$$

if there exists a scalar $\rho \geq 0$ such that

$$P - \rho Q > 0 \, . \qquad\qquad\qquad\qquad\qquad\qquad\qquad \blacksquare$$

Lemma 2 (Finsler's Lemma) (Boyd et al., 1994): Consider a symmetric matrix $P \in R^{n \times n}$ and a matrix $N \in R^{n \times m}$, with *rank(N)*<*n*. The following statements are equivalent:

1) $x^T P x < 0 \quad \forall x \neq 0$ such that $N^T x = 0$
2) $\exists \mu \in R$ such that $P - \mu N N^T < 0$
3) $\exists L \in R^{m \times n}$ such that $P + L^T N^T + N L < 0$. $\qquad\qquad\qquad \blacksquare$

By the particular structure of the switched control system (1), RCLFs of the subsystems in (3) can be chosen as quadratic form and then can be obtained by solving bilinear matrix inequalities (BMIs). From Definition 2, a quadratic function $V_i(x) \equiv x^T P_i x$, with $P_i = P_i^T \in R^{n \times n}$, is a Ω_i-RCLF for the *i*-th subsystem of (3) if

$$x^T P_i x > 0 \text{ for all } x \neq 0 \text{ such that } x^T Q_i x \geq 0 \qquad\qquad (20)$$

and

$$\inf_{u \in R^m} x^T P_i \left(A_i x + C_i f_i(x) \right) + x^T P_i B_i g_i(x) u < 0 \text{ , for all } x \neq 0 \text{ such that } x^T Q_i x \geq 0 \qquad (21)$$

We have the following result.

Theorem 1: There exists a quadratic Ω_i-RCLF for the *i*-th subsystem of (3) if there exist scalars $\rho_i \geq 0$ and $\eta_i \geq 0$, and matrices $L_i \in R^{m \times n}$ and $P_i = P_i^T \in R^{n \times n}$ satisfy the following bilinear matrix inequalities:

$$P_i - \eta_i Q_i > 0 \qquad\qquad\qquad\qquad (22)$$
$$A_i^T P_i + P_i A_i + \rho_i Q_i + L_i^T B_i^T P_i + P_i B_i L_i < 0 \qquad\qquad\qquad (23)$$

In this case, the quadratic function $V_i(x) \equiv x^T P_i x$ is a Ω_i-RCLF for the *i*-th subsystem of (3).
Proof: From (20), (22) and Lemma 1, it is clear that $V_i(x) > 0$ for all $x \in \Omega_i \setminus \{0\}$. Notice that

$$b_i(x) = \frac{\partial V_i(x)}{\partial x} B_i g_i(x) = 2x^T P_i B_i g_i(x) \qquad\qquad\qquad (24)$$

and

$$a_i(x) = \frac{\partial V_i(x)}{\partial x}\left(A_i x + C_i f_i(x)\right) = 2x^T P_i A_i x + 2x^T P_i C_i f_i(x) \tag{25}$$

For $x \in \Omega_i \setminus \{0\}$ such that $b_i(x) = 0$, we have $P_i x \in N(B_i^T)$ since $g_i(x)$ is nonsingular. Then, $P_i x \in N(C_i^T)$ by (2) and therefore $x^T P_i C_i f_i(x) = 0$. From (23), for $x \in \Omega_i \setminus \{0\}$ satisfying $b_i(x) = 0$, we have

$$\begin{aligned}
a_i(x) &= 2x^T P_i A_i x + 2x^T P_i C_i f_i(x) \\
&= 2x^T P_i A_i x \\
&< -x^T(\rho_i Q_i + L_i^T B_i^T P_i + P_i B_i L_i)x \\
&= -\rho_i x^T Q_i x \\
&\le 0
\end{aligned}$$

This proves the result by noting (19). ∎

It should be noted that, by the conditions in Theorem 1, $V_i(x) = x^T P_i x$ is not a classical *control Lyapunov function* for the *i*-th subsystem. This is obvious since the solution P_i may not be positive definite.

By solving (22) and (23) for all $i \in I_S$, if all the subsystems in (3) have their RCLFs, $V_i(x) = x^T P x$, *i*=1,...,*q*, one might think that the switched function $V_{\sigma(x)}(x)$ is a SCLF for switched control system (1). However, this is not true since these RCLFs in general have different values on the switching surfaces. That is, $V_{\sigma(x)}(x)$ will be discontinuous on the switching surfaces. If we use $V_{\sigma(x)}(x)$ as a *control Lyapunov function* for (1), we can find control signal u such that $V_{\sigma(x)}(x)$ decreases between sequel switching times. However, $V_{\sigma(x)}(x)$ may increase at the switching instants (i.e., as the trajectories of (1) pass through the switching surfaces). In this case, the design of stabilizing feedback laws is difficult. To simply the design procedure, an additional continuity requirement is included for SCLFs. In the next theorem we introduce additional constraints in solving the matrix inequalities to guarantee the continuity of SCLFs on the switching surfaces. Moreover, a stabilizing feedback controller is given provided that a SCLF is obtained.

Theorem 2: There exists a piecewise quadratic SCLF for the switched control system (1) if there exist scalars $\rho_i \ge 0$ and $\eta_i \ge 0$, and matrices $L_i \in R^{m \times n}$ and $P_i = P_i^T \in R^{n \times n}$, *i*=1,2,...,*q*, and scalars δ_{ij}, for all $\{i, j\} \in I_A$, satisfy the following matrix inequalities and equalities:

$$P_i - \eta_i Q_i > 0, \ i=1,2,\dots,q, \tag{26}$$

$$A_i^T P_i + P_i A_i + \rho_i Q_i + L_i^T B_i^T P_i + P_i B_i L_i < 0, \ i=1,2,\dots,q, \tag{27}$$

$$P_i - P_j = \delta_{ij}(Q_i - Q_j), \text{ for all } \{i, j\} \in I_A. \tag{28}$$

In this case, the function $V_{\sigma(x)}(x) = x^T P_{\sigma(x)} x$ is a SCLF for (1). In addition,

$$u(x) = h_{\sigma(x)}(x) \tag{29}$$

with ($k > 0$)

$$h_i(x) \equiv \begin{cases} -b_i^T(x) \dfrac{a_i(x) + k\sqrt{a_i^2(x) + \left(b_i(x)b_i^T(x)\right)^2}}{b_i(x)b_i^T(x)}, & \text{if } b_i(x) \neq 0 \\ 0, & \text{if } b_i(x) = 0 \end{cases} \tag{30}$$

is an asymptotically stabilizing feedback law for (1) under the switching rule (7).

Proof: By (28) it is clear that $x^T P_i x = x^T P_j x$ on S_{ij}, for all $\{i, j\} \in I_A$. That is, $V_{\sigma(x)}(x)$ is continuous in all state space. By Definition 1 and Theorem 1 and noting (26) and (27), it is obvious that $V_{\sigma(x)}(x) \equiv x^T P_{\sigma(x)} x$ is a SCLF for (1).

To show that (29) is a stabilizing feedback law, notice that, for each $i \in I_S$, if $x \in \Omega_i \setminus \{0\}$ (and $\sigma(x) = i$) such that $b_i(x) = 0$, we have

$$\begin{aligned}
\dot{V}_{\sigma(x)}(x) = \dot{V}_i(x) &= a_i(x) + b_i(x)u \\
&= a_i(x) + b_i(x)h_i(x) \\
&= a_i(x) \\
&< 0.
\end{aligned}$$

Moreover, if $x \in \Omega_i \setminus \{0\}$ (and $\sigma(x) = i$) such that $b_i(x) \neq 0$, then

$$\begin{aligned}
\dot{V}_{\sigma(x)}(x) = \dot{V}_i(x) &= a_i(x) + b_i(x)u \\
&= a_i(x) + b_i(x)h_i(x) \\
&= -k\sqrt{a_i^2(x) + \left(b_i(x)b_i^T(x)\right)^2} \\
&< 0.
\end{aligned}$$

That is, if no sliding motions occur on the switching surfaces, the closed-loop system is asymptotically stable since $\dot{V}_{\sigma(x)}(x) < 0 \;\; \forall x \neq 0$ (notice that the index set I_S is countable).

In the case that sliding motion occurs, we need to prove the stability of sliding motion. If a sliding motion occurs on S_{ij} for some $\{i, j\} \in I_A$, first suppose that $V_i(x) \leq V_j(x)$ for $x \in \Omega_i$ and $V_i(x) \geq V_j(x)$ for $x \in \Omega_j$ in a neighbourhood of S_{ij}. The existence of a sliding mode on S_{ij} is characterized by the inequalities

$$x^T(P_i - P_j)\left(A_i x + C_i f_i(x) + B_i g_i(x)h_i(x)\right) \geq 0 \tag{31}$$

$$x^T(P_i - P_j)\left(A_j x + C_j f_j(x) + B_j g_j(x)h_j(x)\right) \leq 0. \tag{32}$$

Since a sliding motion occurs on S_{ij}, σ is not uniquely defined on S_{ij}. So let $\sigma(x) = i$ without loss of generality (Liberzon, 2003). Along the corresponding Filippov solution, by (32), we have (for $\alpha \in (0,1)$)

$$
\begin{aligned}
\dot{V}_i(x) &= 2x^T P_i \big(\alpha(A_i x + C_i f_i(x) + B_i g_i(x) h_i(x)) + (1-\alpha)(A_j x + C_j f_j(x) + B_j g_j(x) h_j(x)) \big) \\
&= 2\big(\alpha(x^T P_i A_i x + x^T P_i C_i f_i(x) + x^T P_i B_i g_i(x) h_i(x)) \\
&\quad + (1-\alpha)(x^T P_i A_j x + x^T P_i C_j f_j(x) + x^T P_i B_j g_j(x) h_j(x)) \big) \\
&\leq 2\big(\alpha(x^T P_i A_i x + x^T P_i C_i f_i(x) + x^T P_i B_i g_i(x) h_i(x)) \\
&\quad + (1-\alpha)(x^T P_j A_j x + x^T P_j C_j f_j(x) + x^T P_j B_j g_j(x) h_j(x)) \big) \\
&< 0 \text{, if } x \neq 0 .
\end{aligned}
$$

That is, $V_i(x)$ monotonically decreases along the corresponding Filippov solution. Similarly, if $V_i(x) \geq V_j(x)$ for $x \in \Omega_i$ and $V_i(x) \leq V_j(x)$ for $x \in \Omega_j$ in a neighbourhood of S_{ij}, we can show that $\dot{V}_j(x) < 0$ along the corresponding Filippov solution. This implies that the switched closed-loop system is still asymptotically stable even if sliding motions occur on the switching surfaces. ∎

The formula in (30) is from the well known Sontag's formula (Sontag, 1989). We can also construct the feedback law by the Freeman's formula, see (Krstic et al., 1995).

Remark 1: In the case that the partition of state space is not predetermined and the switching signal is also a design parameter, by the covering property (Pettersson, 2004), some additional matrix inequalities must be included together with (26)-(28) to guarantee the existence of feedback controllers and switching laws for stabilizing the switched control systems. ∎

4. An Illustrative Example

Consider the switched nonlinear control system

$$
\dot{x} = A_{\sigma(x)} x + C_{\sigma(x)} f_{\sigma(x)}(x) + B_{\sigma(x)} g_{\sigma(x)}(x) u , \quad \sigma(x) \in \{1,2\} \tag{33}
$$

with the following system parameters:

$$
A_1 = \begin{bmatrix} 1 & -5 \\ -1 & 0 \end{bmatrix}, \ C_1 = \begin{bmatrix} 0 \\ -3 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ f_1(x) = x_1^2 x_2 , \ g_1(x) = -1 - x_2^2 ,
$$

$$
A_2 = \begin{bmatrix} 3 & -1 \\ 3 & 2 \end{bmatrix}, \ C_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \ B_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \ f_2(x) = x_1 x_2 - |x_2| , \ g_2(x) = 1 + |x_1 - x_2| .
$$

The state space is partitioned as $R^2 = \Omega_1 \cup \Omega_2$, where

$$\Omega_1 \equiv \left\{ x \in R^2 \middle| x^T Q_1 x \geq 0 \right\} \text{ and } \Omega_2 \equiv \left\{ x \in R^2 \middle| x^T Q_2 x \geq 0 \right\}$$

with

$$Q_1 = -Q_2 = \begin{bmatrix} -1 & 1.05 \\ 1.05 & 1 \end{bmatrix}.$$

That is, the switching region (boundaries between Ω_1 and Ω_2) is:

$$S_{12} \equiv \left\{ x \in R^2 \middle| x^T (Q_1 - Q_2) x = 0 \right\}$$

The switching signal is state-dependent and is given by:

$$\sigma(x(t)) = i \text{ , if } x(t) \in Int\Omega_i \text{ , or } x(t) \in \overline{\Omega}_i \text{ and } \sigma(x(t^-)) = i \text{ .} \tag{34}$$

From Theorem 2, we can verify that,

$$\eta_1 = \eta_2 = 0.1 \text{ , } \rho_1 = \rho_2 = 3.4 \text{ , } L_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, L_2 = \begin{bmatrix} -2 & 1 \end{bmatrix},$$

$$\delta_{12} = 1 \text{ , } P_1 = \begin{bmatrix} 1.2 & 1.1 \\ 1.1 & 3.2 \end{bmatrix}, \text{ and } P_2 = \begin{bmatrix} 3.2 & -1 \\ -1 & 1.2 \end{bmatrix}$$

satisfy (26)-(28). Therefore, there exist stabilizing feedback laws for the considered switched control system (33). By (24) and (25), we have

$$a_1(x) = 0.2x_1^2 - 16.2x_1x_2 - 11x_2^2 - (6.6x_1 + 19.2x_2)x_1^2x_2 \text{ ,}$$

$$a_2(x) = 13.2x_1^2 - 9.2x_1x_2 + 6.8x_2^2 + (16.8x_1 - 8.8x_2)(x_1x_2 - |x_2|) \text{ ,}$$

$$b_1(x) = -(2.2x_1 + 6.4x_2)(1 + x_2^2) \text{ ,}$$

$$b_2(x) = (8.4x_1 - 4.4x_2)(1 + |x_1 - x_2|) \text{ .}$$

Moreover, by (30), we have

$$h_1(x) \equiv \begin{cases} -b_1^T(x) \dfrac{a_1(x) + k\sqrt{a_1^2(x) + \left(b_1(x)b_1^T(x)\right)^2}}{b_1(x)b_1^T(x)} \text{ , if } b_1(x) \neq 0 \\ 0, & \text{if } b_1(x) = 0 \end{cases}$$

$$h_2(x) \equiv \begin{cases} -b_2^T(x) \dfrac{a_2(x) + k\sqrt{a_2^2(x) + \left(b_2(x)b_2^T(x)\right)^2}}{b_2(x)b_2^T(x)} \text{ , if } b_2(x) \neq 0 \\ 0, & \text{if } b_2(x) = 0. \end{cases}$$

Then

$$u(x) = h_{\sigma(x)}(x) \tag{35}$$

is a stabilizing feedback controller for (33) under the switching rule (34).

Fig. 1 shows the state trajectories of the closed-loop switched system starting from several different initial conditions with $k = 0.01$. Notice that sliding motions occur.
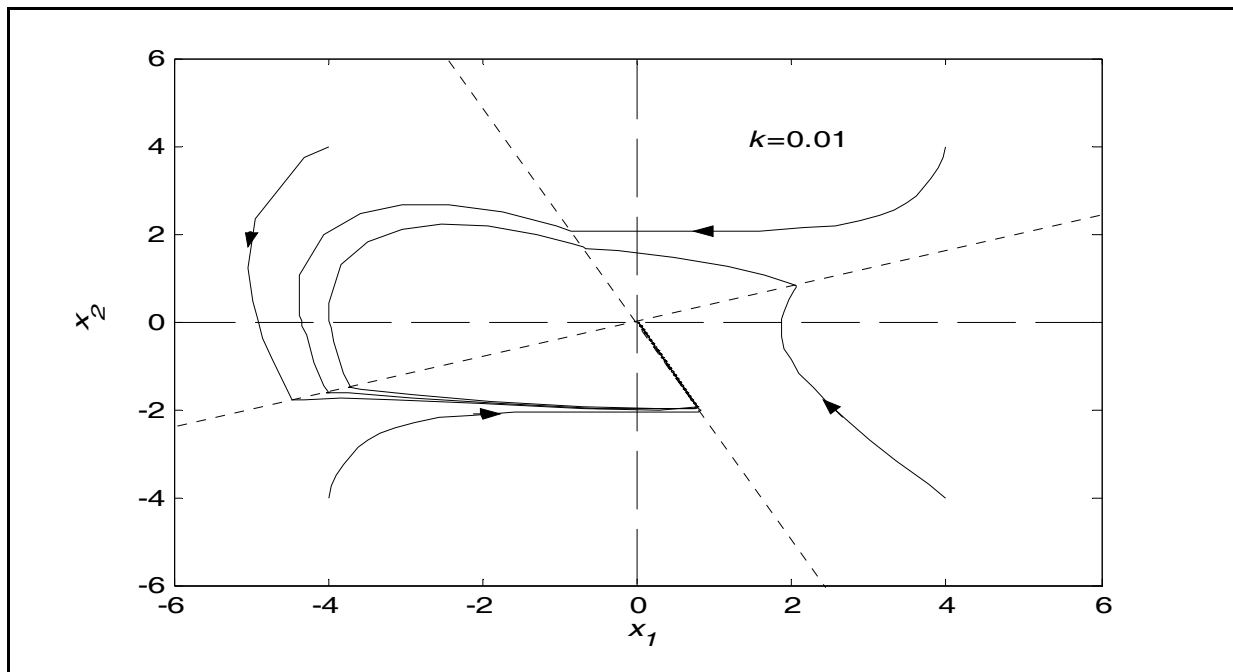

Fig. 1. State trajectories of the closed-loop switched system with k=0.01.

5. Conclusion

In this chapter, based on the use of switched control Lyapunov function approach, it has been shown that the design of stabilizing feedback laws for state-dependent nonlinear control systems in feedback linearizable form can be achieved by solving matrix problems. An example is given to illustrate the success of the method. However, solving the resultant bilinear matrix inequalities with equality constraints is not easy.

Further research topics include the development of feasible and efficient algorithms for solving the resultant matrix problem, the extension of the proposed approach to nonlinear control systems in some more general forms, and the search of stabilizing feedback laws to guarantee the non-existence of sliding motions.

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Switched Systems

Edited by Janusz Kleban

This book presents selected issues related to switched systems, including practical examples of such systems. This book is intended for people interested in switched systems, especially researchers and engineers. Graduate and undergraduate students in the area of switched systems can find this book useful to broaden their knowledge concerning control and switching systems.

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