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Robust Adaptive Control of Switched Systems

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Abstract

In this chapter, a methodology for robust adaptive control design for a class of switched non-linear systems is developed. Under extensions of typical adaptive control assumptions, a leakage-type adaptive control scheme guarantees stability for systems with bounded disturbances and parameters without requiring a priori knowledge on such parameters or disturbances. The problem reduces to an analysis of an exponentially stable and input-to-state stable (ISS) system driven by piecewise continuous and impulsive inputs due to plant parameter switching and variation. As a result, a separation between robust stability and robust performance and clear guidelines for performance optimization via ISS bounds are obtained. The results are demonstrated through example simulations, which follow the developed theory and demonstrate superior robustness of stability and performance relative to non-adaptive and other adaptive methods such as projection and deadzone adaptive controllers.

1. Introduction

Switched and hybrid systems have been gaining considerable interest in both research and industrial control communities. This is motivated by the need for systematic and formal methods to control such systems. These issues arise in systems with discrete changes in energy exchange elements due to intermittent interaction with other systems or with an environment or due to the nature of their constitutive relations. This is common in robotic and mechatronic systems with contact and impact effects, fluidic systems with valves or phase changes, and electrical circuits with switches.

Despite numerous interesting publications on hybrid systems, there is a lack of constructive methods for control of a nontrivial class of switched systems with a priori stability and performance guarantees due to the difficulty of this problem. In terms of stability and response of switched systems, several results have been obtained in recent years, see (10; 2; 25) and references therein. In this context, sufficient conditions for stability such as common Lyapunov functions and average dwell time (10) are the most commonly studied approaches. A corresponding control design requires switching controller gains such that all subsystems are made stable and such that a common Lyapunov function condition is satisfied, which for LTI systems requires system matrices to commute or be symmetric, see (17; 18) for more explicit results. In order to verify that such a condition is met, the system is partitioned into known subsystems and a set of linear matrix inequalities, of increasing order with the number of subsystems, is solved if a solution is feasible. The other class of results requires that

all subsystems are stable (or with some known briefly visited unstable modes) and switching is slow enough on average, *average dwell time condition* (10). The corresponding controller design requires gains to be adjusted to guarantee the stability of each frozen configuration and knowledge of worst case decay rate among subsystems and condition number of Lyapunov matrices in order to compute the maximum admissible switching speed. If plant switching exceeds this switching speed then stability can no longer be guaranteed. Analogous analysis results have been extended for systems with disturbances (22) and with some uncertainties (23) as well as related work for linear-parameter varying (LPV) systems in (20; 12). Thus, there is a need for more explicit methods that can be constructively used to *design* controllers for stable switched systems independent of the success of heuristics or feasibility of complex computational methods.

Adaptive control is another popular approach to deal with system uncertainty. The problem with conventional adaptive controllers is that the transient performance is not characterized and stability with respect to bounded parameter variations or disturbances is not guaranteed. Robust adaptive controllers, (6), developed to address the presence of disturbances and non-parametric uncertainties, are typically based on projection, switching-sigma or deadzone adaptation laws that require a priori known bounds on parameters, and in some cases disturbances as well, in order to ensure state boundedness. Extensions to some classes of time varying systems have been developed in (13; 14; 15; 24). However, the results are restricted to smoothly varying parameters with known bounds and typically require additional restrictive conditions such as slowly varying unknown parameters (24) or constant and known input vector parameters (14), in order to ensure state boundedness. In this case, such a conclusion is of very little practical importance if the error can not be reduced to an acceptable level by increasing the adaptation or feedback gains or using a better nominal estimate of the plant parameters. Furthermore, performance with respect to rejection of disturbances as well as the transient response remain primarily unknown.

However, a leakage-type modification as will be shown in this chapter, achieves internal exponential stability and input-to-state stability (ISS), for the class of systems under consideration, without need for persistence of excitation as required in (6). In this regard, projection and switching-sigma modifications have been favored over fixed-sigma modifications, (6) due to its inability to achieve zero steady-state tracking when parameters are constant and disturbances vanish. However, this is a situation of no interest to this paper since the focus is on time varying switching systems. The developed control methodology, which is a generalization of fixed-sigma modification, yields strong robustness to time varying and switching parameters without requiring a priori known bounds on such parameters, as typically needed in projection and switching-sigma modifications.

In this chapter, the development and formulation of an adaptive control methodology for a class of switched nonlinear systems is presented. Under extensions of typical adaptive control assumptions, a leakage-type adaptive control scheme is developed for systems with piecewise differentiable bounded parameters and piecewise continuous bounded disturbances without requiring a priori knowledge on such parameters or disturbances. This yields a separation between robust stability and robust performance and clear guidelines for performance optimization via ISS bounds.

The remainder of the chapter is organized as follows. Section 2 presents the basic adaptive controller methodology. Analysis of the performance of the control system along with design guidelines is discussed in Section 3. Section 4 gives an example simulation demonstrating the key characteristics of the control system as well as comparing it with other non-adaptive

and adaptive techniques such as projection and dead-zone. Conclusions are given in Section 5. In this chapter, $\bar{\lambda}(\cdot)$ and $\underline{\lambda}(\cdot)$ denote the maximal and minimal eigenvalues of a symmetric matrix, $\|\cdot\|$ the euclidian norm, and $diag(\cdot, \cdot, \dots)$ denotes a block diagonal matrix.

2. Methodology

2.1 Parameterized Switched Systems

A hybrid switched system is a system that switches between different vector fields in a differential equation (or a difference equation) each active during a period of time. In this chapter we consider feedback control of continuous-time switched time varying systems described by:

$$\begin{aligned} \dot{x}(t) &= f_i(x, t, u, d), t_{i-1} \leq t < t_i \\ y(t) &= h_i(x, t), t_{i-1} \leq t < t_i \\ i(t)^+ &= g(i(t), x, t) \end{aligned} \quad (1)$$

where x is the continuous state, d is for disturbances, u is the control input and y is measured output. Furthermore, $i(t) \in \{1, 2, 3, \dots\}$ is a piecewise constant signal with i denoting the i^{th} switched subsystem active during a time interval $[t_{i-1}, t_i)$, where t_i is the i^{th} switching time. The signal $i(t)$, usually referred to as the switching function, is the discrete state of this hybrid system. The discrete state is governed by the discrete dynamics of $g(i(t), x, t)$, which sees the continuous state x as an input. This means switching may be triggered by a time event or a state event, e.g. x reaching certain threshold values, or even memory, i.e. past values for $i(t)$. on state only implicitly with enforced

In this chapter, we view a switching system as one parameterized by a time varying vector of parameters, which is piecewise differentiable, see Equation (2). This is a reasonable representation since it captures many physical systems that undergo switching dynamics, thus we will focus on such systems described by:

$$\begin{aligned} \dot{x} &= f(x, a, u, d) \\ y &= h(x, a) \\ a(t) &= a_i(t), t_{i-1} \leq t < t_i, i = 1, 2, \dots \\ i(t)^+ &= g(i(t), x, t) \end{aligned} \quad (2)$$

Therefore, we embed the switching behavior in the piecewise changes in $a(t)$, which again may be triggered by state or time driven events. $a_i(t) \in C^1$, i.e., at least one time continuously differentiable. This means $a(t)$ is piecewise continuous, with a well defined bounded derivative everywhere except at points t_i where $\dot{a} = da/dt$ consists of dirac-delta functions. Also the points of discontinuity of a , which are distinct and form an infinitely countable set, are separated by a nonzero dwell time, i.e., there are no Zeno phenomena (11; 21). This is a reasonable assumption since this is how most physical systems behave. The main assumptions on the class of systems under consideration are formally stated below:

Assumption 1

For a switched system given by Equation (2) the set of switches associated with a switching sequence $\{(t_i, a_i)\}$ is infinitely countable and \exists a scalar $\mu > 0$ such that $t_i - t_{i-1} \geq \mu \forall i$.

Assumption 2 $d \in \mathbb{R}^k$ is uniformly bounded and piecewise continuous.

Assumption 3 $a \in S_a$ is uniformly bounded and piecewise differentiable, where the set S_a is an admissible, but not necessarily known, set of parameters.

Note that by allowing piecewise changes in a the parametrization allows structural changes in the system if we overparametrize such that all possible structural terms are included. Then some parameters may switch to or from the value of zero as structural changes take place in the system.

2.2 Robust Adaptive Control

In this section, we discuss the basic methodology based on observation of the general structure of the adaptive control problem. In standard adaptive control for linearly-parameterized systems we usually have control and adaptation laws of the form:

$$\begin{aligned} u &= g(x_m, \hat{a}, \dot{\hat{a}}, y_r, t) \\ \dot{\hat{a}} &= f_a(x_m, \hat{a}, y_r, t) \end{aligned} \quad (3)$$

where u is the control signal, \hat{a} is an estimate of plant parameter vector $a \in S_a$, where S_a is an admissible set of parameters, x_m is measured state variables, and y_r is a desired reference trajectory to be followed. This yields the following closed loop error dynamics :

$$\begin{aligned} \dot{e}_c &= f_e(e_c, \tilde{a}, t) + d(t) \\ \dot{\tilde{a}} &= f_a(e_c, \hat{a}, t) - \dot{\hat{a}} \end{aligned} \quad (4)$$

where e_c represents a generalized tracking error vector, which includes state estimation error in general output feedback problems and can depend nonlinearly on the plant states as in backstepping designs, $\tilde{a} = \hat{a} - a$ is parameter estimation error, and d is the disturbance.

In standard adaptive control we typically design the control and adaptation laws, Equation (3), such that $\forall a \in S_a$ we have:

$$e_c^T P f_e + \tilde{a}^T \Gamma(t)^{-1} f_a \leq -e_c^T C e_c \quad (5)$$

where matrices $P > 0$ and $C > 0$ are chosen depending on the particular algorithm, e.g. choice of reference model and the diagonal matrix $\Gamma(t)^{-1} = \text{diag}(\Gamma_o^{-1}, \gamma_\rho^{-1}|b(t)|) > 0$ is an equivalent generalized adaptation gain matrix, where diagonal matrix $\Gamma_o > 0$ and scalar $\gamma_\rho > 0$ are the actual adaptation gains used in the adaptation laws. Whereas, $b(t)$ is a scalar plant parameter, usually the high frequency gain, which appears in Γ in some adaptive designs. The following additional assumption is made for $b(t)$:

Assumption 4 $b(t)$ is an unknown scalar function such that $b(t) \neq 0 \forall t$, and sign of $b(t)$ is known and constant.

This is sufficient to stabilize the system with constant parameters and no disturbances. However, since the error dynamics is not ISS stable, stability is no longer guaranteed in the presence of bounded inputs such as d and $\dot{\hat{a}}$. In order to deal with time varying and switching dynamics, a modification to the adaptation law will be pursued.

Now consider the following modified adaptation law:

$$\dot{\hat{a}} = f_a(e_c, \hat{a}, t) - L(\hat{a} - a^*) \quad (6)$$

with the diagonal matrix $L = \text{diag}(L_o, L_\rho) > 0$ and $a^*(t)$ is an arbitrarily chosen piecewise continuous bounded vector, which is an additional estimate of the plant parameter vector. Then the same system in Equation (4) with the modified adaptation law becomes:

$$\begin{aligned}\dot{e}_c &= f_e(e_c, \tilde{a}, t) + d(t) \\ \dot{\tilde{a}} &= f_a(e_c, \hat{a}, t) - L\tilde{a} + L(a^* - a) - \dot{a}\end{aligned}\quad (7)$$

The modified adaptation law shown above is similar to leakage adaptive laws (6), which have been used to improve robustness with respect to unstructured uncertainties. The leakage adaptation law, also known as fixed-sigma, uses $L_o = \sigma \Gamma_o$, where $\sigma > 0$ is a scalar and the vector $a^*(t)$ above is usually not included or is a constant. In fact, the key contribution from the generalization presented here is not in the algebraic difference relative to leakage adaptive laws (6) but rather in how the algorithm is utilized and proven to achieve new properties for control of rapidly varying and switching systems. In particular, internal exponential and ISS stability of the closed loop system using this leakage-type adaptive controller, without need for persistence of excitation as required in (6), is shown and used to guarantee stability of the state $x_c = [e_c^T, \tilde{a}^T]^T$, see Theorem 1 below.

Theorem 1 *If there exists matrices $P, \Gamma_o, \gamma_\rho, C > 0$ such that (5) is satisfied for $\dot{a} = d = 0$ with $\Gamma(t)^{-1} = \text{diag}(\Gamma_o^{-1}, \gamma_\rho^{-1}|b(t)|) > 0$ and Assumption 2.4 is satisfied then the system given by Equation (7) with $d, \dot{a} \neq 0$ and diagonal $L > 0$ is :*

(i) *Uniformly internally exponentially stable and ISS stable.*

(ii) *If Assumptions (2.1-2.3) are satisfied and $a^*(t)$ is chosen as a piecewise continuous bounded vector then state $x_c = [e_c^T, \tilde{a}^T]^T$ is bounded with*

$$\|e_c(t)\| \leq c_1 \|x_c(t_0)\| e^{-\alpha(t-t_0)} + c_2 \int_{t_0}^t e^{\alpha(\tau-t)} \|v(\tau)\| d\tau$$

where c_1, c_2 are constants, $\alpha = \bar{\lambda}(\text{diag}(P^{-1}C, L))$, and $v = [P^{1/2}d, \Gamma^{-1/2}(L(a^* - a) - \dot{a})]^T$.

The proof of this result is found in Appendix A.

2.3 Remarks

This section presents some remarks summarizing the implications of this result.

- The effect of plant variation and uncertainty is reduced to *inputs* $L(a^* - a)$ and \dot{a} acting on this ISS closed loop system. This, in turn, provides a separation between the robust stability and robust performance control problems.
- The modified adaptation law is a slightly more general version of the leakage modification, also known as fixed-sigma, (6), where $L = \sigma \Gamma$, where $\sigma > 0$ is a scalar and the vector $a^*(t)$ above is usually not included or is a constant. This is a robust adaptive control method that has been less popular than projection and switching-sigma modifications due to its inability to achieve zero steady-state tracking when parameters are constant and disturbances vanish. However, this approach yields stronger stability and performance robustness for time varying switching systems for which the constant parameter case is irrelevant.
- Plant parameter switching no longer affects internal dynamics and stability but enters as a step change in input $L(a^* - a)$ and an impulse in input \dot{a} at the switching instant.

- Controller switching of a^* does not affect internal dynamics but enters as a step change in input $L(a^* - a)$, which is a very powerful feature that can be used to utilize available information about the system.
- Allowed arbitrary time variation and switching in the parameter vector a are for a plant within the admissible set of parameters S_a . This set has not been defined here and will be defined later via design assumptions for the classes of systems of interest.
- The authors believe that the use of this robust adaptive controller is useful for switched systems even in the switched linear uncertainty free plant case, where stability with switched linear feedback is difficult to guarantee based on currently available tools (switching between stable LTI closed loop subsystems does not preserve stability). In this case, knowledge of the switching plant parameter vector $a(t)$ can be used in $a^*(t)$.

3. Performance of the Control System

In this section, the tracking performance of the obtained control system is discussed.

3.1 Dynamic Response

Exponential stability allows for shaping the transient response, e.g. settling time, and frequency response of the system to low/high frequency dynamics and inputs by adjusting the decay rate α , see Theorem 1. This is to be done independent of the parametric uncertainty $a^* - a$, which is contrasted to LTI feedback where closed loop poles change with parametric uncertainty. Thus the response to step and impulse inputs is as we expect for such an exponentially stable system. However, in this case such inputs will not arise from only disturbances but also from parameters and their variation. In particular, switches in parameters $a(t)$ yields step changes in a and impulses in $\dot{a}(t)$. Furthermore, the system display the frequency response characteristics such as in-bandwidth input, disturbances and parametric uncertainty and variations, rejection and more importantly attenuation of high frequency inputs due to roll-off.

3.2 Improving Tracking Error

Since stability and dynamic response of the system to different inputs and uncertainties have been established independent of uncertainty, we are now left with optimizing the control parameters and gains a^* , L , Γ , P , and C for minimal tracking error. Different methods for improving tracking error are described below with reference to the bound in Theorem 1:

1. *Increasing the system input-output gain $\alpha = \lambda(\text{diag}(P^{-1}C, L))$, which as discussed earlier, acts on the overall input uncertainty v . This attenuation, however, increases the system bandwidth, which suggests its use primarily for low/high bandwidth disturbances along the line of frequency response analysis of last section.*
2. *Increasing adaptation gain Γ , which has the effect of attenuating parametric uncertainty and variation independent of system bandwidth (Recall that α is independent of Γ from Theorem 1). This is the case since the size of the input v is reduced by reducing the component $\Gamma^{-1/2}(L(a^* - a) - \dot{a})$. Note that a very large Γ has the effect of amplifying measurement noise, which can be seen from the adaptation law.*
3. *Using a small gain $\Gamma^{-1/2}L$, which is an agreement with increasing adaptation gain matrix Γ mentioned above. However, this differs by the fact that this can be also achieved by simply reducing the size of L . Furthermore, using $\Gamma^{-1/2}L$ is effective mainly for*

parametric uncertainty since the input v contains $\Gamma^{-1/2}(L(a^* - a) - \dot{a})$, which suggests a small $\Gamma^{-1/2}L$ does not necessarily attenuate \dot{a} unless $\Gamma^{-1/2}$ is also small. This is the case since this condition implies having *approximate integral action* in the adaptation law of Equation (7), i.e., approaching integral action in the standard gradient adaptation law.

4. *Adjusting and updating parameter estimate a^** , which can be any piecewise continuous bounded function. This allows for reducing the effect of parametric uncertainty through reducing size of input $a^* - a$ independent of system bandwidth and control gains. In this regard, many of the useful and interesting ideas to monitor, select, and switch between different candidate controllers via multiple models such as those in (1; 16; 7; 26) can be used with switching between a_i^* values playing the role of the i^{th} candidate controller. The difference is that this is to be done without frozen-time instability or switched system instability concerns (verifying dwell time or common Lyapunov function conditions) as $a^*(t)$ is just an input to the closed loop system. Similarly, gain scheduling and Linear Parameter Varying (LPV) control (12; 20) can be applied with a^* playing the role of the scheduled parameter vector to be varied, again with no concerns with instability and transient behavior since $a^* - a$ enter as an input to the system.

3.3 Remarks

- Exponential stability allows for shaping the transient response, e.g. settling time, and frequency response of the system to low/high frequency dynamics and inputs by adjusting the decay rate α , see Theorem 1. This is to be done independent of the parametric uncertainty $a^* - a$, which is contrasted to LTI feedback where closed loop poles change with parametric uncertainty.
- The attenuation of uncertainty by high input-output system gain in this scheme differs from robust control by the fact that ISS stability, the pre-requisite to such attenuation, is never lost due to large parametric uncertainty $a^* - a$. This is the case since it no longer enters as a function of the plant's state but rather as an input $L(a^* - a)$.
- In switching between different a^* values many of the useful and interesting ideas to monitor, select, and switch between different candidate controllers via multiple models such as those in (1; 16) can be used with a_i^* values playing the role of the i^{th} candidate controller. The difference is that this is to be done without frozen-time instability or switched system instability concerns (verifying dwell time or common Lyapunov function conditions) as a^* is just an input to the closed loop system. Similarly, gain scheduling and Linear Parameter Varying (LPV) control (12; 20) can be applied with a^* playing the role of the scheduled parameter vector to be varied, again with no concerns with instability and transient behavior since $a^* - a$ enter as an input to the system.

4. Example Simulation

Consider the following unstable 2^{nd} order plant of relative degree 1 with a 2-mode periodic switching:

$$\begin{aligned}\dot{x}_1 &= a_1 x_1^3 + x_2 + (1 + x_1^2) b_1 u + d \\ \dot{x}_2 &= a_2 x_1 + (1 + x_1^2) b_2 u \\ y &= x_1 + n\end{aligned}$$

where u, d , and n are control signal, disturbance, and measurement noise respectively. Whereas, the plant parameters are given by:

$$\begin{aligned} a_1 &= 3 + 30 \text{square}(2\pi\omega t), a_2 = -2 - 20 \text{square}(2\pi\omega t) \\ b_1 &= 5 + \text{square}(2\pi\omega t), b_2 = 20 + 10 \text{square}(2\pi\omega t) \end{aligned}$$

where square denotes the unity magnitude square wave function and ω is the plant switching frequency is Hz.

4.1 Control System Evaluation

In this section, an adaptive controller, which is based on the design procedure of Section 4. Let us choose the nominal gains $C = 100$ (feedback gain), adaptation filter gain $L = I$, where I is the identity matrix, then we have from Theorem 1 that the decay rate $\alpha = 1$ rad/sec. This should yield a settling time of at most 4 seconds for the closed loop system. Also the nominal value of the adaptation gain $\Gamma = 100I$ will be used. Whereas, a^* is chosen to be a constant vector a_{ave} taking the average values of the parameters a_1, a_2, b_1, b_2 , i.e., when square functions are set to zero.

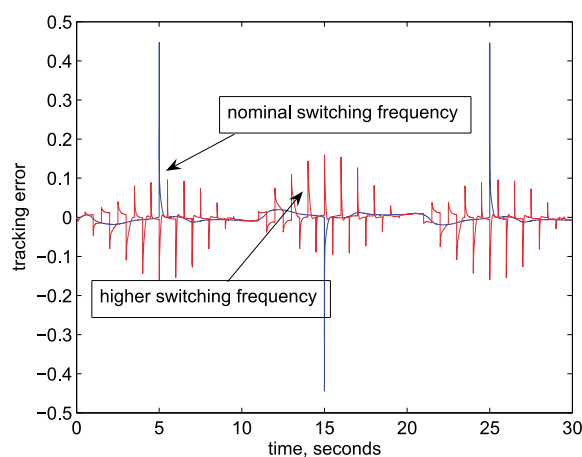


Fig. 1. Tracking error for different plant switching frequencies for developed adaptive controller.

Figure 1 shows the response of the modified adaptive controller for the output of the plant tracking a sinusoidal reference of amplitude 2 and frequency 0.3 rad/sec; the disturbance is set to zero for this case. The response follows the predicted theoretical behavior. The system responds to the corresponding impulse change in \dot{a} and step change in a due to switching in plant parameter vector a with the error settling after exponentially decaying transient according to the system decay rate α . Whereas, by increasing the plant switching frequency, the same trend follows with no concern of instability. In fact, as the suggested by the bound in Theorem 2, plant parametric uncertainty and variation are inputs to the closed loop system. Therefore, increasing the frequency of this input, 6 rads/sec in this case, relative system bandwidth, 1 rads/sec, will lead to attenuation of this input due to system roll-off as in linear systems. This explains why the tracking error is smaller for the higher switching frequency case.

Figure 2 shows the effect of different choices of the additional parameter estimate a^* for the nominal case of Figure 1. The figure shows that the average tracking error is larger when $a^* =$

$10a_{ave}$ and $a^* = 100a_{ave}$, since it corresponds to a larger size of the input $a^* - a$, as predicted by the bound of Theorem 2. The third case in Figure 2 shows the effect of switching the choice of a^* starting from a $a^* = 100a_{ave}$ to $a^* = 10a_{ave}$ at $t = 8$ seconds. Again, the response is that due to step changes in input $a^* - a$ with the transition between these two response takes place within the estimated settling time of 4 seconds based on a designed for decay rate of $\alpha = 1$ rads/ sec. This is a key capability that can be utilized in practice to perform robust and stable gain scheduling and online controller adjustments.

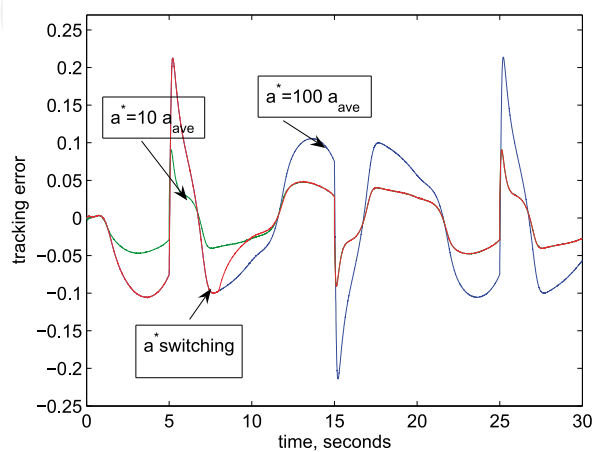


Fig. 2. Effect of parameter estimate a^* on tracking error for developed adaptive controller.

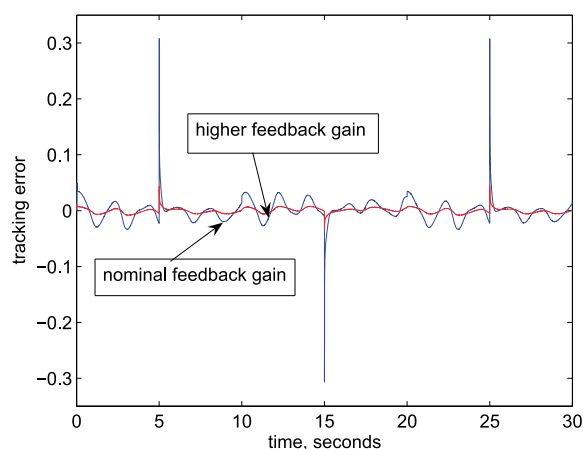


Fig. 3. Effect of feedback gain on tracking error for developed adaptive controller.

Next, Figures 3 and 4 will include the addition of a sinusoidal disturbance $d = 50\sin(\pi t)$ to the nominal case discussed above for switching frequency $\omega = 0.1$ Hz. Figure 3 displays the response of the nominal case of Figure 1 with the addition of a sinusoidal disturbance $d = 50\sin(\pi t)$, which introduces a clear sinusoidal content to the tracking error. Whereas, increasing feedback gain, which corresponds to matrix C in Theorem 1, significantly reduces the tracking error due to both plant switching (jumps and other steady errors) as well as the disturbance-induced error. This is consistent with the discussion in Section IV.B in that increasing system bandwidth α (via feedback gain) attenuates total input(disturbance an parametric uncertainties and variations) as well as speeds up the system bandwidth.

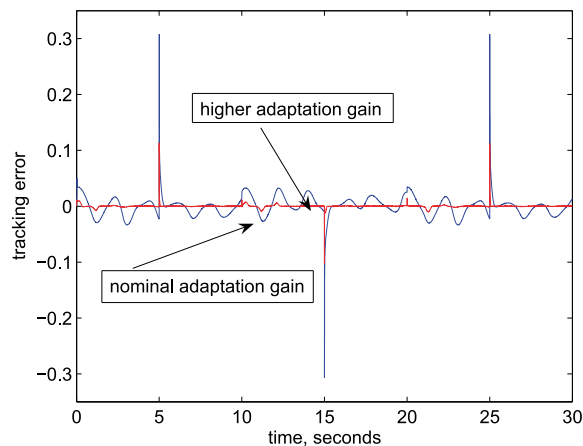


Fig. 4. Effect of adaptation gain Γ on tracking error for developed adaptive controller.

Whereas, Figure 4 considers the same situation in Figure 3 but with increasing adaptation gain instead of feedback gain. Again similar performance improvements are achieved along the lines of the bound in Theorem 1 yet without increasing system bandwidth.

Figures 2-4 show that error can be reduced by adjusting a^* , increasing feedback and adaptation gains, with different levels of effectiveness relative to disturbances, parametric uncertainty, and variation in accordance with the discussion in Section 3. The important message from this case study is not only that the developed control methodology can handle systems with large and rapid switching dynamics but also that this approach yields systematic and practical means to improve performance that follow the developed theory.

4.2 Comparison with Other Techniques

Finally, let us compare the system's response with the developed adaptive controller to other adaptive control techniques. We consider the same system of Section 5.1 with switching frequency $\omega = 1$ Hz case. The system is required to follow a constant reference of amplitude 2. First consider a non-adaptive backstepping controller, where the parameter estimate \hat{a} , in the developed control scheme of is replaced with a fixed value $\hat{a} = a_{ave}$. Figure 5 shows that the non-adaptive controller yields an unstable closed loop despite using the same assumed value of plant parameter vector, which has been used by the modified adaptive controller with $a^* = a_{ave}$.

Next, Figure 6 shows the response of the parameter estimates \hat{a} , when the equivalent standard adaptive controller, Equation (3), is used. This corresponds to setting $L = 0$ in the modified adaptive controller of Equation (6). In this case, some of the parameter estimates \hat{a} grow unbounded, which could yield an unstable system in practical implementation. This is a known issue with standard adaptive control in the presence of parameter variations or even disturbances, which is usually referred to as parameter drift (6). In contrast, the modified adaptive controller for the same situation maintains bounded parameter estimates due to ISS stability of the closed loop, see Figure 7.

The poor robustness of standard adaptive controllers with respect to time varying parameters and disturbances has led to modifying the adaptation law by robust adaptation laws such as deadzone, projection, and leakage modifications (6). Although there have not been any results reporting guaranteed stability and performance characteristics for rapidly varying switching

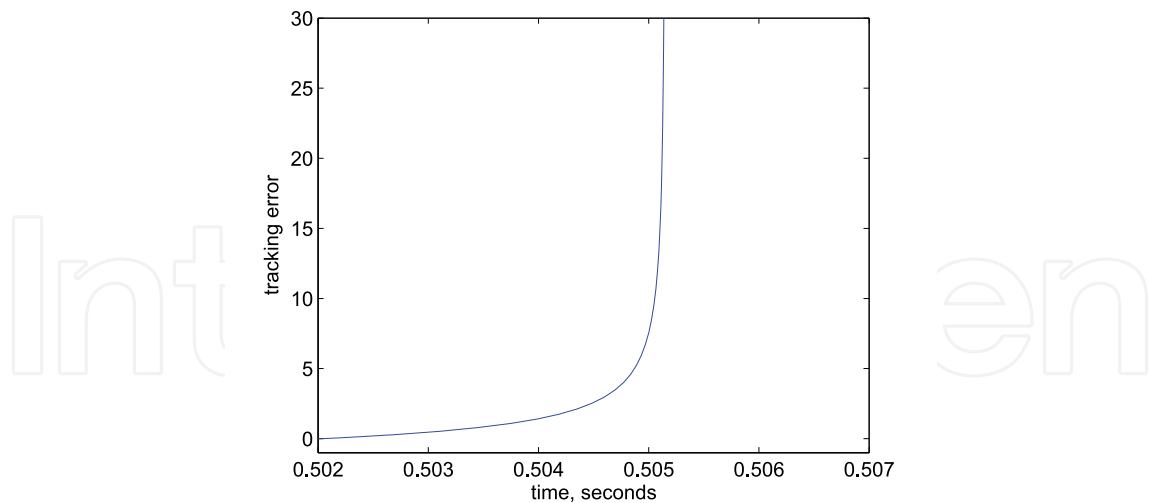


Fig. 5. Tracking error for non-adaptive backstepping controller with $\hat{a} = a_{ave}$.

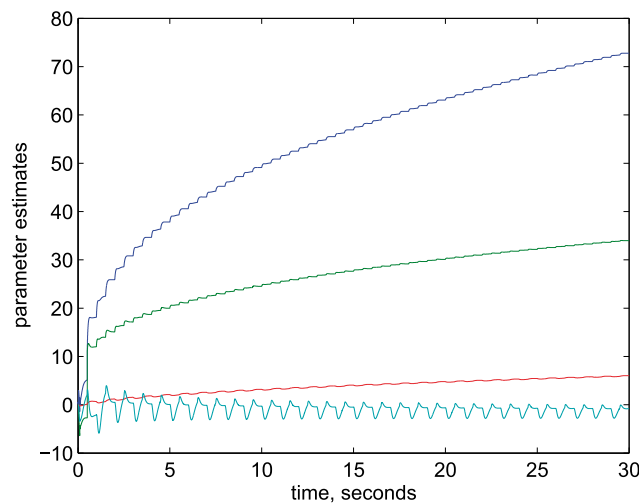


Fig. 6. Parameter estimates \hat{a} for standard adaptive controller with $L = 0$.

systems using these techniques, we will compare the leakage-based modification developed in this chapter with deadzone and projection modifications.

A deadzone modification to the standard adaptation law of Equation (3) can be given by:

$$\hat{a} = \begin{cases} f_a(x_m, \hat{a}, y_r, t) & \text{if } \|e\| > \epsilon \\ 0 & \text{otherwise} \end{cases}$$

This simply means to turn off the adaptation when the tracking error is less than some acceptable threshold ϵ . Figure 8 compares the modified adaptive controller with $a^* = a_{ave}$ to an equivalent deadzone adaptive controller with the same adaptation gain $\Gamma = 10000I$, where I is the identity matrix, and a deadzone threshold of $\epsilon = 0.3$. In this case, the modified adaptive controller outperforms the deadzone adaptive controller in the tracking error. Furthermore, when attempting to reduce the size of the tracking error threshold for the deadzone, ϵ , to allow for improvement in tracking error, the parameter estimates grew unboundedly as in the

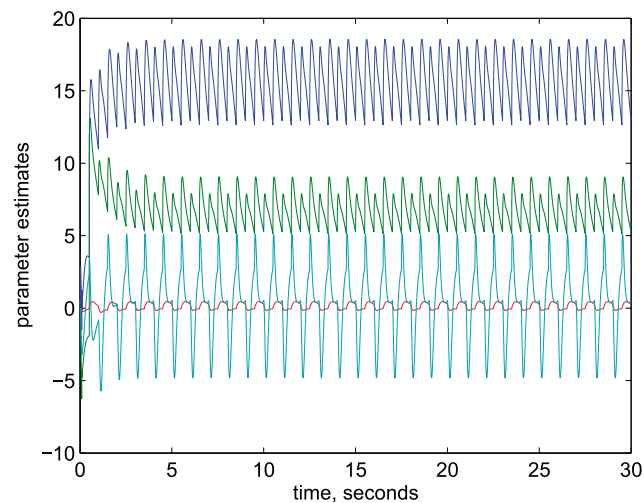


Fig. 7. Parameter estimates \hat{a} for developed adaptive controller.

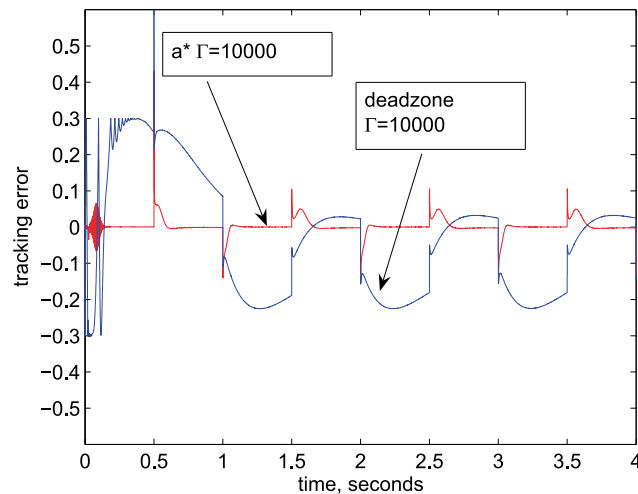


Fig. 8. Tracking error comparison for developed adaptive controller and a deadzone adaptive controller.

standard adaptive controller case of Figure 6. This is expected as the deadzone adaptive controller approaches that of a standard adaptive controller as $\epsilon \rightarrow 0$. Another limitation to the deadzone controller is the lack of systematic dependence on control parameters such as the adaptation gain Γ unlike the modified adaptive controller. Figure 9 shows how increasing the adaptation gain from $\Gamma = 100I$ to $\Gamma = 10000I$ does not necessarily improve tracking but rather yields reduction and increase in tracking at different times and of different signs. This is contrasted with the modified adaptive controller when tested under the same conditions, Figure 10, where a clear reduction in tracking error is observed with increasing Γ , in accordance with the scaling relationship in Section 3.

Next, we consider a parameter projection modification to the standard adaptive controller of

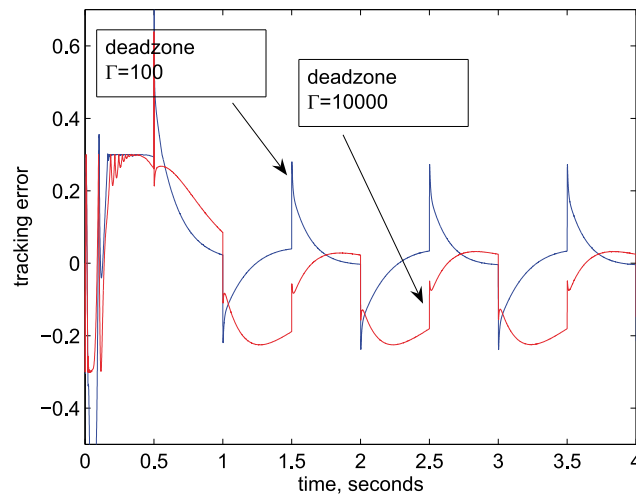


Fig. 9. Effect of adaptation gain Γ on tracking error for deadzone adaptive controller.

Equation (3). The projection modification (6) used here is given by:

$$\hat{a} = \begin{cases} f_a & \text{if } \|\hat{a}\| \leq M \text{ or } \hat{a}^T f_a \leq 0 \\ f_a - \frac{\hat{a}\hat{a}^T}{\|\hat{a}\|^2} \left(\frac{\|\hat{a}\|^2 - M^2}{M^2} \right) f_a & \text{otherwise} \end{cases}$$

Which uses an assumed bound on parameters $\|a\| \leq M$. This assumption is critical to projection algorithms. Figure 11 shows the tracking error growing unbounded when a projection algorithm was implemented with a tight bound $M = 1$. In this case, the assumed bound on parameters was too tight as soon as the system switched to a different mode leading to instability. This is in contrast to the developed adaptive controller, which does not require such information to guarantee stability. This is the case as the assumed parameter vector a^* only affects the size of tracking error for a given choice of control gains.

Nevertheless, it was possible to obtain a choice for the projection bound, $M = 10$, where the system remained stable. Figure 12 compares the tracking error for this projection adaptive controller and the developed adaptive controller with $a^* = a_{ave}$ for the same adaptation gain. Again, the developed adaptive controller achieved smaller tracking error. As was the case with deadzone controller, the projection controller does not display the systematic dependence on the adaptation gain Γ unlike the proposed adaptive controller, see Figure 13. This is the case since both projection and deadzone modification do not achieve a clear bound due to ISS stability as that in Theorem 1. In fact, most results using such techniques to deal with disturbances or parameter variations only conclude boundedness. In this case, such a conclusion is of very little practical importance if the error can not be reduced to an acceptable level by increasing the adaptation gain or using a better nominal estimate of the plant parameters as with using a^* in the proposed adaptation law, see Figure 2.

5. Conclusions

A methodology for robust adaptive control design for a class switched nonlinear systems is presented. Under extensions of typical adaptive control assumptions, a leakage-type adap-

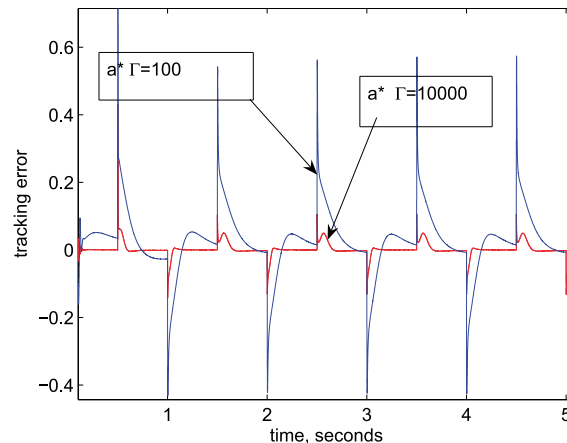


Fig. 10. Effect of adaptation gain Γ on tracking error for developed adaptive controller.

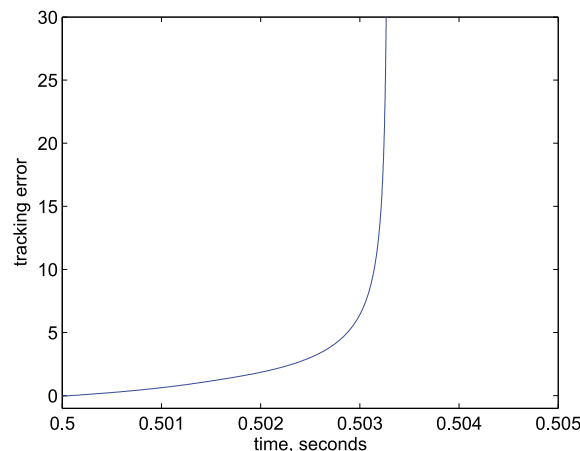


Fig. 11. Tracking error for projection adaptive controller with small parameter projection bound $M = 1$.

tive control scheme guarantees exponential and ISS stability with piecewise differentiable bounded plant parameters and piecewise continuous bounded disturbances without requiring a priori knowledge on such parameters. The effect of plant variation and switching is reduced to piecewise continuous and impulsive inputs acting on this ISS stable closed loop system. This yields a separation between robust stability and robust performance and clear guidelines for performance optimization via ISS bounds. The results are demonstrated through example simulations, which follow the developed theory and demonstrate superior robustness of stability and performance relative to non-adaptive and other adaptive methods such as projection and deadzone adaptive controllers. The authors believe that the use of these type of robust adaptive controllers is useful for switched systems even in the switched linear uncertainty free plant case, where stability with switched linear feedback is difficult to guarantee based on currently available tools.

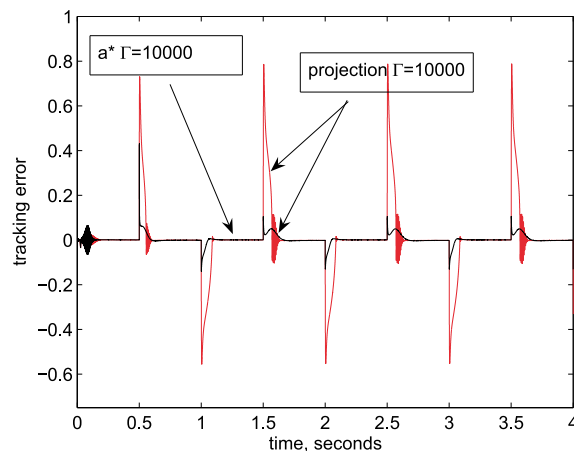


Fig. 12. Tracking error comparison for developed adaptive controller and a projection adaptive controller with large parameter projection bound $M = 10$.

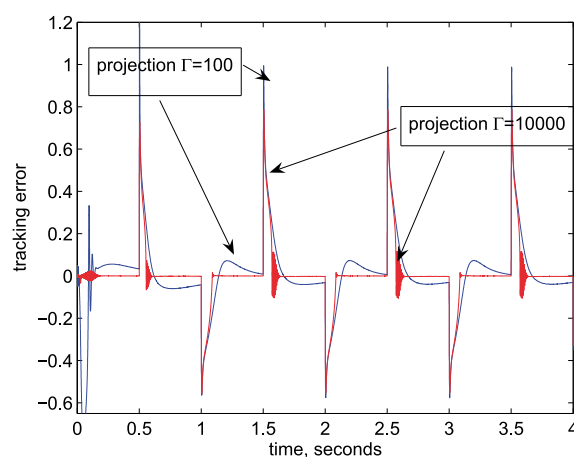


Fig. 13. Effect of adaptation gain Γ on tracking error for projection adaptive controller with large parameter projection bound $M = 10$.

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