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Differential Quadrature Method for Linear Long Wave Propagation in Open Channels

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1. Introduction

The St. Venant equations which are written for the calculation of free surface, gradually varied unsteady flows can only be solved analytically in very special limited problems. Apart from this they can be solved by numerical solution methods. Depending on the problem's characteristics, St. Venant equations can be arranged in different forms such as kinematic wave, dynamic wave, gravity wave, quasi-steady dynamic wave, and non-inertia wave may emerge. In the solution of non-linear St. Venant equations, different numerical solution techniques may be used by linearization of the equations by mathematical approaches. In this study the St. Venant equations made linear have been solved by using the Differential Quadrature Method (DQM), and the results have been compared by the Finite Difference Method (FDM) and Finite Volume Method's (FVM) results.

2. Background

The St Venant equations are used in the calculation of one-dimensional free surfaced flow. This equations, which is gained by writing the equations of continuity and motion, is semi-linear and hyperbolic. Apart from some very special problems, it cannot be directly integrated. For this reason, it can be solved by numerical solution techniques. The numerical solution techniques, which are generally used, are Finite Difference Method, Finite Volume Method and Finite Element Method.

In Finite Differences Method, such as Preismann scheme (Cunge,1975), Holly-Preismann scheme (Yang et al.,1992), McCormack scheme (Aguirre-Pe et al.,1995; Garcia-Navarro & Saviron, 1992; Garcia & Kahawita, 1986; Fennema & Chaudhry, 1986), Lambda scheme (Fennema & Chaudhry, 1986), Gabutti scheme (Fennema & Chaudhry, 1986), Beam-Warming scheme (Jha et al., 1996;1994; Fennema & Chaudhry, 1987; Mingham & Causon, 1998) and more different schemes are used (Glaister, 1988; 1993; Abbot, 1979; Cunge et al., 1980; Fread, 1983; Wang et al., 2000). Hsu and Yeh (2002) are investigated that is iterative explicit simulation of 1D surges and dam-break flows, and, has been summarized studies use different schemes. In addition to these studies, that have different boundary and initial conditions, Kiladze (2009) has investigated the study of the Stability of Finite Difference Schemes to Solve Saint-Venant Equations, Crossley and Wright (2005) has studied the time accurate local time stepping for the unsteady shallow water equations. Chambers (2000) and Hsu and Yeh (2002) are given studies that Finite Element Method has been used.

Additionally, Zhou et al. (2007) are proposed a split-characteristic Finite Element Model for 1-D unsteady flows. In the solution of this method, such as RDKG2 scheme (Kesserwani et al.; 2009), the modified Godunov method (Savic & Holly, 1993) and the Petrov-Galerking finite element scheme (Yang et al. 1993; Khan 2000) has been used. In recently, use of Finite Volume Method is increasing. Very studies are existing that are use of Finite Volume Method with different approximations (Bradford & Sanders, 2002; Mingham & Causon, 1998; Ying, et al., 2004; Erduran et al., 2002; Goutal & Maurel, 2002; Ghidaoui et al., 2001; Lee & Wright, 2009). Besides that conventional methods, in recently, such as Transfer Matrix Method (Daneshfaraz & Kaya, 2008), Incremental Differential Quadrature Method (Hashemi et al., 2006; 2007) and Differential Quadrature Method (Kaya et al., 2010) has been used. In parallel development of computer technology, the numerical methods can be used more effectively.

St. Venant equations can be written in different forms by omitting some terms and approaches such as kinematic wave, dynamic wave, gravity wave, quasi-steady dynamic wave and inertial wave may emerge. A considerable extent of publications regarding the solution of equations using different wave approaches are summarized in Yen and Tsai (2001) and Fan and Li (2006).

DQM has been developed by Richard Bellman (Bellman et al, 1971). This method offers the solution of gained equations in differential form of any given system, including existing boundary/initial conditions in the equations. Shu ve Richards (1992) have studies using the DQM in the area of some applications of fluid mechanics and the bending and twisting of beams. A recent study (Fung, 2001) has emphasized the application principles of DQM in problems we may face in the area of fluid mechanics and heat transfer. Kaya (2010) has investigated the use of DQM on the solution of Advection Diffusion Equation.

Shu et al. (2003) are presented local radial basis function-based differential quadrature method. Shu et al. (2004) examined the DQM was used to simulate the eccentric Couette–Taylor vortex flow in an annulus between two eccentric cylinders with rotating inner cylinder and stationary outer cylinder. Lo et al. (2005) are presented natural convection in a differentially heated cubic enclosure is studied by solving the velocity–vorticity form of the Navier–Stokes equations by a generalized differential quadrature method. Ding et al. (2006) examined, the local multi-quadric differential quadrature method is applied on three-dimensional incompressible flow problems.

3. Differential quadrature method

In the DQM, the partial derivative of a function with respect to a variable at a discrete point is approximated as a weighted linear sum of the function values at all discrete points in the region of that variable. The approximation of the partial derivative can be written as (Civalek, 2004):

$$u_x^{(r)}(x_i) = \left.\frac{\partial^r u}{\partial x^r}\right|_{x=xi} = \sum_{j=1}^{N} A_{ij}^{(r)} u(x_j) \qquad i = 1, 2, ..., N \tag{1}$$

where $u_x^{(r)}$ is the rth order derivative of the function, x_j are the discrete points of the variable x, $u(x_j)$ are the function values at points x_j and $A^{(r)}_{ij}$ are the weight coefficients for the rth order derivative of the function. The DQM is an alternative approach to the standard numerical solution methods such as finite differences and finite elements, for the initial and

boundary value problems encountered in physics and mathematics (Shu, 1992; Shu et al., 2003; Shu & Chew, 1997).

The overall sensitivity of the model especially depends on the location and number of sampling grid points. However, Civalek (2003) points out that the determination of the effective choice of sampling grid points for any problem reduces the analysis time (Civalek, 2003). For instance, previous studies show that for the solution of linear equations with homogeneous boundary conditions, selecting equal intervals between the adjacent grid points are adequate. On the other hand, for the vibration problems, the choice of grid points through the Chebyshev-Gauss-Lobatto method is more reasonable. In time-bound equations and initial value problems, selection of unequal intervals for sampling grid points produces the appropriate solutions (Civalek, 2003). For boundary value problems, DQM performance is highly dependent on the boundary conditions and sampling grid points. The boundary conditions can be easily implemented to DQ system and the common type of boundary conditions, which are Dirichlet, Neumann and/or mixed type function, do not create any difficulty in this implementation process (Bert, 1996; Civalek, 2003).

Determining the weight coefficients is the most crucial step in the use of DQM. Shu and Xue (1997) worked on the selection of the weight coefficients and proposed several solutions in their studies. The weight coefficients change upon approximation function and according to the chosen approximation function, the method takes different names such as Polynomial Differential Quadrature, Fourier Expansion Base Differential Quadrature and Harmonic Differential Quadrature (Civalek, 2004; Shu et al., 2002).

4. St.Venant equations and linearization

For unsteady one-dimensional free surface flows, the continuity (Eq.2) and momentum (Eq.3) equations are to be called the Saint-Venant equations. In use of the Saint-Venant equations, some basic assumptions are made that are (1) the streamline curvature is very small and the pressure distributions are hydrostatic; (2) the flow resistance are the same as for a steady uniform flow for the same depth and velocity; (3) the bed slope is small enough to satisfy : $cos\theta \approx 1$ and $sin\theta \approx tan\theta \approx \theta$; (4) the water density is a constant; and (5) the channel has fixed boundaries, and air entrainment and sediment motion are neglected (Chanson, 2004).

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \tag{2}$$

$$\frac{\partial Q}{\partial t} + \left(\frac{\partial (QV)}{\partial x} - V_x q \right) + gA\frac{\partial y}{\partial x} - gA(S_0 - S_f) = 0 \tag{3}$$

In this equations, the velocities and water depths change with time and longitudinal position, and, Q is flow discharge [L3/T]; y, flow depth [L]; A, flow cross sectional area [L2]; q_L, lateral outflow per unit length along the channel [L2] (infiltration, seepage, precipitation, etc); V_x, x component of velocity of lateral flow [L/T]; S_0, channel bed slope [L/L]; S_f friction slope [L/L]; x, distance in longitudinal direction [L/L]; and t is time [T]. In the solution of St. Venant equations different approximations are used namely dynamic wave, quasi-steady dynamic wave, non-inertia wave, kinematic wave and gravity wave (Yen & Tsai, 2001).

In practical terms, lateral outflows (seepage, infiltration etc) are comparatively low and can be neglected ($q=0$). The linearizing of the basis equation of unsteady and gradually varied flows

in prismatic channels can be done by the transformation $Q=Q_0+Q'$ and $A=A_0+A'$. Here Q_0 and A_0 show the steady initial conditions values. When the linearized St. Venant equations have been arranged and the 3rd degree terms have been neglected from the equation

$$\frac{\partial Q'}{\partial t} + C\frac{\partial Q'}{\partial x} = \left((1-F_0^2) + 2\frac{C}{u_0^2}F_0^2 - \left(\frac{C}{u_0^2}\right)^2 F_0^2 \right) m \frac{\partial^2 Q'}{\partial x^2} \qquad (4)$$

is obtained (Yen & Tsai, 2001). In this equation,

$$m = \frac{u_0 y_0}{2S_0} \qquad (5)$$

$$F_0 = \frac{u_0}{\sqrt{gy_0}} \qquad (6)$$

$$y_0 = \frac{A_0}{B_0} \qquad (7)$$

$$C = -\frac{\partial Sf/\partial A}{\partial Sf/\partial Q}\bigg|_0 u_0 \qquad (8)$$

happens.

$$D_h = \left((1-F_0^2) + 2\frac{C}{u_0}F_0^2 - \left(\frac{C}{u_0}\right)^2 F_0^2 \right) m \qquad (9)$$

impending, as seen this equation can be written as

$$\frac{\partial Q'}{\partial t} + C\frac{\partial Q'}{\partial x} = D_h \frac{\partial^2 Q'}{\partial x^2} \qquad (10)$$

This equation is used in the case of dynamic wave approach. In the other wave approaches because some waves are neglected; in the quasi-steady dynamic wave approach

$$D_h = \left[1 - (1-\frac{C}{u_0^2})F_0^2 \right] m \qquad (11)$$

in the non-inertia wave approach

$$D_h = m \qquad (12)$$

and, in the kinematic wave approach $D_h=0$. Instead of the Q yield St. Venant equations can be arranged by the y value by changing $y=y_0+y'$. In this case

$$\frac{\partial y'}{\partial t} + C\frac{\partial y'}{\partial x} = D_h \frac{\partial^2 y'}{\partial x^2} \qquad (13)$$

equation is gained (Yen & Tsai, 2001).

The celerity used in Eq. (10) depends on the channel cross-section geometry, the flow area and the resistance formula (S_f). For a trapezoidal channel, C is defined as follows:

$$C = \left[1 + \frac{\alpha}{2}\left(1 - \frac{2\sqrt{1+z^2}\,y_0}{b+2\sqrt{1+z^2}\,y_0}\frac{b+zy_0}{b+2zy_0}\right)\right]u_0 \qquad (14)$$

Here, $\alpha=1$ for Chezy formula, $\alpha=4/3$ for the Manning's formula, b is bottom width [L], and z is the side slope [L/L].

5. Application of differential quadrature method

The equation (10) obtained as a result of an arrangement of of the linearized St. Venant equations, can be rewritten for the solution of DQM as seen in Eq. 15

$$\sum_{r=1}^{R} A_{r,s}Q_{i,r} + \left(C\sum_{j=1}^{N} B_{j,i} - D_h\sum_{j=1}^{N} B_{j,i}^{(2)}\right)Q_{j,s} = 0 \qquad i=1,2,...,N;\ \ s=1,2,...,R \qquad (15)$$

where N, is the number of sampling grid points in the x direction; R is the number of sampling grid points in time direction; and A, B, B$^{(2)}$ are weight matrix coefficients.
Determining the boundary conditions, Eq. 15 is solved for the $Q(i,s)$ values. For example, when $Q(x,0)= f_1(t)$ and $Q(0,t)=f_2(x)$, Eq. 15 yields

$$\sum_{r=2}^{R} A_{r,s}Q_{i,r} + \left(C\sum_{j=2}^{N} B_{j,i} - D_h\sum_{j=2}^{N} B_{j,i}^{(2)}\right)Q_{j,s} = -A_{1,s}Q_{i,1} - \left(cB_{1,i} - D_h B_{1,i}^2\right)Q_{1,s} \qquad (16)$$

where, $i = 2,3,…N$ and $s = 2,3,…R$.
Using the properties of Legendre polynomials, the weight coefficient matrix can be written as (Shu et al. 2004):

$$L_i = \prod_{j=1}^{N}(x_j - x_i) \qquad (17)$$

$$B_{i,k} = \frac{L_i}{L_k(x_i - x_k)} \qquad i \neq k \qquad (18a)$$

$$B_{i,i} = -\sum_{k=1}^{N} B_{i,k} \qquad i \neq k \qquad (18b)$$

$$B_{i,k}^{(2)} = 2\left[B_{i,k}B_{i,k}^{)} - \frac{B_{i,k}}{x_i - x_k}\right] \qquad i \neq k \qquad (19a)$$

$$B_{i,i}^{(2)} = -\sum_{k=1}^{N} B_{i,k}^{(2)} \qquad i \neq k \qquad (19b)$$

For the numerical discretization, several different methods are available. It can be selected with equal intervals, or non-equal intervals like Chebyshev-Gauss-Lobatto grid points, or

with the normalization of the routes of Legendre polynomials (Shu, 2000). In the present study, several different approximations have been tested, and, Chebyshev-Gaus-Lobatto grid points have been selected. In the Chebyshev-Gaus-Lobatto approximation, by writing

$$r_i = \frac{1}{2}\left(1 - \cos\frac{i-1}{N-1}\pi\right) \tag{20}$$

the locations as the calculation points can be calculated as follows:

$$x_i = \frac{r_i - r_1}{r_N - r_1} \tag{21}$$

In time domain, this yields

$$t_i = \frac{r_i - r_1}{r_R - r_1} \tag{22}$$

by considering R, in place of N, in Eq.(21) and Eq.(22). A matrix can be written like B matrix as follows.

$$L_i = \prod_{j=1}^{R}(t_j - t_i) \tag{23}$$

$$A_{i,k} = \frac{L_i}{L_k(t_i - t_k)} \qquad i \neq k \tag{24a}$$

$$A_{i,i} = -\sum_{k=1}^{R} A_{i,k} \qquad i \neq k \tag{24b}$$

6. Numerical examples

In the Murumbidgee River in South Wales, Austria, in 1978 and 1983, two serious floods happen (Sivapalan et al. 1997). In the present study, the numerical calculations were performed by use of the flood propagation between Gundagai and Wagga Wagga stations over the river. Gundagai is located upstream of the river with 120 km length and 0.032% longitudinal slope. The Gundagai measurements were undertaken as initial data, and then the flood hydrograph at Wagga Wagga was determined. A comparison was conducted taking into account the DQM, FVM, FDM, actual measurements, and nonlinear FDM solution performed by Sivapalan et al. (1997). Here, the first three analyses (DQM FVM and FDM) were performed for constant C and D_h values.

In the first example, Gundagai station's measurements in 1978 were accepted as boundary data, and the flow rate values at Wagga Wagga station were calculated by use of Implicit Finite Difference Method (IFDM), Explicit Finite Difference Method (EFDM), Implicit Finite Volume Method (IFVM), Explicit Finite Volume Method (EFVM) and DQM. Gundagai and Wagga Wagga stations measurements and IFDM, IFVM and DQM results are given in Fig. 1. An additionally, nonlinear solution at given Sivapalan et al (1997) was examined. The solution of Sivapalan et al. (1997), EFDM, EFVM and DQM results are given in Fig. 2.

The accuracy of these different numerical methods was obtained by using the Root Mean Square (RMS) equation as follows:

$$\|e\|_2 = \sqrt{\frac{1}{R}\sum_{i=1}^{R}|e_i|^2} \qquad e_i = Q_{observed} - Q_{num.method} \qquad (25)$$

where $Q_{observed}$ is the observed value and $Q_{num.method}$ is the numerical solution i.e., DQM, EFDM, IFDM, EFVM, IFVM and Sivapalan et al.(1997) values and R is the number of points in time direction.

RMS error values of FDM, FVM and DQM were given in Fig. 3 and in Fig. 4. RMS errors in Sivapalan et al. (1997) for a nonlinear solution was calculated as approximately 170 m³/sec. RMS error values in FVMs, FDMs and DQM are converged to approximately 165, 137 and 122, respectively.

In the watershed in 1983, a flood had occurred again. Gundagai and Wagga Wagga stations measurements and solution of Sivapalan et al. (1997) at Wagga Wagga were given in Fig. 5. In addition, the results of IFDM, IFVM and DQM are seen in this Figure. EFDM, EFVM results are given in Fig. 6. In Fig. 7 and 8, FDMs, FVMs and DQM RMS error values are given as if in 1978 flood.

RMS error values are converged to 73, 76 and 71 in FDM, FVM and DQM, respectively. This number is 41 in Sivapalan's solution.
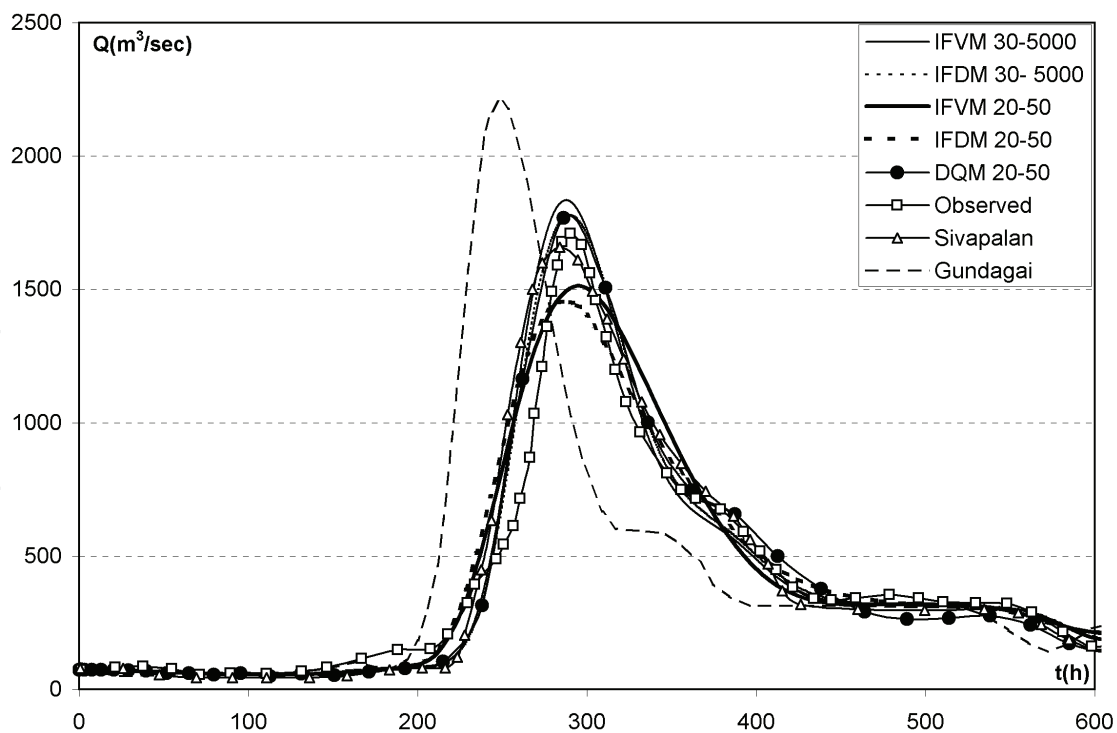
Fig. 1. IFVM, IFDM and DQM results and Sivapalan et al.(1997) solution at Wagga Wagga and actual measurements at Gundagai (upstream boundary) and Wagga Wagga (1978 flood)
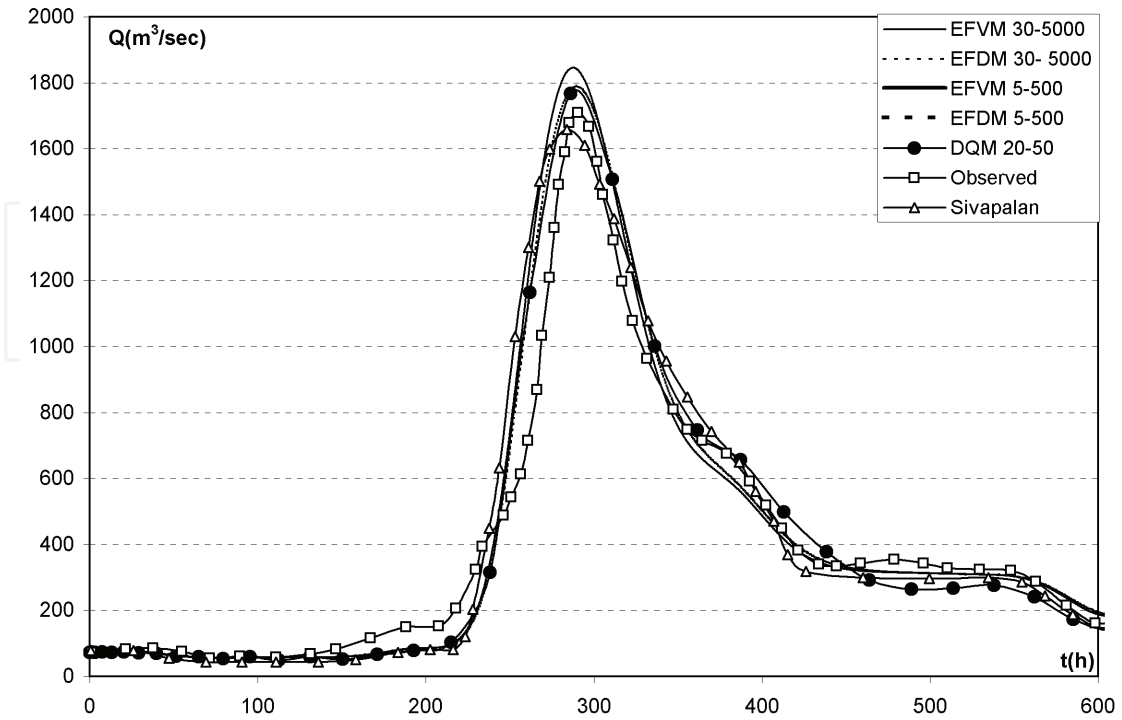
Fig. 2. EFDM, EFVM and DQM results, observed values and Sivapalan et al. (1997) solution at Wagga Wagga (1978 flood)
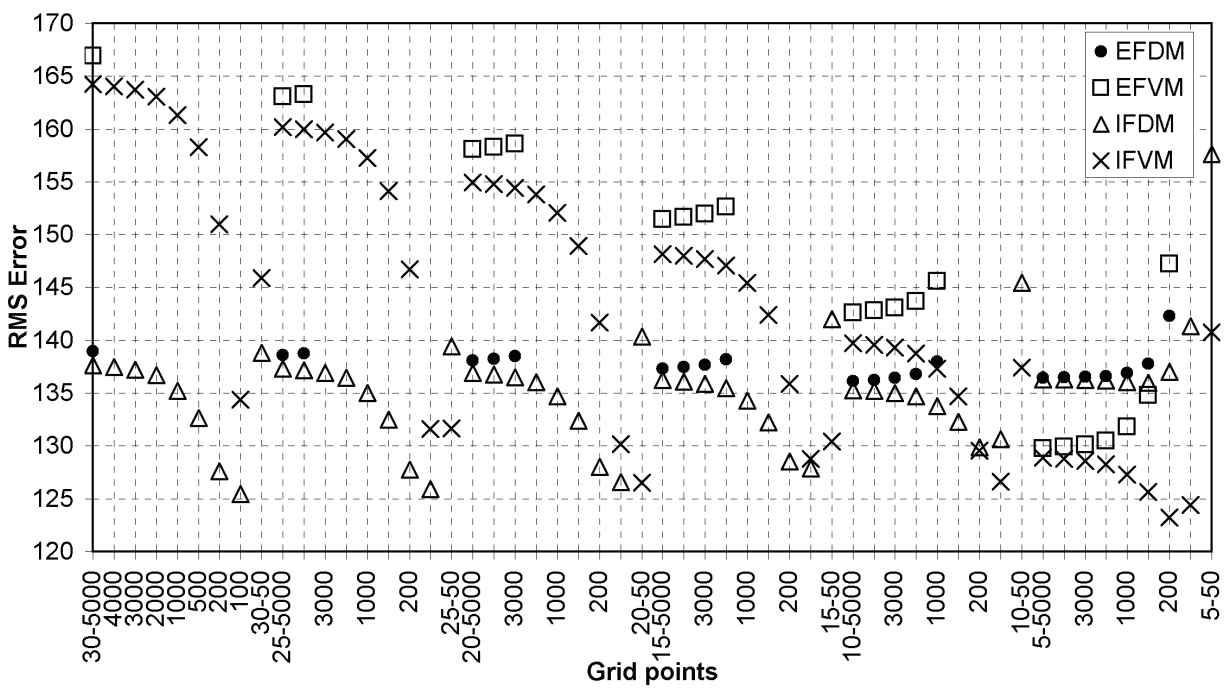
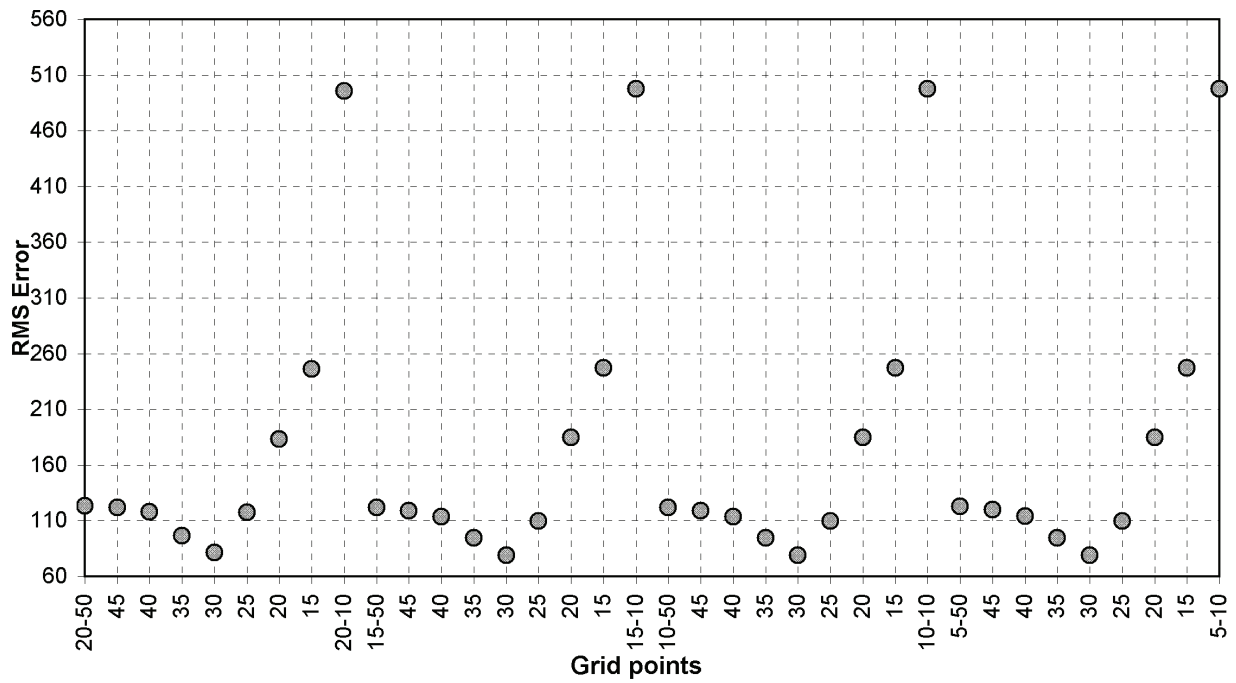Fig. 3. FDMs and FVMs RMS error values for different sampling grid points (1978 flood)

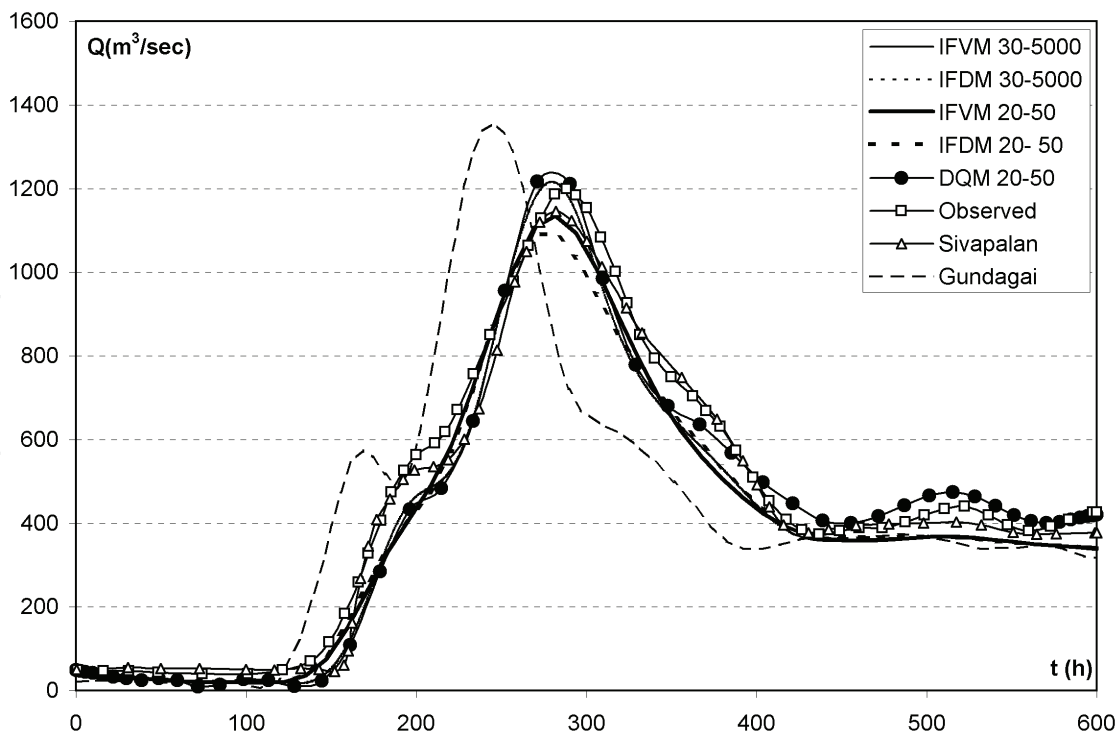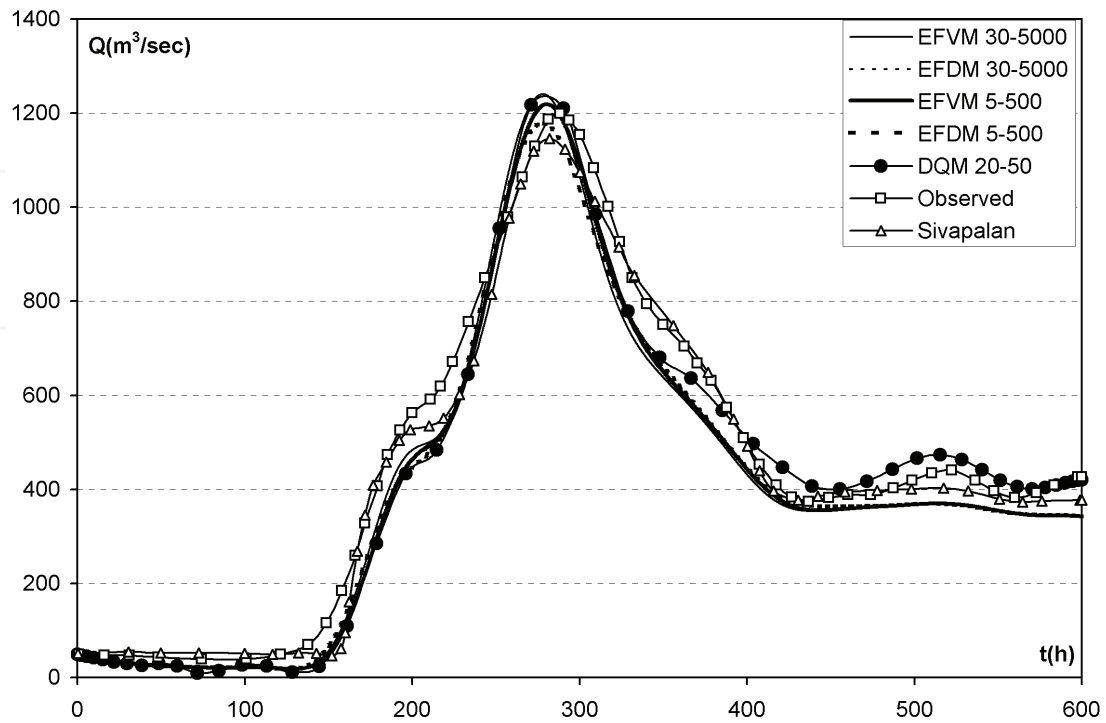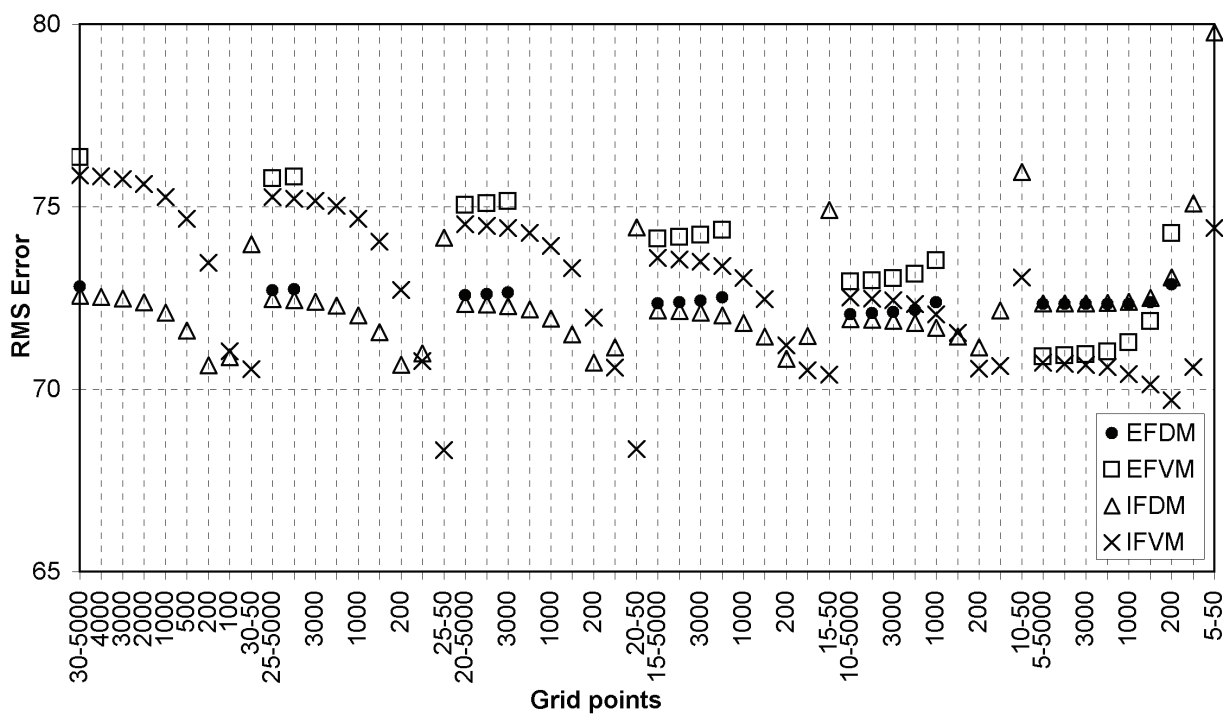Fig. 4. DQM RMS error values for different sampling grid points (1978 flood)

Fig. 5. IFVM, IFDM and DQM results and Sivapalan et al.(1997) solution at Wagga Wagga and actual measurements at Gundagai (upstream boundary) and Wagga Wagga (1983 flood)

Fig. 6. EFDM, EFVM and DQM results, observed values and Sivapalan et al. (1997) solution at Wagga Wagga (1983 flood)

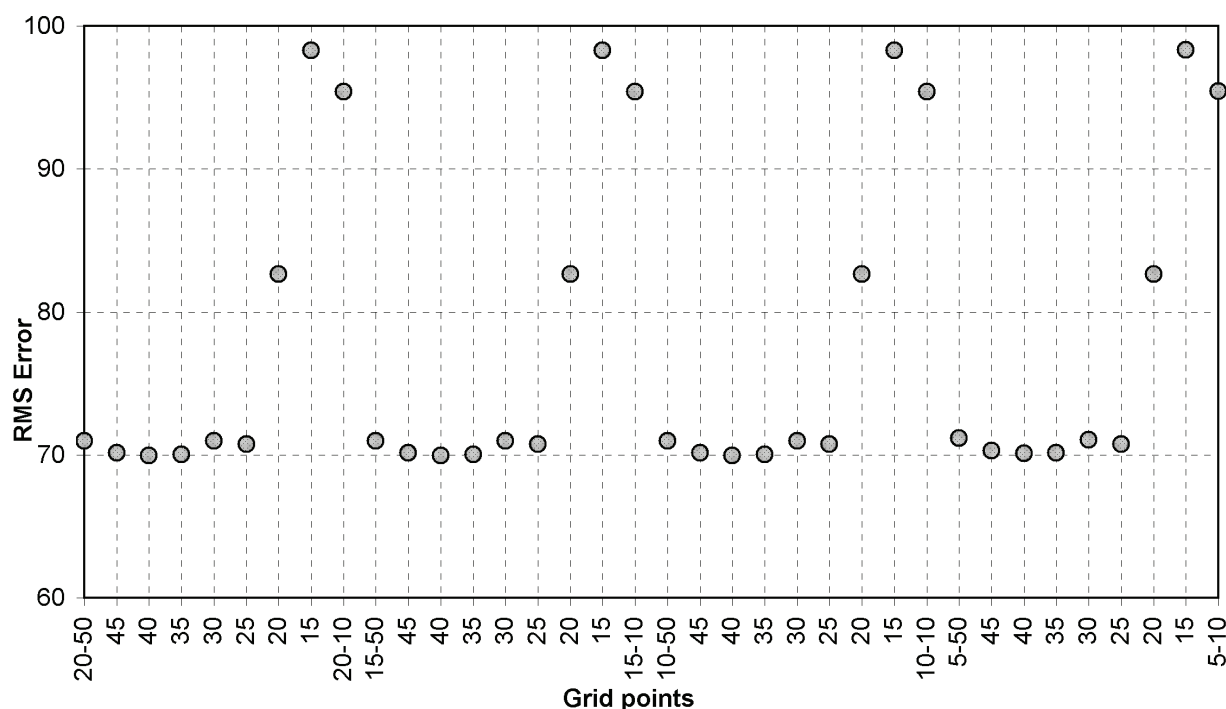Fig. 7. FDMs and FVMs RMS error values for different sampling grid points (1983 flood)

Fig. 8. DQM RMS error values for different sampling grid points (1983 flood)

7. Results and conclusion

DQM has found increasing use in recent years in Hydraulic Engineering, because it is an alternative approach to the standard methods such as Finite Difference Method, Finite Element Method and Finite Volume Method. From the previous applications of DQM in different engineering fields, it is seen that the results of DQM are converged rapidly and closer to analytical solutions than other numerical solutions. A similar consequence is observed in an application of Hydraulic Engineering (Kaya et al., 2010).

The St. Venant equations that are written for an unsteady open channel flow under some acceptance can be translated to linear form, and then can be solved by numerical solution methods. In this study, DQM were employed for one-dimensional, gradually varied, unsteady open-channel flows. The DQM results were compared with the results using the FDMs and FVMs. It is obvious from the examples given above that the DQM results are closer to the measurements than the other numerical methods.

In calculations making use of 1978 Murumbidgee River flood data, the DQM's RMS error values are less than nonlinear solution results in Sivapalan et al. (1997); On the contrary, in calculations of 1983 flood, that is bigger. Yet, for a linear solution performed in this study, the DQM's RMS error values are less than other numerical methods.

In contrast to the FDM and FVM, in the DQM, a small number of grid points are sufficient for a stable solution. The results of FDMs and FVMs are converged from 2000 at t direction, but this value is 25 in DQM.

Determining the weight coefficients is the most crucial step in the use of DQM. In wave propagation problems in open channels, by making use of Legendre polynomials for weight coefficients and Chebyshev-Gauss-Lobatto points for the numerical discretization, the results can be obtained closer to an analytical solution (Kaya, 2010).

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Wave Propagation in Materials for Modern Applications

Edited by Andrey Petrin

In the recent decades, there has been a growing interest in micro- and nanotechnology. The advances in nanotechnology give rise to new applications and new types of materials with unique electromagnetic and mechanical properties. This book is devoted to the modern methods in electrodynamics and acoustics, which have been developed to describe wave propagation in these modern materials and nanodevices. The book consists of original works of leading scientists in the field of wave propagation who produced new theoretical and experimental methods in the research field and obtained new and important results. The first part of the book consists of chapters with general mathematical methods and approaches to the problem of wave propagation. A special attention is attracted to the advanced numerical methods fruitfully applied in the field of wave propagation. The second part of the book is devoted to the problems of wave propagation in newly developed metamaterials, micro- and nanostructures and porous media. In this part the interested reader will find important and fundamental results on electromagnetic wave propagation in media with negative refraction index and electromagnetic imaging in devices based on the materials. The third part of the book is devoted to the problems of wave propagation in elastic and piezoelectric media. In the fourth part, the works on the problems of wave propagation in plasma are collected. The fifth, sixth and seventh parts are devoted to the problems of wave propagation in media with chemical reactions, in nonlinear and disperse media, respectively. And finally, in the eighth part of the book some experimental methods in wave propagations are considered. It is necessary to emphasize that this book is not a textbook. It is important that the results combined in it are taken "from the desks of researchers". Therefore, I am sure that in this book the interested and actively working readers (scientists, engineers and students) will find many interesting results and new ideas.

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