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Applications of Robust Descriptor Kalman Filter in Robotics

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1. Introduction

In this chapter we are interested in designing estimators for the internal variables of two kind of robots, wheeled mobile and robotic leg prosthesis, based on a recently developed robust descriptor Kalman filter. The proposed approach is reasonable since descriptor formulation can cope with algebraic restrictions on system's signals. Further, the recursiveness of this class of filter is useful for on-line applications.

Different procedures have been used to deal with mobile robots localization problem. Measurement systems based on odometric, inertial sensors and ultrasounds are self-contained, simple to use, and able to guarantee a high data rate. However, the problem of these systems is that they integrate the relative increments, and the localization errors considerably grow over time if an appropriate sensor fusion algorithm is not used, see for instance [17], [18] and references therein. The examples developed in these references do not take into account robust approaches, in the line we are proposing here.

In the context of robotic leg prosthesis, we deal with the development of devices for above knee amputees. Robotic prosthesis are devices intended to replace parts of the human body. They should be able to sense the environment and complain with the movement of the body in such a way to aid the user to perform the most common tasks. This is a very interesting and current research topic [7]. Environment sensing is one of the most difficult tasks, mainly in the case of leg prosthesis because of the great diversity of walking conditions and terrains. The use of Electromyographic (EMG) signal processing for detecting the main properties of the walking terrain is the focus of [15]. However, in the case of above knee prosthesis, there is no EMG signal available to allow automatic reorientation of the robotic foot. When the foot of a robotic leg is not in contact with ground, its configuration should be estimated to allow its control with respect to ground. This can be useful for controlling its orientation, mainly in the end of phase where the foot is not in contact with ground. In this chapter, it is shown a solution for this problem using multisensor data fusion by a robust descriptor Kalman filter.

This chapter is divided in three main parts. In the first part we present basic definitions and concepts of descriptor systems and some examples to clarify the use of this kind of approach. In the second part we present three algorithms for the computation of the

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descriptor robust Kalman filter for its filtered, predicted, and time and measurement update forms. Finally we present two practical examples of application in robotics.

2. Descriptor systems and the robust estimation problem

In dynamical systems modeling, it is usually obtained a set of differential equations (by first principles like Newton's law, Ohm's law,...)

$$\begin{aligned} \dot{x}_1(t) &= f_1(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t), t) \\ &\vdots \\ \dot{x}_n(t) &= f_n(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t), t) \end{aligned}$$

and a set of algebraic restrictions (conservative laws, Kirchoff rules,...)

$$\begin{aligned} g_1(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t), t) &= 0 \\ &\vdots \\ g_p(x_1(t), \dots, x_n(t), u_1(t), \dots, u_m(t), t) &= 0 \end{aligned}$$

or in matricial notation

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), t) \\ g(x(t), u(t), t) &= 0. \end{aligned} \tag{1}$$

A dynamic system in this formulation is called descriptor system or differential-algebraic system. These nonlinear sets of dynamic and constraint equations are usually obtained by modeling softwares. Therefore, systems in descriptor formulation frequently arises naturally in the process of modeling of economical systems [21], image modeling [9], robotics [22], and so on.

Note that the descriptor formulation of a dynamical system states that the input $u(t) = [u_1(t) \dots u_m(t)]^T$, the descriptor variable $x(t) = [x_1(t) \dots x_n(t)]^T$ and its dynamics $\dot{x}(t)$ are related implicitly as

$$F(\dot{x}(t), x(t), u(t), t) = 0 \tag{2}$$

where the function $F(*)$ is usually considered differentiable. Due to this alternative general formulation, a descriptor system is also known as implicit system and the descriptor variable is also called internal latent variable, generalized state, semi-state or simply, state variable. In many cases (those called regular), it is possible to obtain some descriptor variables in terms of others and thus obtain a minimum set of equations in state-space form

$$\dot{x}(t) = f(x(t), u(t), t).$$

However, there are cases (those called singular) where it is not possible to reduce to a state-space formulation. Therefore, the descriptor formulation is more general than the state-space formulation in the sense that it can describe some dynamical systems for which state-space description does not exist [31]. Descriptor formulation arises naturally in several situations:

1. **Proportional derivative (PD) controller.** The state derivative feedback leads to a closed loop system

$$(I + BK) \dot{x}(t) = Ax(t)$$

where the matrix $(I + BK)$ is in general nonsingular. [20]

2. **Systems interconnection.** The feedback closed system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

$$u(t) = Ky(t) + v(t)$$

can be written in descriptor form as

$$\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{u}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ C & D & -I \\ 0 & -I & K \end{bmatrix} \begin{bmatrix} x(t) \\ u(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} v(t)$$

The closed loop system is singular if the matrix $\begin{bmatrix} D & -I \\ -I & K \end{bmatrix}$ is singular.

3. **System with unknown inputs.** The system

$$\dot{x}(t) = Ax(t) + d(t)$$

where the input $d(t)$ is considered unknown can be written as

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{d}(t) \end{bmatrix} = \begin{bmatrix} A & I \\ 0 & I \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \end{bmatrix}.$$

For more details on descriptor systems see for instance [2], [19], [5], [30] and references therein, in which an extensive list of applications can be found.

After the first step of obtaining the general descriptor equation (1) or (2), in order to deal with computer implementations, usually it is performed a linearization and time discretization on (2) and it is chosen an output signal z leading to a linear discrete-time descriptor system

$$E_{i+1}x_{i+1} = F_i x_i + G_{u,i} u_i$$

$$z_i = H_i x_i + K_{u,i} u_i.$$

This linear model is suitable in many cases, but its main hypothesis is that the system parameters are exactly known. In most situations the effects of some uncertainties cannot be neglected. For these situations, a more appropriate model can be considered as

$$\begin{aligned} (E_{i+1} + \Delta E_{i+1}) x_{i+1} &= (F_i + \Delta F_i) x_i + (G_i + \Delta G_i) u_i + w_i \\ z_i &= (H_i + \Delta H_i) x_i + (K_i + \Delta K_i) u_i + v_i \end{aligned} \quad (3)$$

where ΔE_{i+1} , ΔF_i , ΔG_i , ΔH_i , and ΔK_i deal with the disturbances in the model parameters (to cope with approximations or nonlinearities) and w_i and v_i are random vectors (to cope with noisy signals).

In the last years, the robust estimation problem of descriptor systems has been considered in the literature by several authors. This chapter deals with a class of robust descriptor filters based on recursive algorithms. They are extensions of the robust filters developed in [25] for standard state space systems.

A. The problem of generalized state estimation

In this subsection, we consider the problem of the general internal variable estimation and its solution. The theoretical presentation is reduced and most proofs are omitted to easy reading. The detailed proofs can be found in the references.

Consider that we have a set of measured or observed signals $z = \{z_0, z_1, \dots, z_k\}$ from a certain real dynamical system in discrete-time. We are interested in obtaining a suitable dynamical model which 'explains' the observed measurements and even obtain some knowledge about the values of next measurements, as z_{k+1} , for example. As a first attempt, we can consider that z is the output of a linear system. An ideal linear dynamical system in its more general form is an implicit or descriptor system

$$E_{i+1}x_{i+1} = F_i x_i, \quad i = 0, 1, 2, \dots \quad (4)$$

$$z_i = H_i x_i$$

where x_i is the descriptor variable which describes the internal behavior of the system; E_{i+1} , F_i , and H_i are real rectangular matrices of appropriated dimensions. Then, we can state a deterministic data fitting problem over the entire trajectory as follows. Suppose that are given a sequence of measurements $\{z_i\}_{i=0}^k$, the matrices E_i , F_i , H_i of appropriate dimensions, and an initial value \bar{x}_0 . For each state sequence $\{x_{0|k}, x_{1|k}, \dots, x_{k|k}, x_{k+1|k}\}$ we can define the following fitting errors

$$\begin{aligned} w_{i|k} &:= E_{i+1}x_{i+1|k} - F_i x_{i|k} \\ v_{i|k} &:= z_i - H_i x_{i|k}, \quad i = 0, 1, \dots, k \\ p_{0|k} &:= E_0 x_{0|k} - F_{-1} \bar{x}_0 \end{aligned} \quad (5)$$

where the matrices E_0 and F_{-1} are supposed of appropriate dimensions. These matrices can deal with the *a priori* information on the initial state x_0 , and usually it is supposed $E_0 = F_{-1} = I$. Now, the deterministic optimal fitting problem is to find a state sequence which minimizes some predefined error functional (the actual expression of the functional is not considered in this chapter. The interested reader can see [12]). Once obtained the minimizing sequence $\{\hat{x}_{i|k}\}$, we can define from (5) the corresponding minimum fitting errors $\hat{w}_{i|k}$, $\hat{v}_{i|k}$, $\hat{p}_{0|k}$ so that the complete model which 'explains' the set of measured signals $z = \{z_i\}_{i=0}^k$ can be written as

$$E_0 \hat{x}_{0|k} = F_{-1} \bar{x}_0 + \hat{p}_{0|k} \quad (6)$$

$$E_{i+1} \hat{x}_{i+1|k} = F_i \hat{x}_{i|k} + \hat{w}_{i|k} \quad (7)$$

$$z_i = H_i \hat{x}_{i|k} + \hat{v}_{i|k}, \quad i = 0, 1, \dots, k. \quad (8)$$

As the model (6)-(8) also gives the next internal value $\hat{x}_{k+1|k}$, we could try to obtain a prevision for the value of the next measurement z_{k+1} . From the process followed until now (with fixed $k + 1$ observations), we do not have $\hat{v}_{k+1|k+1}$ nor $\hat{v}_{k+1|k}$, and so, the best knowledge we have about a possible value for the next measurement is the estimate

$$\hat{z}_{k+1|k} := H_{k+1} \hat{x}_{k+1|k}. \quad (9)$$

This is a reasonable guess since for the last available measurement we have, z_k , we see from (8) that the estimate

$$\hat{z}_{k|k} := H_k \hat{x}_{k|k} \quad (10)$$

differs from the actually measured value by the fitting error $\hat{v}_{k|k}$, which was minimized.

Note that the estimates (10) and (9) are completely determined by the estimates for the present and next descriptor variables. Therefore, our original problem is solved if we consider the following two problems:

1. The filtering problem: obtain the best $x_{k|k}$, $\hat{x}_{k|k}$ from the measurements $\{z_0, z_1, \dots, z_k\}$;
2. The prediction problem: obtain the best $x_{k+1|k}$, $\hat{x}_{k+1|k}$ from the measurements $\{z_0, z_1, \dots, z_k\}$.

The nomenclature 'filtering' is justified by the observation that if $H_k = I$ in the model (6)-(8), then we have from (8)

$$z_k = \hat{x}_{k|k} + \hat{v}_{k|k} \quad (11)$$

and so, if the signal $\hat{x}_{k|k}$ was obtained from the actually measured signal z_k , the error signal $\hat{v}_{k|k}$ has been suppressed.

So far, we have considered a fixed set of measurements $\{z_0, \dots, z_k\}$. However, for many practical dynamical systems, it is usual that the set of measurements is constantly updated forming the sets $\{z_0, \dots, z_k, z_{k+1}\}$, $\{z_0, \dots, z_k, z_{k+1}, z_{k+2}\}$ and so on. Therefore, it is natural to ask if we could obtain $\hat{x}_{k+1|k}$, $\hat{x}_{k+1|k+1}$ based on $\hat{x}_{k|k-1}$, $\hat{x}_{k|k}$. The answer is given by the elegant recursive algorithm of the Kalman filter which can be obtained considering recursively the one-step deterministic optimum data fitting problem

$$\min_{x_i, x_{i+1}} \left[\|x_i - \hat{x}_{i|i}\|_{P_{i|i}}^{-2} + \|E_{i+1}x_{i+1} - F_i x_i\|_{Q_i}^{-2} + \|z_{i+1} - H_{i+1}x_{i+1}\|_{R_{i+1}}^{-2} \right]. \quad (12)$$

The resulting descriptor Kalman filter in filtered form is given by the following theorem (cf. [6], [24], [23], [12]).

Theorem 1: Suppose that $\begin{bmatrix} E_i \\ H_i \end{bmatrix}$ has full column rank for all $i \geq 0$. The Kalman filter to estimate $\hat{x}_{i|i}$ can be obtained from the following recursive algorithm

Step 0: (Initial Conditions): $P_{0|0} := (P_0^{-1} + H_0^T R_0^{-1} H_0)^{-1}$; $\hat{x}_{0|0} := P_{0|0} H_0^T R_0^{-1} z_0$.

Step i : Update $\{\hat{x}_{i|i}, P_{i|i}\}$ to $\{\hat{x}_{i+1|i+1}, P_{i+1|i+1}\}$

$$P_{i+1|i+1} := \left(E_{i+1}^T (Q_i + F_i P_{i|i} F_i^T)^{-1} E_{i+1} + H_{i+1}^T R_{i+1}^{-1} H_{i+1} \right)^{-1}; \quad (13)$$

$$\hat{x}_{i+1|i+1} := P_{i+1|i+1} E_{i+1}^T (Q_i + F_i P_{i|i} F_i^T)^{-1} F_i \hat{x}_{i|i} + P_{i+1|i+1} H_{i+1}^T R_{i+1}^{-1} z_{i+1}. \quad \square \quad (14)$$

Analogously, the predicted estimates recursion can be obtained by solving recursively the following optimization problem

$$\min_{x_i, x_{i+1}} \left[(x_i - \hat{x}_{i|i-1})^T P_{i|i-1}^{-1} (x_i - \hat{x}_{i|i-1}) + (z_i - H_i x_i)^T R_i^{-1} (z_i - H_i x_i) \right. \\ \left. + (E_{i+1} x_{i+1} - F_i x_i)^T Q_i^{-1} (E_{i+1} x_{i+1} - F_i x_i) \right] \quad (15)$$

and is given by the following theorem.

Theorem 2: Suppose that E_i has full column rank for all $i \geq 0$. The optimum predicted estimates $\hat{x}_{i+1|i}$ can be obtained from the following recursive algorithm

Step 0: (Initial Conditions): $P_{0|-1} := P_0$; $\hat{x}_{0|-1} := \bar{x}_0 = 0$.

Step i : Update $\{\hat{x}_{i|i-1}, P_{i|i-1}\}$ to $\{\hat{x}_{i+1|i}, P_{i+1|i}\}$

$$P_{i+1|i} := \left(\begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix}^T \begin{bmatrix} Q_i + F_i P_{i|i-1} F_i^T & -F_i P_{i|i-1} H_i^T \\ -H_i P_{i|i-1} F_i^T & R_i + H_i P_{i|i-1} H_i^T \end{bmatrix}^{-1} \begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix} \right)^{-1}; \quad (16)$$

$$\hat{x}_{i+1|i} := P_{i+1|i} \begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix}^T \begin{bmatrix} Q_i + F_i P_{i|i-1} F_i^T & -F_i P_{i|i-1} H_i^T \\ -H_i P_{i|i-1} F_i^T & R_i + H_i P_{i|i-1} H_i^T \end{bmatrix}^{-1} \begin{bmatrix} F_i \hat{x}_{i|i-1} \\ z_i - H_i \hat{x}_{i|i-1} \end{bmatrix}. \quad \square \quad (17)$$

Remark II.1: The underlying descriptor model in the above theorems is

$$\begin{aligned} E_{i+1} x_{i+1} &= F_i x_i + w_i, \\ z_i &= H_i x_i + v_i. \end{aligned} \quad (18)$$

Remark II.2: The data fitting approach is suitable for general descriptor systems and therefore is valid for state-space systems. However, it is important to remember that what is natural for state-space systems is not directly valid for descriptor systems. For example, in (18), the matrix E_{i+1} restrict the allowed directions of the vectors x_i and w_i . In particular, for some values of x_i and w_i , the value of x_{i+1} may be non-determinable. Therefore, in the descriptor setting, a naive approach of considering w_i a normal random vector can lead to very difficult technical problems as admissible initial conditions and admissible inputs.

Remark II.3: In the fitting functionals, the intuitive notion of (relative) degree of uncertainty or, how big we allow each error to be, is dealt with the introduction of positive definite weighting matrices W_j , V_i , and P_0 to the errors $w_{j|k}$, $v_{i|k}$, and $p_{0|k}$, respectively, for all i and j . This is the deterministic counterpart of covariance matrices.

Remark II.4: Note that it was used one quadratic functional to obtain the best filtered estimate recursion and other quadratic functional to obtain the best predicted estimate recursion. The two functionals have to be considered since for general descriptor systems, the predictor and filtered recursions do not correspond to a same filter transfer matrix as occur in the state-space setting.

B. Robust filtered estimates

The optimum robust fitting problem for the filtered estimates is defined as follows. It is assumed that at step i an estimate for the state x_i , denoted by $\hat{x}_{i|i}$, and there exists a positive-definite weighting matrix $P_{i|i}$ for the state estimation error $x_i - \hat{x}_{i|i}$ along with the new observation at time $i + 1$, i.e., z_{i+1} . To update the estimate of x_i from $\hat{x}_{i|i}$ to $\hat{x}_{i+1|i+1}$, it is proposed the following sequence of robust data fitting problems: Solve

$$\min_{x_0} \max_{\delta H_0} \left[\|x_0\|_{P_0^{-1}}^2 + \|z_0 - (H_0 + \delta H_0)x_0\|_{R_0^{-1}}^2 \right] \quad \text{for } i = 0; \quad (19)$$

$$\min_{\{x_i, x_{i+1}\}} \max_{\{\delta E_{i+1}, \delta F_i, \delta H_{i+1}\}} \left[\|x_i - \hat{x}_{i|i}\|_{P_{i|i}^{-1}}^2 + \|(E_{i+1} + \delta E_{i+1})x_{i+1} - (F_i + \delta F_i)x_i\|_{Q_i^{-1}}^2 \right. \\ \left. + \|z_{i+1} - (H_{i+1} + \delta H_{i+1})x_{i+1}\|_{R_{i+1}^{-1}}^2 \right] \quad \text{for } i > 0 \quad (20)$$

where the uncertainties are assumed modeled with the following structures

$$\delta F_i = M_{f,i} \Delta_i N_{f,i}; \quad \delta E_{i+1} = M_{f,i} \Delta_i N_{e,i+1}; \quad \delta H_i = M_{h,i} \Delta_i N_{h,i}; \quad \|\Delta_i\| \leq 1 \quad (21)$$

where $M_{f,i}$, $M_{h,i}$, $N_{e,i+1}$, $N_{f,i}$, $N_{h,i}$ are known matrices and Δ_i is a bounded matrix (with norm less or equal to 1) but otherwise arbitrary. The robust filtering algorithms we present in this section are based on the following auxiliary functions

$$G(\lambda) := \|x(\lambda)\|_Q^2 + \lambda \|N_a x(\lambda) - N_b\|^2 + \|Ax(\lambda) - b\|_{W(\lambda)}^2; \quad (22)$$

$$x(\lambda) := [Q(\lambda) + A^T W(\lambda) A]^{-1} [A^T W(\lambda) b + \hat{\lambda} N_a^T N_b]; \quad (23)$$

$$Q(\lambda) := Q + \lambda N_a^T N_a; \quad W(\lambda) := W + W H (\lambda I - H^T W H)^\dagger H^T W. \quad (24)$$

The optimum robust filtered estimates $\hat{x}_{i|i}$ resulting from (20) can be obtained recursively from Algorithm 1.

Algorithm 1: Robust Filtered Estimates

Step 0: (Initial Conditions): If $M_{h,0} = 0$ then $P_{0|0} := (P_0^{-1} + H_0^T R_0^{-1} H_0)^{-1}$; $\hat{x}_{0|0} := P_{0|0} H_0^T R_0^{-1} z_0$. Otherwise determine the optimum scalar parameter $\hat{\lambda}_{-1}$ by minimizing over the interval $\lambda > \|M_{h,0}^T R_0^{-1} M_{h,0}\|$ the function $G(\lambda)$ of (22) corresponding to

$$A \leftarrow H_0; b \leftarrow z_0; Q \leftarrow P_0^{-1}; W \leftarrow R_0^{-1}; H \leftarrow M_{h,0}; N_a \leftarrow N_{h,0}; N_b \leftarrow 0. \quad (25)$$

Then set

$$\hat{R}_0 := R_0 - \hat{\lambda}_{-1}^{-1} M_{h,0} M_{h,0}^T; P_{0|0} := (P_0^{-1} + H_0^T \hat{R}_0^{-1} H_0 + \hat{\lambda}_{-1} N_{h,0}^T N_{h,0})^{-1}; \hat{x}_{0|0} := P_{0|0} H_0^T \hat{R}_0^{-1} z_0. \quad (26)$$

Step 1: If $M_{f,i} = 0$ and $M_{h,i+1} = 0$ then $\hat{\lambda}_i := 0$. Otherwise determine the optimum scalar parameter $\hat{\lambda}_i$ by minimizing over the interval

$$\hat{\lambda}_i > \lambda_{l,i} := \|\text{diag}\{M_{f,i}^T Q_i^{-1} M_{f,i}, M_{h,i+1}^T R_{i+1}^{-1} M_{h,i+1}\}\| \quad (27)$$

the function $G(\lambda)$ of (22) corresponding to

$$A \leftarrow \begin{bmatrix} -F_i & E_{i+1} \\ 0 & H_{i+1} \end{bmatrix}; b \leftarrow \begin{bmatrix} F_i \hat{x}_{i|i} \\ z_{i+1} \end{bmatrix}; Q \leftarrow \begin{bmatrix} P_{i|i}^{-1} & 0 \\ 0 & 0 \end{bmatrix}$$

$$W \leftarrow \begin{bmatrix} Q_i^{-1} & 0 \\ 0 & R_{i+1}^{-1} \end{bmatrix}; N_a \leftarrow \begin{bmatrix} -N_{f,i} & N_{e,i+1} \\ 0 & N_{h,i+1} \end{bmatrix}; N_b \leftarrow \begin{bmatrix} N_{f,i} \hat{x}_{i|i} \\ 0 \end{bmatrix}; H \leftarrow \begin{bmatrix} M_{f,i} & 0 \\ 0 & M_{h,i} \end{bmatrix}. \quad (28)$$

Step 2: If $\hat{\lambda}_i \neq 0$, the given parameters $\{Q_i, R_{i+1}, E_{i+1}, F_i, H_{i+1}\}$ are replaced by the corrected parameters

$$\hat{Q}_i := \begin{bmatrix} \hat{Q}_i & 0 \\ 0 & I \end{bmatrix}, \hat{Q}_i := Q_i - \hat{\lambda}_i^{-1} M_{f,i} M_{f,i}^T; \hat{E}_{i+1} := \begin{bmatrix} E_{i+1} \\ \sqrt{\hat{\lambda}_i} N_{e,i+1} \end{bmatrix}; \hat{F}_i := \begin{bmatrix} F_i \\ \sqrt{\hat{\lambda}_i} N_{f,i} \end{bmatrix}$$

$$\hat{R}_{i+1} := \begin{bmatrix} \hat{R}_{i+1} & 0 \\ 0 & I \end{bmatrix}, \hat{R}_{i+1} := R_{i+1} - \hat{\lambda}_i^{-1} M_{h,i+1} M_{h,i+1}^T; \hat{H}_{i+1} := \begin{bmatrix} H_{i+1} \\ \sqrt{\hat{\lambda}_i} N_{h,i+1} \end{bmatrix}.$$

Step 3: If $\text{rank} \begin{bmatrix} \hat{E}_{i+1} \\ \hat{H}_{i+1} \end{bmatrix} = n$, update $\{P_{i|i}, \hat{x}_{i|i}\}$ to $\{P_{i+1|i+1}, \hat{x}_{i+1|i+1}\}$ as follows

$$P_{i+1|i+1} := \left(\hat{E}_{i+1}^T \left(\hat{Q}_i + \hat{F}_i P_{i|i} \hat{F}_i^T \right)^{-1} \hat{E}_{i+1} + \hat{H}_{i+1}^T \hat{R}_{i+1}^{-1} \hat{H}_{i+1} \right)^{-1}, \quad (29)$$

$$\hat{x}_{i+1|i+1} = P_{i+1|i+1} \left(\hat{E}_{i+1}^T \left(\hat{Q}_i + \hat{F}_i P_{i|i} \hat{F}_i^T \right)^{-1} \hat{F}_i \hat{x}_{i|i} + \hat{H}_{i+1}^T \hat{\mathcal{R}}_{i+1} \begin{bmatrix} z_{i+1} \\ 0 \end{bmatrix} \right). \quad (30)$$

Remark II.5: $\begin{bmatrix} -F_i & E_{i+1} \\ 0 & H_{i+1} \end{bmatrix}$ full column rank is a sufficient condition for the existence of the robust filter.

Remark II.6: From (29) and (30), it is easy to verify that for descriptor systems without uncertainties ($M_{f,i} = 0$, $M_{h,i+1} = 0$, $N_{e,i+1} = 0$, $N_{f,i} = 0$, $N_{h,i+1} = 0$), this algorithm collapses to the standard descriptor Kalman filter of Theorem 1.

The next theorem gives necessary and sufficient conditions for the convergence of the Riccati equation (29) and the stability of the filter (30) in steady-state. In the following, for the constant parameters over time, we drop the index i .

Theorem 3: Consider that all system parameters $\{Q, R, E, F, H, M_f, M_h, N_e, N_f, N_h\}$ are constant and that the parameter $\hat{\lambda}$ is a constant value which satisfies (27). If $\text{rank} \begin{bmatrix} \hat{E} \\ \hat{H} \end{bmatrix} = n$ and $P_{0|0} > 0$, then $\text{rank} \begin{bmatrix} z\hat{E} - \hat{F} \\ \hat{H} \end{bmatrix} = n (\forall z \in C, |z| \geq 1)$, is a necessary and sufficient condition for $\lim_{i \rightarrow \infty} P_{i|i} = P$ where $P_{i|i}$ is the solution of (29) and P is the unique solution of

$$P := \left(\hat{E}^T \left(\hat{Q} + \hat{F} P \hat{F}^T \right)^{-1} \hat{E} + \hat{H}^T \hat{\mathcal{R}}^{-1} \hat{H} \right)^{-1}, \quad (31)$$

for which all roots of the state transition matrix $F_p = P \hat{E}^T \left(\hat{Q} + \hat{F} P \hat{F}^T \right)^{-1} \hat{F}$ of the filter (30) are inside the unit circle.

□

The above theorem indicates that for practical implementations, one can use a fixed value for $\hat{\lambda}$. Therefore, if the system's parameters are almost constant, it is expected that even with a constant $\hat{\lambda}$, the filter follows well the input sequence. This is interesting since the computation of $\hat{\lambda}_i$ as indicated in Step 1 of Algorithm 1 can be very time consuming.

C. Robust predictor estimates

The successive robust predicted estimates $\hat{x}_{i+1|i}$ can be obtained recursively from Algorithm 2.

Algorithm 2: Robust Predictor Estimates

Step 0: (Existence and Initial Conditions): Check if E_{i+1} is full column rank, $N_{e,i+1}^T N_{f,i} = 0$ and $N_{e,i+1}^T N_{e,i+1}$ is invertible. Set $P_{0|-1} := P_0$ and $\hat{x}_{0|-1} := \bar{x}_0 = 0$.

Step 1: If $M_{f,i} = 0$ and $M_{h,i} = 0$, then set $\hat{\lambda}_i = 0$. Otherwise determine the optimum scalar parameter $\hat{\lambda}_i$ by minimizing over the interval

$$\hat{\lambda}_i > \lambda_{l,i} := \|\text{diag}\{M_{f,i}^T Q_i^{-1} M_{f,i}, M_{h,i}^T R_i^{-1} M_{h,i}\}\|; \quad (32)$$

the function $G(\lambda)$ of (22) corresponding to

$$A \leftarrow \begin{bmatrix} F_i & E_{i+1} \\ H_i & 0 \end{bmatrix}; b \leftarrow \begin{bmatrix} F_i \hat{x}_{i|i-1} \\ H_i \hat{x}_{i|i-1} - z_i \end{bmatrix}; Q \leftarrow \begin{bmatrix} P_{i|i-1}^{-1} & 0 \\ 0 & 0 \end{bmatrix}; \quad (33)$$

$$W \leftarrow \begin{bmatrix} Q_i^{-1} & 0 \\ 0 & R_i^{-1} \end{bmatrix}; H \leftarrow \begin{bmatrix} M_{f,i} & 0 \\ 0 & M_{h,i+1} \end{bmatrix}; N_b \leftarrow \begin{bmatrix} N_{f,i} \\ N_{h,i} \end{bmatrix} \hat{x}_{i|i-1}; N_a \leftarrow \begin{bmatrix} N_{f,i} & N_{e,i+1} \\ N_{h,i} & 0 \end{bmatrix}. \quad (34)$$

Step 2: If $\hat{\lambda}_i \neq 0$, replace the given parameters $\{Q_i, R_i, P_{i|i-1}, F_i\}$ by the corrected parameters

$$\hat{Q}_i := Q_i - \hat{\lambda}_i^{-1} M_{f,i} M_{f,i}^T; \quad \hat{R}_i := R_i - \hat{\lambda}_i^{-1} M_{h,i} M_{h,i}^T; \quad (35)$$

$$\hat{P}_{i|i-1} := (P_{i|i-1}^{-1} + \hat{\lambda}_i N_{f,i}^T N_{f,i})^{-1}; \quad \hat{F}_i := F_i (I - \hat{\lambda}_i (\hat{P}_{i|i-1}^{-1} + L_i^T J_i^{-1} L_i)^{-1} N_{f,i}^T N_{f,i}). \quad (36)$$

If $\hat{\lambda}_i = 0$, there is no correction: $\{\hat{Q}_i, \hat{R}_i, \hat{P}_{i|i-1}, \hat{F}_i\} := \{Q_i, R_i, P_{i|i-1}, F_i\}$

Step 3: Update $\{P_{i|i-1}, \hat{x}_{i|i-1}\}$ to $\{P_{i+1|i}, \hat{x}_{i+1|i}\}$ as follows

$$P_{i+1|i} := \left(\begin{bmatrix} E_{i+1}^T & 0 \end{bmatrix} \begin{bmatrix} \hat{Q}_i + F_i \hat{P}_{i|i-1} F_i^T & -F_i \hat{P}_{i|i-1} L_i^T \\ -L_i \hat{P}_{i|i-1} F_i^T & J_i + L_i \hat{P}_{i|i-1} L_i^T \end{bmatrix}^{-1} \begin{bmatrix} E_{i+1} \\ 0 \end{bmatrix} + \lambda_i N_{e,i+1}^T N_{e,i+1} \right)^{-1} \quad (37)$$

$$\hat{x}_{i+1|i} := P_{i+1|i} \begin{bmatrix} E_{i+1}^T & 0 \end{bmatrix} \begin{bmatrix} \hat{Q}_i + F_i \hat{P}_{i|i-1} F_i^T & -F_i \hat{P}_{i|i-1} L_i^T \\ -L_i \hat{P}_{i|i-1} F_i^T & J_i + L_i \hat{P}_{i|i-1} L_i^T \end{bmatrix}^{-1} \begin{bmatrix} \hat{F}_i \hat{x}_{i|i-1} \\ z_i \\ 0 \end{bmatrix} - L_i \hat{x}_{i|i-1} \quad (38)$$

$$J_i := \begin{bmatrix} \hat{R}_i & 0 \\ 0 & I \end{bmatrix}; L_i := \begin{bmatrix} H_i \\ \lambda_i N_{h,i} \end{bmatrix}. \quad (39)$$

C.1 Robust descriptor estimator in time and measurement update form

For the examples of applications we are proposing in this chapter, it is important to compute the estimates in two steps, leading to the so-called time and measurement update form. In this section we rewrite the Algorithm 2 in this form. Furthermore, we incorporate the control input and a practical rule to adjust $\hat{\lambda}_i$ in the recursions presented in the Algorithm 3. The underlying uncertain model considered in the Algorithm 3 is given by

$$(E_{i+1} + \delta E_{i+1})x_{i+1} = (F_i + \delta F_i)x_i + G_i u_i + w_i, \quad (40)$$

$$z_i = (H_i + \delta H_i)x_i + v_i, \quad (41)$$

where the perturbations δE_{i+1} , δF_i e δH_i are modeled as in (21).

Algorithm 3: Time and measurement update robust descriptor filter

Initial conditions: $\hat{x}_{0|-1} = 0$ and $P_{0|-1} = P_0$.

Step 1: If $M_{f,i} = 0$ and $M_{h,i} = 0$, then set $\hat{\lambda}_i = 0$. Otherwise, select $\alpha_f > 0$ and set

$$\hat{\lambda}_i = (1 + \alpha_f) \|\text{diag}\{M_{f,i}^T Q_i^{-1} M_{f,i}, M_{h,i}^T R_i^{-1} M_{h,i}\}\|. \quad (42)$$

Step 2: Replace $\{Q_i, R_i, P_{i|i}, E_{i+1}, F_i, H_i\}$ by the corrected parameters:

$$\hat{Q}_i = \begin{bmatrix} \hat{Q}_i & 0 \\ 0 & I \end{bmatrix}, \text{ where } \hat{Q}_i = Q_i - \hat{\lambda}_i^{-1} M_{f,i} M_{f,i}^T,$$

$$\hat{R}_i = \begin{bmatrix} \hat{R}_i & 0 \\ 0 & I \end{bmatrix}, \text{ where } \hat{R}_i = R_i - \hat{\lambda}_i^{-1} M_{h,i} M_{h,i}^T,$$

$$\hat{\mathcal{E}}_{i+1} = \begin{bmatrix} E_{i+1} \\ \sqrt{\hat{\lambda}_i} N_{e,i+1} \end{bmatrix}, \hat{\mathcal{F}}_i = \begin{bmatrix} F_i \\ \sqrt{\hat{\lambda}_i} N_{f,i} \end{bmatrix}, \hat{\mathcal{H}}_i = \begin{bmatrix} H_i \\ \sqrt{\hat{\lambda}_i} N_{h,i} \end{bmatrix}.$$

Step 3: Update $\{\hat{x}_{i|i-1}, P_{i|i-1}\}$ to $\{\hat{x}_{i|i}, \hat{x}_{i+1|i}, P_{i+1|i}\}$ as follows:

$$\hat{x}_{i+1|i} = (\hat{\mathcal{E}}_{i+1}^T \hat{Q}_i^{-1} \hat{\mathcal{E}}_{i+1})^{-1} \hat{\mathcal{E}}_{i+1}^T \hat{Q}_i^{-1} \hat{\mathcal{F}}_i \hat{x}_{i|i} + G_i u_i \quad (43)$$

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} P_{i|i-1}^{-1} + \hat{\mathcal{F}}_i^T \hat{Q}_i^{-1} \hat{\mathcal{F}}_i + \hat{\mathcal{H}}_i^T \hat{R}_i^{-1} \hat{\mathcal{H}}_i & -\hat{\mathcal{F}}_i^T \hat{Q}_i^{-1} \hat{\mathcal{E}}_{i+1} \\ -\hat{\mathcal{E}}_{i+1}^T \hat{Q}_i^{-1} \hat{\mathcal{F}}_i & \hat{\mathcal{E}}_{i+1}^T \hat{Q}_i^{-1} \hat{\mathcal{E}}_{i+1} \end{bmatrix}^{-1} \times \begin{bmatrix} -\hat{\mathcal{F}}_i^T \hat{Q}_i^{-1} \hat{\mathcal{F}}_i \hat{x}_{i|i-1} + \hat{\mathcal{H}}_i^T \hat{R}_i^{-1} (z_i - \hat{\mathcal{H}}_i \hat{x}_{i|i-1}) \\ -\hat{\mathcal{E}}_{i+1}^T \hat{Q}_i^{-1} \hat{\mathcal{F}}_i \hat{x}_{i|i-1} \end{bmatrix} \quad (44)$$

$$P_{i+1|i} = \left(\begin{bmatrix} \hat{\mathcal{E}}_{i+1} \\ 0 \end{bmatrix}^T \begin{bmatrix} \hat{Q}_i + \hat{\mathcal{F}}_i P_{i|i-1} \hat{\mathcal{F}}_i^T & -\hat{\mathcal{F}}_i P_{i|i-1} \hat{\mathcal{H}}_i^T \\ -\hat{\mathcal{H}}_i P_{i|i-1} \hat{\mathcal{F}}_i^T & \hat{R}_i + \hat{\mathcal{H}}_i P_{i|i-1} \hat{\mathcal{H}}_i^T \end{bmatrix}^{-1} \begin{bmatrix} \hat{\mathcal{E}}_{i+1} \\ 0 \end{bmatrix} \right)^{-1} \quad (45)$$

where $Z_i = \begin{bmatrix} z_i \\ 0 \end{bmatrix}$.

Remark II.7: The presented robust descriptor Kalman filters can estimate the state of a rectangular descriptor system (that is, when the matrix E in (3) is rectangular). The proposed filters collapse to the nominal descriptor Kalman filters when the system is not subject to uncertainties. When they are reduced to the standard state space systems, they provide robust Kalman-type recursions.

In the next section, we apply these robust filters in two practical robotic systems. It is possible to use many other estimation algorithms but this chapter is not meant to be exhaustive.

3. Application examples

A. Robust estimation of mobile robot localization

This section provides an example of application for the robust filters aforementioned to estimate mobile robots localization. Measurement systems based on odometric, inertial sensors and ultrasounds are self-contained, simple to use, and able to guarantee a high data rate. However, the problem of these systems is that they integrate the relative increments, and the localization errors considerably grow over time if an appropriate sensor fusion algorithm is not used. Extended Kalman filters (EKF) have been used to solve this kind of problem [3], [4], [8], [16], [26], [29]. They have been widely used mainly for position tracking [28], where the initial robot pose is known, and the problem is to compensate small incremental errors in the robot odometry. The localization approaches developed in [10], [11], [14], [13] are based on the kinematic model of the mobile robot, where the positions are performed through numerical integration of the encoders increments. Due to the several variables involved, the mobile robot localization is subject to several sources of uncertainties. They include deterministic and stochastic factors coming from robots actuators, inaccuracies of the sensors and odometry errors. This set of problems motivates the use of robust estimation methods in order to limit the performance degradation of optimal estimates. In [17] and [18], a robust localization method for mobile robot based on the combination of a Kalman filter with a perturbation estimator was presented with simulated and experimental results. We have observed that all approaches aforementioned were not based on robust filters where the parameters uncertainties are taken into account. The design procedure we present in this section aims to solve this problem.

A.1 Mobile robot and sensors model

The kinematic model of an unicycle mobile robot shown in Fig. 1 is described by:

$$\dot{x}(t) = \nu(t)\cos\theta(t), \quad (46)$$

$$\dot{y}(t) = \nu(t)\sin\theta(t), \quad (47)$$

$$\dot{\theta}(t) = \omega(t), \quad (48)$$

and the nonholonomic kinematic constraint is given by:

$$-\sin\theta(t)\dot{x}(t) + \cos\theta(t)\dot{y}(t) = 0, \quad (49)$$

where x and y are the coordinates between the two driving wheels, θ is the angle between the main axis of the robot and the x -direction, $\nu(t)$ and $\omega(t)$ are, respectively, the displacement and angular velocities of the robot. The discrete displacement and angular velocities $\bar{\nu}_i$ and $\bar{\omega}_i$ are given by odometric measures at time instant $t_i = iT$, where T is the sampling period.

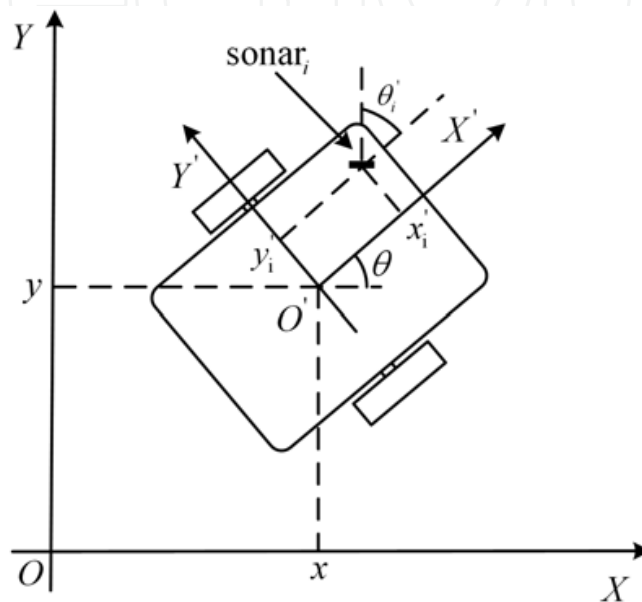


Fig. 1. Geometry of the wheeled mobile robot.

The distances of the sonars readings can be related to the model of the indoor environment and the configuration of the mobile robot as shown in [14] and [13]. Consider a planar distribution of n_s sonar sensors on the robot. Denote x'_m, y'_m, θ'_m the planar position and orientation of the m -th sonar, $m = 1, 2, \dots, n_s$, referring to the coordinate system (O', X', Y') , as reported in Fig. 1. The position x_m, y_m and orientation θ_m of the m -th sonar referred to the inertial coordinate system (O, X, Y) at the time instant t_i , are given by:

$$x_{m,i} = x_i + x'_m \cos\theta_i - y'_m \sin\theta_i, \quad (50)$$

$$y_{m,i} = y_i + x'_m \sin\theta_i + y'_m \cos\theta_i, \quad (51)$$

$$\theta_{m,i} = \theta_i + \theta_m. \quad (52)$$

The walls and the obstacles in indoor environment are represented by a proper set of orthogonal planes to the plane XY of the inertial coordinate system. As shown in Fig. 2, δ is the beam-width of the sonar sensor and each plane P^j , $j = 1, 2, \dots, n_p$ (n_p is the number of planes which describe the indoor environment). These planes are grouped in a vector Π , which is composed of geometric parameters P_r^j , the normal distance of the plane from the origin O ; P_n^j , the angle between the normal line to the plane and the x -direction; and P_v^j , a binary variable. $P_v^j \in \{-1, 1\}$ defines the face of the plane reflecting the sonar beam. Then, if

$P_n^j \in [\theta_m(t_i) - \delta/2, \theta_m(t_i) + \delta/2]$, the distance d_m^j from the m -th sonar to the plane P^j is given by the following equation:

$$d_{m,i}^j = P_n^j (P_r^j - x_{m,i} \cos P_n^j - y_{m,i} \sin P_n^j), \quad (53)$$

otherwise, the sonar measure is rejected.

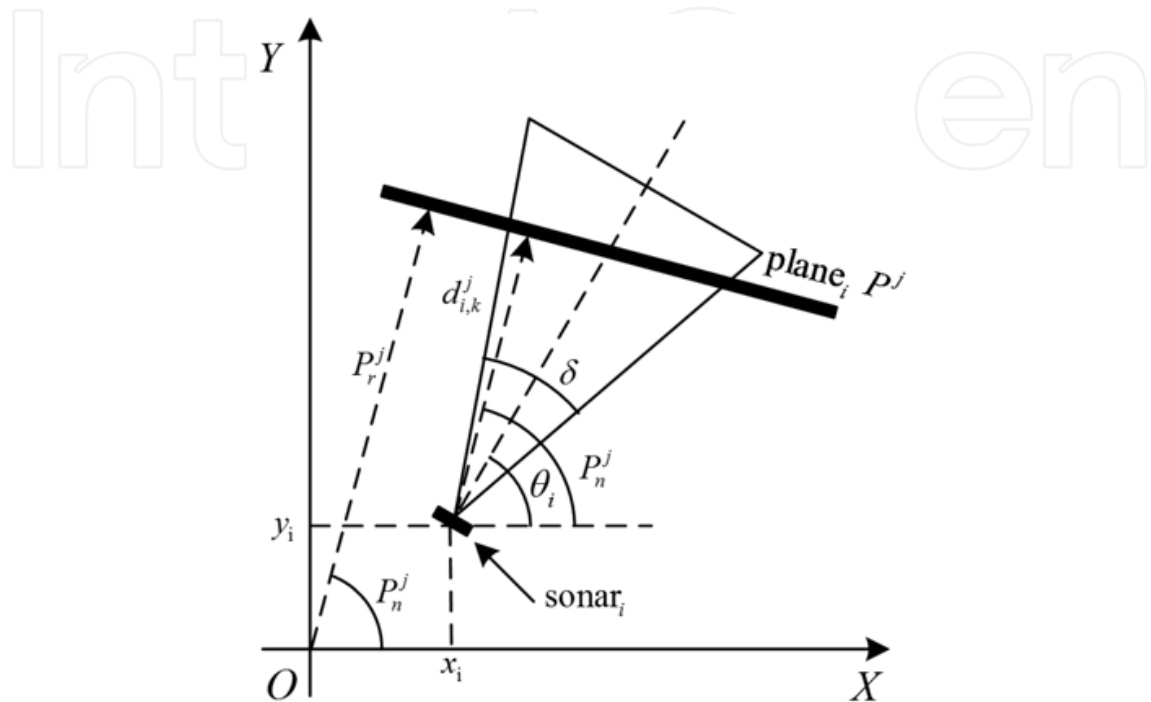


Fig. 2. Sonar measurement.

A.2 Descriptor system

The mobile robot in descriptor form is obtained by the discretization of the equations (46), (47), (48), and (49):

$$E_i \mathbf{x}_{i+1} = F_i \mathbf{x}_i + G_i \mathbf{u}_i + \mathbf{w}_i, \quad (54)$$

where $\mathbf{x}_i := [x_i \ y_i \ \theta_i]^T$ is the robot state, $\mathbf{u}(t) := [\nu_i \ \omega_i]^T$, $\mathbf{w}_i \sim N(0, Q_i)$ is a zero mean white Gaussian noise,

$$E_i = \begin{bmatrix} \frac{1}{T} & 0 & 0 \\ 0 & \frac{1}{T} & 0 \\ 0 & 0 & \frac{1}{T} \\ -\frac{1}{T} \sin \theta_i & \frac{1}{T} \cos \theta_i & 0 \end{bmatrix}, \quad F_i = \begin{bmatrix} \frac{1}{T} & 0 & 0 \\ 0 & \frac{1}{T} & 0 \\ 0 & 0 & \frac{1}{T} \\ -\frac{1}{T} \sin \theta_i & \frac{1}{T} \cos \theta_i & 0 \end{bmatrix}, \quad \text{and } G_i = \begin{bmatrix} \cos \theta_i & 0 \\ \sin \theta_i & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. \quad (55)$$

The sampled nonlinear measurement equation, with sampling period T , is given by:

$$\mathbf{z}_i = \mathbf{s}(\mathbf{x}_i, \Pi) + \mathbf{v}_i \quad (56)$$

where \mathbf{z}_i is the vector containing sonar and odometer measures and $\mathbf{v}_i \sim N(0, R_i)$ is a white sequence. The dimension p_i of \mathbf{z}_i varies, it depends on the number of sonar sensors that are actually used at each time instant. The measurement vector \mathbf{z}_i is composed of two subvectors

$$\mathbf{z}_{1,i} = [z_{1,i} \ z_{2,i} \ z_{3,i}]^T \text{ and}$$

$$\mathbf{z}_{2,i} = [z_{4,i} \ z_{5,i} \ \dots \ z_{p_i,i}]^T,$$

where

$$z_{1,i} = x_i + v_{1,i}, \quad z_{2,i} = y_i + v_{2,i}, \quad z_{3,i} = \theta_i + v_{3,i} \quad (57)$$

are the measurements provided by the odometric device, and

$$z_{(3+m),i} = d_{m,i}^j + v_{(3+m),i} \quad (58)$$

is the distance measure provided by the m -th sonar sensor from the P_j plane with $j \in [1, n_p]$, and $d_{m,i}^j$ given by (53). The environment map provides the information needed to know which plane P_j is in front of the m -th sonar. By definition the measurement vector has the following form:

$$\mathbf{s}(\mathbf{x}_i, \Pi) = \begin{bmatrix} d_{1,i}^{j_1} \\ d_{2,i}^{j_2} \\ \vdots \\ d_{\bar{p}_i,i}^{j_{\bar{p}_i}} \end{bmatrix}, \quad j_1, j_2, \dots, j_{\bar{p}_i} \in [1, n_p] \quad (59)$$

where $\bar{p}_i := p_i - 3$. The number of measures p_i may vary from 3 to $n_s + 3$. In order to present Equation (56) in a form useful to implement the robust filter, it is rewritten as:

$$\mathbf{z}_i = H_i \mathbf{x}_i + \mathbf{v}_i \quad (60)$$

where

$$H_i := \begin{bmatrix} H_{1i} \\ H_{2i} \\ \vdots \\ H_{p_i i} \end{bmatrix}, \quad [H_{1i}^T \ H_{2i}^T \ H_{3i}^T]^T = I_3,$$

$$H_{(m+3)i} = P_\nu^j [-\cos P_n^j \quad -\sin P_n^j \quad x'_i \cos(\theta_i - P_n^j) - y'_i \sin(\theta_i - P_n^j)] \quad (61)$$

$$m = 1, 2, \dots, \bar{p}_i, \leq n_s, \quad j \in [1, n_p].$$

Here a sonar readings selection algorithm is used to reduce the probability of an inadequate interpretation of erroneous sensor data (see [11] for more details): at each step, for each sonar measure z_{3+m} , the residual $\gamma_{m,i} = z_{(3+m),i} - \hat{d}_m^i$ represents the difference between the actual sonar measure $z_{(3+m),i}$ and its expected value \hat{d}_m^i , $m = 1, 2, \dots, p_i - 3$, $j = 1, 2, \dots, n_p$, which is computed by (53) on the basis of the current estimate of \mathbf{x} and on a *priori* knowledge of the environment. As $\gamma_{m,i} \sim N(0, s_{m,i})$, the current read of $z_{(3+m),i}$ is only accepted if $|\gamma_{m,i}| \leq 2\sqrt{s_{m,i}}$. For this chapter $s_{m,i} = 0.15$.

A.3 Simulation

The simulation was performed in MATLAB[®] with nominal parameters of a Pioneer 2 DX mobile robot. The Pioneer has sixteen sonars range finders: 8 on the front and 8 on the back. The locations and directions of the sonar range finders on the Pioneer are listed in Table I and shown in Figure 3.

m	x'_m (m)	y'_m (m)	θ'_m (degrees)
1	0.115	0.130	90
2	0.155	0.115	50
3	0.190	0.080	30
4	0.210	0.025	10
5	0.210	-0.025	- 10
6	0.190	-0.080	- 30
7	0.155	-0.115	- 50
8	0.115	-0.130	- 90
9	-0.115	-0.130	- 90
10	-0.155	-0.115	-130
11	-0.190	-0.080	-150
12	-0.210	-0.025	-170
13	-0.210	0.025	170
14	-0.190	0.080	150
15	-0.155	0.115	130
16	-0.115	0.130	90

Table 1. Mounting positions and directions of the sonar range finders on the Pioneer 2DX.

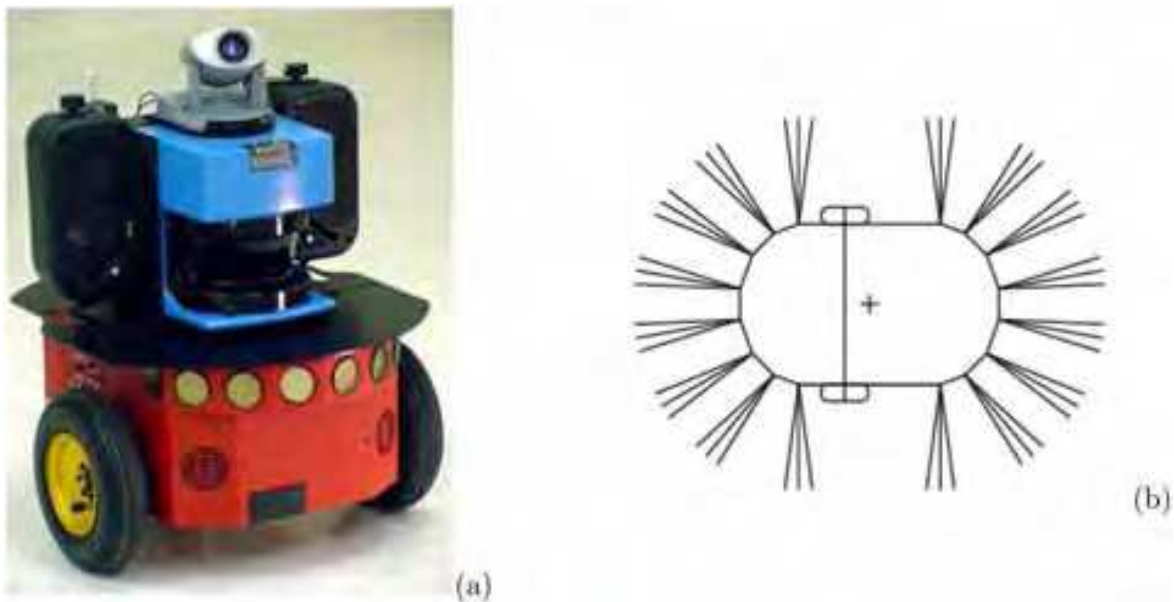


Fig. 3. Pioneer 2 DX: (a) Picture and (b) Locations and directions of the sonars.

A.4 Noise models

There are several kinds of noise typically observed when robots operate in real-world environments. On one hand there is a typical Gaussian noise in the odometry and proximity sensors coming from the inherent inaccuracy of the sensors. On the other hand there are non-Gaussian errors arising from collision with obstacles, or from interference with the sensors. In this chapter, the odometry errors coming from wheel slippage, uneven floors, or different payloads are characterized according to three parameters of the odometry model [27] displayed in Fig. 4.

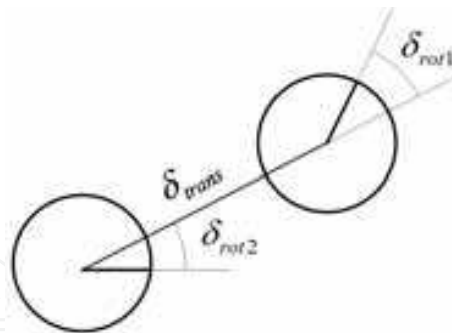


Fig. 4. Odometry Model

The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation δ_{rot1} , followed by a translation δ_{trans} and a second rotation δ_{rot2} . Each element is corrupted by independent noise:

$$\begin{pmatrix} \hat{\delta}_{rot1} \\ \hat{\delta}_{trans} \\ \hat{\delta}_{rot2} \end{pmatrix} = \begin{pmatrix} \delta_{rot1} \\ \delta_{trans} \\ \delta_{rot2} \end{pmatrix} + \begin{pmatrix} \epsilon_{rot1} \\ \epsilon_{trans} \\ \epsilon_{rot2} \end{pmatrix} \quad (62)$$

where

$$\epsilon_{rot1} = (\alpha_1 \delta_{rot1} + \alpha_2 \delta_{trans}), \quad (63)$$

$$\epsilon_{trans} = (\alpha_3 \delta_{trans} + \alpha_4 (\delta_{rot1} + \delta_{rot2})), \quad (64)$$

$$\epsilon_{rot2} = (\alpha_1 \delta_{rot2} + \alpha_2 \delta_{trans}). \quad (65)$$

The α_t , $t = 1 \dots 4$, parameters are the influences of translation and rotation on themselves and on each other. The parameters α_t used in simulation were $\alpha_1 = 0.6$, $\alpha_2 = 0.2$, $\alpha_3 = 0.7$ and $\alpha_4 = 0.0$. Finally, the inaccuracies of the sonars were simulated as:

$$z_{(3+m),i} = d_m^j + \beta p_{random}, \quad (66)$$

where $z_{(3+m),i}$ is the actual sonar measure, d_m^j is computed by (53), β is the inaccuracy of the sonar and p_{random} is pseudo-random value drawn from a normal distribution with mean zero and standard deviation one. The beam width and the imprecision of the sonar considered in this simulation were $\delta = 30^\circ$ and $\beta = 0.03$ m, respectively.

A.5 Descriptor filter

The descriptor robust filter presented in the Algorithm 3 was set with

$$Q_i = \sigma^2 I_4, \text{ com } \sigma = 1000,$$

$$R_i = \begin{bmatrix} 0.0035 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0.0035 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0.0126 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0.03 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0.03 \end{bmatrix}, \quad (67)$$

$$M_{f,i} = I_4, \quad M_{h,i} = I_{p_i},$$

$$N_{e,i} = N_{f,i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.3077 & 0.3869 & 0 \end{bmatrix} \text{ and } N_{h,i} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0_{(\bar{p}_i,1)} & 0_{(\bar{p}_i,1)} & H(4 : p_i, 3)_i \end{bmatrix} \quad (68)$$

and $\alpha_f = 10$. The results obtained are shown in figures 5, 6 and 7.

A.6 Comparative study

The simulation of the Pioneer 2 DX localization (equipped with sonar sensors and odometry) was performed using three different methods: only odometry measures integration; sonar and odometry measures fusion through nominal descriptor filter; and sonar and odometry measures fusion using descriptor robust filter. The following \mathcal{L}_2 norm of the state vector is used to compare the performance of these procedures:

$$\mathcal{L}_2[\tilde{\mathbf{x}}] = \left(\frac{1}{(t_r - t_0)} \int_{t_0}^{t_r} \|\tilde{\mathbf{x}}(t)\|_2^2 dt \right)^2, \quad (69)$$

where $\|\cdot\|_2$ is the Euclidean norm, $\tilde{\mathbf{x}}(t)$ is the state vector error and $t_0 = 0$, $t_r = 30$ (s) is the experimental time interval. As expected, both filters presented better results on tracking the robot's pose than using only odometry integration. However, the Robust filter obtained an average improvement of 16.7% over the EKF in the position error. These results can be seen in Table II. In future works, we intend to perform experiments with an actual Pioneer 2 DX mobile robot.

Algorithms	$\mathcal{L}_2[\tilde{\mathbf{x}}]$
Odometry	0.4618
Nominal Descriptor Filter	0.1315
Robust Descriptor Filter	0.1095

Table 2. Performance Index

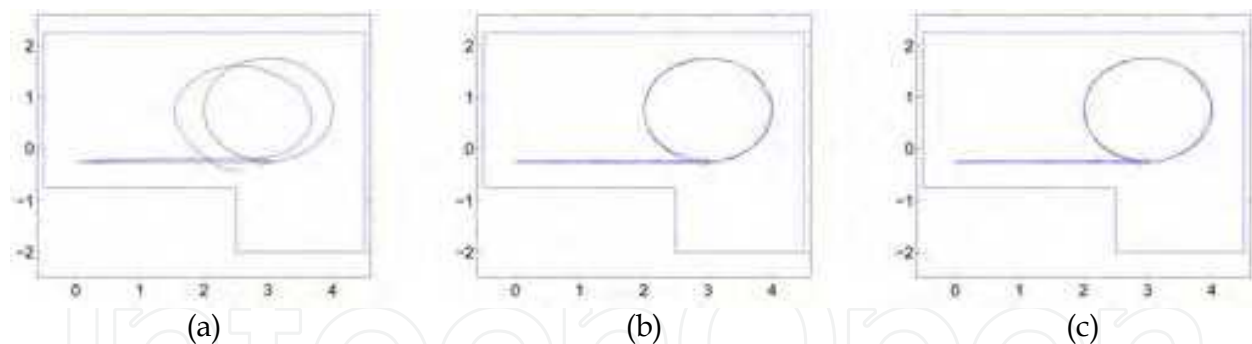


Fig. 5. Robot poses estimation: (a) Only using odometry measures, (b) Nominal filter and (c) Descriptor filter.

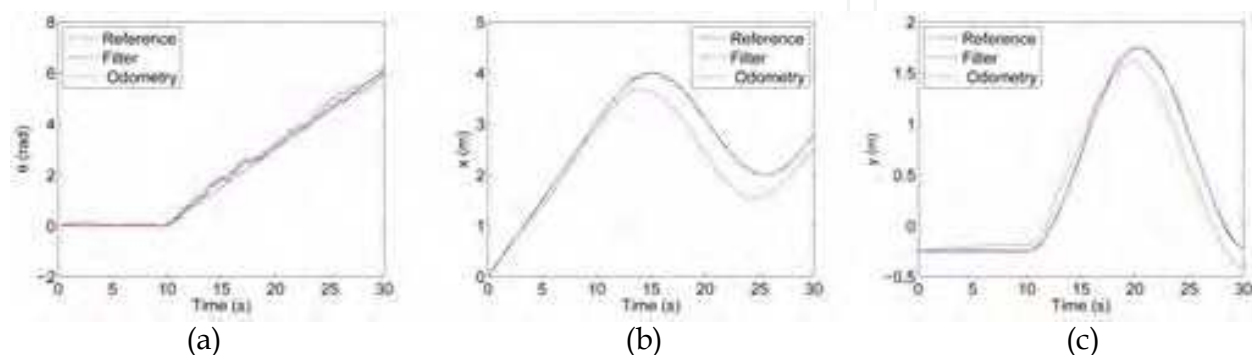


Fig. 6. Robot poses estimation using nominal filter: (a) θ , (b) x and (c) y .

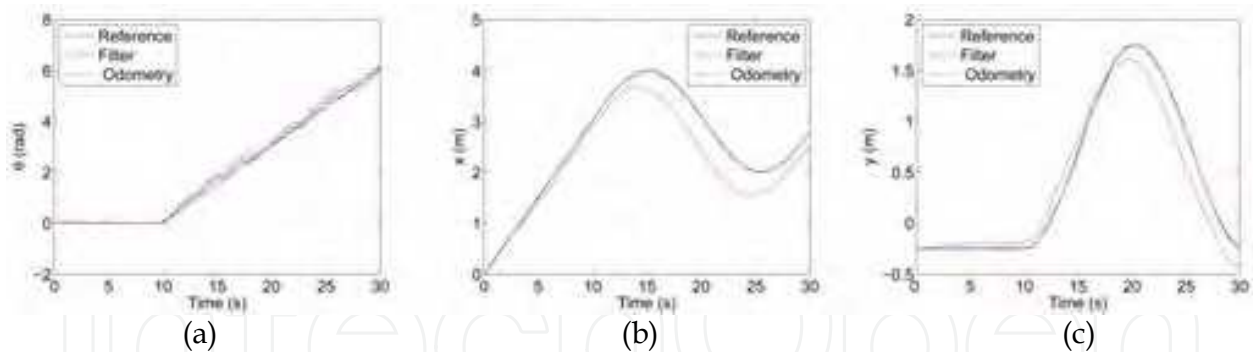


Fig. 7. Robot poses estimation using the descriptor filter: (a) θ , (b) x and (c) y .

B. Estimation of robotic prosthesis configuration with respect to ground

In this section it is presented a new application of stochastic filtering in the field of rehabilitation robotics. It is part of a research project which consists in the development of a robotic prosthesis for above knee amputees. Fig. 8(a) shows a picture of this device, which has three degrees of actuation: one for the knee and two for the ankle. Each joint is controlled by a direct current motor and a switched current source actuator. Concerning the foot, it is equipped with two Analog Devices ADXRS300 gyroscopes, and four Sharp GP2D120 distance measuring infrared sensors mounted underneath the foot (*c.f.*, Fig. 8(b)) [1].

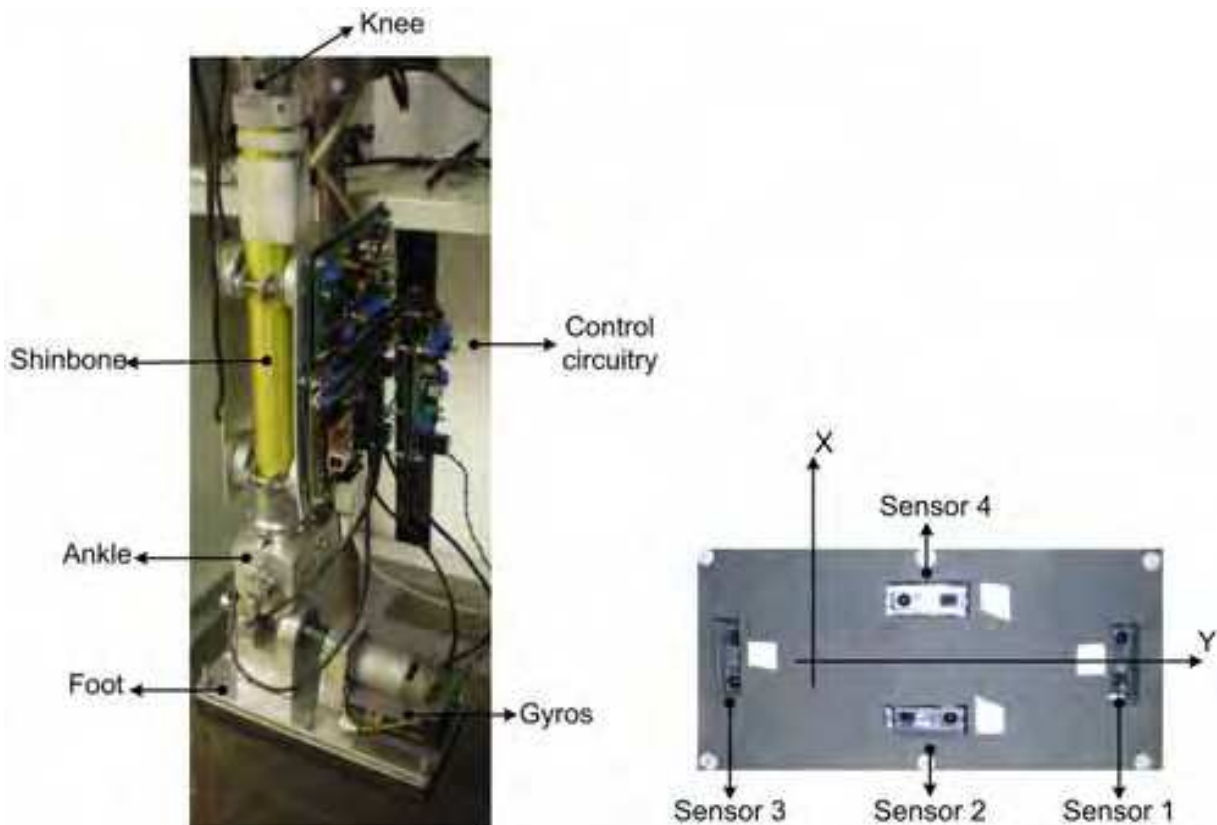


Fig. 8. (a) picture of the prosthesis and (b) infrared sensors placement underneath the foot

The aim of this application is to provide real-time estimates of the prosthesis foot configuration with respect to ground. A solution to this problem would allow the use of the

prosthesis for natural terrain walking, where foot positioning with respect to ground should be controlled. Robust descriptor filter allows to deal with uncertainties in model structure as well as to cope with constraints in estimated variables. Experimental results show the main properties of the system as well as the need of formulating this problem in a descriptor model framework.

B.1 Prosthesis configuration variables and sensing

In order to describe the configuration of the foot with respect to ground, denote $X \times Y \times Z$ the reference cartesian system as depicted in Fig. 9. $X \times Y$ is the foot plan, and Ξ represents the ground plan. Prosthesis foot configuration with respect to ground is described by

$n = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T$, the unit length vector coordinates of the normal to Ξ described in the $X \times Y \times Z$ the reference system, and d is the distance in Z axis of the ground plan to the origin of $X \times Y \times Z$.

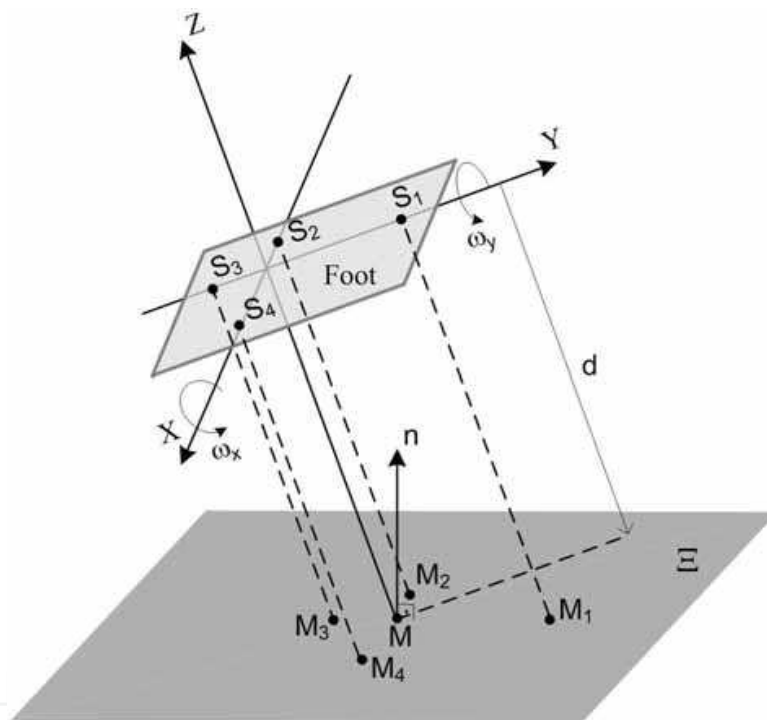


Fig. 9. Foot configuration with respect to ground and sensor measurements

S_1, S_2, S_3 and S_4 are the positioning in foot plan of the infrared sensors. It is considered such sensors make measurements perpendicular to $X \times Y$ plan in the direction of Ξ . Such measurements are taken in points M_1, M_2, M_3 and M_4 , and denoted $d_j, j = 1, \dots, 4$, all of them with the same variance σ_d^2 , meaning that infrared sensors have the same uncertainty. The coordinates m_j of $M_j \in \Xi$ are given by

$$m_j = s_j + \begin{bmatrix} 0 \\ 0 \\ -d_j \end{bmatrix} = \begin{bmatrix} s_{j,x} \\ s_{j,y} \\ -d_j \end{bmatrix}. \quad (70)$$

where $s_j = \begin{bmatrix} s_{j,x} & s_{j,y} & 0 \end{bmatrix}^T$ are the coordinates of S_j corresponding to the placement of the j -th infrared sensor. Since $M \in \Xi$, with coordinates $m = \begin{bmatrix} 0 & 0 & -d \end{bmatrix}^T$, one has

$$n^T(m_j - m) = 0$$

for each infrared sensor measurement, resulting in

$$d_j = \frac{dn_z + s_{j,y}n_y + s_{j,x}n_x}{n_z} \quad (71)$$

as equation relating the j th sensor distance measurement d_j to foot configuration variables n and d .

The two gyroscopes provide angular motion measurements of the foot in X and Y axis. Such measurements are angular velocities ω_x and ω_y as shown in Fig. 9, and take effect in n according to

$$\dot{n} = \frac{\partial n}{\partial \phi_x} \omega_x + \frac{\partial n}{\partial \phi_y} \omega_y$$

where $\omega_x = \frac{d\phi_x}{dt}$ and $\omega_y = \frac{d\phi_y}{dt}$ with ϕ_x and ϕ_y being small rotation angles in axis X and Y , respectively. The partial derivatives are computed as

$$\begin{aligned} \frac{\partial n}{\partial \phi_x} &= \lim_{\Delta \phi_x \rightarrow 0} \frac{\Delta n}{\Delta \phi_x} = \lim_{\Delta \phi_x \rightarrow 0} \frac{R_X(\Delta \phi_x)n - n}{\Delta \phi_x} \\ \frac{\partial n}{\partial \phi_y} &= \lim_{\Delta \phi_y \rightarrow 0} \frac{\Delta n}{\Delta \phi_y} = \lim_{\Delta \phi_y \rightarrow 0} \frac{R_Y(\Delta \phi_y)n - n}{\Delta \phi_y} \end{aligned}$$

with

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) \\ 0 & -\sin(\theta) & \cos(\theta) \end{bmatrix}, \quad R_Y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

being the basic rotations of angle θ about X and Y axis, respectively. Further development results in

$$\dot{n} = \begin{bmatrix} -n_z \omega_y \\ n_z \omega_x \\ -n_y \omega_x + n_x \omega_y \end{bmatrix}. \quad (72)$$

It should be pointed out that the angular velocity about Z axis is considered small when compared to ω_x and ω_y . This is usually the case for general movement of foot.

B.2 System modeling as descriptor model

Let $x_i = \begin{bmatrix} n_i^T & d_i \end{bmatrix}^T = \begin{bmatrix} n_{x,i} & n_{y,i} & n_{z,i} & d_i \end{bmatrix}^T$ the system state variable corresponding to foot configuration at discrete time i . The following model describes the evolution of state variables between measurements:

$$x_{i+1} = \begin{pmatrix} n_{x,i} - Tn_{z,i}\omega_{y,i} \\ n_{y,i} + Tn_{z,i}\omega_{x,i} \\ n_{z,i} - Tn_{y,i}\omega_{x,i} + Tn_{x,i}\omega_{y,i} \\ d_i \end{pmatrix} + w_i \quad (73)$$

In above, concerning evolution of the normal vector n_i , it has been used (72) in a first order Euler approximation with sampling period T . w_i is a Gaussian random noise encompassing gyroscope uncertainty as well as neglecting rotation about Z axis. Further, it has been considered a random evolution for d_i with Gaussian distribution, represented by the last entry of w_i . Rewriting (73) as

$$x_{i+1} = \begin{bmatrix} 1 & 0 & -T\omega_{y,i} & 0 \\ 0 & 1 & T\omega_{x,i} & 0 \\ T\omega_{y,i} & -T\omega_{x,i} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x_i + w_i \quad (74)$$

$$x_{i+1} = F(\omega_{x,i}; \omega_{y,i}) x_i + w_i \quad (75)$$

the evolution of the state variables becomes linear given the pair $(\omega_{x,i}; \omega_{y,i})$. It should be pointed out that (75) does not guarantee n_{i+1} with unit length. Thus, the following constraint should be considered:

$$n_{x,i+1}^2 + n_{y,i+1}^2 + n_{z,i+1}^2 = 1. \quad (76)$$

According to (71), a set $z_i = \begin{bmatrix} d_{1,i} & d_{2,i} & d_{3,i} & d_{4,i} \end{bmatrix}^T$ of measurements from infrared sensors are related to system state as

$$z_i = \begin{pmatrix} \frac{d_i n_{z,i} + s_{1,y} n_{y,i} + s_{1,x} n_{x,i}}{n_{z,i}} \\ \frac{d_i n_{z,i} + s_{2,y} n_{y,i} + s_{2,x} n_{x,i}}{n_{z,i}} \\ \frac{d_i n_{z,i} + s_{3,y} n_{y,i} + s_{3,x} n_{x,i}}{n_{z,i}} \\ \frac{d_i n_{z,i} + s_{4,y} n_{y,i} + s_{4,x} n_{x,i}}{n_{z,i}} \end{pmatrix} + v_i \quad (77)$$

$$z_i = h(x_i) + v_i \quad (78)$$

with $v_i \sim N(0, \sigma_d^2 I_4)$ being the measurement noise. The above equation corresponds to a non-linear measurement model which, when linearized at some given \bar{x}_{i-1} results in

$$z'_i = z_i - h(\bar{x}_{i-1}) + H(\bar{x}_{i-1})\bar{x}_{i-1} \quad (79)$$

$$= H(\bar{x}_{i-1})x_i + v_i \quad (80)$$

with

$$H(x_i) = \begin{bmatrix} \frac{s_{1,x}}{n_{z,i}} & \frac{s_{1,y}}{n_{z,i}} & \frac{s_{1,x}n_{x,i} + s_{1,y}n_{y,i}}{n_{z,i}^2} & 1 \\ \frac{s_{2,x}}{n_{z,i}} & \frac{s_{2,y}}{n_{z,i}} & \frac{s_{2,x}n_{x,i} + s_{2,y}n_{y,i}}{n_{z,i}^2} & 1 \\ \frac{s_{3,x}}{n_{z,i}} & \frac{s_{3,y}}{n_{z,i}} & \frac{s_{3,x}n_{x,i} + s_{3,y}n_{y,i}}{n_{z,i}^2} & 1 \\ \frac{s_{4,x}}{n_{z,i}} & \frac{s_{4,y}}{n_{z,i}} & \frac{s_{4,x}n_{x,i} + s_{4,y}n_{y,i}}{n_{z,i}^2} & 1 \end{bmatrix}. \quad (81)$$

Equations (75), (76), and (80) correspond to a descriptor model when defining a new state variable:

$$x'_i = \begin{bmatrix} x_i^T & u_i \end{bmatrix}^T \quad (82)$$

with $u_i = u_{i-1}$, and $u_0 = 1$ the norm of vector n at all instants i , thus:

$$E_{i+1}x'_{i+1} = F_i x'_i + w'_i \quad (83)$$

$$z'_{i+1} = H_i x'_i + v_i \quad (84)$$

$$E_{i+1} = \begin{bmatrix} I_4 & 0_{4 \times 1} \\ D_{i+1} & 0 \\ 0_{1 \times 4} & 1 \end{bmatrix}, \quad D_{i+1} = \begin{bmatrix} \bar{n}_{x,i-1} & \bar{n}_{y,i-1} & \bar{n}_{z,i-1} & 0 \end{bmatrix} \quad (85)$$

$$F_i = \begin{bmatrix} F(\omega_{x,i}, \omega_{y,i}) & 0_{4 \times 1} \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad w'_i = \begin{bmatrix} w_i \\ 0 \end{bmatrix} \quad (86)$$

$$H_i = \begin{bmatrix} H(\bar{x}_{i-1}) & 0_{4 \times 1} \end{bmatrix} \quad (87)$$

Remark III.1: In this example, we have chosen $\bar{x}_{i-1} = \hat{x}_{i-1|i-1}$, that is, we linearized at the last estimate. Since we have the expression (81), there is no need to perform differentiations in the filter algorithm iterations. We further incorporate the effects of the neglected nonlinearities in the uncertainties δE_{i+1} , δF_i and δH_i .

B.3 Experimental results

In this section, it is presented an experimental evaluation of the proposed foot configuration estimator. The prosthesis shinbone was set in a perpendicular position with respect to floor and several movement signals were sent to the motors responsible for rotation in X and Y axis. Gyroscope and infrared measurements were gathered in real-time during 35 s at 10 ms sampling period. Further, a potentiometer installed at foot provided direct angular measurements in X axis, which can be used to evaluate the performance in estimating the n_x normal vector component. This sensor can be used to partially validate the results of the proposed filter by giving a ground truth reference for n_x . However, it cannot be used in replacement of the proposed system because its measurement is taken with respect to the prosthesis shinbone, which only in this experiment is has approximately known configuration with respect to ground. This is not the case with the prostheses installed in a human patient. The robust descriptor filter of Algorithm 1 presented in Subsection II-B is used to estimate x_i . Equations (83) and (84) are rewritten as an uncertain system based on the following:

- \hat{n}_{i-1} does not represent exactly n_{i+1} , thus the descriptor matrix E_{i+1} is replaced by $(\bar{E}_{i+1} + \delta E_{i+1})$, in order to consider a range of possible n_{i+1} that must satisfy the unit norm constraint (76);
- Gyroscope measurements may have scale factor errors, bias, and also suffer from analog-to-digital conversion noise. Thus, the process matrix F_i is replaced by $(\bar{F}_i + \delta F_i)$ in order to consider the range of possible $F(\omega_{x,i}, \omega_{y,i})$;
- Measurement matrix H_i is the linearized form of Eq. (77) at $\bar{x}_{i-1} = \hat{x}_{i-1|i-1}$, and thus H_i is replaced by $(\bar{H}_i + \delta H_i)$ in order to incorporate linearization errors.

By using

$$M_{d,i} = \begin{bmatrix} 10^{-6} & 10^{-6} & 10^{-8} & 0 \end{bmatrix}, \quad M_{a,i} = 10^{-2} \begin{bmatrix} 0 & 0 & -T\omega_{y,i} & 0 \\ 0 & 0 & T\omega_{x,i} & 0 \\ T\omega_{y,i} & -T\omega_{x,i} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (88)$$

$$M_{f,i} = \begin{bmatrix} M_{a,i} & 0_{4 \times 1} & 0_4 & 0_4 \\ 0_{1 \times 4} & 0 & M_{d,i} & 0_{1 \times 4} \\ 0_{1 \times 4} & 0 & 0_{1 \times 4} & 0_{1 \times 4} \end{bmatrix}, \quad M_{h,i} = 10^{-2} \begin{bmatrix} 1_{4 \times 3} & 0_{4 \times 2} \end{bmatrix} \quad (89)$$

$$N_{e,i} = \begin{bmatrix} 0_4 & 0_{4 \times 1} \\ 0_{1 \times 4} & 1 \\ I_4 & 0_{4 \times 1} \\ 0_4 & 0 \end{bmatrix}, \quad N_{f,i} = \begin{bmatrix} I_4 & 0 \\ 0_{1 \times 4} & 0 \\ 0_4 & 0_{4 \times 1} \\ 0_4 & 0 \end{bmatrix}, \quad N_{h,i} = I_5 \quad (90)$$

$$\lambda = 25 \quad (91)$$

in (29) and (30), the estimated configuration variables are presented in Fig. 10. In the first plot of Fig. 10(a), it is shown the estimated n_x and the corresponding measured projection based on potentiometer readings. It can be seen in Fig. 10(a) that the estimated n_x component is very close to that measured by the potentiometer throughout the whole experiment. A small difference at 25 s and at 30 s is possibly caused by an instantaneous error in gyroscope that could not be completely corrected by the estimator. However, as a robust estimator, that error presented limited influence in the estimate. The obtained robust filter has limited bandwidth according to the parameters used in its design. Otherwise, the estimate could be very sensitive to fast changes in infrared sensor measurements. Thus,

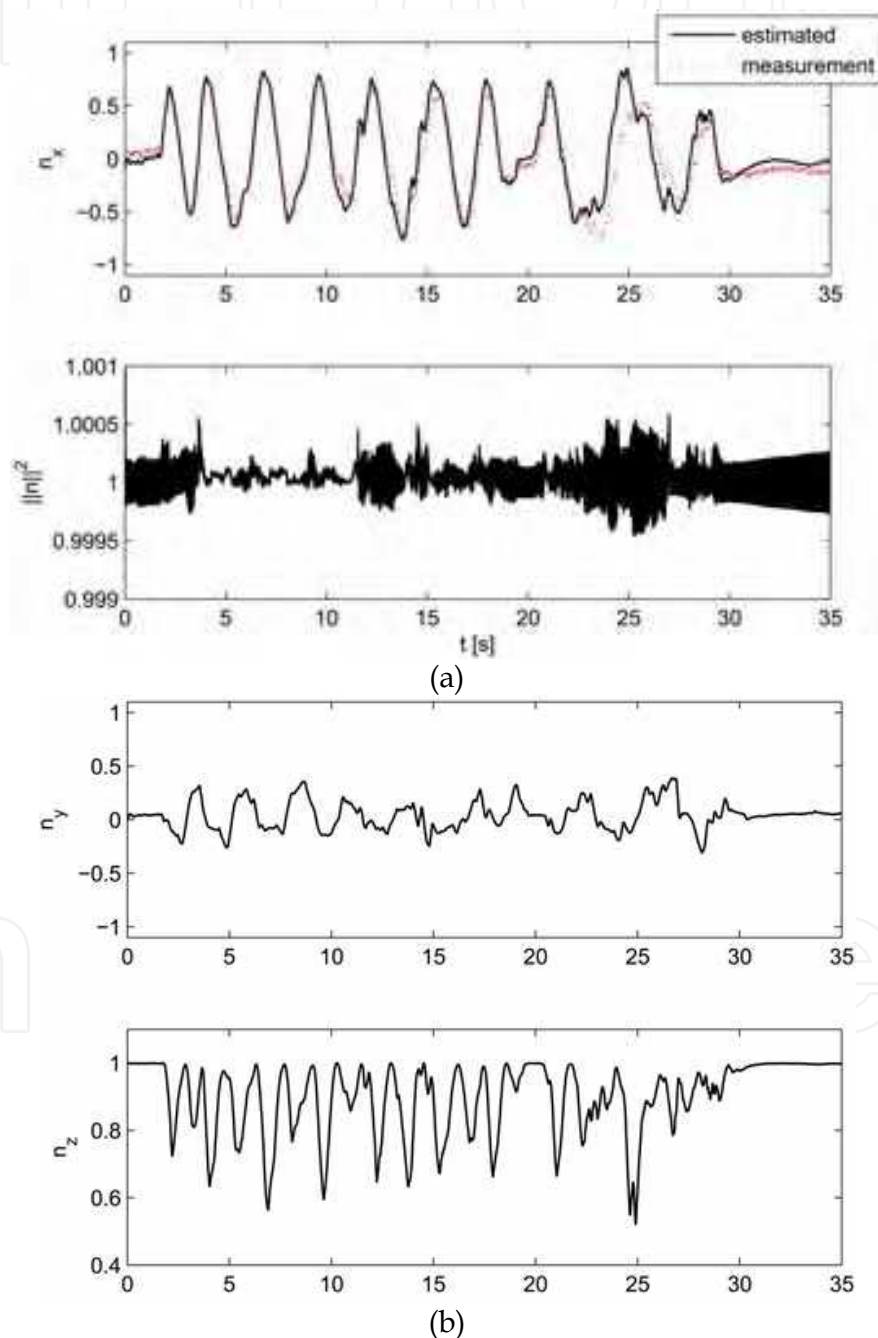


Fig. 10. (a) Vector n component n_x estimated (solid black), measured (dotted red), and unitary norm $\|n\|^2 = n_x^2 + n_y^2 + n_z^2$. (b) Vector n components n_y , n_z estimated

there is a trade off between satisfactory bandwidth and sensitivity to fast changes in infrared sensor measurements due to moving in irregular terrain.

The second plot in Fig. 10(a) presents the norm of vector n . It can be seen that the estimated n has norm very close to unity through the experiment.

Fig. 10(b) presents results on components n_y and n_z . For these estimates there is no ground truth measurements for comparison. The proposed system is currently used to provide foot configuration estimates to a controller responsible for keeping the foot parallel to ground.

4. Conclusion

Many classes of systems, such as systems with fast and low dynamics and systems with equality constraints on the state or inputs can be treated at once in the framework of descriptor systems. In this chapter we presented an extension of the well known Kalman filter to general rectangular descriptor systems subject to uncertainties. We presented the robust *a priori*, *a posteriori* estimates recursions and one version of predictor-corrector recursions. Applications of the filters are illustrated in the robotics field by two practical problems: the robust estimation of the localization of a mobile robot and the robust estimation of the configuration with respect to ground of a robotic prosthesis. The position estimation of a mobile robot is naturally formulated as an estimation problem of a system with algebraic constraints and subject to parametric uncertainties. In the proposed sensing system for the robotic prosthesis, the physical meaning of the signals imposes constraints in the estimates. For both examples, the constraints as well as the model structure uncertainties were fully addressed by the presented robust descriptor Kalman filter.

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