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Innovation Approach Based Sensor FDI in LEO Satellite Attitude Determination and Control System

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1. Introduction

In this study Fault Detection and Isolation (FDI) in the attitude determination and control system of Low Earth Orbit (LEO) satellite is investigated. Attitude determination system uses algebraic method. This method is based on computing any two analytical vectors in the reference frame and measuring these vectors in the body coordinate system (Barishev & Krilov, 1968; Wertz, 1988). As measuring devices, magnetometers and sun sensors are used. The satellite attitude is estimated via Extended Kalman Filter (EKF).

The Kalman filter approach to attitude determination and control is quite sensitive to the any measurement malfunctions (abnormal measurements, sudden shifts in the measurement channel, and other difficulties such as decrease of instrument accuracy, an increase of background noise, etc.). If the condition of operation of the measurement system does not correspond to the models, used in the synthesis of filter, then these changes resulting from some possible failures at the measurement channels significantly decrease the effectiveness of the attitude determination and control system. It is important to achieve fault-tolerance in the design of satellite attitude determination and control systems. For this purpose it is required to perform the sensor FDI in these systems.

Many fault detection methods have been developed to detect and identify faults in dynamic systems by using analytical redundancy (Zhang & Li, 1997; Rago et al., 1998; Larson et al., 2002; Lee & Lyou, 2002). In (Zhang & Li, 1997; Rago et al., 1998) the algorithms for detection and diagnosis of multiple failures in the dynamic systems are described. They are based on the Interacting Multiple-Model (IMM) estimation algorithm, which is one of the most cost-effective adaptive estimation techniques for systems involving structural as well as parametric changes. The proposed algorithms provide an integrated framework for fault detection, diagnosis, and state estimation. In methods, described in these works, the faults are assumed to be known, and the Kalman filters are designed for the known types of faults. As the approach requires several parallel Kalman filters, and the faults should be known, it can be used in limited applications.

In (Larson et al., 2002) an analytical redundancy-based approach for detecting and isolating sensor, actuator, and component (i.e., plant) faults in complex dynamical systems, such as aircraft and spacecraft is developed. The method is based on the use of constrained Kalman filters, which are able to detect and isolate such faults by exploiting functional relationships

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that exist among various subsets of available actuator input and sensor output data. A statistical change detection technique based on a modification of the standard generalized likelihood ratio (GLR) statistic is used to detect faults in real time. The GLR test requires the statistical characteristics of the system to be known before and after the fault occurs. As this information is usually not available after the fault, the method has limited applications in practice.

An integrated robust FDI and fault tolerant control (FTC) scheme for a fault in actuators or sensors of linear stochastic systems subjected to unknown inputs (disturbances) is presented in (Lee & Lyou, 2002). The FDI modules is constructed using banks of robust two-stage Kalman filters, which simultaneously estimate the state and the fault bias, and generate residual sets decoupled from unknown disturbances. All elements of residual sets are evaluated by using a hypothesis statistical test, and the fault is declared according to the prepared decision logic. In this work it is assumed that single fault occurs at a time and the treated fault is of random bias type. The diagnostic method presented in the article is valid only for the control surface FDI.

Fault tolerant attitude control system architecture presented in (Bak, et al., 1996) is based on the sensor reconfiguration. Part of the fault handling is dedicated to the duplicate components. Faults in non-dublicated sensors are detected using analytic redundancy methods based on different sensors. This approach deals with the hardware redundancy and it is very expensive.

In the references (Borairi & Wang, 1998; Alessandri, 2003) the neural network based methods to detect sensor, control surface/actuator failures are developed and discussed. In (Borairi & Wang, 1998) an approach for the fault detection and diagnosis of the actuators and sensors in non-linear systems is presented. First, a known non-linear system is considered, where an adaptive diagnostic model incorporating the estimate of the fault is constructed. Further, unknown nonlinear systems are studied and a feed forward neural network trained to estimate the system under healthy conditions. Genetic algorithms is proposed as a means of optimising the weighting connections of neural network and to assist the diagnosis of the fault.

In (Alessandri, 2003) a neural network based method to detect faults in nonlinear systems is proposed. Fault diagnosis is accomplished by means of a bank of estimators, which provide estimates of parameters that describe actuator, plant, and sensor faults. The problem of designing such estimators for general nonlinear systems is solved by searching for optimal estimation functions. These functions are approximated by feed forward neural networks and the problem is reduced to find the optimal neural weights. The methods based on artificial neural networks and genetic algorithms do not have physical bases. Therefore according to the different data corresponding to the same event, the model gives different solutions. Thus, the model should continuously be trained by using the new data.

To recover the possible malfunctions in the estimation system, the Adaptive Kalman Filters can be used (Sasiadek & Wang, 1999; Zhang & Wei, 2003) The Adaptive KF presented in (Sasiadek & Wang, 1999) has been applied to fuse position signals from the GPS and INS for the autonomous mobile vehicles. The EKF and the noise characteristic have been modified using the Fuzzy Logic Adaptive System. In the paper (Zhang & Wei, 2003), a method of multi-sensor data fusion based on the Adaptive Fuzzy Kalman Filter is presented. This method is applied in fusing position and orientation signals from Dead Reckoning (DR) system and the GPS for landing vehicle navigation**.** The EKF and the characteristics of the

measurement noise are modified by using the Fuzzy Adaptive system, and Fuzzy Adaptive system is based on a covariance matching technique. It has been demonstrated that the Fuzzy Adaptive Kalman Filter gives better results (more accurate) than the EKF (Sasiadek & Wang, 1999; Zhang & Wei, 2003). However, the fuzzy logic is a knowledge-based system operating on linguistic variables. These methods are based on the human experiences and are not widely applicable to the vital systems such as flight control systems.

Faults in multidimensional dynamic systems can be detected with the aid of an innovation sequence of Kalman filter (Mehra & Peschon, 1971; Willsky, 1976; Gadzhiev, 1992; Gadzhiev, 1994). This approach does not require a priori statistical characteristics of the faults, and the computational burden is not very heavy. Generally, fault detection algorithms developed to check the statistical characteristics of the innovation sequence in real-time are based on the following fact. If a system of estimation operates normally, the normalized innovation sequence in the Kalman filter coordinated with a dynamics model, represents the white Gauss noise with zero average value and unitary covariance matrix. Change of indicated statistical characteristics of the normalized innovation sequence are caused by a variety of problems: faults of measuring sensors, anomalous measurements, sudden shifts arising in the measurement channel, changing the statistical characteristics of the object or measurements noises, computer malfunctions, troubles with the deterioration precision of instruments, increasing noise background of instruments, as well as divergence of real process trajectories and divergence of estimations generated by the Kalman filter. The task of efficiently detecting such changes has to be undertaken in real operating conditions in order to correct the estimations. It is also essential to take decisions in a timely manner to change test and operating conditions.

The ways of checking a correspondence of the innovation sequence to the white noise and revealing a change in its mathematical expectation are considered in (Mehra & Peschon, 1971; Willsky, 1976; Hajiyev & Caliskan, 2003). The approaches that verify the covariance matrix of the innovation process are addressed in (Mehra & Peschon, 1971; Gadzhiev, 1992; Gadzhiev, 1994; Hajiyev & Caliskan, 2003).

In this study, fault detection algorithm for LEO satellite attitude determination and control system based on statistic for the mathematical expectation of the spectral norm of the normalized innovation matrix of the Kalman filter is presented. A real-time detection of sensor failures effecting the mean and variance of the innovation sequence, applied to satellite attitude dynamics, is examined and an effective approach to isolate the sensor failures is proposed.

2. Algebraic method based attitude determination and error analysis

2.1 Two-vector algorithms using Sun, Earth's magnetic field and Nadir vectors

The goal of the attitude determination is to find the orientation of the satellite relative to an inertial reference or to some specific object of interest (for example the Earth). In order to do this, there must be one or more available reference vectors, i.e. unit vectors in the known directions with respect to the satellite. Commonly used reference vectors are the Earth's magnetic field and unit vectors in the direction of the Sun, a known star or the centre of the Earth. Given reference vectors, and these vectors' orientations in the frame of the reference of the satellite can be obtained by using the measurements of the attitude sensor. Thus, the orientation of the satellite with respect to these vectors can be computed with some ambiguity (Wertz, 1988; Hajiyev & Bahar, 1998).

As an attitude specification of the satellite in space, Euler angles were selected. An attitude determination procedure that frequently used on three-axis stabilised satellite, is to determine the attitude by measuring the orientation in satellite coordinate system of two reference vectors fixed in inertial space. This is known as two-vector algorithm or algebraic method.

To determine the attitude matrix by using the method, mentioned above, at least components of two vectors have to be known in orbital frame and body frame (Barishev & Krilov, 1968; Wertz, 1988). The direction cosine matrix has to be found also from the orbital frame to the body frame. This gives an opportunity to express the attitude of a satellite in the reference coordinate system. The algorithms' output become "bad" when the reference vectors used in the algorithms are close to parallel or the value of pitch angle (θ) approaches (90°+nπ) degrees.

The aim of this study is to improve these "bad" results as much as possible. As a result, the two-vector algorithm may be used more along the satellite's orbit with better accuracy. To do this, three different algorithms, based on the selected reference vectors (Earth's magnetic field, unit vectors in the direction of the Sun and the center of the Earth), were designed and redundant data processing method was used.

In order to find the expressions of the reference vectors in the reference frame, the satellite's orbital parameters are required. Orbital parameters of a satellite can be determined using $x, y, z, \dot{x}, \dot{y}, \dot{z}$ quantities, which can be obtained by radio-technique measurements (Brandin et al., 1984). The two-vector algorithm includes the following steps (Hajiyev and Bahar, 2002):

1. determination of the orbital parameters,
2. estimation of the orbital parameters,
3. determination of the expressions of the reference vectors in the orbital frame,
4. measurement of the components of these vectors in the body coordinate system,
5. determination of the satellite's attitude.

As it was stated above, the two-vector algorithm requires two reference vectors. Let these vectors be the unit vector in the direction of the Sun (Sun vector) and the Earth's magnetic field. In orbital frame, these vectors are indicated by $\mathbf{S}_0$ and $\mathbf{H}_0$ respectively. In the body frame, these vectors are measured by sun sensors and magnetometers. The resultant measurement vectors are denoted with $\mathbf{S}_k$ and $\mathbf{H}_k$. A transformation matrix between the mentioned coordinate systems has to be formed. If the transformation matrix between these frames is A, and if $\mathbf{n}_0 = \mathbf{S}_0 \times \mathbf{H}_0, \mathbf{n}_k = \mathbf{S}_k \times \mathbf{H}_k$, then the following equalities can be written (Wertz, 1988):

$$\mathbf{S}_k = A\mathbf{S}_0, \mathbf{H}_k = A\mathbf{H}_0, \mathbf{n}_k = A\mathbf{n}_0 \tag{1}$$

Let's form the matrices C and C', which columns are made up of the above vectors:

$$C = [\mathbf{S}_0, \mathbf{H}_0, \mathbf{n}_0] \text{ ve } C' = [\mathbf{S}_k, \mathbf{H}_k, \mathbf{n}_k] \tag{2}$$

Then

$$C' = AC, A = C'C^{-1} \tag{3}$$

equalities can be written ($|\mathbf{n}_0| \neq 0$ and $|\mathbf{n}_k| \neq 0$). So, forming the transformation matrix A, it was found that (Wertz, 1988):

$$[A]=\begin{bmatrix} \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi\cos\psi+\sin\psi\sin\theta\cos\varphi & \cos\varphi\cos\psi+\sin\varphi\sin\theta\sin\psi & \sin\psi\cos\theta \\ \sin\psi\sin\varphi+\sin\theta\cos\psi\cos\varphi & -\cos\varphi\sin\psi+\cos\psi\sin\theta\sin\varphi & \cos\theta\cos\psi \end{bmatrix} \quad (4)$$

Using Eq.(1)-(4) attitude angles pitch (θ), yaw (ψ) and roll (φ) are found as functions of $\mathbf{H}_0$, $\mathbf{S}_0$, $\mathbf{H}_k$, $\mathbf{S}_k$ vectors

$$\theta=f_\theta(\mathbf{H}_0,\mathbf{S}_0,\mathbf{H}_k,\mathbf{S}_k),\ \psi=f_\psi(\mathbf{H}_0,\mathbf{S}_0,\mathbf{H}_k,\mathbf{S}_k),\ \varphi=f_\varphi(\mathbf{H}_0,\mathbf{S}_0,\mathbf{H}_k,\mathbf{S}_k) \quad (5)$$

The detailed expression of the Eq.(5) is given in Appendix-1. These expressions will be used in computing the accuracy of the satellite's attitude angles.

In this study three reference vectors were selected. Thus, three different two-vector algorithms can be designed. The studied algorithms are:

1. algorithm - Earth's magnetic field and Sun vector
2. algorithm - Earth's magnetic field and Nadir vector
3. algorithm - Nadir vector and Sun vector

The first of these algorithms was studied, and the attitude angles were expressed as functions of the reference vectors, included in this algorithm, and their measurements in the body frame. That was done in order to study the accuracy.

To examine the accuracy of the outputs of the other two-vector algorithms, it is necessary to express them like the first algorithm. The results will be similar to these of the first algorithm. Appropriately, adaptation of Eq.(A1) to the new state is sufficient. The result for the second algorithm can be obtained by replacing $S_{x_k}, S_{y_k}, S_{z_k}, S_{x_0}, S_{y_0}$ and S_{z_0} components in Eq.(A1) with $N_{x_k}, N_{y_k}, N_{z_k}, N_{x_0}, N_{y_0}$ and N_{z_k} components respectively. In a similar manner for the third algorithm the components $H_{x_k}, H_{y_k}, H_{z_k}, H_{x_0}, H_{y_0}$ and H_{z_k} taking place in Eq.(A1) have to be changed by $N_{x_k}, N_{y_k}, N_{z_k}, N_{x_0}, N_{y_0}$ and N_{z_k} components respectively.

2.2 Analysis of the LEO satellite attitude determination accuracy

There are a lot of factors affecting the LEO satellite's attitude determination. The most important of these factors are,

- errors due to the determination of the satellite's orbit,
- errors due to the determination and estimation of the satellite's orbital parameters,
- errors due to the models of the reference vectors in the orbital frame,
- errors due to the measurements of the reference vectors in the in body frame,
- errors due to the algorithm itself.

The scheme of the Earth's magnetic field and Sun vector based two-vector algorithm, used for determination of satellite's attitude, is shown in Fig.1.

Determination of attitude angles with two-vector algorithm includes the following procedures:

- determination of the orbit

$$q_{\ddot{o}}=\Phi_1 \mathrm{d},$$

here Φ_1 is the algorithm for determination of the orbital parameters

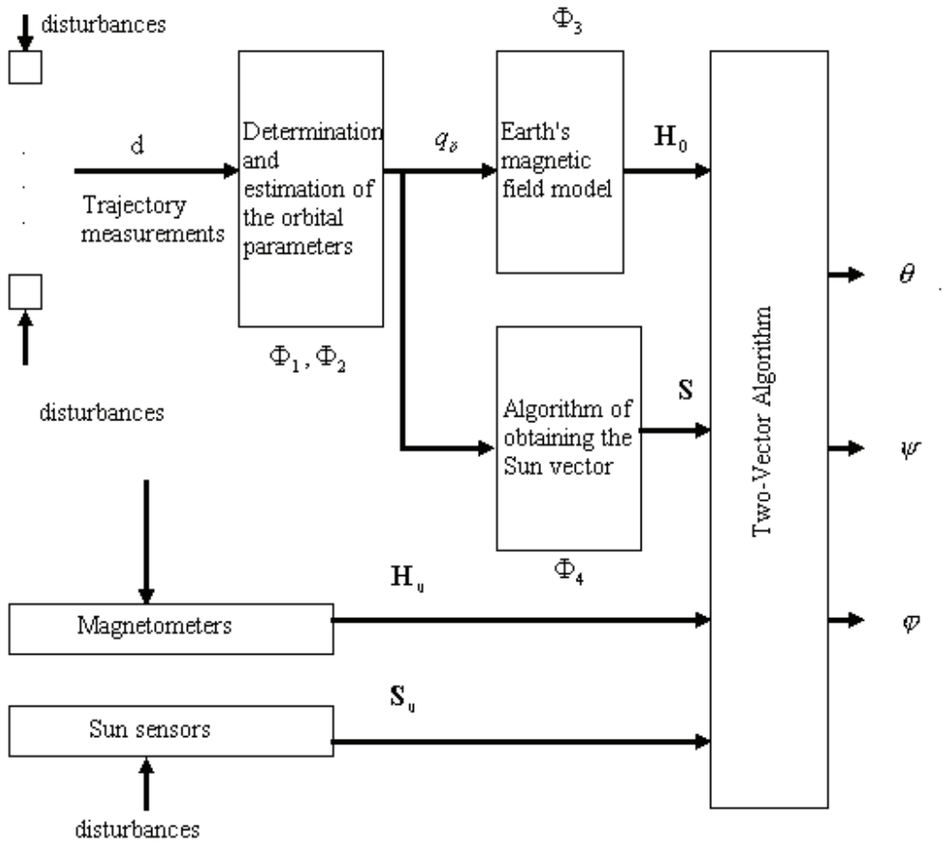


Fig. 1. Earth's magnetic field and Sun vector based two-vector algorithm scheme

- estimation of the orbit

$$q_{ö} = \Phi_2 \mathrm{d}$$

here Φ_2 is the algorithm for estimation of the orbital parameters

- determination of the Earth's magnetic field components in orbital frame

$$\mathbf{H}_0 = \Phi_3 q_{ö},$$

here Φ_3 is the algorithm for Earth's magnetic field determination

- determination of the Sun vector components in orbital frame

$$\mathbf{S}_0 = \Phi_4 q_{ö},$$

here Φ_4 is the algorithm for Sun vector determination

- determination of the satellite's attitude

$$\theta = f_\theta(\mathbf{H}_0, \mathbf{S}_0, \mathbf{H}_k, \mathbf{S}_k),\ \psi = f_\psi(\mathbf{H}_0, \mathbf{S}_0, \mathbf{H}_k, \mathbf{S}_k),\ \varphi = f_\varphi(\mathbf{H}_0, \mathbf{S}_0, \mathbf{H}_k, \mathbf{S}_k)$$

In general, the attitude determination algorithm is a nonlinear function of random variables. Thus, in order to find the error of the algorithm given, it was linearised by expanding to

Taylor series. Correlation between different parameters were ignored. So, after linearization, accuracy (variance) of the algorithm is found as:

$$D_{i\varphi} = \left(\frac{\partial f_{\varphi}}{\partial \mathbf{H}_0}\right)_m^2 D_{i\mathbf{H}_0} + \left(\frac{\partial f_{\varphi}}{\partial \mathbf{S}_0}\right)_m^2 D_{i\mathbf{S}_0} + \left(\frac{\partial f_{\varphi}}{\partial \mathbf{H}_k}\right)_m^2 D_{i\mathbf{H}_k} + \left(\frac{\partial f_{\varphi}}{\partial \mathbf{S}_k}\right)_m^2 D_{i\mathbf{S}_k} \tag{6}$$

here $D_{i\varphi}$ -is φ's computing error variance at step *i*.; $\left(\frac{\partial f}{\partial(\ )}\right)_m$ is the partial differentiation.

The subscript of the Eq.(6) means that, mean values of the parameters have to be used in the equation. In a similar way

$$D_{i\theta} = \left(\frac{\partial f_{\theta}}{\partial \mathbf{H}_0}\right)_m^2 D_{i\mathbf{H}_0} + \left(\frac{\partial f_{\theta}}{\partial \mathbf{S}_0}\right)_m^2 D_{i\mathbf{S}_0} + \left(\frac{\partial f_{\theta}}{\partial \mathbf{H}_k}\right)_m^2 D_{i\mathbf{H}_k} + \left(\frac{\partial f_{\theta}}{\partial \mathbf{S}_k}\right)_m^2 D_{i\mathbf{S}_k} \tag{7}$$

$$D_{i\psi} = \left(\frac{\partial f_{\psi}}{\partial \mathbf{H}_0}\right)_m^2 D_{i\mathbf{H}_0} + \left(\frac{\partial f_{\psi}}{\partial \mathbf{S}_0}\right)_m^2 D_{i\mathbf{S}_0} + \left(\frac{\partial f_{\psi}}{\partial \mathbf{H}_k}\right)_m^2 D_{i\mathbf{H}_k} + \left(\frac{\partial f_{\psi}}{\partial \mathbf{S}_k}\right)_m^2 D_{i\mathbf{S}_k} \tag{8}$$

The equations (6)-(8) can easily be adapted for second and third algorithm.

In the simulation, the satellite's orbital parameters are taken as: inclination i=97°; right ascension of the ascending node λ=15°. The orbit height is h=550 km; the Earth radius is R=6378.140 km; the Earth angular velocity is ω_D=7.28e-5 rad/s; the Earth magnetic field moment is M_{yer}=7.86e+15 Wb.m; the angle between geographical north and magnetic north is δ=11.4°. The accuracy of the orbital parameters i, λ and u are 5e-6 rad, 1e-5 rad and 1.5e-4 rad respectively. The attitude sensors' accuracy are ~1° for magnetometer, 0.1° for sun sensor and 0.36° for horizon sensor (horizon sensor determines the roll and pitch angles). It is assumed that eccentric anomaly is equal to the mean anomaly. Only one orbital period was simulated. In Fig.2 the change of the satellite attitude accuracy throughout the orbit is shown when the first algorithm is used (required accuracy is 1°). As accuracy characteristics pitch (θ), yaw (ψ) and roll (φ) angles' variances are taken.

When the results were examined it was seen that, when the reference vectors became near parallel or the value of the pitch angle θ approaches to (90°+nπ) degrees, the accuracy of the outputs are bellow the requirement. Furthermore, depending on the orbit, the Sun may be out of sight for some periods of orbit (as it is for the most low Earth orbits). It makes the first and third algorithm unusable. Similar results are obtained for the other two-vector algorithms.

It is obvious from the simulation results that the main effect on the accuracy is the magnetometer error, when the first algorithm is used. When the other two algorithms are used, horizon sensor error is more influent on the accuracy of the results.

According to the simulation results, the following conclusions are drawn: a) the accuracy of the spacecraft's attitude is changing in a wide range along the orbit; the accuracy is worst when the reference vectors are close to parallel or the value of the pitch angle θ approaches to 90°+nπ degrees; b) the attitude determination accuracy is affected by different factors in a different manner; the most influent factors on the accuracy are used initial values and the sensor errors; c) to increase the accuracy of attitude determination, redundant data processing methods (statistical methods) can be used (Hajiyev & Bahar, 1998; 2000).

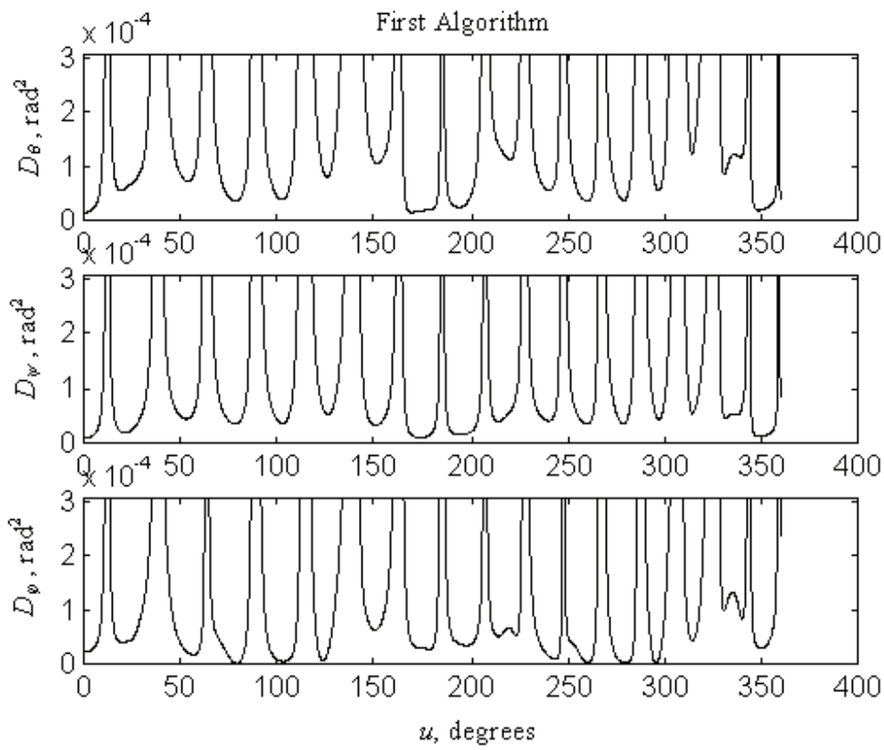


Fig. 2. Change of the variance of the attitude angles, obtained by the first algorithm, along one whole orbit

2.3 Increasing accuracy of the LEO satellite attitude determination using redundancy techniques

In order to increase the attitude determination accuracy, the redundant data processing algorithm, based on the Maximum Likelihood Method (MLM), was used to make the statistical operation on the measurements of the three algorithms mentioned above and appropriate formulas were derived.

Let's assume that the output x of a system is measured simultaneously with n different measurement devices with different measuring principles. Then the measurement equation of the ith device will be

$$z_{x_i} = x + \delta_i, \quad \text{i} = 1..\text{n}$$

here z_{x_i} -is the measurement of the ith device; δ_i -is the measurement error of the ith device. It is assumed that there is no correlation between the measurement errors of the measurement channels. Another assumption is that the measurement errors are subject to normal distribution with zero mean and finite σ_i^2 variance,

$$p(\delta_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left\{ -\frac{\delta_i^2}{2\sigma_i^2} \right\}, i = 1 \ ..n$$

Thus, the distribution density of measurement is known as a function of evaluated parameter,

$$p(z_{x_i} / x) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left\{ -\frac{(z_{x_i} - x)^2}{2\sigma_i^2} \right\}.$$

It is desired to find the optimal value of x which maximizes the Likelihood function. With the assumption that the measurements z_{x_i} (i =1..n) are independent, the Likelihood function $p(z_{x_1}, z_{x_2}, ..., z_{x_n} / x)$ can be expressed as (Hajiyev, 1999),

$$p(z_{x_i}, z_{x_2}, ..., z_{x_n} / x) = \frac{1}{\sqrt{(2\pi)^n} \prod_{i=1}^{n} \sigma_i} \exp\left\{ -\sum_{i=1}^{n} \frac{(z_{x_i} - x)^2}{2\sigma_i^2} \right\}. \tag{9}$$

After mathematical operations the expression for the estimated value that is searched is found as,

$$\hat{x} = \left(\sum_{i=1}^{n} \frac{z_{x_i}}{\sigma_i^2} \right) \Big/ \left(\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \right) \tag{10}$$

which covariance can be shown like,

$$D = \left\{ -E\left[\frac{\partial^2 \ln p(z_{x_1}, z_{x_2}, ... z_{x_n} / x)}{\partial x^2} \right] \right\}^{-1} = \left(\sum_{i=1}^{n} \frac{1}{\sigma_i^2} \right)^{-1}, \tag{11}$$

here E -denotes the operator for mathematical expectation.

Theorem 1. The inequality $D < \sigma_i^2, \forall i \in [1, n]$, is true for the variance of the estimated value (11).

Proof of the theorem is given in (Hajiyev, 1999).

In this study the pitch, roll and yaw angles that characterizes the angular position of the satellite was found with three different algorithms. Which means that, there is more information than required. Thus, adapting the redundant data processing method based on MLM to the case examined, the following results are obtained (Hajiyev & Bahar, 2002):

$$\hat{\varphi} = \frac{D_{\varphi_1} D_{\varphi_2} z_{\varphi_3} + D_{\varphi_3} D_{\varphi_2} z_{\varphi_1} + D_{\varphi_1} D_{\varphi_3} z_{\varphi_2}}{D_{\varphi_1} D_{\varphi_2} + D_{\varphi_1} D_{\varphi_3} + D_{\varphi_2} D_{\varphi_3}}, \tag{12}$$

$$\psi = \frac{D_{\psi_1} z_{\psi_N} + D_{\psi_N} z_{\psi_1}}{D_{\psi_1} + D_{\psi_N}}, \quad D_{\hat{\psi}} = \frac{D_{\psi_1} D_{\psi_N}}{D_{\psi_1} + D_{\psi_N}}, \quad \hat{\theta} = \frac{D_{\theta_1} z_{\theta_N} + D_{\theta_N} z_{\theta_1}}{D_{\theta_1} + D_{\theta_N}}, \quad D_{\hat{\theta}} = \frac{D_{\theta_1} D_{\theta_N}}{D_{\theta_1} + D_{\theta_N}}$$

where, $D_{(\bullet)_i}$ - is the variance of the appropriate angle found via i^{th} algorithm.

In Fig.3 the change of the satellite attitude accuracy is shown, when the redundant data processing method based on MLM is used. It can be seen that the accuracy of the values

found with redundant data processing are better than the values found with each other three algorithms. In order to get good results from the redundant data method, at least one of the three algorithms should produce available output (values with equal or better accuracy than required). The "bad" intervals (intervals where the accuracy is worse than required), formed due to the co-linearity of the reference vectors, can be removed by using redundant data processing method. But the "bad" intervals formed due to the pitch angle's value can not be removed with this method. Because, in that case the three algorithms give unavailable results. So, it can be said that, it is possible to increase the satellite's attitude determination accuracy by using redundant data processing method.

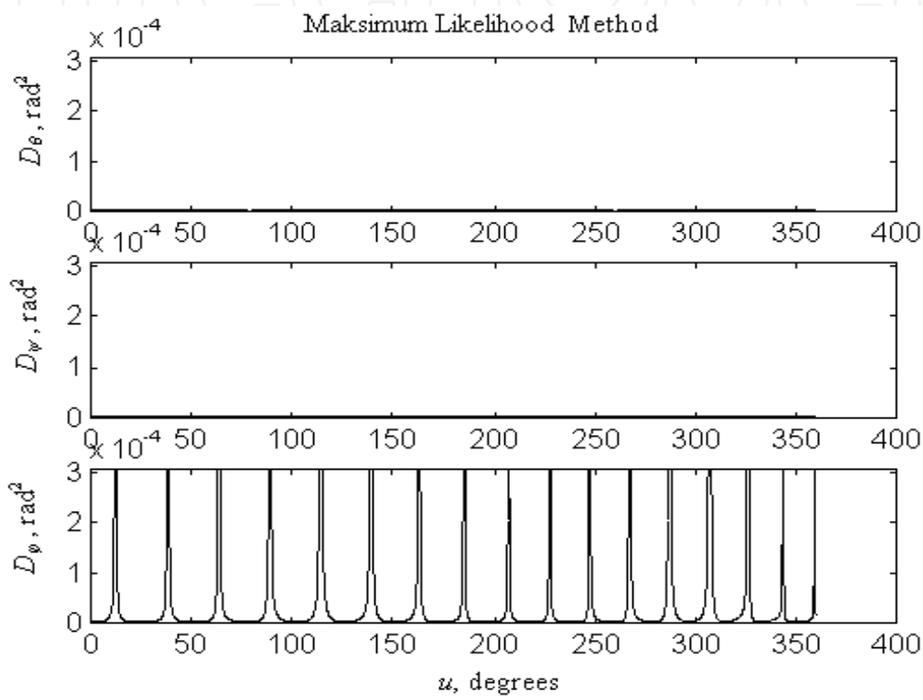


Fig. 3. Change of the variance of the attitude angles obtained by redundant data processing algorithm through the whole orbit

When the results, given in Fig. 2 are compared with results, given in Fig. 3, it is obvious that the result obtained by redundant data processing algorithm are better than all other results given.
In order to get good results from the redundant data processing method, at least one of the other algorithms have to produce available data (equal or better than the required accuracy). The "*bad areas*" (areas where the attitude accuracy is worse than the required one), formed due to the parallelism of the reference vectors, can be removed by using the redundant data processing algorithm. But it is impossible to remove the "*bad areas*", formed due to the pitch angle's value.

3. Satellite attitude estimation via extended Kalman filter

3.1 Extended Kalman filter design

The mathematical model of the satellite's rotational motion about its center of mass, is given bellow

$$\dot{\bar{\omega}}_c = J^{-1}\left\{-\bar{\omega}_c \times \left[J\bar{\omega}_c + \bar{H}_m\right] + \bar{m} + \bar{\xi}\right\} \tag{13}$$

where ω_c is the satellite's angular velocity in body coordinate system; $\bar{H}_m{}^T = \begin{bmatrix} 0 & H_m & 0 \end{bmatrix}$, H_m is the kinetic momentum of the momentum wheel; J is the inertia matrix of the LEO satellite and is equal to $J = \begin{bmatrix} J_{xc} & 0 & 0; & 0 & J_{yc} & 0; & 0 & 0 & J_{zc} \end{bmatrix}$; $\bar{m}$ is the vector of constant unknown disturbances acting on the satellite; $\bar{\xi}$ is the vector containing the random components of the forces acting on the satellite whose mathematical expectation and correlation matrix are as bellow,

$$E\left[\xi\right] = 0\,,\quad E\left[\xi_t \xi_{t-\tau}^T\right] = D_\xi \delta\left(t-\tau\right)$$

it is assumed that D_ξ is known.
The relations among the LEO satellite's rotation angles and angular velocities, are given below

$$\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = T_c \bar{\omega}_c + \bar{\omega}_{orb} \tag{14}$$

where,

$$T_c = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \dfrac{\sin\psi}{\cos\varphi} & \dfrac{\cos\psi}{\cos\varphi} & 0 \\ \sin\psi\ \mathrm{tg}\varphi & \cos\psi\ \mathrm{tg}\varphi & 1 \end{bmatrix},\quad \bar{\omega}_{orb}^T = \begin{bmatrix} 0 & \omega_{orb} & 0 \end{bmatrix}$$

ω_{orb} is the LEO satellite's angular orbit velocity.
The determination model of the angles that are characterizing LEO satellite's attitude, is given bellow,

$$z_{\varphi_i} = \varphi_i + \mathrm{v}_{\varphi_i}\,,\ z_{\theta_i} = \theta_i + \mathrm{v}_{\theta_i}\,,\ z_{\psi_i} = \psi_i + \mathrm{v}_{\psi_i} \tag{15}$$

here $\mathrm{v}_{(\bullet)_i}$ is the error of the attitude angles, determined by the MLM based redundant data processing algorithm which is given above. The mathematical expectations and variances of these errors are $E\left[\mathrm{v}\right] = 0$, $E\left[\mathrm{v}_{\varphi_i}, \mathrm{v}_{\varphi_j}\right] = D_{\varphi_i}\delta_{ij}$, $E\left[\mathrm{v}_{\theta_i}, \mathrm{v}_{\theta_j}\right] = D_{\theta_i}\delta_{ij}$ and $E\left[\mathrm{v}_{\psi_i}, \mathrm{v}_{\psi_j}\right] = D_{\psi_i}\delta_{ij}$; δ_{ij} is Kronecker's delta symbol.
The angular velocities $\omega_x, \omega_y, \omega_z$ of satellite are measured through the rate gyroscopes.
If the general vector

$$\bar{U}^T = \begin{bmatrix} \varphi & \theta & \psi & \omega_x & \omega_y & \omega_z & m_x & m_y & m_z \end{bmatrix}$$

is arranged and the mathematical model of the LEO satellite's rotational motion about its center of mass, is linearized using quasi-linearization method,

$$U_i = f\left(\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}\right) + F_{U_{i-1}}\left(U_{i-1} - \hat{U}_{i-1}\right) + F_{orb}\left(\bar{\omega}_{orb_{i-1}} - \omega_{orb_{i-1}}^{comp}\right) + h_{i-1} \tag{16}$$

where θ_i, ψ_i, φ_i - are pitch, yaw and roll angles respectively; ω_{x_i}, ω_{y_i}, ω_{z_i} - are angular rates; m_x, m_y, m_z - are the unknown constant components of the external moments acting on the satellite; $f\left(\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}\right)$ - is the right hand side of the LEO satellite's rotational motion mathematical model based on estimated values; ω_{orb} - is the satellite's angular orbit velocity; F_{orb}-is the coefficient matrix of the entrance effects;

$$F_U = \left[\frac{\partial f}{\partial U}\right]_{\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}}, F_{orb} = \left[\frac{\partial f}{\partial \omega_{orb}}\right]_{\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}}, h_{i-1} = \left[\frac{\partial f}{\partial \delta}\right]_{\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}}.$$

Minimum of the error's standard deviation was selected as an optimum criterion. It is suggested to derive LEO satellite attitude estimation algorithm using Bayes' method.
The problem of finding the values of the system's parameters and output coordinates, takes us to evaluation of $p\left(U_i / \bar{z}_i, \bar{\omega}_{orb_i}\right)$ conditional probability density. To the Bayes' formula, this probability density can be written as (Hajiyev & Bahar, 2003),

$$p\left(U_i / Z^i, \bar{\omega}_{orb_i}\right) = p\left(U_i / Z^{i-1}, z_i, \bar{\omega}_{orb_i}\right) = \frac{p\left(U_i / Z^{i-1}, \bar{\omega}_{orb_i}\right) p\left(z_i / U_i, Z^{i-1}, \bar{\omega}_{orb_i}\right)}{p\left(z_i / Z^{i-1}\right)}, \tag{17}$$

where $z_i^T = \left[z_{\varphi_i}, z_{\theta_i}, z_{\psi_i}, z_{w_{xi}}, z_{w_{yi}}, z_{w_{zi}}\right]$ is the measurement vector; $Z^i = \{z_1 \quad z_2 \quad \ldots \quad z_i\}$, $Z^{i-1} = \{z_1 \quad z_2 \quad \ldots \quad z_{i-1}\}$.
Finding and substituting terms respectively into Eq.(17) and via taking into consideration that the minimum of the standard deviation, which was chosen as an optimum criterion (in this case the conditional mathematical expectation of the value's a posteriori distribution will be the best value and as for the value's accuracy, the covariance matrix of this distribution will be used) and $p\left(U_i / Z^i, \bar{\omega}_{orb_i}\right)$, is a Gauss distribution, the recursive algorithm for the satellite's attitude estimation is obtained as bellow,

$$\hat{U}_i = f\left(\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}\right) + K_i\left[z_i - Hf\left(\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}}\right)\right], \tag{18}$$

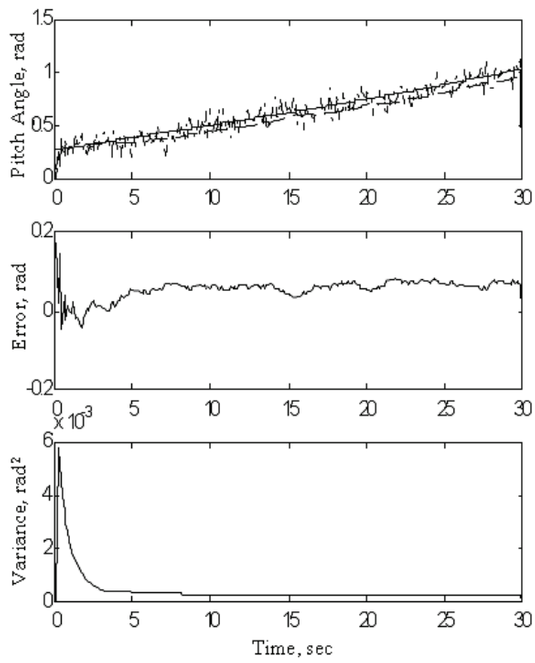
$$P_i = M_i - M_i H^T \left[D_{v_i} + HM_i H^T\right]^{-1} HM_i, \tag{19}$$

$$M_i = F_U P_{i-1} F_U^T + F_{orb} D_{orb_{i-1}} F_{orb}^T + D_{h_{i-1}}. \tag{20}$$

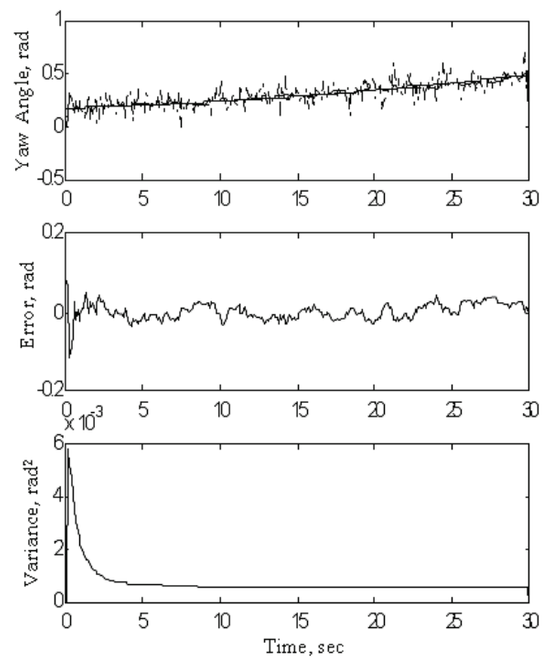
where M_i is the covariance matrix of the extrapolation error, P_i is the covariance matrix of the estimation error, $K_i = P_i H^T D_{v_i}^{-1}$ is the gain matrix of Kalman filter.
The Eqs.(18)-(20) are representing the Extended Kalman Filter (EKF) which fulfils recursive estimation of the satellite's rotational motion parameters about its mass center.

3.2 LEO satellite attitude estimation results

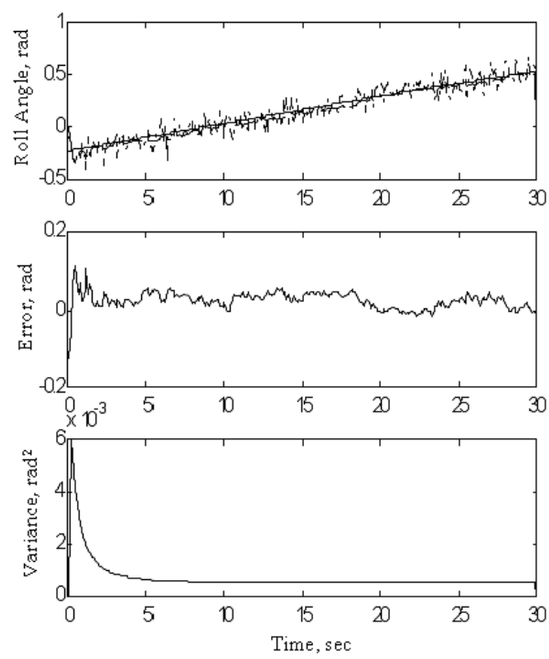
Portion of simulation results are given in Fig. 4 (a,b,c).



(a)



(b)



(c)

Fig. 4. Results by using EKF for a) pitch angle; b) yaw angle; c) roll angle (solid line – actual value; dash and dotted line – measurement value; dashed line – EKF output)

Graphics of the roll, pitch and yaw angles' estimated values, their error variances and the error between the actual values of the attitude angles and their estimated values are shown.

As it is seen from the graphics, for the taken orbit interval, the proposed EKF estimates the satellite rotational motion parameters with high accuracy.

3.3 Structure of attitude estimation and control system

The scheme of the proposed attitude estimation and control system is given in Fig. 5. As it seen, the system includes magnetometers, sun sensors, and horizon sensors to measure the above mentioned vectors. The system also includes three different two-vector algorithms based on the Earth magnetic field vector, nadir vector, and the Sun vector; redundant data processing algorithm based on Maximum likelihood method; EKF, and controller. The controlling action is done with the help of the momentum wheels. The system mentioned gives a possibility to stabilize the satellite through out the orbit.

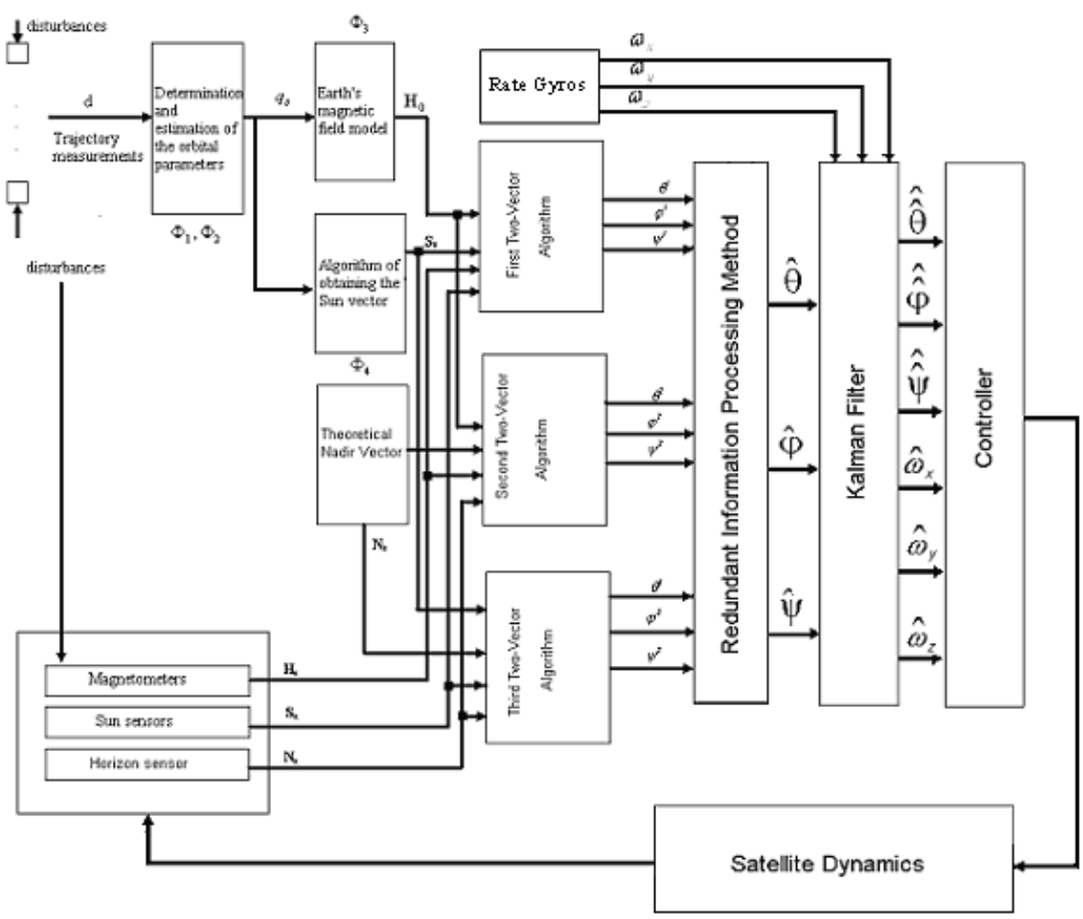


Fig. 5. Attitude estimation and control system scheme of LEO satellite

4. Innovation approach based sensor FDI

4.1 Fault detection via mathematical expectation statistic of spectral norm of normalized innovation matrix

In the Eq.(18)

$$\Delta_i = \left[z_i - Hf\left(\hat{U}_{i-1}, \bar{\omega}_{orb_{i-1}} \right) \right] \tag{21}$$

is the innovation sequence of EKF. If there is no trouble in the estimation system, the normalized innovation sequence

$$\tilde{\Delta}_i = \left[HM_i H^T + D_{v_i} \right]^{-1/2} \Delta_i, \tag{22}$$

of the EKF (18)-(20) accords with the dynamic model representing the white Gauss noise with zero average value and unitary covariance matrix (Mehra & Peschon, 1971):

$$E\left[\tilde{\Delta}_i \right] = 0; E\left[\tilde{\Delta}_k \tilde{\Delta}_j^T \right] = P_{\tilde{\Delta}_i} = I\delta_{kj}.$$

Faults causing abrupt changes in the characteristics of the measurement channel, malfunctions in the computer, as well as divergence of real process trajectories and estimations generated by the Kalman filter (which we shall hereafter name as faults of the estimation system) can cause changes in the demonstrated characteristics of the sequence $\tilde{\Delta}_i$ and can make it different from white noise, displace the zero average value and change the unitary covariance matrix.

It is important to develop an efficient method for simultaneous checking of the mathematical expectation and the variance of the normalized innovation sequence (22), which does not require a priori information on variation values in the fault case and makes it possible to find the faults of the estimation system in real-time.

Let us introduce two hypotheses: γ_0 - the estimation system operates properly; γ_1 - there is a trouble in the estimation system. To find a fault, we build a matrix with the columns of innovation vectors of EKF (18)-(20) and introduce the following definitions (Gadzhiev, 1996).

Definition 1. The innovation matrix of EKF (18)-(20) is rectangular $n \times m$ - matrix (n - dimension of innovation vector; $n \geq 2$; $m \geq 2$), with columns which are innovation vectors Δ_i corresponding to m different moments of time.

Definition 2. The innovation matrix, made up from normalized innovation vectors $\tilde{\Delta}_i$, is named as the normalized innovation matrix of EKF (18)-(20).

Hereafter for simplicity, we shall use the normalized innovation matrices A consisting of a finite number of innovation vectors. If the check is realized in real time, it is reasonable to form the matrix A_i at the i- th instant ($i \geq m$) of time from finite number m ($m \geq 2$) of sequential innovation vectors $\underbrace{\cdots \tilde{\Delta}_{i-2}, \tilde{\Delta}_{i-1}, \tilde{\Delta}_i}_{m}$. In order to check the hypotheses γ_0 and γ_1, we use a spectral norm of matrix A_i built in this way. As it is known (Horn and Johnson, 1986), the spectral norm $\|\cdot\|_2$ of the real matrix A_i is defined by the formula $\|A_i\|_2 \equiv \max\left\{ (\lambda_l \left[A^T{}_i A_i \right])^{1/2} \right\}$, where $\lambda_l \left[A^T{}_i A_i \right]$ are eigenvalues of the matrix $A^T{}_i A_i$. Square roots from eigenvalues of the matrix $A^T{}_i A_i$ i.e. values $(\lambda_l \left[A^T{}_i A_i \right])^{1/2}$ are named as singular values of the matrix A_i. Hence the spectral norm of the matrix A_i is equal to its maximum singular value. The singular values are real and non-negative (Horn and Johnson, 1986). By that reasoning, a determination

of singular values and, consequently, spectral norm represents a simpler problem in computing than determination of eigenvalues for arbitrary matrix. It explains the choice of the controlled scalar measure for the spectral norm of normalized innovation matrix of Kalman filter. In order to check the hypotheses γ_0 and γ_1, one-dimensional statistic for mathematical expectation of spectral norm of the matrix A_i for large values of k is introduced:

$$E\left\{\|A_i\|_2\right\} \approx \overline{\|A_i\|}_2 = \frac{1}{k}\sum_{j=1}^{k}\|A_j\|_2 . \tag{23}$$

As it is clear from (23), the mathematical expectation of spectral norm of the matrix A_i is substituted by its average arithmetical estimate. For determining upper and lower limit $E\left\{\|A_i\|_2\right\}$ use results obtained in (Hansen, 1988), where a number of bounds have been found for the mathematical expectation of spectral norm of random matrix $A_i \in R^{n \times m}$, constituted of random Gaussian values, having zero mathematical expectation and σ standard deviation. Let us consider some of them. Assume, r_k^T and a_j are rows and columns of the matrix A. Introduce maximum row-column norm

$$\mu \equiv \max\left[\|r_k\|_2, \|a_j\|_2\right], \tag{24}$$

where $\|r_k\|_2$ and $\|a_j\|_2$ are corresponding Euclid vector norms. The following bounds for $E\left\{\|A_i\|_2\right\}$ have been obtained in (Hansen, 1988) by means of norm μ introduced:

$$E\{\mu\} \le E\left\{\|A_i\|_2\right\} \le \left[\max(n,m)\right]^{1/2} E\{\mu\}. \tag{25}$$

Using the formula (25) in practical calculations represents a complex problem, because of the difficulty of estimation of $E\{\mu\}$. So, the value $E\{\mu\}$ is replaced by its lower bound

$$\sigma\sqrt{\max(n,m)} = \max\left[E\left\{\|r_i\|_2\right\}, E\left\{\|a_j\|_2\right\}\right] \le E\{\mu\}. \tag{26}$$

Then the equation (25) can be written as follows:

$$\sigma\sqrt{\max(n,m)} \le E\left\{\|A_i\|_2\right\} \le f(\max(n,m))\sigma\sqrt{\max(n,m)}, \tag{27}$$

where f is an unknown function to be determined. It is shown in (Hansen, 1988) by means of computer simulation, the value $\sigma\sqrt{\max(n,m)}$ is good lower bound for $E\left\{\|A_i\|_2\right\}$. It is also shown by numeric calculations that function f asymptotically approaches value 2 as $n=m\to\infty$, and f is always between values 1 and 2. So the value 2 is suggested to be used for estimating function f. Taking the above mentioned fact into consideration the following simple bounds are obtainable for $E\left\{\|A_i\|_2\right\}$:

$$\sigma\sqrt{\max(n,m)} \le E\left\{\|A\|_2\right\} \le 2\sigma\sqrt{\max(n,m)} . \tag{28}$$

The expression (28) characterizes the connection between the standard deviation σ of elements of the random matrix A and its spectral norm.

The normalized innovation matrix A_i, used for finding the troubles in the estimation system consists of the Gaussian random elements with zero mathematical expectation and finite variance $a_{kj} \in N(0,1)$. The inequality (28) can be applied for solving the diagnostic problem formulated in this study. Thus it is possible to say, if elements a_{kj} of the controlled normalized innovation matrix of EKF are subordinated to distribution $N(0,1)$, the inequality (28) is fulfilled. Nonfulfilment of the inequality (28) indicates a shifting zero average value of elements a_{kj}, changing the unitary variance or that $\{a_{kj}\}$ is other than white noise.
The algorithm offered for real system operation conditions is reduced to the following sequence of calculations to be executed at every step of measurements.

1. The EKF evaluating system state vector and vector value of the normalized innovation sequence on given step i are calculated by means of expressions (18)-(22).
2. The normalized innovation matrix of the EKF is formed for given $n \geq 2$ and $m \geq 2$. The eigenvalues of the matrix $A^T{}_i A_i$ as roots of equation

$$\det[A_i^T A_i - \lambda I] = 0 \tag{29}$$

and the spectral norm

$$\|A_i\|_2 \equiv \max\left\{(\lambda_l\left[A_i^T A_i\right])^{1/2}\right\} \tag{30}$$

are determined.
3. The statistic of mathematical expectation of spectral norm of the matrix A_i is calculated by means of (23).
4. The fulfilment of inequality (28) is checked and the solution is made according to the faulty operation of system.
5. The sequence of calculations is repeated as from the operation 1 for the following moment of time $i+1$.

It is necessary to note that the offered algorithm does not permit the realization of checking the nondiagonal elements of the covariance matrix of the normalized innovation sequence, but permits checks only on its mathematical expectation and variance. In spite of this fact, the given approach (due to its simplicity and ease of application) can bring good results when deciding the problems of check and diagnostics under conditions of relatively limited computer memory.

4.2 Sensor failure isolation based on innovation sequence

If the sensor fault is detected, then it is necessary to determine what sensor is faulty. For this purpose, the s-dimensional sequence $\tilde{\Delta}$ is transformed into n one-dimensional sequences to isolate the faulty sensor, and for each one-dimensional sequence $\tilde{\Delta}_i\ (i = 1, 2, \ldots, n)$ corresponding monitoring algorithm is run. The statistic of the faulty sensor is assumed to be affected much more than those of the other sensors. Let the statistics is denoted as $\xi_i(k)$. When $\max\{\xi_i(k)/i = 1,2,\ldots,n\} = \xi_p(k)$ for $i \neq j$, and $\xi_i(k) \neq \xi_j(k)$, it is judged that p-th control channel has failed.

Let the statistics, which is a rate of sample and theoretical variances; $\hat{\sigma}_i^2 / \sigma_i^2$ be used to verify the variances of one-dimensional innovation sequences $\tilde{\Delta}_i(k), i = 1,2,\ldots,n$. When $\tilde{\Delta}_i \sim N(0,\sigma_i)$ it is known that,

$$\frac{v_i}{\sigma_i^2} \sim \chi^2_{M-1}, \forall i, i = 1,2,...,n \tag{31}$$

where

$$v_i = (M-1)\hat{\sigma}_i^2 . \tag{32}$$

As $\sigma_i^2 = 1$ for normalized innovation sequence, it follows that,

$$v_i \sim \chi^2_{M-1}, \forall i, i = 1,2,...,n . \tag{33}$$

By selecting α level of significance as,

$$P\{\chi^2 > \chi^2_{\alpha,M-1}\} = \alpha \; ; 0 < \alpha < 1$$

So from the equation above, the threshold value $\chi^2_{\alpha,M-1}$ will be determined.
When a fault affecting the variance of the innovation sequence, occurs in the system, the statistics v_i exceeds the threshold value $\chi^2_{\alpha,M-1}$ depending on the confidence probability (1-α), and degree of freedom $(M-1)$. Using (33) it can be proved that any change in the mean of the normalized innovation sequence can be detected. Let a change in the mean of the innovation sequence occur at the time τ, and let $\tilde{\Delta}^*(k)$ denote the unchanged normalized innovation sequence, then the changed normalized innovation sequence is given by,

$$\tilde{\Delta}(k) = \tilde{\Delta}^*(k) \qquad k = 1,2,...,\tau-1 \tag{34}$$

$$\tilde{\Delta}(k) = \tilde{\Delta}^*(k) + \mu(k-\tau) \qquad k = \tau, \tau+1,... \tag{35}$$

where $\mu(.)$ is an unknown change and may vary with respect to time, but there exists a quantity $L > 0$ such that $|\mu(j)| < L$, for $\forall j$. (34) and (35) yield,

$$\tilde{\Delta}(k) \sim N(0,1) \quad k = 1,2,...,\tau-1 \tag{36}$$

$$\tilde{\Delta}(k) \sim N(\mu(k-\tau),1) \quad k = \tau, \tau+1,... \tag{37}$$

Let the number of shifted values from $j = k - M + 1$ to k in a window be denoted by N. When $k<\tau$ it can be easily shown that the mathematical expectation of investigated statistic (32) is $E[v_i] = M-1$. When a fault occurs, the mathematical expectation of (32) can be determined by the following theorem.
Theorem 2. When $k \geq \tau$, i.e. the hypothesis H_1 is true, the following equation is also true,

$$E[\upsilon(k)] = (M-1)\sigma^2 + E\left\{ \sum_{j=k-M+1}^{k} \left[\mu(j-\tau) - \frac{\sum_{j=k-M+1}^{k} \mu(j-\tau)}{M} \right]^2 \right\} \tag{38}$$

where

$$\mu(j-\tau)=\begin{cases} 0 & j<\tau \\ \mu^* = \text{constant} & j\geq\tau \end{cases}$$

The proof is given in (Hajiyev, 2006).
Let the number of shifted innovation values from $j=k-M+1$ to k in a window be denoted by N. Two distinct cases may be considered;

a. N=M, in this case,

$$E\left\{\sum_{j=k-M+1}^{k}\left[\mu(j-\tau)-\frac{\sum_{j=k-M+1}^{k}\mu(j-\tau)}{M}\right]^2\right\}=0 \tag{39}$$

and so, $E[\nu(k)]=(M-1)\sigma^2$. When the values $\tilde{\Delta}(j)$ have shifted by the same amount $\mu(j-\tau)$ in a window, it is impossible to detect the change by using (32).

b. $N<M$, in this case

$$\left[\mu(j-\tau)-\frac{\sum_{j=k-M+1}^{k}\mu(j-\tau)}{M}\right]^2=\left[\mu(j-\tau)-\frac{N\mu^*}{M}\right]^2\geq 0 \tag{40}$$

and a shift in the innovation sequence will cause an asymptotic increase in the expected value of the statistic $\nu(k)$, and $\nu(k)$ will exceed the threshold $\chi^2_{\alpha,M-1}$. The larger μ^* the faster detection is.
The sample variances $\hat{\sigma}_i$ are the diagonal components of the sample covariance matrix $S(k)$. Therefore there is no need to make heavy additional computation in the existent algorithm, but only the diagonal components of the matrix $S(k)$ are multiplied by $(M-1)$, and compared with $\chi^2_{\alpha,M-1}$ and with one another at each iteration. The decision making for isolation is done as follows; if the hypothesis H_1 is true and $S_{ii}(k)\neq S_{jj}(k), i\neq j$ and $\max\{S_{ii}(k)/i=1,2,...,n\}=S_{pp}(k)$ where $S_{ii}(k)$ is the ii th component of $S(k)$, then it is judged that there is a fault in the pth channel.

4.3 Simulation results of FDI algorithms

To test the proposed algorithm, it is applied to the mathematical model of the LEO satellite's rotational motion about its center of mass. It is demonstrated that the faults in a measurement channel can be detected by checking the mathematical expectation and the variance of the EKF innovation sequence. Under computer simulation of the above specified problem, as the estimation of system state vector is calculated, the values of normalized innovation sequence were determined by means of the expression (22). The spectral norm of matrix A_i for the case $n=6$, $m=6$ was determined by means of expression (30); the mathematical expectation of spectral norm $\|A_i\|_2$ was determined by

means of (23). Decisions on finding a system fault were made on the basis of inequality (28), written for the case *n=6, m=6.* If the case is *σ=1, n=6* and *m=6* ,the inequality (28) can be written in a simpler form

$$\sqrt{6} \le E\left\{\|A\|_2\right\} \le 2\sqrt{6} \tag{41}$$

The results of calculations are shown in Figures 5-7.

One can see in Fig.5 that the values of statistic $E\left\{\|A_i\|_2\right\}$ fall within the permissible domain (between lower and upper thresholds) when no sensor fault occurs. The graphs of the values of statistic $E\left\{\|A\|_2\right\}$ are shown in Fig.6 when a shift occurs in the pitch rate gyroscope at the step 30.The behavior of the appropriate normalized innovation sequences $\tilde{\Delta}_{\omega_y}(k)$ is presented in the Fig. 7.

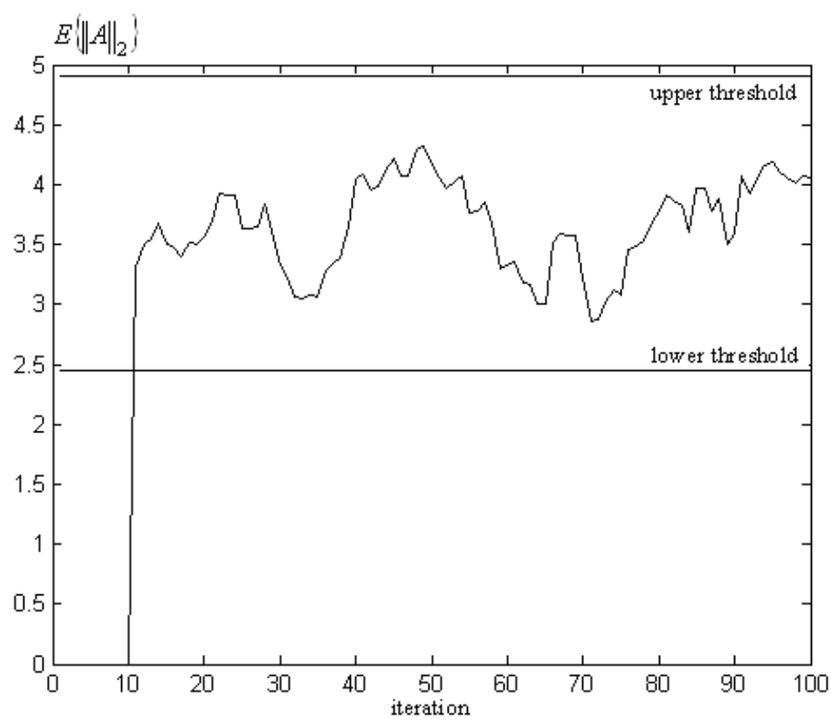


Fig. 5. The behavior of the statistic $E\left\{\|A\|_2\right\}$ for a normal operating system

This fault causes a change in the mean of the innovation sequence. As seen in Fig.6, when there is no sensor fault the values of statistic $E\left\{\|A\|_2\right\}$ fall within the permissible domain, and when a fault occurs in the pitch rate gyroscope $E\left\{\|A\|_2\right\}$ grows rapidly and after 1 steps it exceeds the upper threshold. Hence γ_1 hypotheses is judged to be true.

The Fig.8 shows detection of faults changing the noise variance of the pitch rate gyroscope.

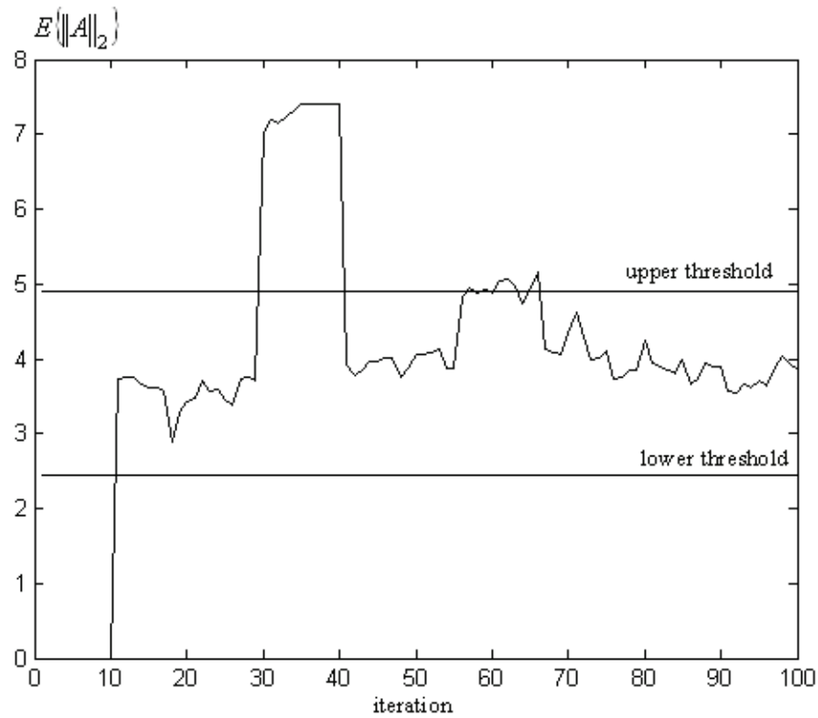


Fig. 6. The behavior of the statistic $E\{\|A\|_2\}$ in case of shift in the pitch rate gyroscope (the moment of the shift appears at k=30, the moment reveals the shift at k=31)

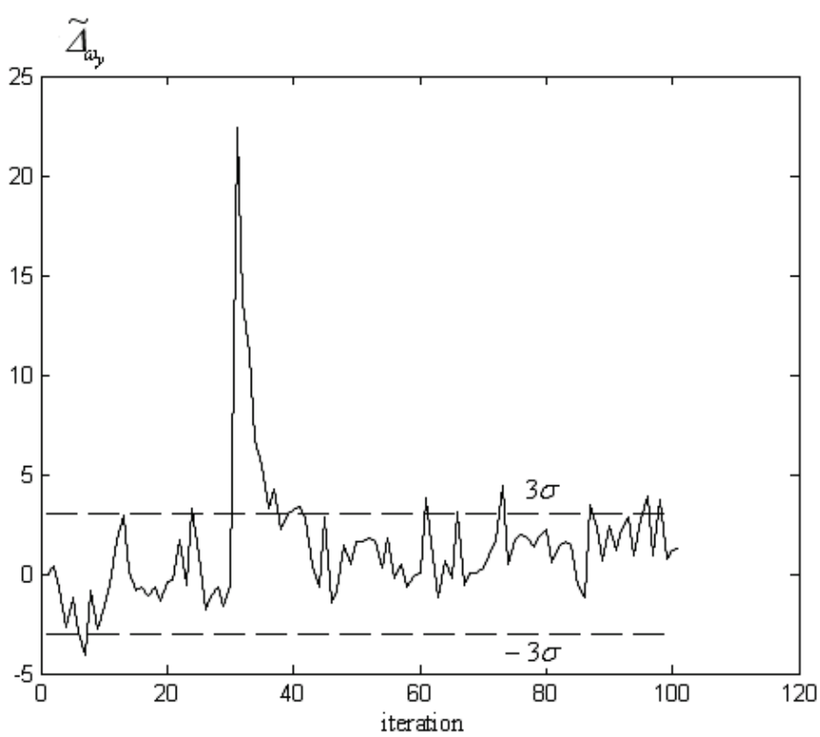


Fig. 7. Behavior of the normalized innovation sequence $\tilde{\Delta}_{\omega_y}(k)$ in case of shift in the pitch rate gyroscope

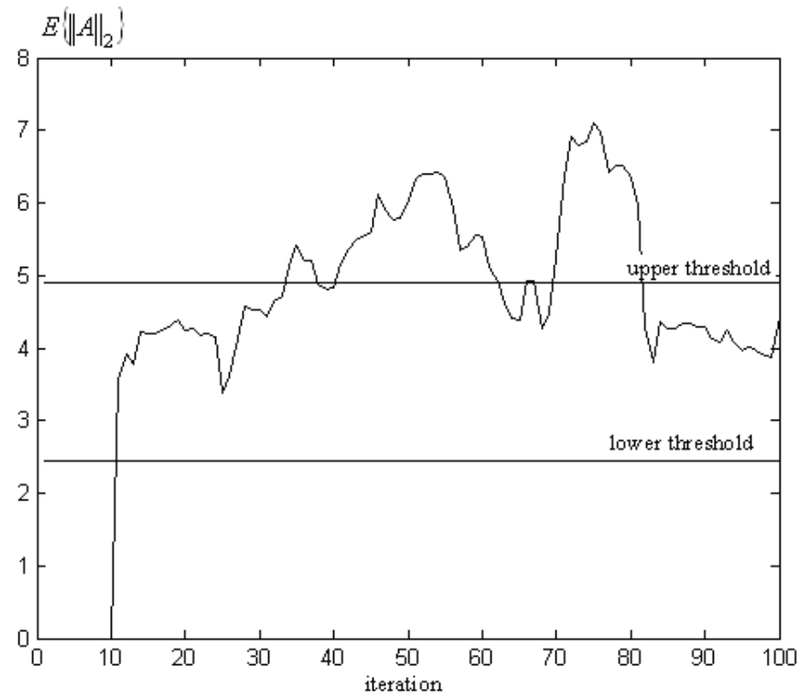


Fig. 8. The behavior of the statistic $E\{\|A\|_2\}$ in case of changes in noise variance of the pitch rate gyroscope (the moment of variance changes at k=30, the moment of revealing variance changes at k=34)

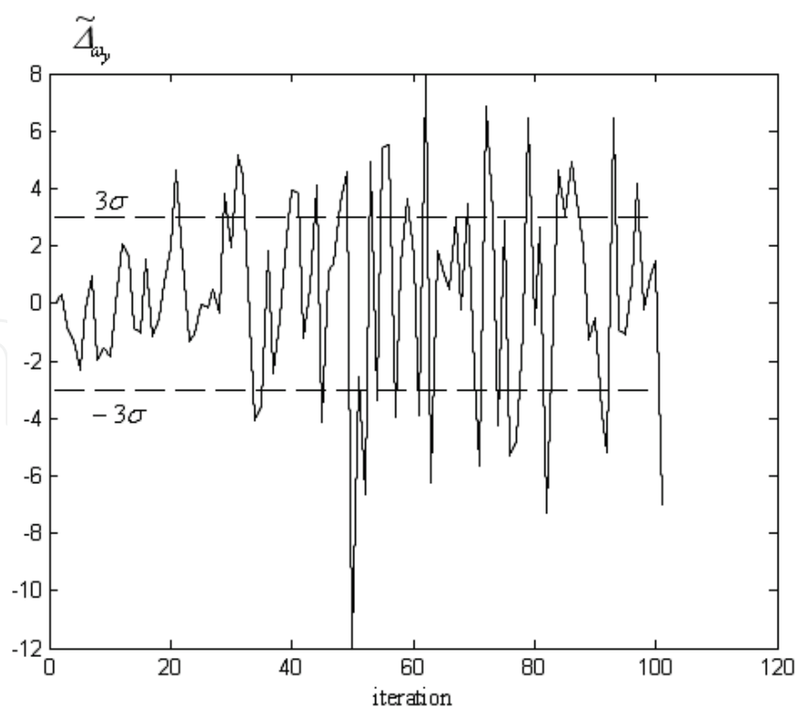
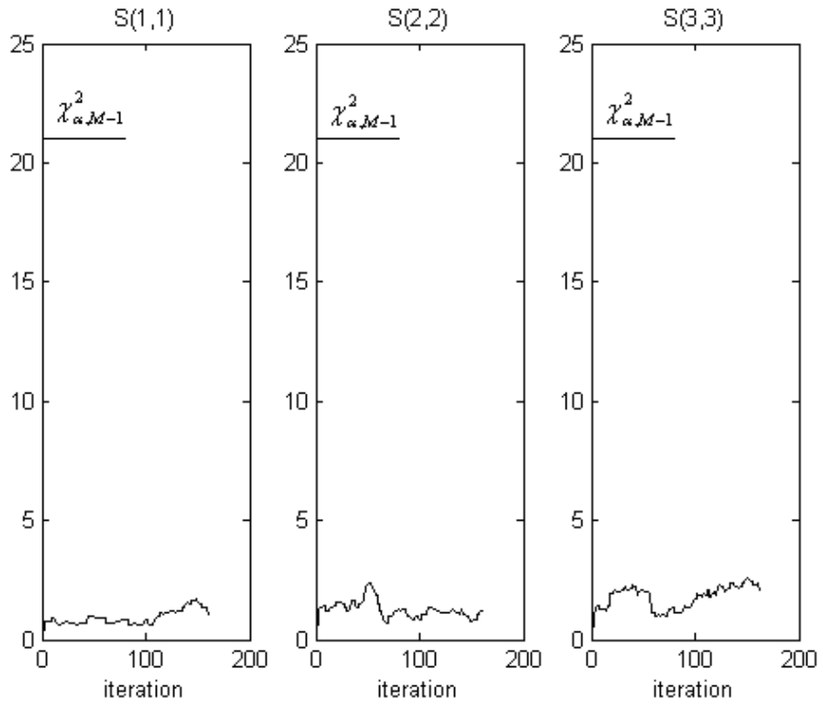


Fig. 9. Behavior of the normalized innovation sequence $\tilde{\Delta}_{\omega_y}(k)$ in case of changes in noise variance of the pitch rate gyroscope

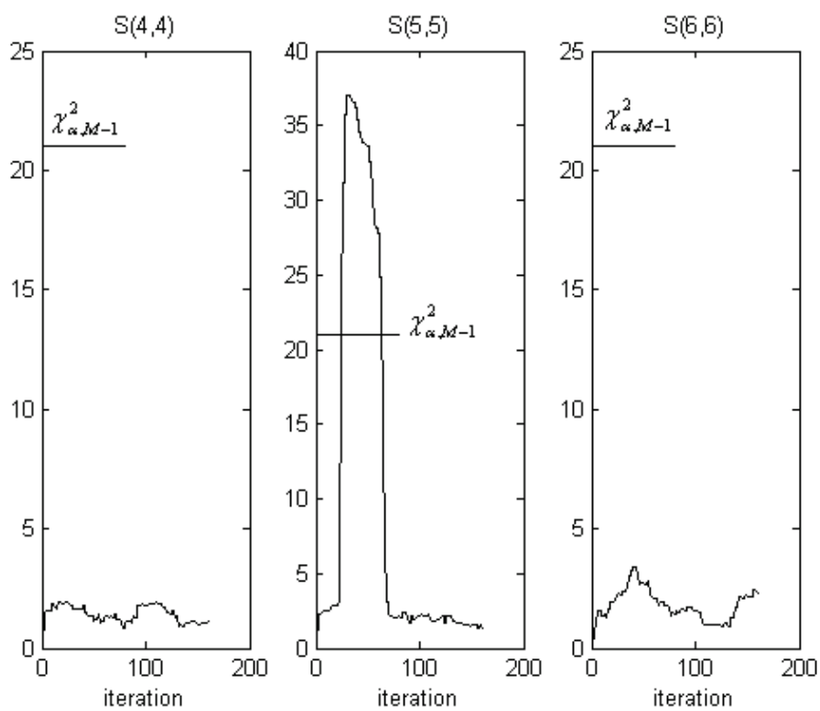
In this case, the mean value of the innovation sequence does not change, but the variance changes. The graphs of the values of statistic $E\{\|A\|_2\}$ are shown in Fig.8 when a fault occurs in the pitch rate gyroscope at the step 30. This fault causes a change in the variance of the innovation sequence. As seen in Fig.8, when there is no sensor fault $E\{\|A\|_2\}$ fall between lower threshold and upper threshold lines, and when a fault occurs in the pitch rate gyroscope $E\{\|A\|_2\}$ grows rapidly and after 4 steeps it exceeds the threshold. Hence γ_1 hypotheses is judged to be true. The behavior of the appropriate normalized innovation sequences $\tilde{\Delta}_{\alpha_i}(k)$ is presented in the Fig. 9.
The results of computer simulation have confirmed the practical possibility of simultaneous real-time check of mathematical expectation and variance of normalized innovation sequence with the aid of the statistic introduced (32).
Sensor failure isolation results in case of shift in the pitch rate gyroscope are given in Fig. 10(a,b) and in case of changes in noise variance of the pitch rate gyroscope in Fig. 11(a,b). As it is shown from presented figures, only the *(5,5)* element of the covariance matrix S ($S(5,5)$) exceeds the threshold $\chi^2_{\alpha,M-1}$ (for M=15 and $\alpha = 0.1$ the threshold value $\chi^2_{\alpha,M-1} = 21.1$) which indicates a failure in the pitch rate gyro. $S(j,j)$, $j \neq 5$ elements do not exceed the thresholds.

5. Conclusion

Fault detection and isolation algorithms for LEO satellite attitude determination and control system using an approach for checking the statistical characteristics of EKF innovation sequence are proposed. The fault detection algorithm is based on statistic for the mathematical expectation of the spectral norm of the normalized innovation matrix of the EKF. This approach permits simultaneous real-time checking of the mathematical expectation and the variance of the innovation sequence and does not require a priori information about the faults and statistical characteristics of the system in fault cases.
In this study an attitude estimation and control system for LEO satellite is proposed. To determine the attitude of the satellite, this system use algebraic method (two-vector algorithm). As a reference direction, the unit vectors toward the Sun, the Earth's center, and the Earth magnetic field are used. Thus, it includes three different two-vector algorithms based on using the Earth's magnetic field – the Sun vector, the Earth's magnetic field – nadir vector, and nadir vector – the Sun vector couples. In order to increase the attitude determination accuracy, the redundant data processing algorithm, based on the Maximum Likelihood Method, is used.
An extended Kalman filter has been developed for nonlinear rotational dynamics estimation of LEO satellite. Failures in the sensors affect the characteristics of the innovation sequence of the EKF. The failures that affect the mean and variance of the innovation sequence have been considered. The application of the proposed fault detection algorithm to the LEO satellite attitude determination and control system has shown that, sensor fault detection by the presented algorithm is possible in real time.
Assuming that the effect of the faulty sensor on its channel is more significant than on the other channels, a sensor isolation method is presented by transforming *n*-dimensional innovation process to n one-dimensional processes. The simulations, carried out on a nonlinear dynamic model of the rotational motion of LEO satellite, confirm the theoretical results.
The future work is to investigate the faults affecting the attitude dynamics e.g., actuator faults and to perform the integrated sensor/actuator FDI via innovation approach.

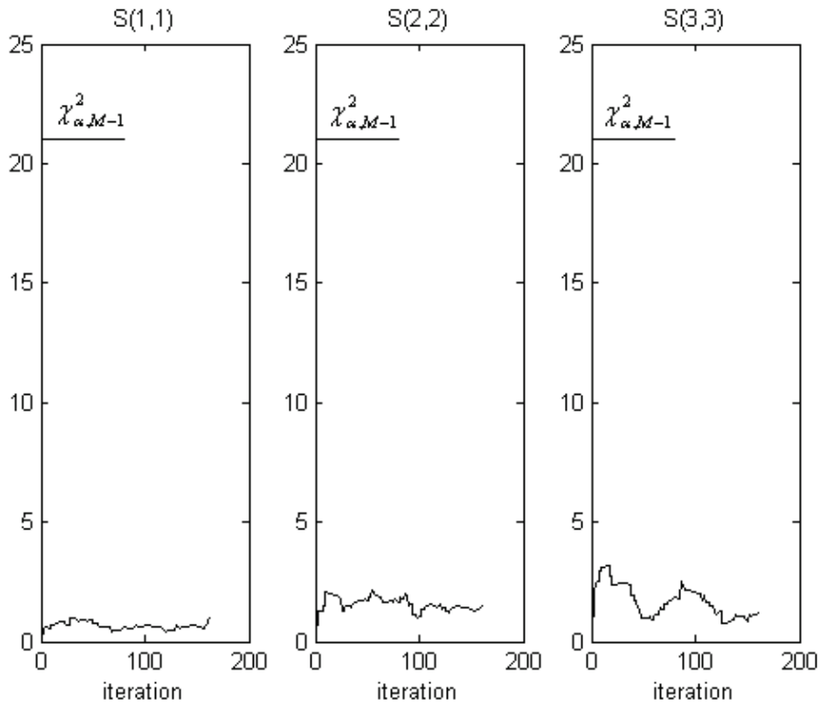


(a)

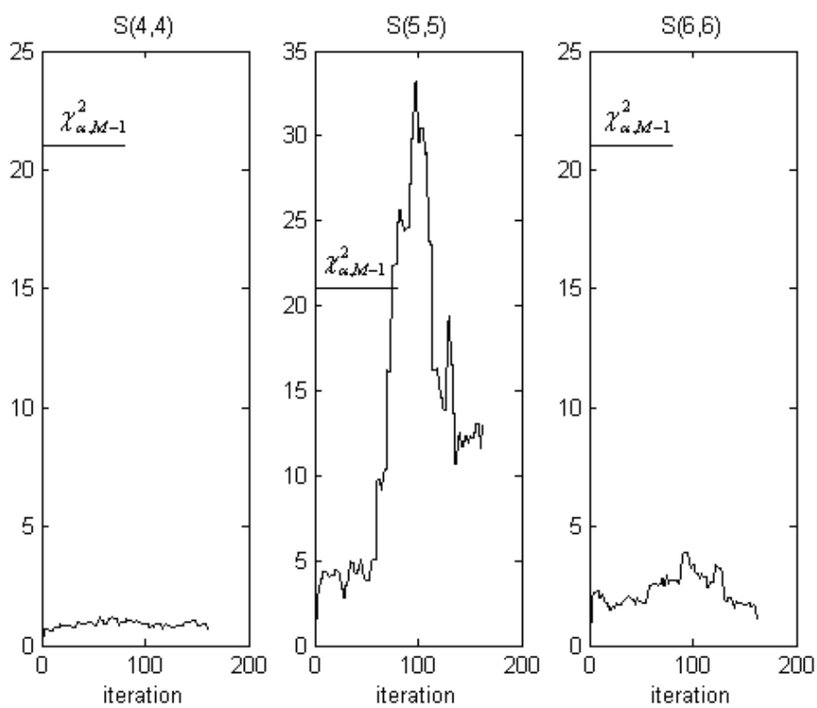


(b)

Fig. 10. Sensor failure isolation in case of shift in the pitch rate gyroscope



(a)



(b)

Fig. 11. Sensor failure isolation in case of changes in noise variance in the pitch rate gyroscope

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Appendix-1

It is possible to express the attitude angles with the components of the vectors used in the algorithm. For that purpose, first it have to be found the inverse of matrix *C*. Then using the Eq.(3) the transformation matrix *A* could be formed. Once the transformation matrix is formed, the following relations can be written,

$$\theta = \sin^{-1}\left(R'/P'\right)$$
$$\varphi = \tan^{-1}\left(R''/P''\right)$$
$$\psi = \tan^{-1}\left(R'''/P'''\right)$$

here,

$$\begin{aligned} R' = {} & H_{z_0} S_{x_0}^2 H_{x_k} - H_{x_0} S_{x_0} S_{z_0} H_{x_k} - H_{y_0} S_{y_0} S_{z_0} H_{x_k} + H_{z_0} S_{y_0}^2 H_{x_k} + H_{y_0}^2 S_{z_0} S_{x_k} - \\ & - H_{y_0} H_{z_0} S_{y_0} S_{x_k} - H_{x_0} H_{z_0} S_{x_0} S_{x_k} + H_{x_0}^2 S_{x_0} S_{x_k} + H_{x_0} S_{y_0} H_{y_k} S_{z_k} - H_{y_0} S_{x_0} H_{y_k} S_{z_k} - \\ & - H_{x_0} S_{y_0} H_{z_k} S_{y_k} + H_{y_0} S_{x_0} H_{z_k} S_{y_k} \end{aligned}$$

$$\begin{aligned} P' = {} & H_{x_0}^2 S_{y_0}^2 + H_{x_0}^2 S_{z_0}^2 + H_{y_0}^2 S_{x_0}^2 + H_{y_0}^2 S_{z_0}^2 + H_{z_0}^2 S_{x_0}^2 + H_{z_0}^2 S_{y_0}^2 - 2H_{x_0} H_{y_0} S_{x_0} S_{y_0} - \\ & - 2H_{x_0} H_{z_0} S_{x_0} S_{z_0} - 2H_{y_0} H_{z_0} S_{y_0} S_{z_0} \end{aligned}$$

$$
\begin{aligned}
R'' = {} & H_{y_0} H_{x_k} S_{z_0}^2 - H_{z_0} H_{x_k} S_{y_0} S_{z_0} - H_{x_0} H_{x_k} S_{x_0} S_{y_0} + H_{y_0} H_{x_k} S_{x_0}^2 + H_{x_0}^2 S_{y_0} S_{x_k} - \\
& - H_{x_0} H_{y_0} S_{x_0} S_{x_k} - H_{y_0} H_{z_0} S_{z_0} S_{x_k} + H_{z_0}^2 S_{y_0} S_{x_k} + H_{z_0} H_{y_k} S_{x_0} S_{z_k} - H_{x_0} H_{y_k} S_{z_0} S_{z_k} - \\
& - H_{z_0} H_{z_k} S_{x_0} S_{y_k} + H_{x_0} H_{z_k} S_{z_0} S_{y_k}
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
P'' = {} & H_{x_0} H_{x_k} S_{y_0}^2 - H_{y_0} H_{x_k} S_{x_0} S_{y_0} - H_{z_0} H_{x_k} S_{x_0} S_{z_0} + H_{x_0} H_{x_k} S_{z_0}^2 + H_{z_0}^2 S_{z_0} S_{x_k} - \\
& - H_{x_0} H_{z_0} S_{z_0} S_{x_k} - H_{x_0} H_{y_0} S_{y_0} S_{x_k} + H_{y_0}^2 S_{x_0} S_{x_k} + H_{y_0} H_{y_k} S_{z_0} S_{z_k} - H_{z_0} H_{y_k} S_{y_0} S_{z_k} - \\
& - H_{y_0} H_{z_k} S_{z_0} S_{y_k} + H_{z_0} H_{z_k} S_{y_0} S_{y_k}
\end{aligned}
$$

$$
\begin{aligned}
R''' = {} & H_{z_0} H_{y_k} S_{x_0}^2 - H_{x_0} H_{y_k} S_{x_0} S_{z_0} - H_{y_0} H_{y_k} S_{y_0} S_{z_0} + H_{z_0} H_{y_k} S_{y_0}^2 + H_{y_0}^2 S_{z_0} S_{y_k} - \\
& - H_{y_0} H_{z_0} S_{y_0} S_{y_k} - H_{x_0} H_{z_0} S_{x_0} S_{y_k} + H_{x_0}^2 S_{z_0} S_{y_k} + H_{x_0} H_{z_k} S_{y_0} S_{x_k} - H_{y_0} H_{z_k} S_{x_0} S_{x_k} - \\
& - H_{x_0} H_{x_k} S_{y_0} S_{z_k} + H_{y_0} H_{x_k} S_{x_0} S_{z_k}
\end{aligned}
$$

$$
\begin{aligned}
P''' = {} & H_{z_0} H_{z_k} S_{x_0}^2 - H_{x_0} H_{z_k} S_{x_0} S_{z_0} - H_{y_0} H_{z_k} S_{y_0} S_{z_0} + H_{z_0} H_{z_k} S_{y_0}^2 + H_{y_0}^2 S_{z_0} S_{z_k} - \\
& - H_{y_0} H_{z_0} S_{y_0} S_{z_k} - H_{x_0} H_{z_0} S_{x_0} S_{z_k} + H_{x_0}^2 S_{z_0} S_{z_k} + H_{x_0} H_{x_k} S_{y_0} S_{y_k} - H_{y_0} H_{x_k} S_{x_0} S_{y_k} - \\
& - H_{x_0} H_{y_k} S_{y_0} S_{x_k} + H_{y_0} H_{y_k} S_{x_0} S_{x_k}
\end{aligned}
$$

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The aim of this book is to provide an overview of recent developments in Kalman filter theory and their applications in engineering and scientific fields. The book is divided into 24 chapters and organized in five blocks corresponding to recent advances in Kalman filtering theory, applications in medical and biological sciences, tracking and positioning systems, electrical engineering and, finally, industrial processes and communication networks.

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