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### Optimal Control of Underactuated Underwater Vehicles with Single Actuator

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#### 1. Introduction

The research on underwater systems has gained an immense interest during the last decades with applications taken place in many fields such as exploration, investigation, repair, construction, etc. Hereby, control of underwater systems has emerged as a growing field of research. Underwater vehicles, in fact, accounted for 21% of the total number of service robots by the end of 2004, and are the most expensive class of service robots (UNECE/IFR, 2005). Typically, underwater vehicles can be divided into three underwater systems, namely, the manned submersibles, remotely operated vehicles (ROV) and autonomous underwater vehicles (AUV). ROVs and AUVs are mostly utilized in the oil and gas industries, and for scientific and military applications. AUVs, especially are of great importance due to their ability to navigate in abyssal zones without necessitating a tether that limits the range and maneuverability of the vehicle. However, their autonomy property directly affects the design of the control system. This requires advanced controllers and specific control schemes for given tasks.

Almost all AUVs are six degrees-of-freedom (DOF) systems, and various types of actuator configurations are available in the industry for the vehicles ranging from fully-actuated vehicles to underactuated ones. The vehicle of interest here falls into the class of underactuated AUVs. Any mechanical system having fewer actuators than its degrees of freedom is defined as an underactuated system. Some examples of underactuated systems include manipulators; (Arai et al., 1998), (Oriolo & Nakamura, 1991), (Yabuno et al., 2003), marine vehicles; (Reyhanoglu, 1997), (Pettersen & Egeland, 1996), space robots; (Tsiotras & Luo, 1997), and the examples given in (Fantoni & Lozano, 2002).

Controlling all of the DOF of underactuated mechanical systems is an arduous task compared to the fully actuated systems since the mathematical analysis of the system renders it difficult. Determining whether an underactuated system is controllable is one of these difficulties encountered. Control synthesis is also another challenge in this field and is still accepted as an open problem. The techniques used for fully actuated systems cannot be used directly for underactuated systems. However, there are some potential benefits over fully actuated systems depending on the efficiency of control and the task. In case of actuator failures, a fully actuated mechanical system falls into the class of underactuated systems and might still be controlled if a successful control scheme can be designed. Besides that, reduction of the weight and cost, and the increase of reliability can be considered as

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advantages of underactuated systems. On the other hand, underactuation may take place by design as in helicopters, ships, underwater vehicles, satellites, hovercrafts, etc.

Control problem of underactuated systems has been generally studied as a control problem of a class of nonholonomic systems, although the relation between underactuated systems and nonholonomic systems has not been clear yet (Kolmanovsky & McClamroch, 1995). Nonholonomic systems are known as the mechanical systems of nonholonomic constraints which cannot be integrated to obtain the equations describing the position of the system. Control of nonholonomic systems poses a difficult problem requiring a special control approach depending on the nature of the mechanical system, and the modelling of nonholonomic systems as state equations is another difficulty (Sampei et al., 1999).

In spite of having six effective DOF, the vehicle has one controllable DOF since it has just one actuator (the propeller). The vehicle does not have any other control element save for the thrust provided by the propeller. The propeller produces the main thrust. Consequently, the reaction of the body to the load torque of the propeller produces a moment with respect to its rotational axis. Thus, the vehicle is considered underactuated because it has fewer actuators than the degrees of freedom of the system. The vehicle is a nonlinear system: all equations of motion of the system include coupled terms. Some equations of the motion of the system appear as second-order nonholonomic constraints, and they cannot be integrated to obtain position. Therefore, such underwater vehicle is pertained as a nonholonomic system.

In control of underactuated autonomous underwater vehicles (UUVs), optimal control approach has not been widely applied. Jeon et al. (Jeon et al., 2003) proposed an optimal linear quadratic controller for a 6-DOF underwater vehicle with four thrusters to distribute the thrust optimally. Additionally, some motion planning approaches were discussed in (Bøerhaug et al., 2006) and (Bullo & Lynch, 2001).

Fukushima (Fukushima, 2006) proposes a novel control method of solving optimal control problems including nonlinear systems. In this study, this control method is used. His method proposes a wide range of applicability and simplicity. It can be applied to both linear systems and nonlinear systems including the systems to be controlled in real-time. Although this method has similarities with the classical optimal control theory, it can be seen as a radical contribution in the control engineering field, rather than the extension of the existing optimal control theory.

Fukushima's method is fundamentally based on the employment of the energy generation, storage and dissipation of the controlled system. The total system energy stored in the system boundary is the sum of each energy. The criteria function consists of the controlperformance, which is determined for a given task, the input energy, and the energy equation. First derivatives of the energy equation and the performance measures constitute a scalar function. The minimization of the scalar function yields the optimal control law. The necessary condition for the minimization is the Euler equation. The use of energy equation in the criteria function enables the optimal control law to have efficient dissipation characteristics. Obtaining the control law is a simple process and is not much mathematically involved. As one of the important properties of his control method, the control-performance can be of any form. There is no restriction in determining it, whereas the classical optimal control theory works well with performance measures of quadratic form.

20

#### 2. Mathematical modelling of the UUV

The mathematical model of the UUV is derived in this section. The kinematics and dynamics of the UUV are studied assuming it as a rigid body. The kinematic equations, dynamic equations for the rigid body, and fluid-body dynamics are needed in the discussion of the UUV's motion. Combining those equations, the nonlinear model of the UUV as 6-DOF equations of motion is obtained. In this study, derivation of the mathematical model is not discussed in full detail. The reader is referred to (Meirovitch, 1970) and (Fossen, 1994) for a detailed treatment.

#### 2.1 Description of the system

To obtain the mathematical model, the specifications and the assumptions which enable the formulation of kinematic and dynamic equations are needed. The simple model of the UUV holds the essential dynamical properties. A hull having a scalene ellipsoidal shape is shown in Fig.1. As illustrated in Fig.1, the UUV has a flat type of hull. For simulations, the UUV is assumed as neutrally buoyant and completely rigid. The fluid, which the vehicle interacts with, is considered as an ideal fluid (unbounded, irrotational, inviscid, and incompressible) and is chosen as sea water in simulations. Additionally, added mass related contributions in the equations of motion are neglected.

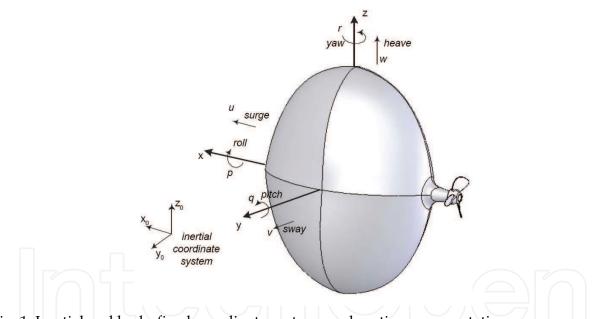


Fig. 1. Inertial and body-fixed coordinate systems and motion representations

#### 2.2 Nonlinear model representation of the dynamic equations of motion

The motion of the UUV in space needs to be defined with respect to some certain coordinate frames. One of the coordinate frames can be chosen to be fixed to the vehicle and is called the body-fixed reference frame. The advantage of defining the motion of the UUV in terms of the linear and angular motion components about the orthogonal body axes leads to define the body-fixed frame, *xyz*. The body-fixed frame is chosen so as to coincide with the center of buoyancy (CB), which is the volumetric center of the fluid displaced. It implies that the CB vector is zero vector,  $r_b = [x_b, y_b, z_b]^T = [0, 0, 0]^T$ . Another orthogonal coordinate system is defined to describe the motion of the moving body-fixed frame relative to an inertial frame.

The earth-fixed reference frame,  $x_0y_0z_0$ , is assumed as fixed in Earth and accepted as inertial. These right-handed frames are shown in Fig.1.

The rotation of a rigid-body can be represented in many ways. The well-known and mostly used representation is Euler angles. This representation is practical, popular and has intuitive physical meaning. However, Euler angle parameterization causes some singularities (Ang & Tourassis, 1987) and inaccuracies in calculations, e.g. discontinuous changes may occur in the attitude when the rotation is changed incrementally. A singularity-free and well-suited quaternion parameterization is preferred for accurate calculations.

The transformation between the body-fixed frame and the earth-fixed frame is given by (Fossen, 1994):

$$\dot{\eta}_{\rm E} = {\rm E}(\eta_{\rm E}) v , \qquad (1)$$

where *E* is the transformation matrix and  $\eta_E = [x, y, z, \varepsilon_1, \varepsilon_2, \varepsilon_3, \eta]^T$ . Here,  $e = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \eta]$  is a unit quaternion vector. The unit quaternion vector represents the rotation with respect to an axis of rotation,  $\varepsilon = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \eta]^T$ , and a rotation of angle,  $\eta$ , about that axis. The linear and angular velocities of the vehicle are described as  $v = [u, v, w, p, q, r]^T$ .

The dynamic equations of motion for the UUV can be written as follows (Fossen, 1994):

$$M\dot{v} + C(v)v + D(v)v = \tau , \qquad (2)$$

where  $\dot{v}$  is the time derivative of the velocity vector, and  $\tau$  is propulsion forces and moments vector. The simplified inertia matrix, *M*, is written as

$$M = diag\left\{m, m, m, I_X, I_y, I_z\right\},$$
(3)

where *m* is the mass of the UUV, and  $I_x$ ,  $I_y$ , and  $I_z$  are the moments of inertia. Coriolis and centripetal matrix, C(v), is given as

$$C(v) = \begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & mw & -mv & 0 & I_{z}r & -I_{y}q \\ -mw & 0 & mu & -I_{z}r & 0 & I_{x}p \\ mv & -mu & 0 & I_{y}q & -I_{x}p & 0 \end{bmatrix}.$$
 (4)

D(v) is the damping matrix which includes only terms of quadratic drag:

$$D(v) = -diag \left\{ X_{u|u|} |u|, Y_{v|v|} |v|, Z_{w|w|} |w|, K_{p|p|} |p|, M_{q|q|} |q|, N_{r|r|} |r| \right\},$$
(5)

where,  $-X_{uu}$  is the coefficient of the drag which the vehicle experience due to the motion along the x-axis, and  $-N_{rr}$  is the coefficient of the hydrodynamic torque due to the rotational motion of the vehicle with respect to the z-axis. Other coefficients can be described similarly.

The effect of environmental disturbances such as waves and ocean currents are neglected in this study. As a result, following six differential equations describing the 6-DOF equations of motion in surge, sway, heave, roll, pitch, and yaw, respectively, are obtained:

$$m(\dot{u} + qw - rv) - X_{uu}u|u| = F_{X}$$

$$m(\dot{v} + ru - pw) - Y_{vv}v|v| = 0$$

$$m(\dot{w} + pv - qu) - Z_{ww}w|w| = 0$$

$$I_{X}\dot{p} + (I_{Z} - I_{Y})qr - K_{pp}p|p| = k_{1}F_{X}$$

$$I_{Y}\dot{q} + (I_{X} - I_{z})rp - M_{qq}q|q| = 0$$

$$I_{X}\dot{p} + (I_{Z} - I_{Y})qr - K_{pp}p|p| = k_{1}F_{X}.$$
(6)

The main thrust is denoted by  $F_{x}$ , thus, the propulsion forces and moments vector can be written as  $\tau = [F_{x}, 0, 0, k_1 F_{x}, 0, 0]^T$ , where  $k_1$  is a coefficient relating the ratio of the thrust to the rolling moment of the body. The load torque of the propeller causes the body to react with an equal torque and to rotate in opposite direction. The reaction torque is another control input to the system, but, obviously, it is dependent on the thrust. Since the mechanical model does not have control fins, (6) does not include any input for control surface forces and moments. In (6), except for the equations in surge and roll, four equations can be identified as second-order nonholonomic constraints which expose the non-integrable velocity relationships. These constraints imply that possible displacements of the body in each direction are not independent, but are mutually connected.

#### 3. Control method

Fukushima proposed a control method of solving optimal control problems including nonlinear systems. Although, in Fukushima's method, there are similarities with the classical optimal control theory, it introduces a new optimal control approach which is fundamentally based on the employment of the energy generation, storage and dissipation of the controlled system. The total system energy stored in the system boundary is the sum of each energy. The rate of change of the instantaneous energy yields the net power flow of the dynamical system. Hence, the general power balance equation for a controlled system can be represented as follows:

$$P = u^{T} \dot{q} + v^{T} \dot{q} - \ddot{q}^{T} M(q) \dot{q} - d^{T} (q, \dot{q}) \dot{q} - e^{T} (q, z) \dot{q} = 0,$$
(7)

where *u* is the input force vector, *v* is the input disturbance force vector,  $M(q) \in \mathbb{R}^{n^{x_n}}$  is the symmetric positive-definite inertia matrix with *n*, the number of the DOF of the controlled system, *d* is combination of the damping force vector, and Coriolis and centrifugal force

vector, e is the potential force vector, z is the input disturbance displacement vector, and q is the generalized coordinates vector. The power equation of the system has dynamic characteristics of the controlled system. It is unique to the system and has an important role in the design of the optimal control system.

In optimal control theory, it is aimed to obtain a control law which satisfies the given constraints and extremizes the performance measure. A performance measure is mostly a combination of some scalar functions. The functional below is the performance measure used in Fukushima's control method:

$$J = \int (g(q, \dot{q}, \ddot{q}) + r'u^{T}\dot{q})dt, \qquad (8)$$

where *g* is the performance criterion (control performance) to be selected for the given control problem. Combination of these two terms is known as the performance measure for a general optimal control system. In (8), *g* might represent the linear combination of more than one performance criterion. The use of multiple performance criteria depends on the selection of performance measures for the control objective. In (8), the integrand of the last term represents the power delivered to actuators; here  $u^T$  is the transpose of the input force vector, and *r*' is a weighting factor.

According to the fundamental theorem of the calculus of variations (Elsgolc, 1961), the necessary condition for minimizing the performance measure is that the first variation of the functional must be zero. In Fukushima's method, the scalar function, *L*, is composed of the power equation (7) and the differential of the performance measure (8):

$$L = \kappa (u^{T} \dot{q} + v^{T} \dot{q} - \ddot{q}^{T} M(q) \dot{q} - d^{T} (q, \dot{q}) \dot{q} - e^{T} (q, z) \dot{q}) + g(q, \dot{q}, \ddot{q}) + r' u^{T} \dot{q}, \qquad (9)$$

where  $\kappa$  is an undecided constant. The included power balance equation is zero because it satisfies the energy conservation law. As the performance measure is minimized by means of the calculus of variations, *L* can be also minimized. The Euler equation is a necessary condition for minimization. After applying the Euler equation, the optimal control law is obtained.

#### 4. Optimal control of the UUV

In this section, application of the introduced control method is discussed. Before formulating the optimal control problem, the constraints, performance criterion and the criteria function is given. The application of the method to the control problem yields the optimal control law.

#### 4.1 Constraints

In this section, the constraints on the state and control values are defined. Let  $t_0$  is the initial time and  $t_f$  is the final time, then, the state constraints are given as:

$$\begin{bmatrix} x_0(t_0) \\ y_0(t_0) \\ z_0(t_0) \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}; \qquad \begin{bmatrix} x_0(t_f) \\ y_0(t_f) \\ z_0(t_f) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (10)$$

24

where  $x_i$ ,  $y_i$ , and  $z_i$  are initial positions. There are no constraints imposed on attitude and velocities of the UUV. Control constraints are imposed on the physical systems to be controlled due to the limitations of actuators. Force or torque inputs are bounded by some upper limit. In control of the UUV, it is assumed that there is no constraint on control, since the aim of this study is to show the possibility of control of such a challenging mechanical system. However, the simulations are also done for considering the constraints on the thrust:

 $-T_{c} \leq F_{x} \leq T_{c}$ 

where  $T_c$  is the thrust value.

#### 4.2 Performance criterion

As a performance criterion, the minimum distance between two points has chosen to transfer the system from a point to another. The optimal control will try to minimize this measure. For this performance, only position of the UUV is of interest. When missions of underwater vehicles are considered, mostly, the positioning of the underwater vehicle is carried out. Thus, the performance criterion describing the minimum distance between two points in 3-D space can be written as a terminal-error function:

$$J_{c} = (x_{f}(t) - x_{d})^{2} + (y_{f}(t) - y_{d})^{2} + (z_{f}(t) - z_{d})^{2}.$$
(12)

 $J_c$  represents the deviation of the actual path of the system from its desired path. The components of the final position vector,  $x_d$ ,  $y_d$ , and  $z_d$ , are chosen as constant. For the calculation of the functions,  $x_f(t)$ ,  $y_f(t)$ , and  $z_f(t)$ , which give the actual position of the vehicle, the relation (1) is used.

#### 4.3 Criteria function

As was explained above, the criteria function consists of the control-performance (performance criterion), the performance measure describing the control effort, and the energy equation of the controlled system is given by

$$J = J_{c} + \int r' u^{T} v dt + \int P dt, \qquad (13)$$

where the second term on the right-hand side of the equation is the power delivered to the actuator, and r' is the weighting factor which is determined as equal to the number of actuators. This control effort term can be denoted as  $J_e$  and written as:

$$J_{\rho} = r' \int F_{\chi} u dt \tag{14}$$

The last term in (13) is the energy equation. The total power equation, *P*, is the sum of all power flows in the system:

$$P = P_{in} + P_{generated} - (P_{dissipated} + P_{stored} + P_{out}) = 0.$$
(15)

Since each equation corresponding to each DOF in the equations of motion, (6), represents a force or moment equality, each equation can be also formulated so as to represent a power

(11)

equality. If each term of each equation in (6) is multiplied by the corresponding velocity, the power equalities are obtained. The sum of all equations yields the total power equation:

$$F_{x} (u + k_{1} p) - m(\dot{u}u + \dot{v}v + \dot{w}w) - I_{x} \dot{p}p - I_{y} \dot{q}q - I_{z} \dot{r}r + X_{uu}u^{2} |u| + Y_{vv}v^{2} |v| + Z_{ww}w^{2} |w| + K_{pp}p^{2} |p| + M_{qq}q^{2} |q| + N_{rr}r^{2} |r| = 0$$
(16)

In the total power equation, the remaining terms after summation are the input, stored, and dissipated power terms. Among these terms, the dissipation terms are of great importance since they appear in the control law and have contribution on the stability of the system.

#### 4.4 Optimal control problem

Consider finding an admissible control  $u_{opt}$  which causes the system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \tau) \,, \tag{17}$$

to follow an admissible trajectory,  $x^*$ , that minimizes the criteria function

$$J = J_{c} + J_{\rho} + \int \kappa P dt , \qquad (18)$$

so that  $u_{opt}$  is called the optimal control. In (18), the term  $\kappa$  is a weighting factor which is determined in search of the minimum *J*.

Let criteria function be formulated as follows:

$$J = \int (r'F_{\chi}u + P + \frac{dJ_{c}}{dt})dt,$$

$$J = \int L dt$$
(19)

where the scalar function *L* is written as

$$L = 2(x_{f} \dot{x}_{f} + y_{f} \dot{y}_{f} + z_{f} \dot{z}_{f}) + F_{X}r'u + \kappa P.$$
(20)

In the calculus of variations, it is well known that a necessary condition for  $x^*$  to be an optimal of the functional given by (19) is (Naidu, 2003)

$$\frac{\partial L}{\partial x} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) = 0.$$
(21)

This equation is called Euler equation. Application of Euler equation to (19) yields the optimal control law:

$$u_{opt} = \rho_1 \frac{3\kappa X_{uu} u | u |}{\kappa + r'} + \rho_2 \frac{3K_{pp} p | p |}{k_1} - \rho_1 \frac{2T_e}{\kappa + r'}$$
(22)

with

$$T_e = x_f (1 - 2(\varepsilon_2^2 + \varepsilon_3^2)) + y_f (\varepsilon_1 \varepsilon_2 + \varepsilon_3 \eta) + z_f (\varepsilon_1 \varepsilon_3 - \varepsilon_2 \eta), \qquad (23)$$

where  $\rho_1$  and  $\rho_2$  are weighting factors. These weighting factors and  $\kappa$  are computed by carrying out a multivariable optimization for the minimization of the terminal error function (12). Choice of the best control history for different values of these weighting factors leads to the determination of their values. When control constraint is not imposed on the system, it is obtained more than one admissible control history for different values of weighting factors. If control constraint is imposed on the system, there might still be admissible control histories.

In the optimal control law, (22), two points are of importance: the dissipation terms and the control-performance related terms. The terms coming from the power equation are the dissipation terms:  $3X_{uu}u | u |$  and  $3K_{pp}p | p |$ . There is not any other term reflecting system dynamics in the control law. The input to the UUV is effective on its two DOF. The surge and roll motions are controlled by the thrust. Therefore, the input related dissipation terms appear in (22). The second important term is  $T_e$  and it is dominantly effective than dissipation terms in positioning the vehicle.

#### 5. Simulation results

Using the derived control law (22), numerical simulations of the described ellipsoidal UUV were performed for two cases via *Matlab/Simulink*<sup>TM</sup> software tool. In the first case, the control input is not constrained, whereas the second case shows the simulations with the constrained input. In the second case, the initial position has been chosen different than the first case. For both cases, initial velocities of the underwater vehicle are given as:  $[u(0),v(0),w(0),p(0),q(0),r(0)]^T = [0,0,0,0,0,1]^T$ . The values of the parameters used in the simulations are given in Table 1.

	Parameter	Value	Unit	Description	
	а	0.2	т	Equatorial radius(X)	
	b	0.15	т	Equatorial radius(Y)	
	С	0.25	т	Polar radius(Z)	
	т	32	kg	Vehicle mass	
	ρ	1030	kg/m³	Seawater density	
	$I_x$	0.55	kgm <sup>2</sup>	Moment of inertia	
	$I_y$	0.66	kgm <sup>2</sup>	Moment of inertia	
	$I_z$	0.4	kgm <sup>2</sup>	Moment of inertia	
	$\rho_1$	- 1.15	n/a	Weighting factor	$\smile$ 7
	$\rho_2$	- 0.007	n/a	Weighting factor	
	к	0.0065	n/a	Weighting factor	
	$k_1$	0.025	т	Weighting factor	
	r'	1	n/a	Weighting factor	

Table 1. Simulation parameters

#### 5.1 Case-1

In this case, the initial position and attitude of the vehicle with respect to the earth-fixed frame are  $[x_0(0), y_0(0), z_0(0), \varepsilon_1(0), \varepsilon_2(0), \varepsilon_3(0), \eta(0)]^T = [50, 50, 50, 0, 0, 0, 1]^T$  at time  $t_0=0$ . The target point is the origin,  $[x_d, y_d, z_d]^T = [0, 0, 0]^T$ . There is no constraint on the control input.

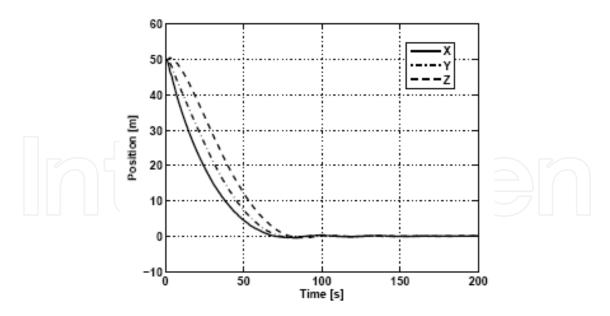


Fig. 1. Time evolution of the position of the UUV

As shown in Fig.1, the UUV achieved to reach the target point at about t=80. After reaching the target, the vehicle could keep its position very close to the origin as the variation of linear and angular velocities indicate in Fig.3. The control law, Fig.2, has oscillating and damping characteristics, and the frequency of the control signal is observed as f=0.5Hz at most.

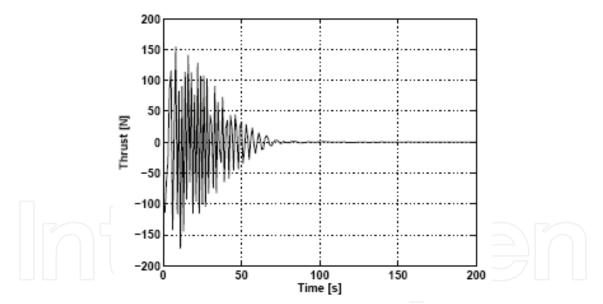


Fig. 2. Time evolution of the applied input

#### 5.2 Case-2

In this case, the initial position and attitude of the vehicle with respect to the earth-fixed frame are  $[x_0(0), y_0(0), z_0(0), \varepsilon_1(0), \varepsilon_2(0), \varepsilon_3(0), \eta(0)]^T = [-40, 25, -15, 0, 0, 0, 1]^T$  at time  $t_0=0$ . The target point is the origin,  $[x_d, y_d, z_d]^T = [0, 0, 0]^T$ . The following thrust constraint (in Newtons) has been imposed on the control input:

$$-25 \le F_X \le 25$$
 ,

28

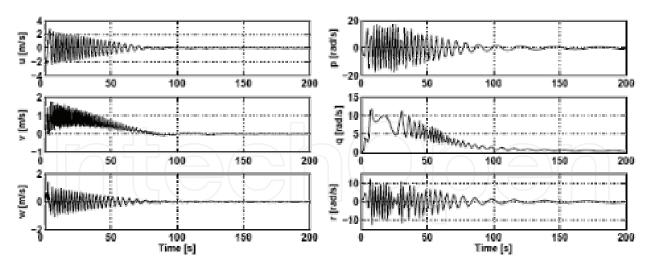


Fig. 3. Time evolution of the linear and angular velocities

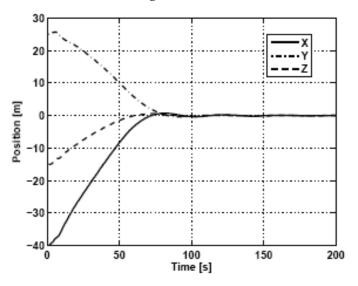


Fig. 4. Time evolution of the position of the UUV

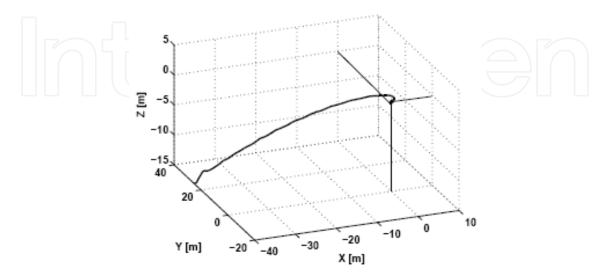


Fig. 5. 3-D flight path of the UUV

In this case, the only difference between applying an unconstrained input and constrained input is that the operation time is longer in the latter case. The constrained control, Fig.6, has a similar characteristic with that of the previous case.

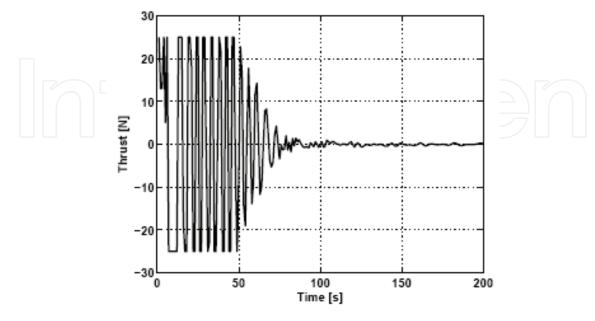


Fig. 6. Time evolution of the applied input

#### 6. Conclusion

In this study, position control of the UUV as the application of a novel optimal control was discussed. The dynamics and control of the 6-DOF UUV are presented and the new optimal control method was introduced. To the best knowledge of the authors, there is no 6-DOF underwater vehicle that can be controlled with one actuator in literature. Based on this control approach, the criteria function determined included a terminal error function. The terminal error function plays an important role in control of the UUV. After the minimization of the criteria function, the terms coming from the terminal error function appeared in the control law. After simulations, it was observed that these terms are crucial in the maneuver of the vehicle. It is understood that they dominantly, with respect to the other terms, determine the value and direction of the force applied to the UUV.

It has been proved that, the control of the UUV necessitates the initial conditions as either pitch velocity or yaw velocity at time  $t_0=0$ . The roll velocity is produced by the rolling moment. Gyroscopic moments and centrifugal forces occur when the pitch or yaw velocity has an initial value. Existence of these forces and moments make the control possible. Even though the use of only one actuator seems to be a big challenge for a 6-DOF underwater vehicle, the UUV has succeeded to reach the target point by application of the energy-based control method.

At present, attitude control of the UUV has not been achieved. Research for controlling the attitude variables is ongoing. Even the vicinity of the target state is attained, the vehicle may still have small velocities. The body needs forces in each direction for a necessary maneuver. Since the nonholonomic constraints do not allow every possible motion, motion to minimize the position errors needs other necessary forces to be produced by only  $F_x$ . Therefore, the

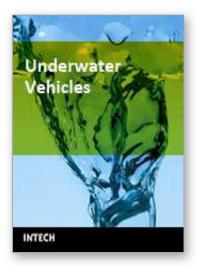
body moves undesired directions recurrently. If attitude control would be achieved, this hovering motion is thought to be prevented.

#### 7. References

- Ang, J. M. H. & Tourassis, V. D. (1987). Singularities of euler and roll-pitch-yaw representations, IEEE Transactions on Aerospace and Electronic Systems, vol. AES-23, no. 3, pp. 317–324
- Arai, H.; Tanie, K. & Shiroma, N. (1998). Nonholonomic control of a three-DOF planar underactuated manipulator, IEEE Transactions on Robotics and Automation, vol. 14, no. 5, pp. 681–695
- Bøerhaug, E.; Pettersen, K.Y. & Pavlov, A. (2006). An optimal guidance scheme for crosstrack control of underactuated underwater vehicles, 14th Mediterranean Conference on Control and Automation, pp.1-5, June 2006
- Bullo F. & Lynch, K.M. (2001). Kinematic controllability for decoupled trajectory planning in underactuated mechanical systems, *IEEE Transactions on Robotics and Automation*, vol.17, no.4, pp.402-412
- Elsgolc, L. E. (1961). Calculus of Variations, Pergamon Press
- Fantoni I. & Lozano, R. (2002). Non-lienar Control for Underactuated Mechanical Systems, Springer-Verlag, London
- Fossen, T. I. (1994). Guidance and Control of Ocean Vehicles. John Wiley & Sons, New York
- Fukushima, N. (2006). Optimal control of mechanical system based on energy equation, *Trans. of JSME(C)*, vol. 72, no. 722, pp.3106–3114
- Jeon, B.-H.; Lee, P.-M.; Li, J.-H.; Hong, S-W; Kim, Y.-G. & Lee J. (2003). Multivariable optimal control of an autonomous underwater vehicle for steering and diving control in variable speed, *Proceedings of OCEANS* 2003, vol.5, pp.2659-2664
- Kolmanovsky, I. & McClamroch, N. H. (1995). Developments in nonholonomic control problems, *IEEE Control Systems*, vol. 15, pp. 20–36
- Meirovitch, L. (1970). Methods of Analytical Dynamics, McGraw-Hill
- Naidu, D. S. (2003). Optimal Control Systems, CRC Press, London
- Oriolo, G. & Nakamura, Y. (1991). Control of mechanical systems with second-order nonholonomic constraints: Underactuated manipulators, *Proceedings of IEEE Conf. Decision and Control*, pp. 2398-2403, Brighton, U.K.
- Pettersen K. & O. Egeland (1996). Position and attitude control of an underactuated autonomous underwater vehicle, *Proceedings of the 35th Conference on Decision and Control*, pp. 987–991, Kobe, Japan
- Reyhanoglu, M. (1997). Exponential stabilization of an underactuated autonomous surface vessel, *Automatica*, vol. 33, no. 12, p. 2249-2254
- Sampei, M.; Kiyota, H. & Ishikawa, M. (1999). Control strategies for mechanical systems with various constraints-control of non-holonomic systems, *IEEE International Conference on Systems, Man, and Cybernetics*, vol. 3, pp. 158–165
- Tsiotras, P. & Luo, J. (1997). Reduced-effort control laws for underactuated rigid spacecraft, *Journal of Guidance, Control, and Dynamics,* vol. 20, pp. 1089–1095
- UNECE/IFR (2005). World robotics survey 2005, press release, ECE/STAT/05/P03, Geneva, 1 October 2005.

Yabuno, H.; Matsuda, T. & Aoshima, N. (2003). Motion control of an underactuated manipulator without feedback control, *Proceedings of 2003 IEEE Conference on Control Applications*, vol. 1, pp. 700–705





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For the latest twenty to thirty years, a significant number of AUVs has been created for the solving of wide spectrum of scientific and applied tasks of ocean development and research. For the short time period the AUVs have shown the efficiency at performance of complex search and inspection works and opened a number of new important applications. Initially the information about AUVs had mainly review-advertising character but now more attention is paid to practical achievements, problems and systems technologies. AUVs are losing their prototype status and have become a fully operational, reliable and effective tool and modern multi-purpose AUVs represent the new class of underwater robotic objects with inherent tasks and practical applications, particular features of technology, systems structure and functional properties.

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