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# Applying Fuzzy Linear Programming to Supply Chain Planning with Demand, Process and Supply Uncertainty

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## 1. Introduction

A Supply Chain (SC) is a dynamic network of several business entities that involve a high degree of imprecision. This is mainly due to its real-world character where uncertainties in the activities extending from the suppliers to the customers make SC imprecise (Fazel Zarandi et al., 2002).

Several authors have analysed the sources of uncertainty present in a SC, readers are referred to Peidro et al. (2008) for a review. The majority of the authors studied (Childerhouse & Towill, 2002; Davis, 1993; Ho et al., 2005; Lee & Billington, 1993; Mason-Jones & Towill, 1998; Wang & Shu, 2005), classified the sources of uncertainty into three groups: demand, process/manufacturing and supply. Uncertainty in supply is caused by the variability brought about by how the supplier operates because of the faults or delays in the supplier's deliveries. Uncertainty in the process is a result of the poorly reliable production process due to, for example, machine hold-ups. Finally, demand uncertainty, according to Davis (Davis, 1993), is the most important of the three, and is presented as a volatility demand or as inexact forecasting demands.

The coordination and integration of key business activities undertaken by an enterprise, from the procurement of raw materials to the distribution of the end products to the customer, are concerned with the SC planning process (Gupta & Maranas, 2003), one of the most important processes within the SC management concept. However, the complex nature and dynamics of the relationships among the different actors imply an important degree of uncertainty in the planning decisions. In SC planning decision processes, uncertainty is a main factor that can influence the effectiveness of the configuration and coordination of supply chains (Davis, 1993; Jung et al., 2004; Minegishi & Thiel, 2000) and tends to propagate up and down along the SC, affecting its performance appreciably (Bhatnagar & Sohal, 2005).

Most of the SC planning research (Alonso-Ayuso et al., 2003; Guillen et al., 2005; Gupta y Maranas, 2003; Lababidi et al., 2004; Santoso et al., 2005; Sodhi, 2005) models SC uncertainties with probability distributions that are usually predicted from historical data. However, whenever statistical data are unreliable or are even not available, stochastic models may not be the best choice (Wang y Shu, 2005). The fuzzy set theory (Zadeh, 1965)

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and the possibility theory (Dubois & Prade, 1988; Zadeh, 1978) may provide an alternative simpler and less-data demanding than probability theory to deal with SC uncertainties (Dubois et al., 2003).

Few studies address the SC planning problem on a medium-term basis (tactical level) which integrate procurement, production and distribution planning activities in a fuzzy environment (see Section 2. Literature review). Moreover, models contemplating the different sources of uncertainty in an integrated manner are lacking. Hence in this study, we develop a tactical supply chain model in a fuzzy environment in a multi-echelon, multi-product, multi-level, multi-period supply chain network. In this proposed model, the demand, process and supply uncertainties are contemplated simultaneously.

In the context of fuzzy mathematical programming, two very different issues can be addressed: fuzzy or flexible constraints for fuzziness, and fuzzy coefficients for lack of knowledge or epistemic uncertainty (Dubois et al., 2003). Our proposal jointly considers the possible lack of knowledge in data and existing fuzziness.

The main contributions of this paper can be summarized as follows:

- Introducing a novel tactical SC planning model by integrating procurement, production and distribution planning activities into a multi-echelon, multi-product, multi-level and multi-period SC network.
- Achieving a model which contemplates the different sources of uncertainty affecting SCs in an integrated fashion by jointly considering the possible lack of knowledge in data and existing fuzziness.
- Applying the model to a real-world automobile SC dedicated to the supply of automobile seats.

The rest of this paper is arranged as follows. Section 2 presents a literature review about fuzzy applications in SC planning. Section 3 proposes a new fuzzy mixed-integer linear programming (FMILP) model for the tactical SC planning under uncertainty. Then in Section 4, appropriate strategies for converting the fuzzy model into an equivalent auxiliary crisp mixed-integer linear programming model are applied. In Section 5, the behaviour of the model in a real-world automobile SC has been evaluated and, finally, the conclusions and directions for further research are provided.

## 2. Literature review

In Peidro et al. (2008) a literature survey on SC planning under uncertainty conditions by adopting quantitative approaches is developed. Here, we present a summary, extracted from this paper, about the applications of fuzzy set theory and the possibility theory to different problems related to SC planning:

SC inventory management: Petrovic et al. (1998; 1999) describe the fuzzy modelling and simulation of a SC in an uncertain environment. Their objective was to determine the stock levels and order quantities for each inventory during a finite time horizon to achieve an acceptable delivery performance at a reasonable total cost for the whole SC. Petrovic (2001) develops a simulation tool, SCSIM, for analyzing SC behaviour and performance in the presence of uncertainty modelled by fuzzy sets. Giannoccaro et al. (2003) develop a methodology to define inventory management policies in a SC, which was based on the echelon stock concept (Clark & Scarf, 1960) and the fuzzy set theory was used to model uncertainty associated with both demand and inventory costs. Carlsson and Fuller (2002) propose a fuzzy logic approach to reduce the bullwhip effect. Wang and Shu (2005) develop

a decentralized decision model based on a genetic algorithm which minimizes the inventory costs of a SC subject to the constraint to be met with a specific task involving the delivery of finished goods. The authors used the fuzzy set theory to represent the uncertainty of customer demands, processing times and reliable deliveries. Xie et al. (2006) present a new bilevel coordination strategy to control and manage inventories in serial supply chains with demand uncertainty. Firstly, the problem associated with the whole SC was divided into subproblems in accordance with the different parts that the SC it was made up of. Secondly, for the purpose of improving the integrated operation of a whole SC, the leader level was defined to be in charge of coordinating inventory control and management by amending the optimisation subproblems. This process was to be repeated until the desired level of operation for the whole SC was reached.

Vendor Selection: Kumar et al. (2004) present a fuzzy goal programming approach which was applied to the problem of selecting vendors in a SC. This problem was posed as a mixed integer and fuzzy goal programming problem with three basic objectives to minimize: the net cost of the vendors network, rejects within the network, and delays in deliveries. With this approach, the authors used triangular membership functions for each fuzzy objective. The solution method was based on the intersection of membership functions of the fuzzy objectives by applying the min-operator. Then, Kumar et al. (2006a) solve the same problem using the multi-objective fuzzy programming approach proposed by (Zimmermann, 1978). Amid et al. (2006) address the problem of adequately selecting suppliers within a SC. For this purpose, they devised a fuzzy-based multi-objective mathematical programming model where each objective may be assigned a different weight. The objectives considered were related to cost cuts, increased quality and to an increased service of the suppliers selected. The imprecise elements considered in this work were to meet both objectives and demand. Kumar et al. (2006b) analyze the uncertainty prevailing in integrated steel manufacturers in relation to the nature of the finished good and the significant demand by customers. They proposed a new hybrid evolutionary algorithm named endosymbioticpsychoclonal (ESPC) to decide what and how much to stock as an intermediate product in inventories. They compare ESPC with genetic algorithms and simulated annealing. They conclude the superiority of the proposed algorithm in terms of both the quality of the solution obtained and the convergence time.

Transport planning: Chanas et al. (1993) consider several assumptions on the supply and demand levels for a given transportation problem in accordance with the kind of information that the decision maker has: crisp values, interval values or fuzzy numbers. For each of these three cases, classical, interval and fuzzy models for the transportation problem are proposed, respectively. The links among them are provided, focusing on the case of the fuzzy transportation problem, for which solution methods are proposed and discussed. Shih (1999) addresses the problem of transporting cement in Taiwan by using fuzzy linear programming models. The author uses three approaches based on the works by Zimmermann (1976). Chanas (1983) and Julien (1994), who contemplate: the capacities of ports, the fulfilling demand, the capacities of the loading and unloading operations, and the constraints associated with traffic control. Liu and Kao (2004) develop a method to obtain the membership function of the total transport cost by considering this as a fuzzy objective value where the shipment costs, supply and demand are fuzzy numbers. The method was based on the extension principle defined by Zadeh (1978) to transform the fuzzy transport problem into a pair of mathematical programming models. Liang (2006) develops an

interactive multi-objective linear programming model for solving fuzzy multi-objective transportation problems with a piecewise linear membership function.

Production-distribution planning: Sakawa et al. (2001) address the real problem of production and transport related to a manufacturer through a deterministic mathematical programming model which minimizes costs in accordance with capacities and demands. Then, the authors develop a mathematical fuzzy programming model. Finally, they present an outline of the distribution of profits and costs based on the game theory. Liang (2007) proposes an interactive fuzzy multi-objective linear programming model for solving an integrated production-transportation planning problem in supply chains. Selim et al. (2007) propose fuzzy goal-based programming approaches applied to planning problems of a collaborative production-distribution type in centralized and decentralized supply chains. The fuzzy elements that the authors consider correspond to the fulfilment of different objectives related to maximizing profits for manufacturers and distribution centers, retailer cost cuts and minimizing delays in demand in retailers. Aliev et al. (2007) develop an integrated multi-period, multi-product fuzzy production and distribution aggregate planning model for supply chains by providing a sound trade-off between the fillrate of the fuzzy market demand and the profit. The model is formulated in terms of fuzzy programming and the solution is provided by genetic optimization.

Procurement-production-distribution planning: Chen and Chang (2006) develop an approach to derive the membership function of the fuzzy minimum total cost of the multi-product, multi-echelon, and multi-period SC model when the unit cost of raw materials supplied by suppliers, the unit transportation cost of products, and the demand quantity of products are fuzzy numbers. Recently, Tarabi and Hassini (2008) propose a new multi-objective possibilistic mixed integer linear programming model for integrating procurement, production and distribution planning by considering various conflicting objectives simultaneously along with the imprecise nature of some critical parameters such as market demands, cost/time coefficients and capacity levels. The proposed model and solution method are validated through numerical tests.

As mentioned before, models contemplating the different sources of uncertainty in an integrated manner are lacking and few studies address the SC planning problem on a medium-term basis which integrate procurement, production and distribution planning activities in a fuzzy environment. Moreover, the majority of the models studied are not applied in supply chains based on real world cases.

### 3. Problem description

This section outlines the tactical SC planning problem. The overall problem can be stated as follows:

Given:

- A SC topology: number of nodes and type (suppliers, manufacturing plants, warehouses, distribution centers, retailers, etc.)
- Each cost parameter, such as manufacturing, inventory, transportation, demand backlog, etc.
- Manufacture data, processing times, production capacity, overtime capacity, BOM, production run, etc.
- Transportation data, such as lead time, transport capacity, etc.
- Procurement data, procurement capacity, etc.

- Inventory data, such as inventory capacity, etc.
- Forecasted product demands over the entire planning periods.

To determine:

- The production plan of each manufacturing node.
- The distribution transportation plan between nodes.
- The procurement plan of each supplier node.
- The inventory level of each node.
- The sales and demand backlog.

The target is to centralize the multi-node decisions simultaneously in order to achieve the best utilization of the resources available in the SC throughout the time horizon so that customer demands are met at a minimum cost.

### 3.1 Fuzzy model formulation

The fuzzy mixed integer linear programming (FMILP) model for the tactical SC planning proposed by Peidro et al. (2007) is adopted as the basis of this work. Sets of indices, parameters and decision variables for the FMILP model are defined in the nomenclature (see Table 1). Table 2 shows the uncertain parameters grouped according to the uncertainty sources that may be presented in a SC.

Set of indices	
$T$ :	Set of planning periods ( $t = 1, 2 \dots T$ ).
$I$ :	Set of products (raw materials, intermediate products, finished goods) ( $i = 1, 2 \dots I$ ).
$N$ :	Set of SC nodes ( $n = 1, 2 \dots N$ ).
$J$ :	Set of production resources ( $j = 1, 2 \dots J$ ).
$L$ :	Set of transports ( $l = 1, 2 \dots L$ ).
$P$ :	Set of parent products in the bill of materials ( $p = 1, 2 \dots P$ ).
$O$ :	Set of origin nodes for transports ( $o = 1, 2 \dots O$ ).
$D$ :	Set of destination nodes for transports ( $d = 1, 2 \dots D$ ).
Objective function cost coefficients	
$V\tilde{P}C_{ijn}$ :	Variable production cost per unit of product $i$ on $j$ at $n$ in $t$ .
$O\tilde{T}C_{njt}$ :	Overtime cost of resource $j$ at $n$ in $t$ .
$U\tilde{T}C_{njt}$ :	Undertime cost of resource $j$ at $n$ in $t$ .
$RMC_{int}$ :	Price of raw material $i$ at $n$ in $t$ .
$T\tilde{C}_{odlt}$ :	Transport cost per unit from $o$ to $d$ by $l$ in $t$ .
$I\tilde{C}_{int}$ :	Inventory holding cost per unit of product $i$ at $n$ in $t$ .
$D\tilde{B}C_{int}$ :	Demand backlog cost per unit of product $i$ at $n$ in $t$ .
General Data	
$B_{pint}$ :	Quantity of $i$ to produce a unit of $p$ at $n$ in $t$ .
$MP\tilde{R}C_{nt}$ :	Maximum procurement capacity from supplier node $n$ in $t$ .
$\tilde{D}_{int}$ :	Demand of product $i$ at $n$ in $t$ .
$M\tilde{O}T_{njt}$ :	Overtime capacity of resource $j$ at $n$ in $t$ .
$M\tilde{P}C_{njt}$ :	Production capacity of resource $j$ at $n$ in $t$ .

$I0_{in}$ :	Inventory amount of $i$ at $n$ in period 0.
$PR_{injt}$ :	Production run of $i$ on $j$ at $n$ in $t$ .
$MPR_{injt}$ :	Minimum production run of $i$ on $j$ at $n$ in $t$ .
$DB0_{int}$ :	Demand backlog of $i$ at $n$ in period 0.
$SR0_{iodlt}$ :	Shipments of $i$ received at $d$ from $o$ by $l$ at the beginning of period 0.
$SIP0_{iodlt}$ :	Shipments in progress of $i$ from $o$ to $d$ by $l$ at the beginning of period 0.
$\tilde{P}T_{injt}$ :	Processing time to produce a unit of $i$ on $j$ at $n$ in $t$ .
$\tilde{T}LT_{odlt}$ :	Transport lead time from $o$ to $d$ by $l$ in $t$ .
$V_{it}$ :	Physical volume of product $i$ in $t$ .
$\tilde{M}TC_{nt}$ :	Maximum transport capacity of $l$ in $t$ .
$\tilde{M}IC_{nt}$ :	Maximum inventory capacity at $n$ in $t$ .
$\chi_{odlt}^1$ :	0-1 function. It takes 1 if $\tilde{T}LT_{odlt} > 0$ and 0 otherwise.
$\chi_{odlt}^2$ :	0-1 function. It takes 1 if $\tilde{T}LT_{odlt} = 0$ and 0 otherwise.
Decision Variables	
$P_{injt}$ :	Production amount of $i$ on $j$ at $n$ in $t$ / $\tilde{P}T_{injt} > 0$ .
$k_{injt}$ :	Number of production runs of $i$ produced on $j$ at $n$ in $t$ .
$S_{int}$ :	Supply of product $i$ from $n$ in $t$ .
$DB_{int}$ :	Demand backlog of $i$ at $n$ in $t$ / $\tilde{D}BC_{int} > 0$ .
$TQ_{iodlt}$ :	Transport quantity of $i$ from $o$ to $d$ by $l$ in $t$ / $o \neq d, \tilde{T}C_{odlt} > 0, \tilde{I}C_{i,n=d,t} > 0$ .
$SR_{iodlt}$ :	Shipments of $i$ received at $d$ from $o$ by $l$ at the beginning of period $t$ / $o \neq d, \tilde{T}C_{odlt} > 0, \tilde{I}C_{i,n=d,t} > 0$ .
$SIP_{iodlt}$ :	Shipments in progress of $i$ from $o$ to $d$ by $l$ at the beginning of period $t$ / $o \neq d, \tilde{T}C_{odlt} > 0, \tilde{I}C_{i,n=d,t} > 0, \tilde{T}LT_{odlt} > 0$ .
$\tilde{F}TLT_{iodlt}$ :	Transport lead time for $i$ from $o$ to $d$ by $l$ in $t$ (only used in the fuzzy model).
$I_{int}$ :	Inventory amount of $i$ at $n$ at the end of period $t$ .
$PQ_{int}$ :	Purchase quantity of $i$ at $n$ in $t$ / $\tilde{R}MC_{int} > 0$ .
$OT_{njt}$ :	Overtime for resource $j$ at $n$ in $t$ .
$UT_{njt}$ :	Undertime for resource $j$ at $n$ in $t$ .
$YP_{injt}$ :	Binary variable indicating whether a product $i$ has been produced on $j$ at $n$ in $t$ .

Table 1. Nomenclature (fuzzy parameters are shown with tilde: ~).

FMILP is formulated as follows:

Minimize  $z =$

$$\begin{aligned} & \sum_{i=1}^I \sum_{n=1}^N \sum_{j=1}^J \sum_{t=1}^T (\tilde{V}PC_{injt} \cdot P_{injt}) + \sum_{n=1}^N \sum_{j=1}^J \sum_{t=1}^T (\tilde{O}TC_{njt} \cdot OT_{njt} + \tilde{U}TC_{njt} \cdot UT_{njt}) + \\ & + \sum_{i=1}^I \sum_{n=1}^N \sum_{t=1}^T (\tilde{R}MC_{int} \cdot PQ_{int} + \tilde{I}C_{int} \cdot I_{int} + \tilde{D}BC_{int} \cdot DB_{int}) + \sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D \sum_{l=1}^L \sum_{t=1}^T (\tilde{T}C_{odlt} \cdot TQ_{iodlt}) \end{aligned} \quad (1)$$

Subject to

$$\sum_{i=1}^I (P_{injt} \cdot \tilde{P}T_{injt}) \lesssim \tilde{M}PC_{njt} + \tilde{M}OT_{njt} \quad \forall n, j, t \quad (2)$$

Source of uncertainty in supply chains	Fuzzy coefficient	Formulation
Demand	Product demand Demand backlog cost	$\tilde{D}_{int}$ $D\tilde{B}C_{int}$
Process	Processing time Production capacity Production costs Inventory holding cost Maximum inventory capacity	$P\tilde{T}_{injt}$ $M\tilde{P}C_{njt}, M\tilde{O}T_{njt}$ $V\tilde{P}C_{injt}, O\tilde{T}C_{njt}, U\tilde{T}C_{njt}$ $I\tilde{C}_{int}$ $M\tilde{I}C_{nt}$
Supply	Transport lead time Transport cost Maximum transport capacity Maximum procurement capacity	$T\tilde{L}T_{odlt}$ $T\tilde{C}_{odlt}$ $M\tilde{T}C_{nt}$ $M\tilde{P}\tilde{R}C_{nt}$

Table 2. Fuzzy parameters considered in the model.

$$P_{injt} = k_{injt} \cdot PR_{injt} \quad \forall i, n, j, t \tag{3}$$

$$P_{injt} \cdot PT_{injt} \lesssim M\tilde{P}C_{njt} \cdot YP_{injt} + M\tilde{O}T_{njt} \cdot YP_{injt} \quad \forall i, n, j, t \tag{4}$$

$$P_{injt} \geq MPR_{injt} \cdot YP_{injt} \quad \forall i, n, j, t \tag{5}$$

$$I_{int} = I_{in,t-1} + \sum_{j=1}^J P_{injt} + \sum_{o=1}^O \sum_{l=1}^L SR_{io,d=n,lt} + PQ_{int} - \sum_{d=1}^D \sum_{l=1}^L TQ_{i,o=n,dlt} - S_{int} - \sum_{p=1}^P (B_{pint} \cdot \sum_{j=1}^J P_{i=p,njt}) \quad \forall i, n, t \tag{6}$$

$$SR_{iodlt} = SR0_{iodlt} + TQ_{iodl,t-T\tilde{L}T} \quad \forall i, o, d, l, t \tag{7}$$

$$SIP_{iodlt} = SIP0_{iodlt} + SIP_{iodl,t-1} + TQ_{iodlt} - SR_{iodlt} \quad \forall i, o, d, l, t \tag{8}$$

$$\sum_{i=1}^I I_{int} \cdot V_{it} \lesssim M\tilde{I}C_{nt} \quad \forall n, t \tag{9}$$

$$\sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D SIP_{iodlt} \cdot V_{it} \cdot \chi^1_{odt} + \sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D TQ_{iodlt} \cdot V_{it} \cdot \chi^2_{odt} \lesssim M\tilde{T}C_{lt} \quad \forall l, t \tag{10}$$

$$\sum_{i=1}^I PQ_{int} \lesssim M\tilde{P}\tilde{R}C_{nt} \quad \forall n, t \tag{11}$$



$$DB_{int} \approx DB_{in,t-1} + \tilde{D}_{int} - S_{int} \quad \forall i, n, t \quad (12)$$

$$OT_{njt} \approx \sum_{i=1}^I P_{injt} \cdot P\tilde{T}_{injt} - M\tilde{P}C_{njt} + UT_{njt} \quad \forall n, j, t \quad (13)$$

$$\sum_{n=1}^N \sum_{t=1}^T S_{int} \lesssim \sum_{n=1}^N \sum_{t=1}^T (\tilde{D}_{int} + DB0_{int}) \quad \forall i \quad (14)$$

$$P_{injt}, k_{injt} \geq 0 \quad \forall i, n, j, t \quad (15)$$

$$S_{int}, DB_{int}, I_{int}, PQ_{int} \geq 0 \quad \forall i, n, t \quad (16)$$

$$SR_{iodlt}, SIP_{iodlt}, TQ_{iodlt} \geq 0 \quad \forall i, o, d, l, t \quad (17)$$

$$OT_{njt}, UT_{njt} \geq 0 \quad \forall n, j, t \quad (18)$$

Eq. (1) shows the total cost to be minimized. The total cost is formed by the production costs with the differentiation between regular and overtime production. The costs corresponding to idleness, raw material acquisition, inventory holding, demand backlog and transport are also considered. Most of these costs cannot be measured easily since they mainly imply human perception for their estimation. Therefore, these costs are considered uncertain data and are modelled by fuzzy numbers. Only the raw material cost is assumed to be known.

The production time per period could never be higher than the available regular time plus the available overtime for a certain production resource of a node (2). Symbol  $\lesssim$  represents the fuzzy version of  $\leq$  and means "essentially less than or similar to". This constraint shows that the planner wants to make the left-hand side of the constraint, the production time per period, smaller or similar to the right-hand side, the maximum production time available, "if possible". The production time and the production capacity are only known approximately and are represented by fuzzy numbers. On the other hand, the produced quantity of each product in every planning period must always be a multiple of the selected production lot size (3).

Eq. (4) and (5) guarantee a minimum production size for the different productive resources of the nodes in the different periods. These equations guarantee that  $P_{injt}$  will be equal to zero if  $YP_{injt}$  is zero.

Eq. (6) corresponds to the inventory balance. The inventory of a certain product in a node, at the end of the period, will be equal to the inputs minus the outputs of the product generated in this period. The inputs concern the production, transport receptions from other nodes, purchases (if supplying nodes) and the inventory of the previous period. The outputs are related to shipments to other nodes, supplies to customers and the consumption of other products (raw materials and intermediate products) that are necessary to produce in the node.

Eqs. (7) and (8) control the shipment of products among nodes. The receptions of shipments for a certain product will be equal to the programmed receptions plus the shipments carried out in previous periods. In constraint (7), the transport lead time are considered uncertainty data. On the other hand (8), the shipments in progress will be equal to the initial shipments

in progress plus those from the previous period, plus the new shipments initiated in this period minus the new receptions.

Both the transports and inventory levels are limited by the available volume (known approximately). Thus according to Eq. (9), the inventory level for the physical volume of each product must be lower than the available maximum volume for every period (considered uncertainty data). The inventory volume depends on the period to consider the possible increases and decreases of the storage capacity over time. Additionally, the physical volume of the product depends on the time to cope with the possible engineering changes that can occur and affect the dimensions and volume of the different products.

On the other hand, the shipment quantities in progress of each shipment in every period multiplied by the volume of the transported products (if the transport time is higher than 0 periods), plus the initiated shipments by each transport in every period multiplied by the volume of the transported products (if the transport time is equal to 0 periods), can never exceed the maximum transport volume for that period (10). The reason for using a different formulation in terms of the transport time among nodes ( $TLT_{odt}$ ) is because the transport in progress will never exist if this value is not higher than zero because all the transport initiated in a period is received in this same period if  $TLT_{odt} = 0$ . Finally, the transport volume depends on the period to consider the possible increases and decreases of the transport capacity over time.

Eq. (11) establishes an estimated maximum of purchase for each node and product per period. Eq. (12) contemplates the backlog demand management over time. The backlog demand for a product and node in a certain period will be equal (approximately) to the backlog demand of the previous period plus the difference between supply and demand.

Eq. (13) considers the use of overtime and undertime production for the different productive resources. The overtime production for a productive resource of a certain node in one period is equal (approximately) to the total production time minus the available regular production time plus the idle time.  $OT_{njt}$  and  $UT_{njt}$  will always be higher or equal to zero if the total production time is higher than the available regular production time,  $UT_{njt}$  will be zero as it does not incur in added costs, and  $OT_{njt}$  will be positive. On the contrary, if the total production time is lower than the available regular production time,  $UT_{njt}$  will be positive and  $OT_{njt}$  will be zero.

Conversely, Eq. (14) establishes that the sum of all the supplied products is essentially lower or equal to demand plus the initial backlog demand. At any rate, the problem could easily consider that all demand is served at the end of last planning period by transforming this inequality equation into an equality equation. Finally, Eqs. (15), (16), (17) and (18) guarantee the non negativity of the corresponding decision variables.

#### 4. Solution methodology

In this section, we define an approach to transform the fuzzy mixed-integer linear programming model (FMILP) into an equivalent auxiliary crisp mixed-integer linear programming model for tactical SC planning under supply, process and demand uncertainties. According to Table 2, and in order to address the fuzzy coefficients of the FMILP model, it is necessary to consider the fuzzy mathematical programming approaches that integrally consider the fuzzy coefficients of the objective function and the fuzzy constraints: technological and right-hand side coefficients. In this context, several research works exist in the literature, and readers are referred to them (Buckley, 1989; Cadenas &

Verdegay, 1997; Carlsson & Korhonen, 1986; Gen et al., 1992; Herrera & Verdegay, 1995; Jiménez et al., 2007; Lai & Hwang, 1992; Vasant, 2005). In this paper, we adopt the approach by Cadenas and Verdegay (1997; 2004). The authors propose a general model for fuzzy linear programming that considers fuzzy cost coefficients, fuzzy technological coefficients and fuzzy right-hand side terms in constraints. Fuzziness is also considered in the inequalities that define the constraints. This general fuzzy linear programming model is as follows:

$$\begin{aligned} \text{Max } z &= \sum_{j=1}^n \tilde{c}_j x_j \\ \text{s.t.} & \\ & \sum_{j=1}^n \tilde{a}_{ij} x_j \lesssim \tilde{b}_i \\ & x_j \geq 0, i \in M, j \in N \end{aligned} \quad (19)$$

where the fuzzy elements are given by:

- For each cost  $\exists \mu_j \in F(\mathfrak{R})$  so that  $\mu_j: \mathfrak{R} \rightarrow [0,1], j \in N$ , which defines the fuzzy costs.
- For each row  $\exists \mu_i \in F(\mathfrak{R})$  so that  $\mu_i: \mathfrak{R} \rightarrow [0,1], i \in M$ , which defines the fuzzy number in the right-hand side of constraints.
- For each  $i \in M$  and  $j \in N \exists \mu_{ij} \in F(\mathfrak{R})$  so that  $\mu_{ij}: \mathfrak{R} \rightarrow [0,1]$ , which defines the fuzzy number in the technological matrix.
- For each row  $\exists \mu_i \in F[F(\mathfrak{R})]$  so that  $\mu_i: F(\mathfrak{R}) \rightarrow [0,1], i \in M$  which provides the accomplishment degree of the fuzzy number for each  $x \in \mathfrak{R}^n$

$$\tilde{a}_{i1}x_1 + \tilde{a}_{i2}x_2 + \dots + \tilde{a}_{in}x_n, i \in M$$

with regard to the  $i$ th constraint, that is, the adequacy between this fuzzy number and the one  $\tilde{b}_i$  in relation to the  $i$ th constraint.

Cadenas and Verdegay (1997) define a solution method which consists of substituting (19) by a convex fuzzy set through a ranking function as a comparison mechanism of fuzzy numbers.

Let  $A, B \in F(\mathfrak{R})$ ; a simple method for ranking fuzzy numbers consists of defining a ranking function mapping each fuzzy number into the real line,  $g: F(\mathfrak{R}) \rightarrow \mathfrak{R}$ . If this function  $g(\cdot)$  is known, then:

$$\begin{aligned} g(A) < g(B) &\Leftrightarrow A \text{ is less than } B \\ g(A) > g(B) &\Leftrightarrow A \text{ is greater than } B \\ g(A) = g(B) &\Leftrightarrow A \text{ is equal to } B \end{aligned}$$

Usually,  $g$  is called a linear ranking function if:

$$\forall A, B \in F(\mathfrak{R}), g(A + B) = g(A) + g(B)$$

$$\forall r \in \mathfrak{R}, r > 0, g(rA) = rg(A), \forall A \in F(\mathfrak{R})$$

To solve the problem, (19) define: let  $g$  be a fuzzy number linear ranking function and given the function,  $\Psi: F(\mathfrak{R}) \times F(\mathfrak{R}) \rightarrow F(\mathfrak{R})$  so that:

$$\psi(\tilde{a}_i x, \tilde{b}_i) = \begin{cases} \tilde{t}_i, & \tilde{a}_i x \leq_g \tilde{b}_i \\ \tilde{t}_i(-) \tilde{a}_i x(+) \tilde{b}_i & \tilde{b}_i \leq_g \tilde{a}_i x \leq_g \tilde{b}_i(+) \tilde{t}_i \\ 0, & \tilde{a}_i x \leq_g \tilde{b}_i(+) \tilde{t}_i \end{cases}$$

Where  $\tilde{t}_i \in F(\mathfrak{R})$  is a fuzzy number in such a way that its support is included in  $\mathfrak{R}^+$ , and  $\leq_g$  is a relationship that measures that  $A \leq_g B, \forall A, B \in F(\mathfrak{R})$ , and  $(-)$  and  $(+)$  are the usual operations among fuzzy numbers.

According to Cadenas and Verdegay (2004), the membership function associated with the fuzzy constraint  $\tilde{a}_i x \leq \tilde{b}_i$  with  $\tilde{t}_i$  a fuzzy number giving the maximum violation of the  $i$ th constraint is:

$$\mu^i : F(\mathfrak{R}) \rightarrow [0,1] / \mu^i(\tilde{a}_i x, \tilde{b}_i) = \frac{g(\psi(\tilde{a}_i x, \tilde{b}_i))}{g(\tilde{t}_i)} \tag{20}$$

where  $g$  is a linear ranking function.

Given the problem (19),  $\lesssim$  with the membership function (20) and using the Decomposition Theorem (Cadenas, 1993; Negoita & Ralescu, 1975) for fuzzy sets, the following is obtained:

$$\begin{aligned} \mu^i(\tilde{a}_i x, \tilde{b}_i) \geq \alpha &\Leftrightarrow \frac{g(\psi(\tilde{a}_i x, \tilde{b}_i))}{g(\tilde{t}_i)} \geq \alpha \Leftrightarrow \frac{g(\tilde{t}_i(-) \tilde{a}_i x(+) \tilde{b}_i)}{g(\tilde{t}_i)} \geq \alpha \Leftrightarrow \\ g(\tilde{t}_i) - g(\tilde{a}_i x) + g(\tilde{b}_i) &\geq g(\tilde{t}_i) \alpha \Leftrightarrow g(\tilde{a}_i x) \leq g(\tilde{b}_i(+) \tilde{t}_i(1 - \alpha)) \Leftrightarrow \\ \tilde{a}_i x \leq_g \tilde{b}_i + \tilde{t}_i(1 - \alpha) \end{aligned}$$

where  $\leq_g$  is the relationship corresponding to  $g$ .

Therefore, an equivalent model to solve (19) is the following:

$$\begin{aligned} \text{Max } z &= \sum_{j=1}^n \tilde{c}_j x_j \\ \text{s.t.} & \\ \sum_{j=1}^n \tilde{a}_{ij} x_j &\leq_g \tilde{b}_i + \tilde{t}_i(1 - \alpha), \\ x_j &\geq 0, i \in M, j \in N, \alpha \in [0,1] \end{aligned} \tag{21}$$

To solve (21), the different fuzzy numbers ranking methods can be used in both the constraints and the objective function, or ranking methods can be used in the constraints and  $\alpha$ -cuts in the objective, which will lead us to obtain different traditional models, which allows to obtain a fuzzy solution (Cadenas & Verdegay, 2004).

Specifically in this paper and for illustration effects of the method, we apply a linear ranking function for the constraints (the first index of Yager (1979; 1981)) and  $\beta$ -cuts in the objective, although the approach could be easily adapted to the use of any other index.

Thus, if we effect  $\beta$ -cuts in the coefficients of the objective and we apply the first index of Yager as a linear ranking function to the constraint set, we obtain the following  $\alpha, \beta$ -parametric auxiliary problem.

$$\begin{aligned} \text{Max } z &= \sum_{j=1}^n \left[ (c_j - \beta \cdot d'_{c_j}), (c_j + \beta \cdot d_{c_j}) \right] x_j \\ \text{s.t.} & \\ \sum_{j=1}^n \left( a_{ij} + \frac{d_{a_{ij}} - d'_{a_{ij}}}{3} \right) x_j &\leq \left( b_i + \frac{d_{b_i} - d'_{b_i}}{3} \right) + \left( t_i + \frac{d_{t_i} - d'_{t_i}}{3} \right) (1 - \alpha), \\ x_j &\geq 0, i \in M, j \in N, \alpha, \beta \in [0,1] \end{aligned} \quad (22)$$

where, for instance,  $d_{c_j}$  and  $d'_{c_j}$  are the lateral margins (right and left, respectively) of the triangular fuzzy number central point  $c_j$  (see Fig. 1).

Solving Eq. (22) by weighting objectives ( $w_1, w_2 / w_1 + w_2 = 1$ ) the FLP problem defined in Eq. (21) is transformed into the crisp equivalent linear programming problem defined in Eq. (23) (Cadenas and Verdegay, 1997).

$$\begin{aligned} \text{Max } z &= w_1 \cdot \sum_{j=1}^n \left[ (c_j - \beta \cdot d'_{c_j}) x_j \right] + w_2 \cdot \sum_{j=1}^n \left[ (c_j + \beta \cdot d_{c_j}) x_j \right] \\ \text{s.t.} & \\ \sum_{j=1}^n \left( a_{ij} + \frac{d_{a_{ij}} - d'_{a_{ij}}}{3} \right) x_j &\leq \left( b_i + \frac{d_{b_i} - d'_{b_i}}{3} \right) + \left( t_i + \frac{d_{t_i} - d'_{t_i}}{3} \right) (1 - \alpha), \\ x_j &\geq 0, i \in M, j \in N, \alpha, \beta \in [0,1], w_1 + w_2 = 1 \end{aligned} \quad (23)$$

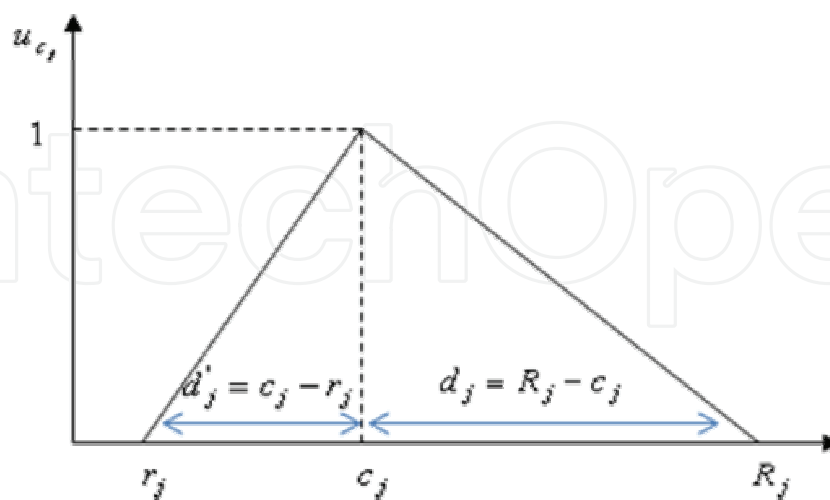


Fig. 1. Triangular fuzzy number

Consequently, by applying this approach to the previously defined FMILP model, we would obtain an auxiliary crisp mixed-integer linear programming model (MILP) as follows:

Minimize  $z =$

$$\begin{aligned}
 & \left[ \sum_{i=1}^I \sum_{n=1}^N \sum_{j=1}^J \sum_{t=1}^T \left[ (VPC_{ijn} - \beta \cdot d'_{VPC}) \cdot P_{ijn} \right] + \right. \\
 & \left. \sum_{n=1}^N \sum_{j=1}^J \sum_{t=1}^T \left[ (OTC_{njt} - \beta \cdot d'_{OTC}) \cdot OT_{njt} + (UTC_{njt} - \beta \cdot d'_{UTC}) \cdot UT_{njt} \right] + \right. \\
 & \left. \sum_{i=1}^I \sum_{n=1}^N \sum_{t=1}^T \left[ RMC_{int} \cdot PQ_{int} + (IC_{int} - \beta \cdot d'_{IC}) \cdot I_{int} + (DBC_{int} - \beta \cdot d'_{DBC}) \cdot DB_{int} \right] + \right. \\
 & \left. \sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D \sum_{l=1}^L \sum_{t=1}^T \left[ (TC_{odlt} - \beta \cdot d'_{TC}) \cdot TQ_{odlt} \right] \right] + \\
 & w_1 \cdot \left[ \sum_{i=1}^I \sum_{n=1}^N \sum_{j=1}^J \sum_{t=1}^T \left[ (VPC_{ijn} - \beta \cdot d'_{VPC}) \cdot P_{ijn} \right] + \right. \\
 & \left. \sum_{n=1}^N \sum_{j=1}^J \sum_{t=1}^T \left[ (OTC_{njt} - \beta \cdot d'_{OTC}) \cdot OT_{njt} + (UTC_{njt} - \beta \cdot d'_{UTC}) \cdot UT_{njt} \right] + \right. \\
 & \left. \sum_{i=1}^I \sum_{n=1}^N \sum_{t=1}^T \left[ RMC_{int} \cdot PQ_{int} + (IC_{int} - \beta \cdot d'_{IC}) \cdot I_{int} + (DBC_{int} - \beta \cdot d'_{DBC}) \cdot DB_{int} \right] + \right. \\
 & \left. \sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D \sum_{l=1}^L \sum_{t=1}^T \left[ (TC_{odlt} - \beta \cdot d'_{TC}) \cdot TQ_{odlt} \right] \right] \quad (23)
 \end{aligned}$$

Subject to

$$\begin{aligned}
 & \sum_{i=1}^I \left[ P_{ijn} \cdot \left( PT_{ijn} + \frac{d_{PT} - d'_{PT}}{3} \right) \right] \leq MPC_{ijn} + \frac{d_{MPC} - d'_{MPC}}{3} + MOT_{ijn} + \frac{d_{MOT} - d'_{MOT}}{3} + \\
 & \left( t_1 + \frac{d_{t_1} - d'_{t_1}}{3} \right) (1 - \alpha) \quad \forall n, j, t \quad (24)
 \end{aligned}$$

$$\left( PT_{ijn} + \frac{d_{PT} - d'_{PT}}{3} \right) \cdot PT_{ijn} \leq \left( MPC_{ijn} + \frac{d_{MPC} - d'_{MPC}}{3} \right) \cdot YP_{ijn} + \quad \forall i, n, j, t \quad (25)$$

$$\begin{aligned}
 & \left( MOT_{ijn} + \frac{d_{MOT} - d'_{MOT}}{3} \right) \cdot YP_{ijn} + \left( t_3 + \frac{d_{t_3} - d'_{t_3}}{3} \right) (1 - \alpha) \\
 & SR_{iodlt} = SR0_{iodlt} + TQ_{iodl,t-FTLT} \quad \forall i, o, d, l, t \quad (26)
 \end{aligned}$$

$$\sum_{i=1}^I I_{int} \cdot V_{it} \leq \left( MIC_{nt} + \frac{d_{MIC} - d'_{MIC}}{3} \right) + \left( t_8 + \frac{d_{t_8} - d'_{t_8}}{3} \right) (1 - \alpha) \quad \forall n, t \quad (27)$$

$$\begin{aligned}
 & \sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D SIP_{iodlt} \cdot V_{it} \cdot \chi^1_{odlt} + \sum_{i=1}^I \sum_{o=1}^O \sum_{d=1}^D TQ_{iodlt} \cdot V_{it} \cdot \chi^2_{odlt} \leq \\
 & \left( MTC_{lt} + \frac{d_{MTC} - d'_{MTC}}{3} \right) + \left( t_9 + \frac{d_{t_9} - d'_{t_9}}{3} \right) (1 - \alpha) \quad \forall l, t \quad (28)
 \end{aligned}$$

$$\sum_{i=1}^I PQ_{int} \leq \left( MPC_{nt} + \frac{d_{MPC} - d'_{MPC}}{3} \right) + \left( t_{10} + \frac{d_{t_{10}} - d'_{t_{10}}}{3} \right) (1 - \alpha) \quad \forall n, t \quad (29)$$

$$DB_{int} - DB_{in,t-1} + S_{int} \leq \left( D_{int} + \frac{d_D - d'_D}{3} \right) + \left( t_{11} + \frac{d_{t_{11}} - d'_{t_{11}}}{3} \right) (1 - \alpha) \quad \forall i, n, t \quad (30)$$

$$DB_{int} - DB_{in,t-1} + S_{int} \geq \left( D_{int} + \frac{d_D - d'_D}{3} \right) - \left( t_{12} + \frac{d_{t_{12}} - d'_{t_{12}}}{3} \right) (1 - \alpha) \quad \forall i, n, t \quad (31)$$

$$\sum_{i=1}^I P_{injt} \cdot PT_{injt} + UT_{njt} - OT_{njt} \leq \left( MPC_{njt} + \frac{d_{MPC} - d'_{MPC}}{3} \right) + \left( t_{13} + \frac{d_{t_{13}} - d'_{t_{13}}}{3} \right) (1 - \alpha) \quad \forall n, j, t \quad (32)$$

$$\sum_{i=1}^I P_{injt} \cdot PT_{injt} + UT_{njt} - OT_{njt} \geq \left( MPC_{njt} + \frac{d_{MPC} - d'_{MPC}}{3} \right) - \left( t_{14} + \frac{d_{t_{14}} - d'_{t_{14}}}{3} \right) (1 - \alpha) \quad \forall n, j, t \quad (33)$$

$$\sum_{n=1}^N \sum_{t=1}^T S_{int} \leq \sum_{n=1}^N \sum_{t=1}^T \left( D_{int} + \frac{d_D - d'_D}{3} + DB0_{int} \right) + \left( t_{15} + \frac{d_{t_{15}} - d'_{t_{15}}}{3} \right) (1 - \alpha) \quad \forall i \quad (34)$$

$$FTLT_{odlt} \leq \left( TLT_{odlt} + \frac{d_{TLT} - d'_{TLT}}{3} \right) + \left( t_{16} + \frac{d_{t_{16}} - d'_{t_{16}}}{3} \right) (1 - \alpha) \quad \forall o, d, l, t \quad (35)$$

$$FTLT_{odlt} \geq \left( TLT_{odlt} + \frac{d_{TLT} - d'_{TLT}}{3} \right) - \left( t_{17} + \frac{d_{t_{17}} - d'_{t_{17}}}{3} \right) (1 - \alpha) \quad \forall o, d, l, t \quad (36)$$

$$FTLT_{odlt} \geq 0 \quad \forall o, d, l, t \quad (37)$$

The non fuzzy constraints (3), (5), (6), (8), (15), (16), (17) and (18) are also included in the model in a similar way.

In order to solve the problem and according to Eq. (22)  $\alpha$ ,  $\beta$  is settled parametrically to obtain the value of the objective function for each of these  $\alpha$ ,  $\beta \in [0, 1]$ . The result is a fuzzy set and the SC planner has to decide which pair  $(\alpha, \beta, z)$  is more adequate to obtain a crisp solution. Although the decomposition theorem could be applied in different scales to the objective and to the constraint set (the decision maker's aspirations on the objective could be different from his/her satisfaction degree on the accomplishment of the constraints), in this work, the auxiliary crisp mixed-integer linear programming model (MILP) presented before is solved by using the same values for the parameters  $\alpha$  and  $\beta$ .

## 5. Application to an automobile supply chain

The proposed model has been evaluated by using data from an automobile SC which comprises a total of 47 companies (see Figure 2). In fact, these companies constitute a

segment of the automobile SC. Specifically, this SC segment supplies a seat model to an automobile assembly plant. The nodes that form the SC are a seat assembly company, its first tier suppliers, a manufacturing company of foams for seats and a second tier supplier that supplies chemical components for foam manufacturing. The automobile assembly plant weekly transmits the demand information (automobile seats) with a planning horizon for six months. However, these demand forecasts are rarely precise (Mula et al., 2005). This section validates whether the proposed fuzzy model for SC planning can be a useful tool for improving the decision-making process in an uncertain decision environment.

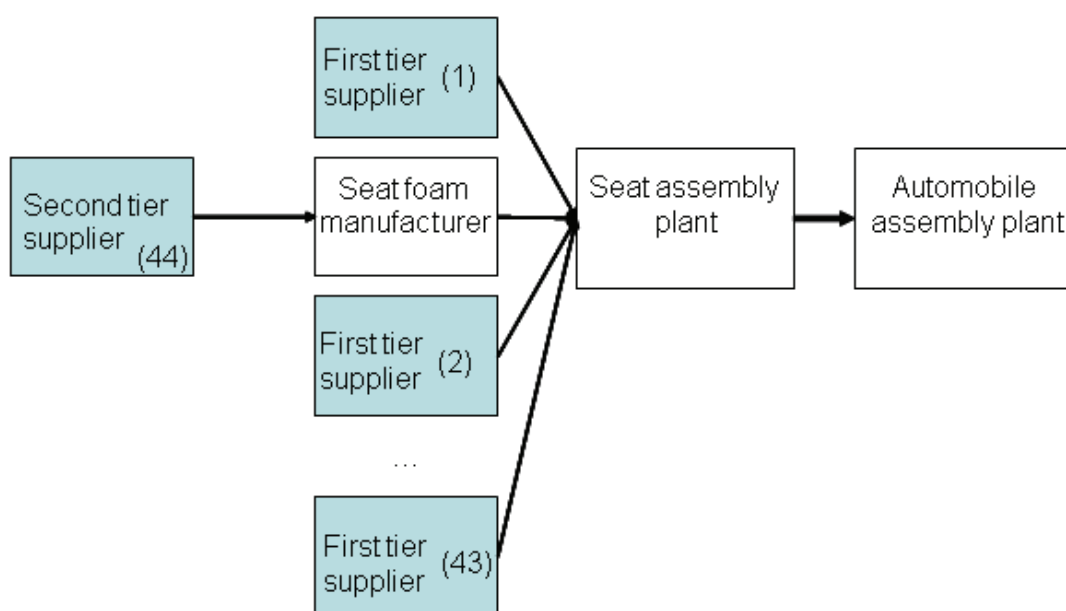


Fig. 2. Supply chain

### 5.1 Implementation and resolution

The model has been developed with the modelling language MPL (Maximal Software Incorporation, 2004) and solved by the CPLEX 9.0 solver (ILOG Incorporation, 2003). The input and output data are managed through a MS SQL Server database.

The model has been executed for a rolling horizon over a total of 17 weekly periods. These periods correspond to 17 different demand forecast programs, which are transmitted weekly by the automobile assembly plant. The total set of planning periods considered by the demand forecast programs is 42 weeks. Figure 3 depicts the execution of the models based on the rolling horizon technique. Each model calculation in the different planning horizon periods updates the data for the period being considered, and the results of the decision variables for the remaining periods are ruled out. Some of the stored decision variables are used as input data to solve the model in the following periods. These data include: demand backlog, shipments received, shipments in progress and inventory. This process is repeated for all the rolling horizon planning periods. The results of the model are evaluated from the data of the decision variables stored in each model execution. The experiments were run in an Intel Xeon PC, at 2.8 Ghz and with 1GB of RAM memory.



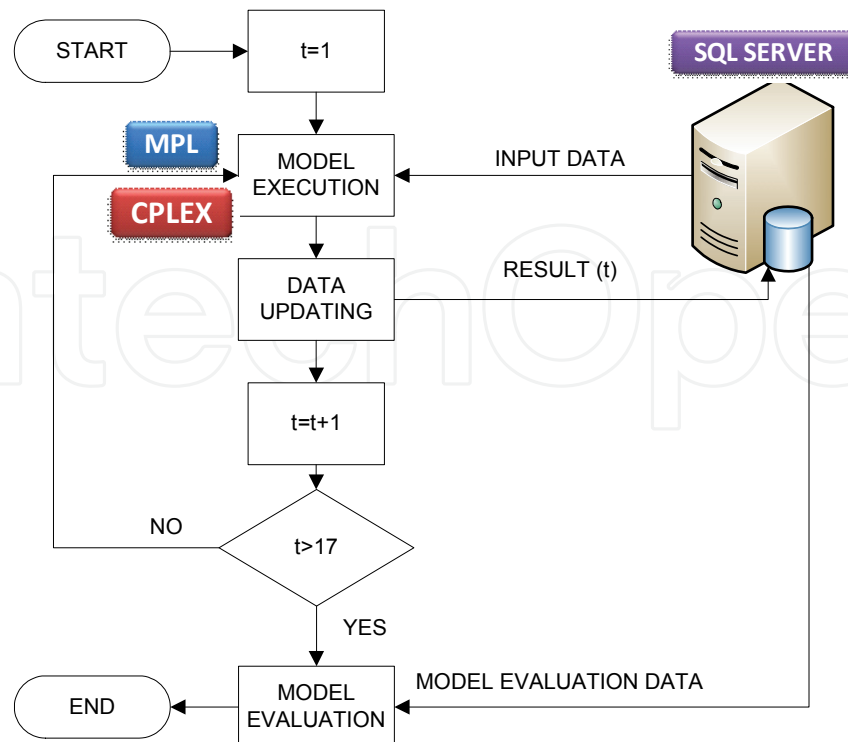


Fig. 3. Computational experiment diagram

## 5.2 Assumptions

The main characteristics and assumptions used in the experiment are presented below:

- The study considers a representative single finished good, i.e. a specific seat which can be considered to be a standard seat. The bill of materials of the standard seat is composed of 53 elements arranged in a three-level structure.
- The decision variables  $S_{intr}$ ,  $k_{injtr}$ ,  $DB_{intr}$ ,  $TQ_{iodltr}$ ,  $SR_{iodltr}$ ,  $SIP_{iodltr}$ ,  $PQ_{intr}$  and  $P_{injtr}$  are considered integer. Therefore, a mixed integer linear programming model is required to be solved.
- Only the finished good has external demand.
- The demand backlog for the finished good is considered but with a high penalization cost since the service level required by a sequenced and synchronized automobile seat supplier is 100%.
- A single productive resource restricts the capacity of the productions nodes (i.e. by focusing on the bottleneck resource).
- Triangular fuzzy numbers were defined by the decision makers involved in the planning process from the deviation percentages on the crisp value. These percentages range from an average 5% to 30%, depending on the parameter to be evaluated.
- A maximum violation of 5% is contemplated on the right-hand side of fuzzy constraints.
- The demand values for the first period of each model run, according to the rolling planning horizon, are considered to be firm. This means that the fuzzy intervals of the demand for this period will be the equivalent to a crisp number. The same happens for all the demand values of the last program. Thus, all the models will have the same net requirements to fulfill.
- A maximum calculation time of 100 CPU seconds is set.

### 5.3 Evaluation of the results

Here, we compare the behaviour of the proposed fuzzy model with its deterministic version. The aim is to determine the possible improvements that can provide the fuzzy model, which incorporates the uncertainties that may be presented in a SC.

Table 3 shows the computational efficiency of the deterministic model and the fuzzy SC planning model proposed. The data are related to the iterations, number of constraints, variables, integers, non zero elements, calculation time and the average density of the array of constraints for the set of the 17 planned executions of the models. Although the fuzzy model obtains higher values for these parameters, the CPU time has not markedly increased.

	<i>Deterministic</i>	<i>Fuzzy</i>
Iterations	636,128	717,377
Constraints	4,475,429	4,759,207
Variables	4,840,812	5,853,474
Integers	5,823,864	6,836,526
Non zero elements	15,793,161	23,251,766
Array density (%)	12.35 %	16.25 %
CPU time (seconds)	1,298.50	1,494.73

Table 3. Efficiency of the computational experiments.

Table 4 summarizes the evaluation results with the different  $\alpha$  and  $\beta$  values, according to a group of parameters defined in (Mula et al., 2006): (i) the average service level, (ii) the inventory levels, (iii) planning nervousness in relation to the planned period and planned quantity and (iv) the total costs.

i. The average service level for the finished good is calculated as follows:

$$Average\ service\ level\ (\%) = \sum_{i=1}^T \frac{\left( 1 - \frac{DB_{int}}{\sum_{t'=1}^t D_{int'}} \right) \times 100}{T} \quad \forall i, n \quad (38)$$

ii. The inventory level is calculated as the sum of the total quantity of inventory of the finished good and parts at the end of each planning period  $T= (1, \dots, 42)$ . Then the following rules are applied to determine which model presents, on average, the minimum and maximum inventory levels:

- If for each model the minimum inventory level is presented, it is assigned the value of 1, while a null value is assigned to the rest. The model which obtains the highest number will have the minimum levels of inventory. The maximum inventory levels can be determined in a similar way but by assigning the value of 1 to the maximum inventory level for item and model.

iii. Planning nervousness with regard to the planned period. "Nervous" or unstable planning refers to a plan which undergoes significant variations when incorporating the demand changes between what is foreseen and what is observed in successive plans, as defined by Sridharan et al. (1987). Planning nervousness can be measured according to the demand changes in relation to the planned period or to the planned quantity. The demand changes in the planned period measure the number of times that a planned

order is rescheduled, irrespectively of the planned quantity (Heisig, 1998). The next rule proposed by Donselaar et al. (2000) is summarized as follows:

At time  $t$  we check for each period  $t + x$  ( $x = 0, 1, 2, \dots, T-1$ ):

- If there is a planned order in  $t + x$ , and this order is not planned in the next planning run, we increase the number of reschedules by 1.
- If there was no planned order in  $t + x$ , and there is one in the next planning run, we increase the number of reschedules by 1.

Planning nervousness with regard to the planned period measures the demand changes in the planned quantity as the number of times that the quantity of a planned order is modified (De Kok and Inderfurth 1998). The rule is described as follows:

In the period  $t = 1, \dots, T$ , where  $T$  is the number of periods that forms the planning horizon, it is checked for every period  $t + x$  ( $x = 0, 1, 2, \dots, T-1$ ):

- If a planned order exists in the period  $t + x$ , then if the quantity of the planned order is not the same as in the next planning run, we increase the number of reschedules by 1.

In the computation of planning nervousness, we measure the number of changes. Another way to compute it would be to take into account the rate of the changes.

- Total costs are the sum of all the costs that are generated in every period of the considered planning horizon, and derived from the procurement, production and distribution plans provided by the model.

$\alpha=\beta$	Service level (%)	Number of min/max inventory levels	Planning nervousness (period)	Planning nervousness (quantity)	Total cost (€)
0	98.32%	10/11	1.31	20,69	4,528,053.0
0.1	98.31%	12/12	1.31	20,56	4,565,027.9
0.2	98.28%	8/10	1.31	20,56	4,601,774.4
0.3	98.28%	9/6	1.31	20,56	4,624,121.9
0.4	98.28%	9/12	1.31	20,56	4,640,859.9
0.5	98.28%	7/9	1.31	20,63	4,655,773.8
0.6	98.24%	11/11	1.31	20,56	4,698,218.3
0.7	98.24%	13/8	1.31	20,56	4,711,902.7
0.8	98.24%	15/8	1.31	20,56	4,737,356.4
0.9	98.21%	12/9	1.31	20,56	4,769,438.2
1	98.21%	12/9	1.31	20,56	4,769,438.2
<i>Deterministic model</i>	98.21%	8/7	1.31	20.56	4,768,579.1

Table 4. Evaluation of results

As seen in Table 4, all the fuzzy models, in general, obtain better results than the deterministic model. Only those models whose  $\alpha$  values come close to 1 obtain similar results to the deterministic model. This situation is logical because the closer the  $\alpha$  value comes to 1, the more similar the triangular fuzzy number model will be to a deterministic model. As seen in Figure 4, the fuzzy models obtain service levels that are better than or equal to the deterministic model, and these fuzzy models have better adapted to the existing

uncertainties in the demand forecasts considered in this work because these demand forecasts in this sector are rarely precise (Mula et al., 2005), as previously mentioned.

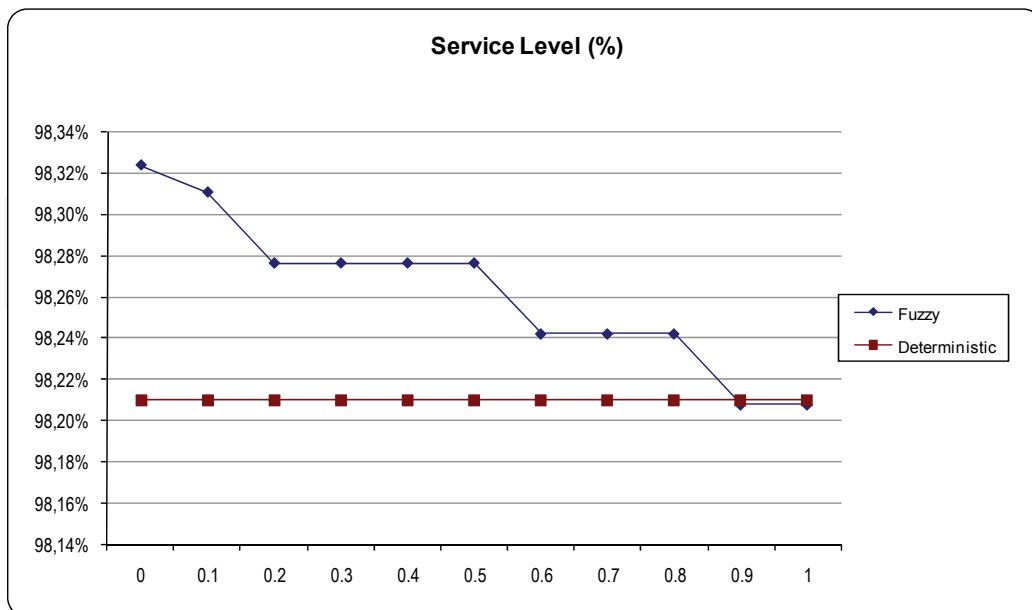


Fig. 4. Service Level (%)

With regard to inventory levels (see Fig. 5), the fuzzy model obtains better results for the number of minimum inventory levels for almost all the  $\alpha$  values. It is important to highlight that for  $\alpha > 0.3$ , the fuzzy model generates better results for minimum and maximum inventory levels. Besides, the levels of nervousness of fuzzy model are similar to those of the deterministic model. Finally, all the fuzzy models (see Fig. 6) obtain lower or similar costs than the deterministic model. This is because the demand backlog in this work is very heavily penalized, which means that those models with higher service levels achieve lower costs.

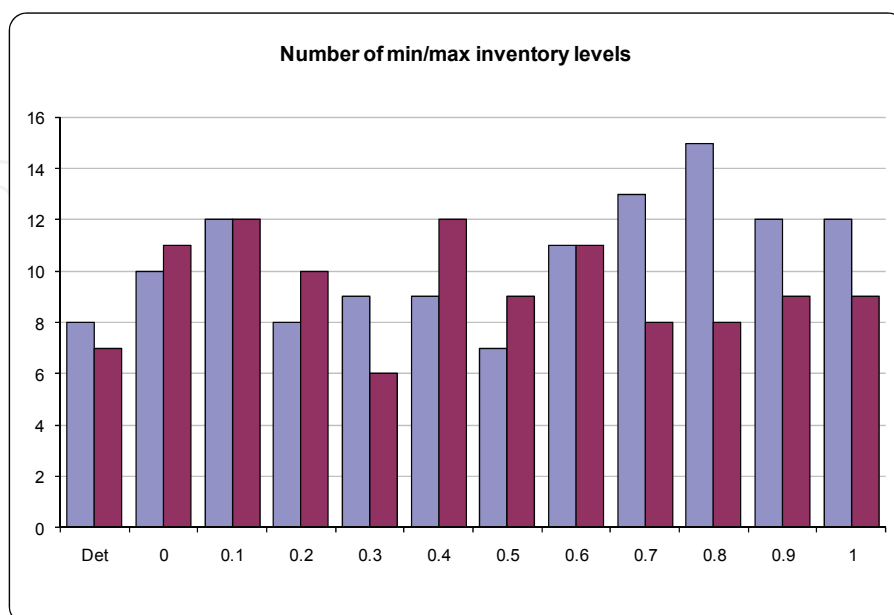


Fig. 5. Number of min/max inventory levels

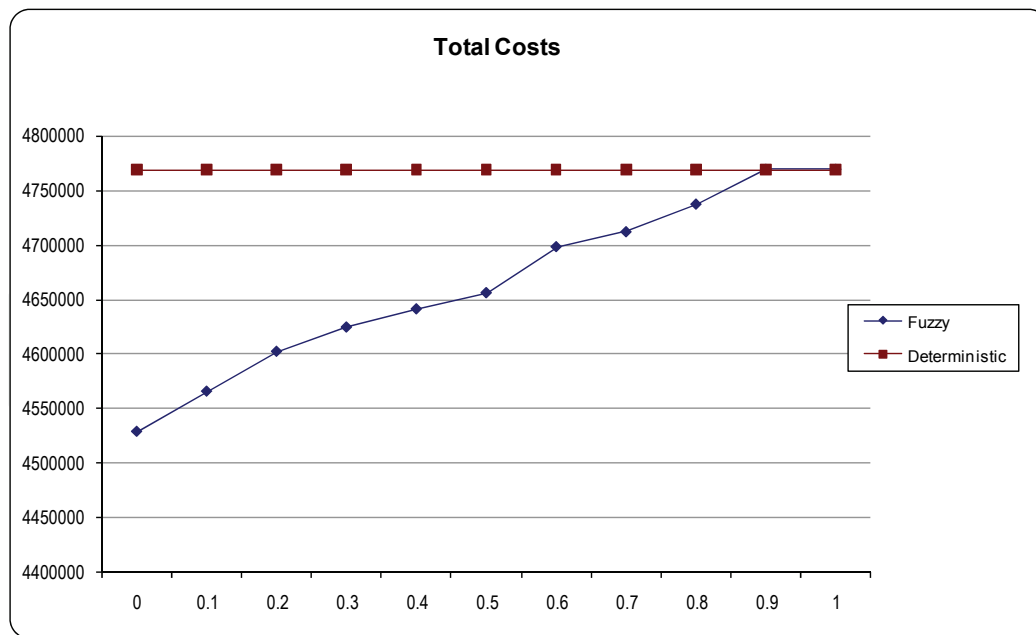


Fig. 6. Total Costs

## 6. Conclusions

This paper has proposed a novel fuzzy mixed integer linear programming (FMILP) model for the tactical SC planning, by integrating procurement, production and distribution planning activities into a multi-echelon, multi-product, multi-level and multi-period SC network. The fuzzy model integrally handles all the epistemic uncertainty sources identified in SC tactical planning problems given lack of knowledge (demand, process and supply uncertainties). This model has been tested by using data from a real-world automobile SC applying the rolling horizon technique over a total of 17 weekly periods. The evaluation of the results has demonstrated the effectiveness of a fuzzy linear programming approach for SC planning under uncertainty. The proposed fuzzy formulation is more effective than the deterministic methods for handling the real situations where precise or certain information is not available for SC planning. Additionally, the fuzzy model behaviour has been clearly superior to the deterministic model, as previously shown. Furthermore, the fuzzy model has not generated an excessive increment of the computational efficiency.

Finally, further research will consider: (1) other fuzzy mathematical programming-based approaches; (2) to design an expert system to solve the problem in which each decision maker, according to its aspirations, experiences and business, could have that index for ranking fuzzy numbers that better is adapted to its requirements; (3) the use of evolutionary computation in order to solve the fuzzy multi-objective, non linear SC planning problems; and (4) the application of hybrid models based on the integration of analytical and simulation models as an interesting option to integrate the best capacities of both types of models for SC planning problems.

## 7. Acknowledgment

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## **Supply Chain the Way to Flat Organisation**

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With the ever-increasing levels of volatility in demand and more and more turbulent market conditions, there is a growing acceptance that individual businesses can no longer compete as stand-alone entities but rather as supply chains. Supply chain management (SCM) has been both an emergent field of practice and an academic domain to help firms satisfy customer needs more responsively with improved quality, reduction cost and higher flexibility. This book discusses some of the latest development and findings addressing a number of key areas of aspect of supply chain management, including the application and development ICT and the RFID technique in SCM, SCM modeling and control, and number of emerging trends and issues.

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