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Distributed Supply Chain Planning for Multiple Companies with Limited Local Information

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1. Introduction

With rapid progress in market liberalization of a variety of products, intermediates, or electrical power energy, many companies are trying to integrate their enterprises with other organizations by optimizing planning and distribution for multiple companies. Supply chain coordination for Business to Business (B2B) is remarkably increasing for several enterprises at different industry levels with the progress of deregulation of trading products in recent years. Partner companies are reducing distribution costs while keeping high service levels to vendors. For example, in the petroleum industry, several companies at a chemical complex depend on other companies to provide raw materials or to deliver intermediates through a shared pipeline connected to multiple companies at lower delivery costs (Taneda, 2003).

In most companies, strategic instructions are given to each section in the company. Each company is thoroughly capable in regard to its own production planning and scheduling. The supply and demand planning must be coordinated by mutual negotiations across supply chain. Such coordination has been performed by the communications among human operators. However, in recent years, the decision making for each company is becoming increasingly complex with huge number of alternative planning for a number of companies. Conventional planning system has been configured to obtain a near optimal planning with detailed information about multiple companies. Organizations generally have their own private coordination methods, and that accessing others' private information or intruding on their decision-making authority should be avoided. In a practical situation, a plan must be created without sharing such confidential information as production cost, inventory holding costs, or price of products for competitive companies. In this chapter, a framework for distributed supply chain planning system without requiring all of information for multiple companies is proposed. Planning coordination can be efficiently automated by the proposed method.

Various types of supply chain models have been proposed in the literature (Vidal & Goetschalckx, 1997). A midterm planning model involving maximization of total profits and minimization of production, inventory, and transportation costs has been developed within a company (McDonald & Karimi, 1997). Planning coordination problems for production and distribution for multiple organizations are studied by Gaonkar and Viswanadham, 2002,

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Jackson and Grossmann, 2003 and Jayaraman and Pirkul, 2001. Schedule coordination problem for multiple organizations has been studied in Luh et al. 2003. This work concentrates on a planning model for multiple organizations from different companies.

Typical approach for supply chain planning is to use simulation-based approach combined with discrete optimization methods or to solve integer programming methods. Conventional supply chain planning systems have been configured to obtain near-optimal plans incorporating an information sharing for overall supply chain with detailed data-exchanging with multiple companies. Simulation-based methods for supply chain management (Julka et al. 2002, Tu et al. 2003) require detailed and precise information for all entities to analyze the performance of the entire supply chain. However, in practice, such information as production costs, inventory holding costs, or price of products are considered confidential for competing companies. For such reasons of confidentially, a distributed planning system with partial information sharing is preferable for supply chain planning for multiple companies. It requires the development of a distributed planning system in which each company can generate its own planning with partial information sharing with other companies.

In this chapter, a distributed supply chain planning system for multiple companies with partial information sharing is studied. Supply chain planning problems for multiple companies are formulated as mixed integer programming problems. A Lagrangian relaxation method is applied to decompose the overall problem by relaxing interconnection constraints between suppliers and vendors. Lagrangian relaxation is an optimization method that derives a lower bound by removing complicating constraints from constraint sets and replaces them with a penalty term in the objective function that can be decomposed into multiple solvable subproblems (Fisher, 1973),. Scheduling methods based on Lagrangian relaxation methods have been widely used to improve computation efficiency with near-optimal solution for jobshop problems (Hoitomt et al., 1993). The Lagrangian decomposition and coordination method has been applied to an asynchronous distributed decision making problems (Androulakis & Reklaitis, 1999). In this method, improvements of the Lagrangian multiplier value and generation of a solution of each subproblem are iteratively repeated. For planning problems, the method has been applied to supply chain planning problems in which machine capacity constraints are relaxed (Gupta & Maranus, 1999). Supply chain coordination problems for multiple organizations have been extensively studied by Luh et al. 2003. The problem for overall organizations is decomposed into individual organization-based subproblems relaxing precedence constraints. The performance of a price-based Lagrangian relaxation method is compared with an auction-based method.

The decomposition techniques for supply chain planning problem have been addressed before. For conventional decomposition methods, the solution derived in a distributed manner is coordinated by the heuristic procedure using the entire information for multiple entities. These heuristics are often problem-dependent to generate near-optimal solution. Thus it is difficult to construct good heuristics for general problems.

In this study, an augmented Lagrangian approach with a quadratic penalty function is used to decompose the original problem to eliminate duality gap. The augmented Lagrangian approach has recently extensively studied in short-term hydrothermal coordination (Beltran & Herdia, 1999), unit commitment (Beltran & Herdia, 2002, Georges, 1994) in power systems. However, the approach has not ever been applied to distributed algorithm in supply chain. The main difficulty is that the quadratic penalty term is not decomposable.

Firstly, the original problem is decomposed into several subproblems for each company with an objective function with a weighted penalty function. This method is called multiplier method for continuous convex optimization problems. By applying it to mixed integer programming problems, a feasible solution is expected to be derived by gradually increasing the weighting factor without using the construction procedure of feasible solutions. By adopting the approach, it may be possible to coordinate the entire plan without sharing confidential information among competing companies. The properties and the performance of the proposed approach for supply chain is investigated for a simple delivery/receiving planning problem for multiple companies in a petroleum complex, and a mid-term planning problem for multiple companies for more realistic model.
The rest of the chapter is organized as follows. In Section 2, the supply chain planning problem for multiple companies in a petroleum complex is formulated as a mixed integer linear programming problem. The formulation for distributed optimization using the augmented Lagrangian decomposition and coordination method is introduced in Section 3. Computational results including the discussion of the optimality of the solution by the proposed method, and the comparison of the proposed method with other distributed optimization methods, are shown in Section 4. Section 5 describes our conclusions and future research direction.

2. Supply chain planning for multiple companies

2.1 Problem definition and formulation

In this section, the supply chain planning problem in a petroleum complex is formulated as a mixed integer linear programming problem (MILP). Two layers of supply chain consisting of suppliers and manufacturers are treated in this work. Consider both of supplier companies and vendor companies in a petrochemical complex equipped with a shared pipeline for the delivery of intermediate products. The pipeline has a restricted capacity only for the delivery of one type of product at a time during a time period. Each company has its own demanded supply and demand plan for products. The supply chain planning problem is to determine a supply and demand planning for each supplier and vendor companies satisfying the constraints for the shared pipeline.
The following conditions (i)-(iv) are assumed:

i. Supply/demand quantity is restricted by the capacity of the shared pipeline.
ii. No more than two supplier companies can deliver/receive products into more than two vendor companies during the same time period.
iii. Production delivery/receiving cannot be interrupted for a pre-specified *K* time period once it has started.
iv. Total delivery/receiving quantity for the companies must be equal to the pre-determined quantity of products during total time horizon *H*.

Each company can adopt any optimization model. In this study, the following optimization model is adopted.
Let z_S and z_C denote a set of supplier companies and a set of vendor companies treating a set of products *P*. Supplier company $c \in Z_S$ have a demanded delivery plan $D_{i,t}^c$ $(i \in P; t = 1,\dots,H)$, and vendor company $c \in Z_C$ have a demanded receiving plan $D_{i,t}^c$ $(i \in P; t = 1,\dots,H)$, respectively. Each company has its own objective function $f^c\,(c \in Z_S \cup Z_C)$. To simplify the expression, but to formulate general problems, it is

assumed for all companies that the objective function for each company f^c consists of the sum of the following costs:

Penalty costs for deviation of delivery/receiving plan $S^c_{i,t}$ from the demanded delivery/receiving quantity $D^c_{i,t}$ that corresponds to inventory holding costs or to due date penalties.

i. For chemical complex or manufacturing systems, it is desirable that the delivery/receiving quantity is almost the same as that of previous periods due to changeover costs for flow rate. Thus, penalty costs are imposed if the delivery/receiving quantity in time period *t* is different from that in time period $(t-1)$.

ii. Transportation costs imposed at each company that correspond to pipeline usage costs.

The objective function f^c for company $c \in Z = \{Z_S \cup Z_C\}$ is given by the following equation.

$$f^c(S^c_{i,t}, X^c_{i,t}, Y^c_{i,t}) = \mu^c_{i,t} \mid D^c_{i,t} - S^c_{i,t} \mid + e^c_i X^c_{i,t} + d^c_{i,t} Y^c_{i,t} \tag{1}$$

The supply chain planning problem for multiple companies is formulated as a mixed integer linear programming problem.

$$(P_0) \ \min \sum_{c \in Z} \sum_{i \in P} \sum_t f^c(S^c_{i,t}, X^c_{i,t}, Y^c_{i,t}) \tag{2}$$

subject to

$$\sum_{c \in Z_S} S^c_{i,t} = \sum_{c \in Z_C} S^c_{i,t} \quad (\forall i \in P; \forall t = 1, \ldots, H) \tag{3}$$

$$\sum_t S^c_{i,t} = m^c_i \quad (\forall c \in Z; \forall i \in P) \tag{4}$$

$$\sum_{i \in P} Y^c_{i,t} \le 1 \quad (\forall c \in Z; \forall t = 1, \ldots, H) \tag{5}$$

$$S^c_{i,t} \le s^{\max}_i Y^c_{i,t} \quad (\forall c \in Z; \forall i \in P; \forall t = 1, \ldots H) \tag{6}$$

$$S^c_{i,t} \ge s^{\min}_i Y^c_{i,t} \quad (\forall c \in Z; \forall i \in P; \forall t = 1, \ldots H) \tag{7}$$

$$Y^c_{i,t} - \sum_{i \in P} Y^c_{i,t-1} \le Y^c_{i,t+1} \quad (\forall c \in Z; \forall i \in P; \forall t = 2, \ldots, H) \tag{8}$$

$$(K+1)(1 - Y^c_{i,t-1}) + KY^c_{i,t} \ge \sum_{i \in P} \sum_{t'=0}^{K-1} Y^c_{i,t+t'} \quad (\forall c \in Z; \forall i \in P; \forall t = 2, \ldots, H-K+1) \tag{9}$$

$$X^c_{i,t} = \begin{cases} 1 \ (S^c_{i,t} \ne S^c_{i,t-1}) \\ 0 \ (S^c_{i,t} = S^c_{i,t-1}) \end{cases} \quad (\forall c \in Z; \forall i \in P; \forall t = 2, \ldots H) \tag{10}$$

$$Y^c_{i,t} = \begin{cases} 1 \ (S^c_{i,t} > 0) \\ 0 \ (S^c_{i,t} = 0) \end{cases} \quad (\forall c \in Z; \forall i \in P; \forall t = 1, \ldots, H) \tag{11}$$

where s_i^{max}, s_i^{min} : minimum/maximum quantity of delivery for product *i* in time period *t*,
$D_{i,t}^c$: demanded delivery/receiving quantity of delivery for product *i* in time period *t* for company *c*,
$d_{i,t}^c$: transportation cost of the usage of pipeline for the delivery for product *i* in time period *t* for company *c*,
e_i^c : penalty incurred by the difference of delivery/receiving quantity of product *i* between the previous time period for company *c*,
K : set up time duration,
m_i^c : total delivery/receiving quantity for of product *i* during the total time horizon for company *c*,
$\mu_{i,t}^c$: penalty for deviation from demanded delivery/receiving quantity in time period *t* for company *c*,
$S_{i,t}^c$: delivery/receiving quantity of product *i* for company *c* in time period *t*,
$X_{i,t}^c$: binary variable which takes a value of 1 if delivery/receiving quantity of product *i* in time period *t* is different from that in time period (*t* – 1) for company *c*, and 0 otherwise,
$Y_{i,t}^c$: binary variable which takes a value of 1 if product *i* is delivered/received in time period *t* for company *c*.

The overall objective function given by (1) and (2) consists of sum of the penalty for deviating from the demanded delivery/receiving plan for each company, and the penalty for difference of delivery/receiving quantity, and transportation costs for delivery/receiving. (3) represents that the total delivery/receiving quantity from suppliers is equal to the total quantity of demand for vendor companies at each time period. (4) specifies that the total delivery/receiving quantity must be equal to a pre-determined set point from condition (iv). (5) implies that each company can deliver/receive only one type of product during a time period from condition (ii). (6) and (7) restrict delivery/receiving quantity must be less than s_i^{max} when $Y_{i,t}^c = 1$ and it is greater than s_i^{min} in each time period from condition (i). (8) denotes delivery/receiving duration constraints from condition (iii). It indicates that delivery/receiving cannot be interrupted at least two conservative time periods (if $(\sum_{i \in P} Y_{i,t-1}^c = 0) \wedge (Y_{i,t}^c = 1)$, then $(Y_{i,t+1}^c = 1)$ where the operation $\wedge$ stands for conjunction. (9) describes setup time constraints indicating that K conservative time periods are necessary for set up when the delivered/received product type is changed if $(Y_{i,t-1}^c = 1) \wedge (Y_{i,t}^c = 0)$, then $\sum_i \sum_{t'=1}^{K-1} Y_{i,t+t'}^c = 0$. $X_{i,t}^c$ in (10) is a binary variable which takes a value of 1 if the delivery/receiving quantity in time period t is different from that in time period $(t-1)$ and 0 otherwise. It can be realized by (12) and (13).

$$S_{i,t}^c + UX_{i,t}^c \geq S_{i,t-1}^c \quad (\forall c \in Z; \forall i \in P; \forall t = 2, \ldots, H) \tag{12}$$

$$S_{i,t-1}^c + UX_{i,t}^c \geq S_{i,t}^c \quad (\forall c \in Z; \forall i \in P; \forall t = 2, \ldots, H) \tag{13}$$

where U is a sufficiently large constant. (12) ensures $S_{i,t}^c \geq S_{i,t-1}^c$ when $X_{i,t}^c = 0$, on the other hand, (13) indicates $S_{i,t-1}^c \geq S_{i,t}^c$ when $X_{i,t}^c = 0$. If the delivery quantity in time period t is different from that in time period $(t-1)$, then $X_{i,t}^{c}=1$, otherwise $X_{i,t}^{c}=0$. (11) represents binary variable which takes a value of 1 if product i is delivered in time period t, then $Y_{i,t}^{c}=1$, and $Y_{i,t}^{c}=0$ otherwise.

3. Distributed optimization using Lagrangian decomposition and coordination method

3.1 Decomposable formulation

The original problem (P_0) can be easily solved by a commercial MILP (Mixed Integer Linear Programming) solver if the problem size is sufficiently small by using all of the information for the companies. However, when competing companies participate in the supply chain, such confidential information as cost data or product prices for other companies cannot be collected. In such situation, it is required to generate a feasible plan in a distributed environment without requiring all of the information. The distributed optimization without using all of information is necessary. From that viewpoint, a distributed optimization method using Lagrangian decomposition and coordination method is explained. The problem (P_0) is decomposed into several subproblems for each company by Lagrangian decomposition and coordination method (LDC method).

The LDC method is considered as a distributed optimization method to derive near-optimal solution efficiently by relaxing several constraints such that the original problem can be decomposed into subproblems. The constraints given by (3) in the original problem (P_0) are relaxed by Lagrangian multiplier $\lambda_{i,t}$, then the Lagrangian relaxation problem (R_0) to minimize Lagrangian function L with fixed Lagrangian multipliers, is described as Lagrangian relaxation problem (R_0).

$$(R_0)\ \min L$$

$$\begin{aligned} L &= \sum_{c\in Z}\sum_{i\in P}\sum_{t} f^{c}(S_{i,t}^{c}, X_{i,t}^{c}, Y_{i,t}^{c}) - \sum_{i\in P}\sum_{t}\lambda_{i,t}\left(\sum_{c\in Z_S} S_{i,t}^{c} - \sum_{c\in Z_C} S_{i,t}^{c}\right) \\ &= \sum_{c\in Z_S}\sum_{i\in P}\sum_{t}\left\{ f^{c}(S_{i,t}^{c}, X_{i,t}^{c}, Y_{i,t}^{c}) - \lambda_{i,t}S_{i,t}^{c}\right\} + \sum_{c\in Z_C}\sum_{i\in P}\sum_{t}\left\{ f^{c}(S_{i,t}^{c}, X_{i,t}^{c}, Y_{i,t}^{c}) + \lambda_{i,t}S_{i,t}^{c}\right\} \end{aligned} \tag{14}$$

The Lagrangian function L of (14) is additive for each company. Therefore the original problem (P_0) can be decomposed into subproblems for individual company. The subproblem for supplier $c \in Z_S$ and the subproblem for vendor company $c \in Z_C$ are represented as (15) and (16) when the multipliers are fixed.

For supplier company $c \in Z_S$

$$(SP_{0c}^{s})\quad \min \sum_{i\in P}\sum_{t}\left\{ f^{c}(S_{i,t}^{c}, X_{i,t}^{c}, Y_{i,t}^{c}) - \lambda_{i,t}S_{i,t}^{c}\right\} \tag{15}$$

For vendor company $c \in Z_C$

$$(SP_{0c}^{c})\quad \min \sum_{i\in P}\sum_{t}\left\{ f^{c}(S_{i,t}^{c}, X_{i,t}^{c}, Y_{i,t}^{c}) + \lambda_{i,t}S_{i,t}^{c}\right\} \tag{16}$$

subject to (1), (4)- (11).

3.2 Solving Lagrangian dual problem

The dual problem (D_0) for the original problem is represented as the following equation.

$$(D_0) \quad \max_{\{\lambda_{i,t}\}} q(\{\lambda_{i,t}\}) \quad \text{where } q(\{\lambda_{i,t}\}) = \min L \tag{17}$$

To solve the dual problem, subgradient optimization method is used in most cases. The Lagrangian multipliers are updated according to (18). The steps of the solving subproblems and the updating Lagrangian multipliers are iteratively repeated until dual solution has not been updated.

$$\lambda_{i,t} = \begin{cases} \lambda_{i,t} + \Delta\lambda \ (\sum_{c\in Z_S} \overline{S^c_{i,t}} < \sum_{c\in Z_C} \overline{S^c_{i,c}}) \\ \lambda_{i,t} - \Delta\lambda \ (\sum_{c\in Z_S} \overline{S^c_{i,t}} > \sum_{c\in Z_C} \overline{S^c_{i,c}}) \end{cases} \tag{18}$$

$S^c_{i,t}$ represents tentative delivery quantity derived by solving a subproblem in a previous iteration. $\overline{\lambda_{i,t}}$ represents Lagrangian multiplier in a previous iteration (tentative value of Lagrangian multipliers). If all of the information for other companies is available, a step size of Lagrangian multiplier $\Delta\lambda$ is calculated by (19).

$$\Delta\lambda = \gamma \frac{\overline{L} - \underline{L}}{(\sum_{c\in Z_S} \overline{S^c_{i,t}} - \sum_{c\in Z_C} \overline{S^c_{i,t}})^2} \tag{19}$$

where γ is a positive coefficient satisfying $0 < \gamma < 2$, $\overline{L}$ is upper bound of the original problem, and $\underline{L}$ is lower bound obtained by calculating L for the solution of subproblems.

The algorithm of the LDC method is described in the following steps.

Step 1: Initialization of multipliers.

Step 2: Generation of the solution of subproblem for each company with fixed multipliers. The lower bound $\underline{L}$ is calculated.

Step 3: Generation of a feasible solution using a solution derived at Step 2. The upper bound $\overline{L}$ is calculated.

Step 4: Evaluation of convergence. The condition for convergence is that the duality gap calculated by $\frac{\overline{L} - \underline{L}}{\underline{L}}$ has not been updated at a pre-specified number of times.

Step 5: Update of the multipliers by (18), (19) and return to Step 2.

The solution of dual problem is not always feasible for nonconvex optimization problem when the problem includes the setup costs depending on product type in the objective function. In order to obtain a feasible solution at Step 3, the construction of a feasible solution is necessary for LDC method to calculate $\overline{L}$ by modifying the solution of subproblems using a heuristic procedure. Simple priority-based heuristics such like FIFO rules, etc. backward or backward-forward heuristics are often used to generate a feasible solution. The performance of LDC method highly depends on the performance of heuristics to modify the dual solution into a feasible one requiring all of the information corrected to apply these heuristic procedures. However, it is difficult to construct a heuristic to obtain a good upper bound. Moreover, the solution oscillations often occur if the dual solution is not identical to the primal optimal solution that makes the algorithm to find a feasible solution. To obtain a feasible solution, we applied an augmented Lagrangian approach (Rockafellar, 1974) without using heuristic procedure. The method is called as a multiplier method (Bertsekas, 1976), which is commonly used for continuous optimization problems. The supply chain planning problem for multiple companies is solved by the distributed optimization approach.

3.3 Decomposable reformulation for augmented Lagrangian approach

The main drawback of the augmented Lagrangian approach is that the quadratic penalty term introduced by the augmented Lagrangian is not separable into each subproblem for a company. To make the problem decomposable, a linear approximation technique of the cross penalty terms around a tentative solution has been proposed (Androulakis and Reklaitis, 1999, Cohen and Zhu, 1984, Stephanopoulas and Westerberg, 1975). Let us consider a simple supply chain planning problem for supplier company a and vendor company b treating product $i \in P$. An augmented Lagrangian function with the quadratic penalty term is given by the following equation.

$$L_r = \sum_{i \in P} \sum_t \left\{ f^a + f^b + \lambda_{i,t} (S^b_{i,t} - S^a_{i,t}) \right\} + r \sum_{i \in P} \sum_t (S^b_{i,t} - S^a_{i,t})^2 \tag{20}$$

The Lagrangian function of (20) cannot be decomposed because the cross-product term $S^a_{i,t} S^b_{i,t}$ is included in the penalty term. To keep the problem decomposable, a first order Taylor series of expansion around the tentative solution $(\overline{S^a_{i,t}}, \overline{S^b_{i,t}})$ is used. The augmented Lagrangian function can be reformulated as:

$$\begin{aligned} L_r &= \sum_i \sum_t (f^a - \lambda_{i,t} S^a_{i,t}) + \sum_i \sum_t (f^b + \lambda_{i,t} S^b_{i,t}) \\ &+ r \sum_i \sum_t \left\{ (S^a_{i,t})^2 + (S^b_{i,t})^2 - 2 S^a_{i,t} \overline{S^b_{i,t}} - 2 S^b_{i,t} \overline{S^a_{i,t}} + 2 \overline{S^a_{i,t}}\, \overline{S^b_{i,t}} \right\} \\ &= \sum_i \sum_t (f^a - \lambda_{i,t} S^a_{i,t}) + r \sum_i \sum_t \left\{ (S^a_{i,t})^2 - 2 S^a_{i,t} \overline{S^b_{i,t}} + \overline{S^a_{i,t}}\, \overline{S^b_{i,t}} \right\} \\ &+ \sum_i \sum_t (f^b - \lambda_{i,t} S^b_{i,t}) + r \sum_i \sum_t \left\{ (S^b_{i,t})^2 - 2 S^b_{i,t} \overline{S^a_{i,t}} + \overline{S^a_{i,t}}\, \overline{S^b_{i,t}} \right\} \end{aligned} \tag{21}$$

r is a positive scalar parameter. (21) states that the problem to minimize the Lagrangian function L of (20) with fixed multipliers can be decomposed into subproblems for each company by using the tentative solution at each iteration. The decomposed function consists of sum of the objective function for each company, a multiplier penalty term, and a quadratic penalty term. The minimization problem for each company is a mixed integer quadratic programming (MIQP) problem which is difficult to be solved in reasonable computation time. Therefore, in our study we replaced the quadratic penalty term by a linear penalty term shown in (22). By using this reformulation, we do not have to use nonlinear optimization methods for solving MIQP problem.

The new decomposed function for company a: L'_a can be given by:

$$L_a^{'} = \sum_{i \in P} \sum_t (f^a - \lambda_{i,t} S_{i,t}^a) + r \sum_{i \in P} \sum_t | S_{i,t}^a - \overline{S_{i,t}^b} | \quad (22)$$

The penalty parameter *r* for a linear penalty term is gradually increased in each iteration. By applying the linear approximation technique around a tentative solution for the proposed method, the solutions derived by solving subproblem for each company cannot provide an exact lower bound of the original problem.

3.4 Coordination of supply chain planning among multiple companies

A sequence of optimization problems E_0^k can be given by (23) where L is given by (14).

$$(E_0^k) \quad \min (L + r_k | \sum_{c \in Z_S} S_{i,t}^c - \sum_{c' \in Z_C} S_{i,t}^{c'} |^2) \quad (23)$$

The decomposed subproblem for each company is reformulated as (24) and (25) by applying the first order Taylor series of expansion around a tentative solution.

For supplier company $c \in Z_S$

$$(EP_{0c}^k) \quad \min \sum_{i \in P} \sum_t \left\{ f^c(S_{i,t}^c, X_{i,t}^c, Y_{i,t}^c) - \lambda_{i,t} S_{i,t}^c \right\} + r_k \sum_{i \in P} \sum_t | S_{i,t}^c + \sum_{c' \in Z_S / \{c\}} S_{i,t}^{c'} - \sum_{c' \in Z_C} S_{i,t}^{c'} | \quad (24)$$

For vendor company $d \in Z_C$

$$(EP_{0c}^k) \quad \min \sum_{i \in P} \sum_t \left\{ f^d(S_{i,t}^d, X_{i,t}^d, Y_{i,t}^d) + \lambda_{i,t} S_{i,t}^d \right\} + r_k \sum_{i \in P} \sum_t | S_{i,t}^d + \sum_{c' \in Z_C / \{d\}} S_{i,t}^{c'} - \sum_{c' \in Z_S} S_{i,t}^{c'} | \quad (25)$$

subject to (1), (4)-(11)

The subproblem for each company is an MILP problem, which can be solved by a commercial solver. r_k represents a weighting factor for penalty function. To derive near-optimal solution for the proposed method, the weighting factor r_k must be gradually increased according to the following equation.

$$r_{k+1} = r_k + \Delta r \quad (26)$$

Δ r is the step size parameter for penalty weighting coefficient which should be determined by preliminary tests. Even though the objective function includes a linear penalty function for each subproblem, a lower bound of the original problem can be obtained by calculating *L* for the solution of subproblem when r_k is set to zero.

3.5 Scenario of planning coordination for multiple companies

The system generates near-optimal plan in the following steps.

Step 1: Initialization

$k \leftarrow 0$. The multipliers $\lambda_{i,t}$ and the weighting factor r_k are set to an initial value (*e.g.* set to zero).

Step 2: Generation of an initial plan

A manager for each company inputs the demanded delivery/receiving plan at each time period $D_{i,t}^c$ and the total delivery/receiving quantity for each product during time horizon. Each company solves each subproblem and generates a tentative plan with the fixed multipliers.

Step 3: Data exchange of tentative solution

Each company exchanges the data of tentative delivery/receiving quantity of products $\overline{S_{i,t}^{c}}$ derived at each company.

Step 4: Evaluating the convergence

If the plan generated at Step 6 or Step 2 for initial iteration satisfies the following conditions, the algorithm is considered as convergence. Then no more calculation is made and the derived plan is regarded as a final plan.

i. The solution derived at Step 6 is the same as that generated at Step 6 in a previous iteration.
ii. The solution derived at Step 6 satisfies the constraints (3).
iii. The solutions of all other companies also satisfy both of conditions (i) and (ii).

Step 5: Update of the multiplier and the weighting factor

The weighting factors are updated by (26) and the multipliers are updated by (18).

Step 6: Solving subproblems

A company solves its subproblem while the solution of other company is fixed. Then, the tentative solution $S_{i,t}^{c}$ is updated and return to Step 3. If some of the companies derive its solutions concurrently in parallel at Step 6, the same solution is generated cyclically because tentative solution of a previous iteration is used, that makes the convergence of the algorithm more difficult. Skipping heuristic (Nishi et al., 2002) is effective to avoid such situations. Skipping heuristic is a procedure that the Step 6 for each company is randomly skipped. If the proposed method is implemented on a parallel processing system, the procedure must be added to avoid cyclic generation of solutions. Our numerical experiments used a sequential computation that the Step 6 for each company is sequentially executed to avoid the difficulty of convergence without skipping heuristic.

The data exchanged among companies is tentative supply and demand quantity in each time period. This information is not directly concerned with confidential information for each company. The multipliers are updated by (27) without using the information of $\overline{L}-\underline{L}$ for the step size because the upper bound is not calculated for augmented Lagrangian approach.

$$\lambda_{i,t}=\begin{cases}\lambda_{i,t}^{*}+\Delta\lambda & (\overline{S_{i,t}^{c}}<\overline{S_{i,t}^{c'}})(c\in Z_S;\, c'\in Z_C)\\ \lambda_{i,t}^{*}-\Delta\lambda & (\overline{S_{i,t}^{c}}>\overline{S_{i,t}^{c'}})(c\in Z_S;\, c'\in Z_C)\\ \lambda_{i,t}^{*} & (\overline{S_{i,t}^{c}}=\overline{S_{i,t}^{c'}})(c\in Z_S;c'\in Z_C)\end{cases} \tag{27}$$

$S_{i,t}^{c}$ represents a tentative solution obtained by solving subproblem for company c. $\Delta\lambda$ is the step size given as a scalar parameter, and $\lambda_{i,t}^{*}$ is the value of multipliers at a previous iteration. For the proposed system, $\Delta\lambda$ is considered as a constant step size without generation of a feasible solution for the entire company. All of the information that is exchanged at each iteration during the optimization is the tentative delivery quantity $\overline{S_{i,t}^{c'}}\,(c'\in Z_S\cup Z_C\setminus\{c\})$ derived at other companies. Each company has the same value of its own multipliers and updates the value of them for itself. Thus the dual problem can be solved in a distributed environment without exchanging such confidential data as cost information for the proposed method.

4. Computational experiments

4.1 Supply chain planning for 1 supplier and 2 vendor companies

An example of supply chain planning problem for 1 supplier (A) and 2 vendor companies (B, C) treating with 2 types of products is solved. The total time horizon is 30 time periods. The parameters for the problem are generated by random numbers on uniform distribution in the interval shown in Table 1. The demanded delivery/receiving plan which is input data for each company is illustrated in Fig. 1. The result obtained by the proposed method is also shown in Fig. 2. The numbers printed in the figure indicate the delivery and receiving quantity for each company. The program is coded by C++ language. A commercial MILP solver, CPLEX8.0 ILOG(C) is used to solve subproblems. A Pentium IV 2AGHz processor with 512 MB memory was used for computation.

The optimality of solution is minimized when $\Delta r = 0.01$ and $\Delta\lambda = 0.1$ from several preliminary tests. These parameters are used for computation in the following example problems.

	Supplier company $c \in Z_S$	Vendor company $c \in Z_C$
$D_{i,t}^c$	0 – 200	0 – 180
$\mu_{i,t}^c$	1 – 10	1 – 10
$d_{i,t}^c$	1 – 10	1 – 10
e_i^c	10 – 30	10 – 30
$m_{i,t}^c$	1500 – 4000	750 – 2000

Table 1. Parameters for the example problems

Augmented Lagrangian decomposition method (ALDC)

$\Delta\lambda = 0.1$, $\Delta r = 0.01$

Lagrangian decomposition method (LDC)

$\gamma = 0.1$

Penalty method (PM)

$\Delta r = 0.01$ (Case 1, Case 2), $\Delta r = 0.1$ (Case 3)

Table 2. Parameters for the distributed optimization method

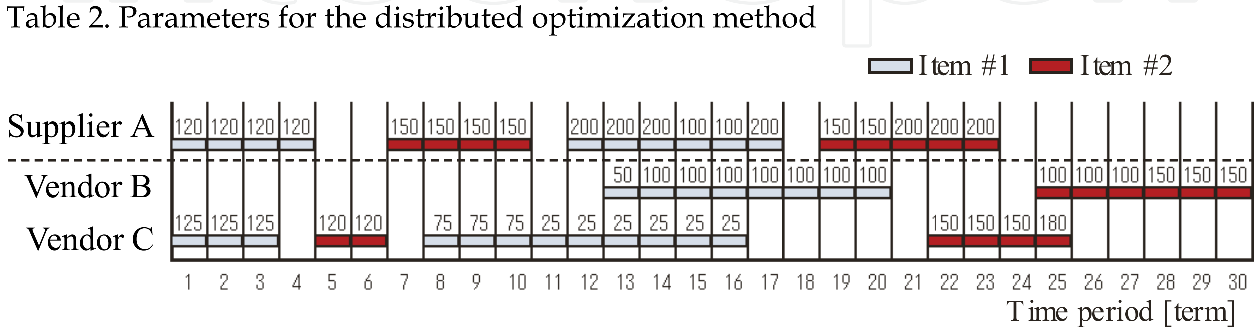

Fig. 1. An initial request for the plan (1 supplier and 2 vendor companies)

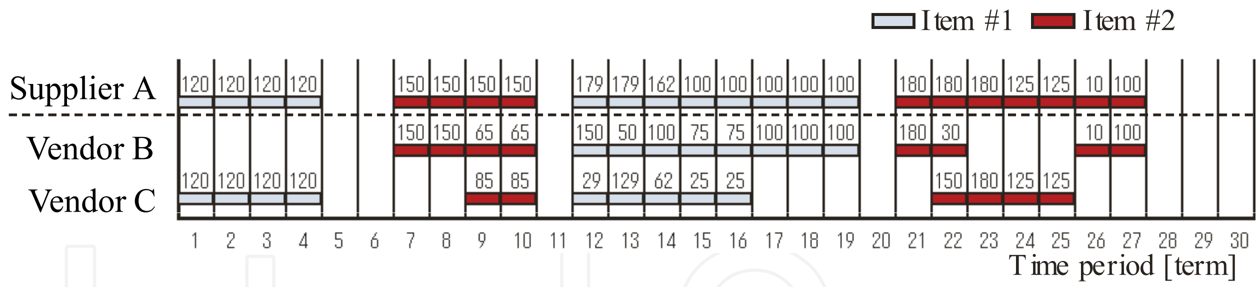

Fig. 2. Result of distributed supply chain planning by the proposed method (after 72 times of data exchanges)

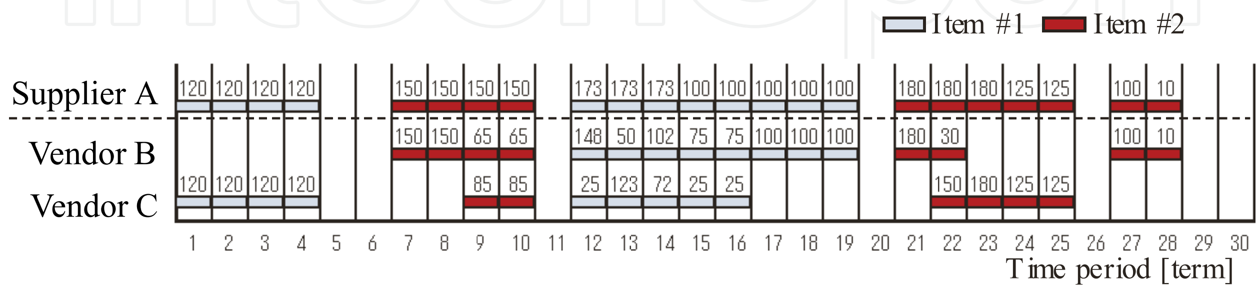

Fig. 3. The optimal solution derived by CPLEX solver

The proposed method generates a feasible solution for the problem after 72 iterations using the parameters shown in Table 2. The result is shown in Fig. 2. An optimal solution derived by commercial solver is also shown in Fig. 3. The result obtained by the proposed method is almost the same as that of an optimal solution. The transition of the value of L_r and the decomposed function $L_r^{'}$ for each company c is shown in Fig. 4. The condition for evaluating convergence is that the difference of the delivery and receiving quantity is less than 0.01 for all products and for all time periods. The optimal value of the objective function of (2) obtained by the proposed method is 9,979. The value for the optimal solution obtained by the commercial MILP solver with all of the information is 9,960. The gap between the derived solution and the optimal solution is 0.18%. It demonstrates that the proposed method can derive near-optimal solution without requiring all of the information for other companies.

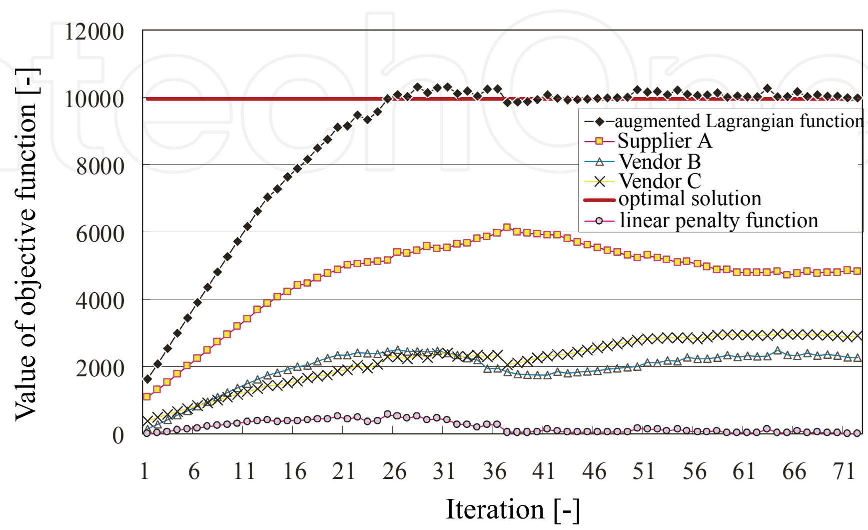

Fig. 4. Transition of the value of objective function for the proposed method

4.2 Comparison with other distributed optimization methods

To investigate the performance of the proposed method, the performance of the proposed method (ALDC method) is compared with other distributed optimization methods: a penalty method (PM method) that the terms of Lagrangian multipliers are removed from (24) and (25), and an ordinary LDC method (LDC method).

For the LDC method, the dual problem D_0 is solved by standard Lagrangian function. The dual solution is modified to generate a feasible solution with the following heuristic procedure at each iteration. The heuristic procedure is constructed so that the constraint violation is checked in forward and the solution is modified to satisfy three types of constraints of (5), (6), (7) and (8), (9) successively satisfying (3).

Step i) Receiving quantity for vendor companies is modified to satisfy the delivery quantity for suppliers. Set

$$S_{i,t}^{c} \leftarrow \overline{S_{i,t}^{c}}\,(\forall c \in Z_S; \forall i \in P; \forall t = 1,\ldots,H);$$

$$S_{i,t}^{d} \leftarrow \frac{m_i^d}{\sum_{y \in Z_C} m_i^d} \sum_{c \in Z_S} \overline{S_{i,t}^{c}}\,(\forall d \in Z_C; \forall i \in P; \forall t = 1,\ldots,H).$$

Step ii) Find a time period t in forward in which (5) is violated. For a plan in time period t, one type of product is allocated and allocation of other types of products are moved to a neighbour time period e.g. (t -1) or (t +1). If (3) and (5) are not satisfied, then return to step i). Otherwise go to step iii).

Step iii) Find a time period t in forward in which (6) or (7) is violated. For a plan in time period t, the violated delivery/receiving quantity is modified to allocate into a neighbour time period e.g. (t -1) or (t +1). If (3) and (5)-(7) are not satisfied, then return to step i). Otherwise go to step iv).

Step iv) Find a time period t in forward when (8) or (9) is violated. For a plan in time period t, the allocation of delivery/receiving quantity is modified to allocate a neighbour time period e.g. (t -1) or (t +1). If (3) and (5)-(9) are not satisfied, return to step i). Otherwise the heuristic procedure is completed.

Three cases of the supply chain planning problem for 1 supplier and 2 vendor companies are solved by the proposed method, LDC method and PM method. For each case, ten types of problems are generated by using random numbers on uniform distribution with different seeds in the range shown in Table 1. The parameters used for each method are shown in Table 2. The average objective function (Ave. obj. func.), average gap between the solution and an optimal solution (Ave. gap), average number of iterations to converge (Ave. num. iter.), and average computation time (Ave. comp. time) for ten times of calculations for each case are summarized in Table 3. The centralized MILP method uses a branch and bound method to obtain an optimal solution by CPLEX 8.0 using Pentium IV 2GHz processor with 512MB memory.

Computational results of Table 3 show that the ALDC method can generate better solutions than any other distributed optimization methods. The gap between the optimal solutions is within 3% for all cases. This indicates that the proposed method can generate near-optimal solution without using the entire information for each company. The total computation time for ALDC method to derive a feasible solution is shorter than that of MILP method, however, it is larger than that of PM method. The MILP solver cannot derive a solution within 100,000 seconds of computation time for Case 3 (3 types of products). This is why the computational complexity for the problem grows exponentially with number of products. The petroleum complex usually treats multi-products more than 3 types of products. Thus it is very difficult to apply the conventional MILP solver for supply chain planning for multiple companies. The optimality performance of the LDC method is not better than the other methods. This is because the heuristic procedure to generate a feasible solution is not effective for large-sized problems. The LDC method cannot derive a feasible solution by the current heuristic procedure. This is due to the difficulty of finding a feasible solution to satisfy all of such constraints as setup time constraints, and delivery duration constraints. The computation time of penalty method (PM method) is shorter than the proposed method, however, the optimality performance is not better than that of the proposed method. This result implies that the use of Lagrangian multipliers is effective to improve the optimality performance. Even though the proposed method needs a number of iterations to converge to a feasible solution than that of PM method, it is demonstrated that near-optimal solution with less than 3% of gap from the optimal solution can be obtained by the proposed method.

Case 1	Problem for 1 type of product			
Method	MILP	ALDC	LDC	PM
Ave. obj. func. [-]	10,829	10,976	13,297	11,153
Ave. gap [%]	0.00	1.37	23.0	2.90
Ave. num. iter.	-	180	90	53
Ave. comp. time[s]	1783	110	85	27
Case 2	Problem for 2 types of products			
Method	MILP	ALDC	LDC	PM
Ave. obj. func. [-]	48,700	49,975	50,562	51,005
Ave. gap [%]	0.00	2.71	4.06	4.66
Ave. num. iter.	-	137	80	39
Ave. comp. time[s]	16,142	246	149	41
Case 3	Problem for 3 types of products			
Method	MILP	ALDC	LDC	PM
Ave. obj. func. [-]	-	78280	-	79089
Ave. num. iter.	-	827	-	104
Ave. comp. time[s]	-	110	-	30

Table 3. Comparison of the performances of MILP and the distributed optimization methods

5. Conclusion and future works

A distributed supply chain planning system for multiple companies using an augmented Lagrangian relaxation method has been proposed. The original problem is decomposed into several sub-problems. The proposed system can derive a near optimal solution without using the entire information about the companies. By using a new penalty function, the proposed method can obtain a feasible solution without using a heuristic procedure. This is also a predominant characteristic of the proposed algorithm and the improvement of the conventional Lagrangian relaxation methods. It is demonstrated from numerical tests that a near optimal solution within a 3% of gap from an optimal solution can be obtained with a reasonable computation time. The applicability of the augmented Lagrangian function to the various class of supply chain planning problems is one of our future works.

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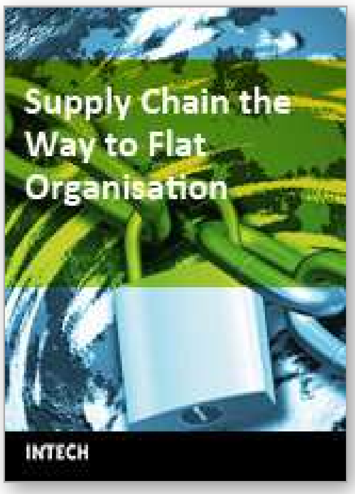

Supply Chain the Way to Flat Organisation
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With the ever-increasing levels of volatility in demand and more and more turbulent market conditions, there is a growing acceptance that individual businesses can no longer compete as stand-alone entities but rather as supply chains. Supply chain management (SCM) has been both an emergent field of practice and an academic domain to help firms satisfy customer needs more responsively with improved quality, reduction cost and higher flexibility. This book discusses some of the latest development and findings addressing a number of key areas of aspect of supply chain management, including the application and development ICT and the RFID technique in SCM, SCM modeling and control, and number of emerging trends and issues.