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Operational Management of Supply Chains: A Hybrid Petri Net Approach

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1. Introduction

The emergence of Supply Chains (SCs) is an outcome of the recent advances in logistics and information technology. SCs are complex networks interconnecting different independent manufacturing and logistics companies integrated with material, information and financial flows (Viswanadham & Raghavan, 2000). Typically, SC management decisions are classified into three hierarchical levels according to the time horizon of decisions: strategic (longterm), tactical (medium-term), and operational (short-term, real-time) (Chopra & Meindl, 2001; Shapiro, 2001). Accordingly, different models have to be defined at each level of the decision hierarchy to describe the multiple aspects of the SC. While the development of formal models for SC design at strategic and tactical levels was addressed in the related literature (Dotoli et al., 2005; Dotoli et al., 2006; Gaonkar & Viswanadham, 2001; Luo et al., 2001; Vidal & Goetschalckx, 1997; Viswanadham & Gaonkar, 2003), research efforts are lagging behind in the subject of modeling and analyzing the SC operational performance.

At the operational level, SCs can be viewed as Discrete Event Dynamical Systems (DEDSs), whose dynamics depends on the interaction of discrete events, such as customer demands, departure of parts or products from entities, arrival of transporters at facilities, start of assembly operations at manufacturers, arrival of finished goods at customers etc (Viswanadham & Raghavan, 2000). Among the available DEDS analytical formalisms, Petri Nets (PNs) may be singled out as a graphical and mathematical technique to model systems concurrency and synchronization. Moreover, PNs are able to capture precedence relations and structural interactions and may be executed in standard engineering software packages simply implementing their dynamics via the corresponding matrix equations. However, most SC models based on PNs proposed in the related literature share the limitation that products are modelled by means of discrete quantities, called tokens (Desrochers et al., 2005; Dotoli & Fanti, 2005; Elmahi et al., 2003; Viswanadham & Raghavan, 2000; Von Mevius & Pibernik, 2004; Wu & O'Grady, 2005). This assumption is not realistic in large systems with a huge amount of material flow: the state space of the SC model generally turns out to be excessively large, so that inconveniences in the simulation and performance optimization often arise, leading to large computational efforts. Since SCs are DEDSs whose number of reachable states is very large, PN formalisms using fluid approximations provide an

aggregate formulation to reduce the state space dimension (Alla & David, 1998; Silva & Recalde, 2004). Hence, hybrid PNs may be employed to describe SCs efficiently and effectively at the operational level. In this context, First Order Hybrid Petri Nets (FOHPNs) (Balduzzi et al., 2000) are an emerging formalism.

This chapter focuses on SC operational management: we employ a FOHPN model recently proposed by the authors (Dotoli et al., 2008) and implement and compare two standard operational management strategies. The model is built by a modular approach based on the bottom-up methodology (Zhou & Venkatesh, 1998): manufacturers are described by continuous transitions, buffers are continuous places and products are continuous flows routing from manufacturers, buffers and transporters. Transporters are stochastic transitions with a triangular distribution for the transportation time. Moreover, discrete places and transitions describe the financial and information flows that enable, inhibit or change the material flow. Discrete exponential transitions model information about demands and occurrence of unpredictable events, e.g. the blocking of a supply or an accident in a transportation facility. The FOHPN model allows us to address two issues: system management and optimal mode of operation. The SC management is realized by PN structures that synthesize two well-known policies, namely Make-To-Stock and Make-To-Order, and standard inventory control rules. While the SC optimization models presented in the related literature determine the decision parameters off-line (e.g. see Gaonkar & Viswanadham, 2001; Vidal & Goetschalckx, 1997; Viswanadham, 1999; Viswanadham & Gaonkar, 2003) in order to design and manage the SC, the task of the considered model is selecting some operational SC parameters in a short time, based on knowledge of the system state and of the occasional uncontrollable events. More precisely, the optimal mode of operation is obtained by the computation of control variables, i.e., the instantaneous firing speeds of continuous transitions, solving a linear programming problem optimizing a chosen performance index. A representative SC example shows the technique effectiveness under the two operational management policies and by a standard inventory control rule, considering in each case a different optimal operative condition. Simulation results show that the selected formalism leads to an effective SC operational management, as well as to the possibility of choosing important system control parameters, e.g. the production rates. Future research includes implementing additional management policies and inventory rules in the chosen formalism.

2. The system description

2.1 The SC structure

The SC structure is typically described by a set of facilities with materials that flow from the sources of raw materials to subassembly producers and onwards to manufacturers and consumers of finished products. Moreover, feedback paths may be present if demanufacturers or recyclers are included in the SC. The SC facilities are connected by transporters of materials, semi-finished goods and finished products. More precisely, the entities of a SC are the following:

1. *Suppliers*: a supplier is a facility that provides raw materials, components and semifinished products to manufacturers that make use of them.
2. *Manufacturers and assemblers*: manufacturers and assemblers are facilities that transform input raw materials/components into desired output products.

3. *Distributors*: distributors are intermediate nodes of material flows representing agents with exclusive or shared rights for the marketing of an item.
4. *Retailers or customers*: retailers or customers are sink nodes of material flows.
5. *De-manufacturers or recyclers*: entities of the de-manufacturing stage feed recovered material, components or energy back to suitable upstream SC facilities.
6. *Logistics and transporters*: storage systems and transporters play a critical role in distributed manufacturing. The attributes of logistics facilities are storage and handling capacities, transportation times, as well as operation and inventory costs.

Here, part of the logistics, such as storage buffers, is considered pertaining to manufacturers, suppliers and customers. Moreover, transporters connect the different stages of the production process.

The SC dynamics is traced by the flow of products between facilities (i.e., entities of types 1-5) and transporters (i.e., entities of type 6). Because of the large amount of material flowing in the system, we model a SC as a hybrid system: the continuous dynamics models the flow of products in the SC, the manufacturing and the assembling of different products and its storage in appropriate buffers. Hence, resources with limited capacities are represented by continuous states describing the amount of fluid material that the resource stores.

Moreover, we consider also discrete events occurring stochastically in the system, such as:

a. the blocking of the raw material supply, e.g. modeling the occurrence of labour strikes, accidents or stops due to the shifts;
b. the blocking of the transport operations due to the shifts or to unpredictable events such as jamming of transportation routes, accidents, strikes of transporters etc.;
c. the start of a request from the retailers.

2.2 An example of SC

We describe an example of SC whose target product is a desktop computer system (Dotoli et al., 2008), inspired from a case study reported in (Dotoli et al., 2006; Luo et al., 2001). Figure 1 depicts the SC network, comprising three suppliers, two manufacturers, one distributor, two retailers and one de-manufacturer. Moreover, twelve transporters connect the facilities. Each edge represents the flow of the material and is labelled by the parts/products that are transported between the connected facilities: the Personal Computer or PC, the central processing unit or C, the hard disk driver or H, the keyboard or K and the monitor or M. In particular, the last four types of products are semi-finished products obtained by the suppliers S1, S2 and S3, while the PC is produced by manufacturer M1 (M2) with a bill of materials of C, H, M and K obtained from suppliers S1 and S2 (S3). Moreover, retailers R1 and R2 obtain from distributor D1 the product PC. In addition, the de-manufacturer DM1 obtains the finished product PC from the retailers and supplies manufacturer M1 (M2) with the semi-finished product H (C). Note that the SC scheme includes two inter-twined productive chains with a remarkable advantage: if a transportation link is temporarily unavailable the productive cycle does not stop.

2.3 SC management and inventory control rules

The operational SC dynamics depends on the considered planning and management methodology, which specifies the business model and determines the paths for the information and material flow in the SC, and on the corresponding inventory control rules governing each SC facility (Viswanadham & Raghavan, 2000).
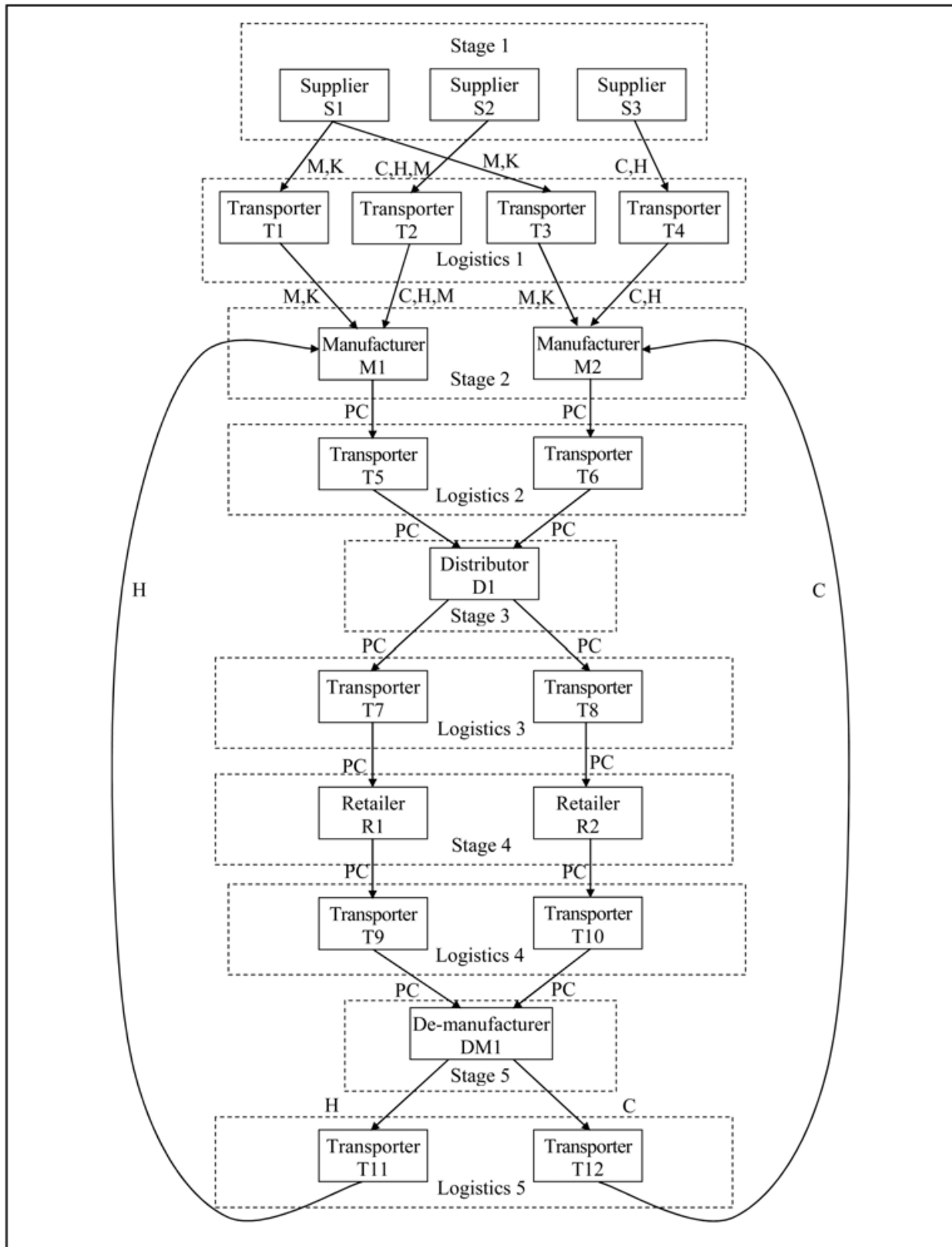
Fig. 1. The structure of the considered example SC.

According to the Wortmann classification (Wortmann, 1983), three SC managing policies are followed in practice: Make-To-Stock (MTS), Make-To-Order (MTO) and Assemble-To-Order (ATO) or Build-To-Order (BTO). In particular, in order to deliver on time the produced goods to end-users, the MTS strategy governs the system initiating production before the actual occurrence of demands, so that end customers are satisfied from stocks of inventory of finished goods. Often, under MTS management, stocks in the SC are governed by a reorder point based policy and inventory is replenished as soon as a preset reorder level is reached, so that the target level is maintained. On the other hand, in the MTO technique customer orders trigger the flow of materials and the requirements at each production stage of the SC. Hence, under such a management strategy, a lower level of material and product inventory is maintained. Furthermore, the ATO or BTO policy can be viewed as a hybrid of the former two strategies, basically applying MTS in the first stages of the SC and MTO in the last stages (Viswanadham & Raghavan, 2000). An additional production management choice is made between push and pull strategies (Hopp & Spearman, 2004): a pull production policy explicitly limits the amount of work-in-process that can be in the system, while in a push one no explicit limit on the amount of work-in-process is defined. Wellknown push systems are the Material Requirements Planning (MRP) approach and the Manufacturing Resources Planning technique (also known as MRP II) (Hopp & Spearman, 2004). Well-established pull strategies are the just-in-time technique based on Kanbans and the CONWIP (or CONstant Work-In-Process) strategy (Hopp & Spearman, 2004; Spearman et al., 1990), which is a generalized form of the Kanban policy: while the latter procedure establishes a fixed limit on work-in-process in a part of the system via the limited number of Kanban cards, the CONWIP strategy limits the total number of parts allowed in the whole system at any time, so that the SC is controlled at a constant level of work-in-process.

Together with the operational planning and management policy, inventory systems play a very important role in SC management. Inventory management addresses two fundamental issues: when a stock should replenish its inventory (order timing choice) and how much it should order from suppliers for each replenishment (order size choice) (Chen et al., 2005). These choices have to be adequately made in order to protect the SC from uncertainties, such as variations from their nominal values of demand quantity and mix, of production and transportation capacities, of quality and reliability of deliveries etc. (Rota et al., 2002). In particular, the numerous inventory management models proposed in the related literature may be mainly classified in four types, depending on order frequency and quantity. More precisely, order frequency may be either fixed, as in periodic review systems (T), or variable (R); similarly, order quantity may be either fixed, as in continuous review systems (Q), or variable (L). Accordingly, we may distinguish the following categories of inventory management rules (Vollmann et al., 2004):

i. (T,Q), in which orders are emitted with given frequency 1/T and ordered quantities are fixed and equal to Q, as in the well known economic order quantity model;

ii. (R,Q), where fixed quantities of parts that are Q in number are ordered any time the stock level drops below the reorder point R, as in the reorder point based rules;

iii. (T,L), where at every time step T a variable quantity of material is ordered so as to reach the preset desired level L, as in the reorder level rules;

iv. (R,L), where variable quantities are ordered to reach the preset level L each time the inventory level drops below the reorder point R.

Summing up, under the chosen SC management technique, a customer order for a product triggers a series of activities in the SC entities, and these have to be synchronized, so that the consumer demand and the selected inventory control rules are simultaneously satisfied.

This chapter focuses on SCs governed either by the MTS policy, which is typical of standardized products with high volumes (and reasonably accurate forecasts), or by the MTO strategy, which is characteristic of customized goods with low volumes. For the sake of brevity the ATO or BTO strategy is not considered in detail, although the presented SC model could be straightforwardly adapted to systems governed by such a strategy. Moreover, the (R,Q) inventory control rule is applied to manage the inventory of buffers that are governed as follows. Any time a withdrawal is made, a control system tracks the remaining inventory level of the buffer of products to determine whether it is time to reorder: in practice, thanks to automation and information systems, these reviews are continuous. At each review, the inventory level is compared with the pre-set reorder point R. In case the inventory level is higher than R, then no change in the inventory occurs. On the contrary, if the inventory level is lower than R, then a fixed quantity Q of products or lots of the considered items is ordered upstream, i.e., Q products or lots are manufactured if the considered stock level refers to an output product, or else they are ordered from an upstream facility in the SC.

3. First order hybrid Petri nets

In this section we briefly outline the basics of the FOHPN formalism (Balduzzi et al., 2000).

3.1 The FOHPN structure and marking

A FOHPN is a bipartite digraph described by the seven-tuple $PN=(P, T, \textbf{\textit{Pre}}, \textbf{\textit{Post}}, \Delta, F, RS)$. The set of places $P=P_d \cup P_c$ is partitioned into a set of discrete places P_d (represented by circles) and a set of continuous places (represented by double circles).

The set of transitions $T=T_d \cup T_c$ is partitioned into a set of discrete transitions T_d and a set of continuous transitions T_c (represented by double boxes). Moreover, the set of discrete transitions $T_d=T_I \cup T_S \cup T_D$ is further partitioned into a set of immediate transitions T_I (represented by bars), a set of stochastic transitions T_S (represented by boxes and including exponentially distributed transitions as well as transitions with triangular distribution) and a set of deterministic timed transitions T_D (represented by black boxes). We also denote $T_t=T_S \cup T_D$, indicating the set of timed transitions.

The matrices $\textbf{\textit{Pre}}$ and $\textbf{\textit{Post}}$ are the pre-incidence and the post-incidence matrices, respectively, of dimension $|P| \times |T|$. Note that the symbol $|A|$ denotes the cardinality of set A. Such matrices specify the net digraph arcs and are defined as follows:

$$\textbf{\textit{Pre}}, \textbf{\textit{Post}} : \begin{cases} P_c \times T \to \mathbb{R}^+ \\ P_d \times T \to \mathbb{N} \end{cases}.$$

We require that for all $t \in T_c$ and for all $p \in P_d$ it holds $\textbf{\textit{Pre}}(p,t)=\textbf{\textit{Post}}(p,t)$ (well-formed nets).

The function $\Delta: T_t \to \mathbb{R}^+$ specifies the timing associated to timed transition. In particular, we associate to each $t_j \in T_S$ the average firing delay $\Delta(t_j)= \delta_j=1/\lambda_j$, where λ_j is the average firing rate of the transition. More precisely, in case the transition is exponential δ_j represents the expected value of the associated distribution, while in case it is triangular δ_j represents the

modal value of such a distribution and we assume that the minimum and maximum values of the range in which the firing delay varies equal respectively $d_{\delta j}=0.8 \ \delta_j$ and $D_{\delta j}=1.2 \ \delta_j$. In addition, each $t_j \in T_D$ is associated the constant firing delay $\Delta(t_j)= \delta_j$. Moreover, the function F: $T_c \rightarrow \mathbb{R}^+ \times \mathbb{R}^+_\infty$ specifies the firing speeds associated to continuous transitions (we denote $\mathbb{R}^+_\infty = \mathbb{R}^+ \cup \{+\infty\}$). For any continuous transition $t_j \in T_c$ we let $F(t_j)=(V_{mj}, V_{Mj})$, with $V_{mj} \le V_{Mj}$, where V_{mj} represents the minimum firing speed and V_{Mj} the maximum firing speed of the generic continuous transition. Finally, the function RS: $T_d \rightarrow \mathbb{R}^+$ associates a probability value called random switch to conflicting discrete transitions.

Given a FOHPN and a transition $t \in T$, the following place sets may be defined: $\bullet t = \{p \in P$: $Pre(p,t)>0\}$ (pre-set of t); $t \bullet = \{p \in P$: $Post(p,t)>0\}$ (post-set of t). Moreover, the corresponding restrictions to continuous or discrete places are defined as $^{(d)}t = \bullet t \cap P_d$ or $^{(c)}t = \bullet t \cap P_c$. Similar notations may be used for pre-sets and post-sets of places. The incidence matrix of the net is defined as $C=Post-Pre$. The restriction of C to P_X and T_Y (with X,Y$\in \{c, d\}$) is denoted by C_{XY}.

A marking $\mathbf{m} : \begin{cases} P_d \rightarrow \mathbb{N} \\ P_c \rightarrow \mathbb{R}^+ \end{cases}$ is a function that assigns to each discrete place a non-negative

number of tokens, represented by black dots, and to each continuous place a fluid volume; m_i denotes the marking of place p_i. The value of a marking at time τ is denoted by $\mathbf{m}(\tau)$. The restrictions of $\mathbf{m}$ to P_d and to P_c are denoted by $\mathbf{m}_d$ and $\mathbf{m}_c$, respectively. A FOHPN system $\langle PN, \mathbf{m}(\tau_0) \rangle$ is a FOHPN with initial marking $\mathbf{m}(\tau_0)$.

The following statements rule the firing of continuous and discrete transitions:

1. a discrete transition $t \in T_d$ is enabled at $\mathbf{m}$ if for all $p_i \in \bullet t$, $m_i > \boldsymbol{Pre}(p_i,t)$;
2. a continuous transition $t \in T_c$ is enabled at $\mathbf{m}$ if for all $p_i \in ^{(d)}t$, $m_i > \boldsymbol{Pre}(p_i,t)$.

Moreover, we say that an enabled transition $t \in T_c$ is strongly enabled at $\mathbf{m}$ if for all places $p_i \in ^{(c)}t$, $m_i > 0$; we say that transition $t \in T_c$ is weakly enabled at $\mathbf{m}$ if for some $p_i \in ^{(c)}t$, $m_i = 0$.

In addition, for any continuous transition $t_j \in T_c$ its IFS is indicated by v_j and it holds:

1. if t_j is not enabled then $v_i=0$;
2. if t_i is strongly enabled, then it may fire with any firing speed $v_j \in [V_{mj}, V_{Mj}]$;
3. if t_j is weakly enabled, then it may fire with any firing speed $v_j \in [V_{mj}, V_j]$, where $V_j \le V_{Mj}$ depends on the amount of fluid entering the empty input continuous place of t_i.

We denote by $v(\tau)=[v_1(\tau) \ v_2(\tau) \dots \ v_{|Tc|}(\tau)]^T$ the IFS vector at time τ. Hence, any admissible IFS vector v at $\mathbf{m}$ is a feasible solution of the following set of linear constraints:

$$
\begin{aligned}
V_{Mj} - v_j &\ge 0 & \forall t_j &\in T_\varepsilon(\mathbf{m}) \\
v_j - V_{mj} &\ge 0 & \forall t_j &\in T_\varepsilon(\mathbf{m}) \\
v_j &= 0 & \forall t_j &\in T_\upsilon(\mathbf{m}) \\
\sum_{t_j \in T_\varepsilon(\mathbf{m})} C(p,t_j)v_j &\ge 0 & \forall p &\in P_\varepsilon(\mathbf{m}),
\end{aligned}
\tag{1}
$$

where $T_\varepsilon(\mathbf{m}) \subset T_c$ ($T_\upsilon(\mathbf{m}) \subset T_c$) is the subset of continuous transitions that are enabled (not enabled) at $\mathbf{m}$ and $P_\varepsilon(\mathbf{m}) = \{p_i \in P_c \ | \ m_i=0\}$ is the subset of empty continuous places. In particular, the first three constraints in (1) follow from the firing rules of continuous transitions, while the last constraint in (1) imposes that if a continuous place is empty then its fluid content does not become negative.

The set of all feasible solutions of (1) is denoted as $S(PN,\mathbf{m})$.

3.2 The FOHPN dynamics

The dynamics of the hybrid net combines both time-driven and event-driven dynamics. We define *macro-events* the events that occur when:

i.   a discrete transition fires or the enabling/disabling of a continuous transition takes place;

ii.   a continuous place becomes empty.

The equation that governs the time-driven evolution of the marking of a place $p_i \in P_c$ is:

$$\dot{m}_i(\tau) = \sum_{t_j \in T_c} C(p_i, t_j) v_j(\tau) \tag{2}$$

Now, if τ_k and τ_{k+1} are the occurrence times of two subsequent macro-events, we assume that within the time interval $[\tau_k, \tau_{k+1})$ (macro-period) the IFS vector $v(\tau_k)$ is constant. Then the continuous behavior of an FOHPN for $\tau \in [\tau_k, \tau_{k+1})$ is described by:

$$\begin{aligned} \mathbf{m}^c(\tau) &= \mathbf{m}^c(\tau_k) + \mathbf{C_{cc}} v(\tau_k)(\tau - \tau_k) \\ \mathbf{m}^d(\tau) &= \mathbf{m}^d(\tau_k) \end{aligned} \tag{3}$$

The evolution of the net at the firing of a discrete transition $t_j \in T_d$ at $\mathbf{m}(\tau_k-)$ yields the following marking:

$$\begin{aligned} \mathbf{m}^c(\tau_k) &= \mathbf{m}^c(\tau_k^-) + \mathbf{C_{cd}} \boldsymbol{\sigma}(\tau_k) \\ \mathbf{m}^d(\tau_k) &= \mathbf{m}^d(\tau_k^-) + \mathbf{C_{dd}} \boldsymbol{\sigma}(\tau_k), \end{aligned} \tag{4}$$

where $\sigma(\tau_k)$ is the firing count vector associated to the firing of transition t_j at time τ_k.

Moreover, we associate to each timed transition $t_j \in T_t$ a timer v_j and we call $\mathbf{v}(\tau_k)$ the vector of timers associated to timed transitions at time τ_k. Hence, the timer evolution within the macro-period $[\tau_k, \tau_{k+1})$ for each transition $t_j \in T_t$ is as follows:

$$v_j(\tau_{k+1}) = \begin{cases} v_j(\tau_k) = 0 & \text{if } t_j \text{ is not enabled,} \\ v_j(\tau_k) + (\tau - \tau_k) & \text{if } t_j \text{ is enabled,} \end{cases} \quad \text{for } j=1,\ldots,|T_t|. \tag{5}$$

Whenever t_j is disabled or it fires, its timer is reset to zero.

Equations (3)-(4)-(5) describe the dynamics of the FOHPN model. The overall state of the system at time τ_k is given by the marking of all places and by the values of all timers and is hence indicated by $\mathbf{x}(\tau_k) = \begin{bmatrix} \mathbf{m}^c(\tau_k) \\ \mathbf{m}^d(\tau_k) \\ \mathbf{v}(\tau_k) \end{bmatrix}$.

Moreover, the system input is vector $\mathbf{u}(\tau_k) = \begin{bmatrix} \tau_{k+1} - \tau_k \\ \boldsymbol{\sigma}(\tau_{k+1}) \end{bmatrix}$, collecting the length of the current macro-period and the transition (if any) that will fire at the end of such macro-period. Note that this input vector depends on the current state vector $\mathbf{x}(\tau_k)$ and on the next macro-event occurring at the end of the current macro-period. Consequently, a FOHPN system (3)-(4)-(5) can be described by a linear discrete-time time-varying state variable model, so that an efficient simulation algorithm determining the state vector at the beginning of each macroperiod, given the initial state $\mathbf{x}(\tau_k)$, may be straightforwardly derived.

3.3 The FOHPN control

Once the set of all admissible IFS vectors has been defined, a procedure is required to select one among them and thus let equation (3) of the net dynamics be univocally determined. In other words, each IFS vector $v \in S(PN, \mathbf{m})$ solving (1) represents a particular mode of operation of the system described by the FOHPN. Consequently, the designer may choose the best operative condition according to a given objective and solving the corresponding optimization problem with the constraint set (1). In this chapter the following two cases are considered.

1) Maximize flows. We may consider as optimal the solution v^* of (1) that maximizes the performance index $J = \mathbf{1}^T \cdot v$, which is intended to maximize the sum of all the flow rates. In the manufacturing domain, this corresponds to maximizing resource utilization.

2) Minimize stored fluid. We may choose as optimal the vector v^* solving (1) that minimizes the derivative of the marking of each place $p \in P_c$. This can be done by minimizing the performance index $J = \mathbf{c}^T \cdot v$ where $c_j = C(p, t_j)$ if $t_j \in p^{(c)} \cup {}^{(c)}p$ and 0 otherwise. In the manufacturing domain, this corresponds to minimizing the work-in-process.

3.4 An example of FOHPN

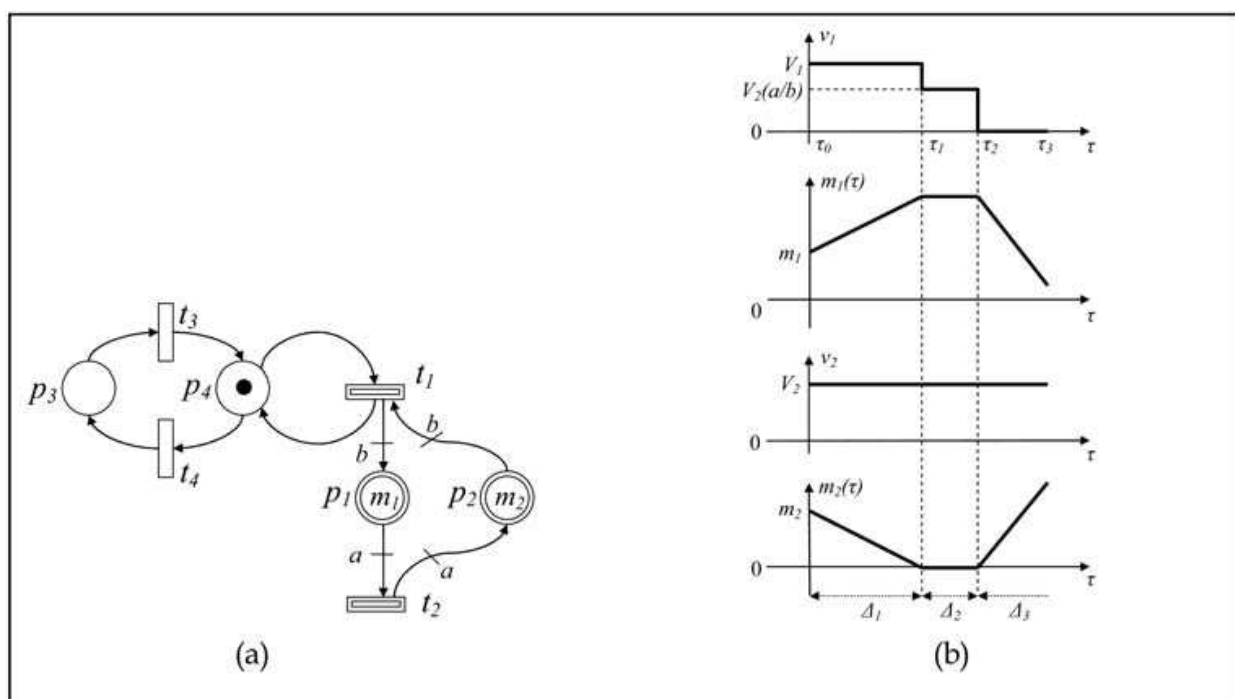In this section we describe an example of FOHPN in order to clarify its dynamics.

Fig. 2. An example of FOHPN (a) and its evolution (b).

Consider the net in Fig. 2a. Places p_1 and p_2 are continuous places and places p_3 and p_4 are discrete places. Transitions t_1 and t_2 are continuous transitions with firing speeds $v_1 \in [0, V_1]$ and $v_2 \in [0, V_2]$, respectively. We assume $V_1 b > V_2 a$ (here a and b are the arc weights in Fig. 2a). In addition, the discrete transitions t_3 and t_4 are exponentially distributed timed transitions with average firing rates λ_3 and λ_4, respectively.

The net dynamics, depicted in Fig. 2b, is described as follows. Since place p_4 is initially marked, transition t_1 is enabled. Moreover, the initial markings of the continuous places are $m_1(\tau_0)>0$ and $m_2(\tau_0)>0$ so that transitions t_1 and t_2 are both strongly enabled and may fire according to the set of constraints (1):

$$\begin{cases} V_1 - v_1 \geq 0 \\ V_2 - v_2 \geq 0 \\ v_1, v_2 \geq 0. \end{cases} \quad (6)$$

We assume $v_1=V_1$ and $v_2=V_2$. By (3), the continuous marking of the net during this first macro-period Δ_1 is $\boldsymbol{m}^c(\tau) = \begin{cases} m_1(\tau) = m_1(\tau_0) - (V_2 a - V_1 b)(\tau - \tau_0) \\ m_2(\tau) = m_2(\tau_0) - (V_1 b - V_2 a)(\tau - \tau_0) \end{cases}$ for $\tau > \tau_0$ until the subsequent macro-event. Moreover, by (5) the timer vector is $\boldsymbol{v}(\tau) = [0 \ \tau - \tau_0]^T$ for $\tau > \tau_0$, since t_3 is disabled and t_4 is enabled. Figure 2b shows the corresponding marking evolution and the IFSs of the net continuous transitions. In particular, we remark that the marking m_1 increases while m_2 decreases since it holds $V_1 b > V_2 a$.

At time τ_1 a macro-event occurs because place p_2 becomes empty. Consequently, t_1 becomes weakly enabled and the set of constraints (1) has to be re-written as follows:

$$\begin{cases} V_1 - v_1 \geq 0 \\ V_2 - v_2 \geq 0 \\ v_1, v_2 \geq 0 \\ v_2 a - v_1 b \geq 0. \end{cases} \quad (7)$$

Since t_2 remains strongly enabled, its firing speed is assumed $v_2=V_2$. On the other hand, we choose the firing speed of t_1 as $v_1=V_2(a/b)$. Therefore, during the macro-period Δ_2, by (3) the continuous marking is expressed by $\boldsymbol{m}^c(\tau) = \begin{cases} m_1(\tau) = m_1(\tau_1) \\ m_2(\tau) = 0 \end{cases}$ for $\tau > \tau_1$ until the subsequent macro-event (see Fig. 2b). Moreover, by (5) it holds $\boldsymbol{v}(\tau) = [0 \ \tau - \tau_1]^T$ for $\tau > \tau_1$.

Next, suppose that at time τ_2 transition t_4 fires and the macro-event updates the discrete markings to $m_3(\tau_2)=1$ and $m_4(\tau_2)=0$. Hence, t_1 is disabled, i.e., $v_1=0$, while t_2 remains strongly enabled and we assume $v_2=V_2$. Then, during the macro-period Δ_3 the marking is given, as in (5), by $\boldsymbol{m}^c(\tau) = \begin{cases} m_1(\tau) = m_1(\tau_2) - V_2 a(\tau - \tau_2) \\ m_2(\tau) = m_2(\tau_2) + V_2 a(\tau - \tau_2) \end{cases}$ (see Fig. 2b). Moreover, by (5) it holds $\boldsymbol{v}(\tau) = [\tau - \tau_2 \ 0]^T$ for $\tau > \tau_2$.

4. The SC model

Based on the idea of the bottom-up approach (Zhou & Venkatesh, 1998), this section reviews a modular FOHPN model to describe a SC previously proposed by the authors (Dotoli et al., 2008). Such a method can be summarized in two steps: decomposition and composition. Decomposition consists in partitioning a system into several subsystems. In SCs this subdivision can be performed based on the determination of distributed system entities (i.e., suppliers, manufacturers, distributors, customers and transporters). All these subsystems are modelled by FOHPNs modules. On the other hand, composition involves the interconnections of these sub-models into a complete model, representing the whole SC.

In particular, manufacturers are described by continuous transitions, buffers are continuous places and products are represented by continuous flows (fluids) routing from manufacturers, buffers and transporters. Moreover, transporters are described by discrete stochastic transitions with a triangular distribution and the customers demand is modelled by exponential transitions. In addition, exponential transitions model discrete events occurring stochastically in the system, such as the blocking of a raw material supply or of a transport operation due to unpredictable events. Hence, the state of the SC model at the beginning of each macro-period is a vector $\mathbf{x}(\tau_k)$ that includes the following sub-vectors:

- the sub-vector $\mathbf{m}^c(\tau_k)$, collecting the markings of the continuous places, i.e., the buffer places and the associated capacity places (absent for infinite capacity buffers);
- the sub-vector $\mathbf{m}^d(\tau_k)$, collecting the markings of the discrete places, i.e., the places modeling choices, constraints and the operative states of entities;
- the timers vector $\mathbf{v}(\tau_k)$, collecting the values of the timers of discrete timed transitions, i.e.,
- the transitions associated to customer demands or transporters and the transitions modeling
- the blockings of supplies or transports due to unpredictable or external events.

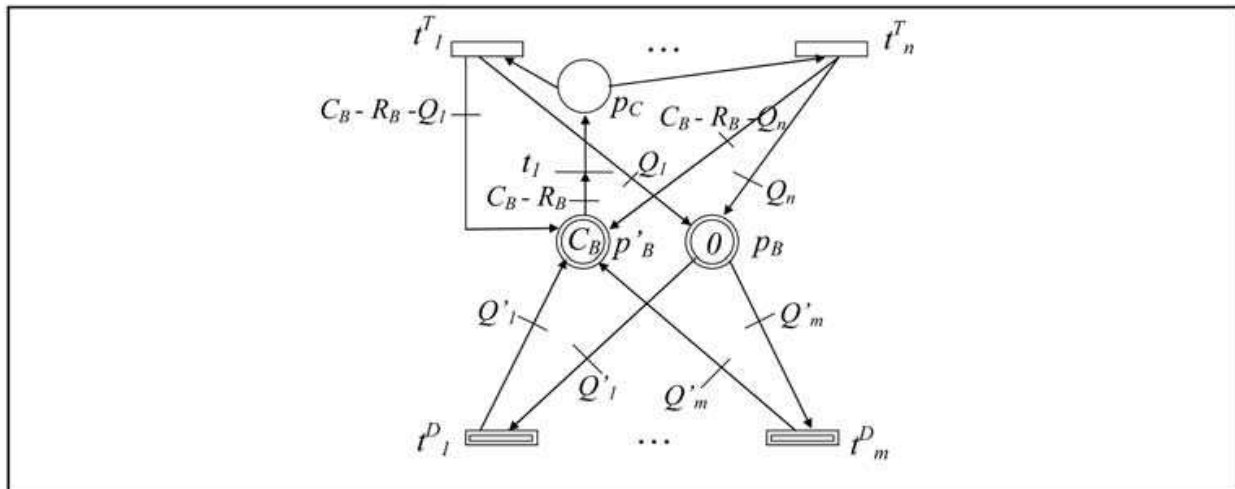The following FOHPN modules model the individual subsystems composing the SC (Dotoli et al., 2008).

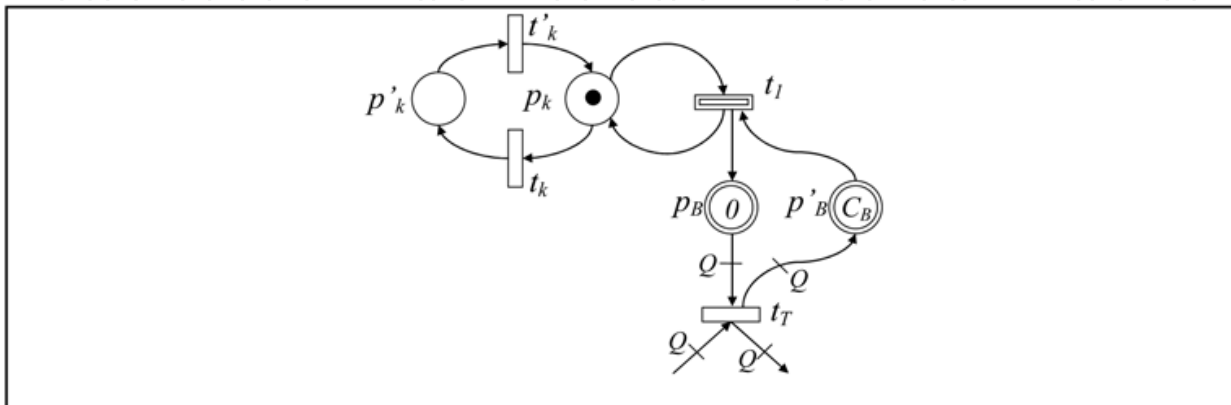Fig. 3. The FOHPN modeling the input buffers.

Fig. 4. The FOHPN modeling the suppliers.

4.1 The inventory management model of the input buffers

In this section we describe the model of the input buffers of manufacturers and distributors managed by the (R,Q) policy. On the other hand, the output buffers are not managed by the (R,Q) policy since they are devoted just to providing the requested material. The basic quantities of the (R,Q) inventory management strategy are: the *fixed order quantity Q*; the *lead time*, i.e., the time between placing an order and receiving the goods in stock; the *demand D*, i.e., the number of units to be supplied from stock in a given time period; the *reorder level R*, i.e., the new orders take place whenever the stock level falls to *R*.

Figure 3 shows the FOHPN model for the input buffers managed by the (R,Q) policy (Furcas et al., 2001). The continuous place p_B denotes the input buffer of finite capacity C_B. The complementary place p'_B models the available buffer space so that at each time instant it holds $m_B + m'_B = C_B$. Here and in the following models the assumed initial marking corresponds to empty buffers. We assume that the buffer can receive demands from different facilities and can require the goods from different transporters. Hence, each demand is modelled by a continuous transition $t^D{}_i$ with $i = 1, \ldots, m$ so that the demand to be fulfilled is $D_i = v^D{}_i Q'{}_i$. When $m_B > 0$ a transition $t^D{}_i$ with $i \in \{1, \ldots, m\}$ may fire at the firing speed v_{Di}, reducing the marking of the place p_B with a constant slope $v^D{}_i Q'{}_i$. As soon as m_B falls below the level R_B (or, equivalently, the marking m'_B goes over $C_B - R_B$), the immediate transition t_1 is enabled. When t_1 fires, the choice place $p_C \in P_d$ becomes marked and performs the choice of the input facility. Hence, new materials/products are requested by enabling one of the transitions $t^T{}_i$ according to the value of the random switches RS($t^T{}_i$) with $i = 1, \ldots, n$. If a particular transition t_{Ti} with $i \in \{1, \ldots, n\}$ is selected and fires after the lead time of average$\Delta(t^T{}_i) = \delta_i = 1/\lambda_i$, Q_i products are received in the buffer and $C_B - R_B - Q_i$ units are restored in the buffer capacity. Typically, transitions t_{Ti} can represent a transport operation.

4.2 The inventory management model of the SC entities

The supplier model. Suppliers are modelled as a continuous transition and two continuous places (see Fig. 4). The continuous place p_B represents the raw material output buffer of finite capacity C_B and the complementary place p'_B represents the available buffer space. Moreover, the continuous transition t_1 models the arrival of raw material into the system. In addition, we consider the possibility that the providing of raw material is blocked. This situation is represented by a discrete event modelled by two exponentially distributed transitions and two discrete places. In particular, place $p_k \in P_d$ models the operative state of the supplier and $p'_k \in P_d$ is the non-operative state. The blocking and the restoration of the raw material supply correspond to the firing of exponential transitions t_k and t'_k, respectively. For the sake of clarity, Fig. 4 depicts the transition $t_T \in T_S$ that, as discussed later, models the transport operation. Here and in the following models the initial marking assumes that the entity is operative.

The manufacturer and assembler model. Manufacturers and assemblers are modelled by the FOHPN shown by Fig. 5. More precisely, the continuous places p_{Bi} and p'_{Bi} with $i = 2, \ldots n$ describe the input buffers and the corresponding available capacities, respectively. Each buffer stores the input goods of a particular type. Analogously, the continuous places p_{B1} and p'_{B1} model the output buffer and its capacity, respectively. The production rate of the facility is modelled by the continuous transition t_1 with the assigned firing speed $v_1 \in [V_{m1}, V_1]$.

The transporter model. The transporters connecting the different facilities are modelled by a set of timed transitions $t^T{}_i$ with triangular distributions for $i = 1, \ldots, n$ (see Fig. 6), according to

(Kelton et al., 2004; Law & Kelton, 2000). Each transition describes the transport of items of a particular type from an upstream facility to a downstream one in an average time interval $\Delta(t^T_i) = \delta_i = 1/\lambda_i$. The control places $p_{Ci} \in P_d$ with $i=1,\dots,n$ determine the choice of only one type of material to transport among the available set by the firing of the corresponding immediate transition t_i with $i=1,\dots,n$ modeling the replenishment request to the transporter by a downstream SC entity. In addition, place $p_1 \in P_d$ disables the remaining transitions. Moreover, the random stop of the material transport is represented by two places p_k, $p'_k \in P_d$ and two exponentially distributed transitions $t_k, t'_k \in T_S$. The transporter capacity is Q and places $p_{Bi} \in P_b$ and $p'_{Bi} \in P'_b$ with $i=1,\dots n$ in Fig. 4 describe the n input buffers of the downstream facility (e.g., a manufacturer, a distributor, a retailer) and the corresponding available capacities, respectively. The initial marking shown assumes that no material has yet been selected for transportation.
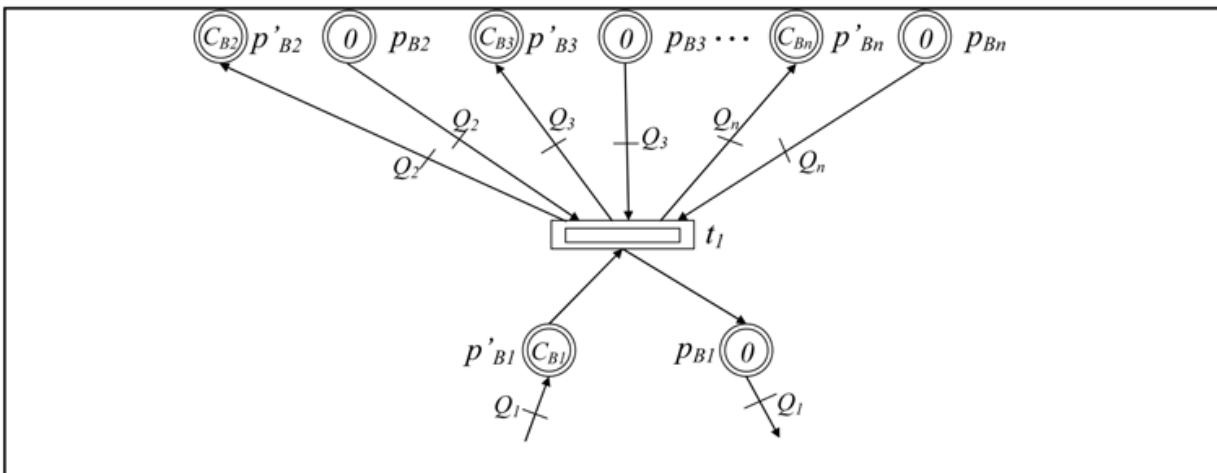

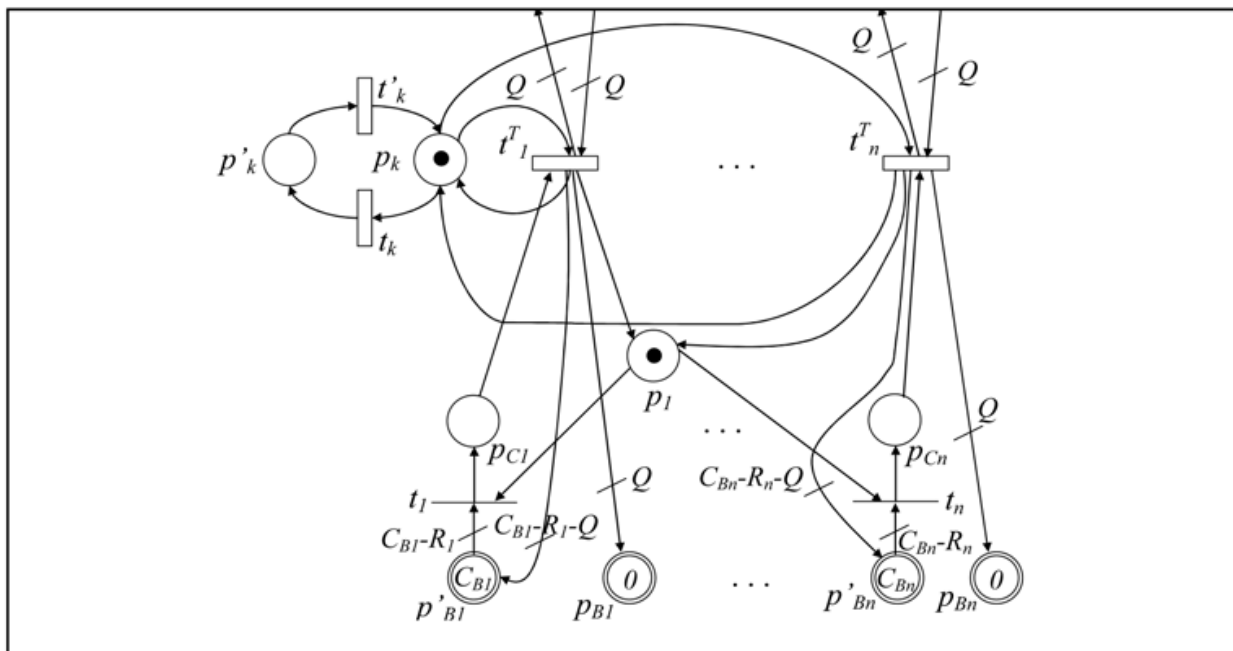Fig. 5. The FOHPN modeling manufacturers and assemblers.
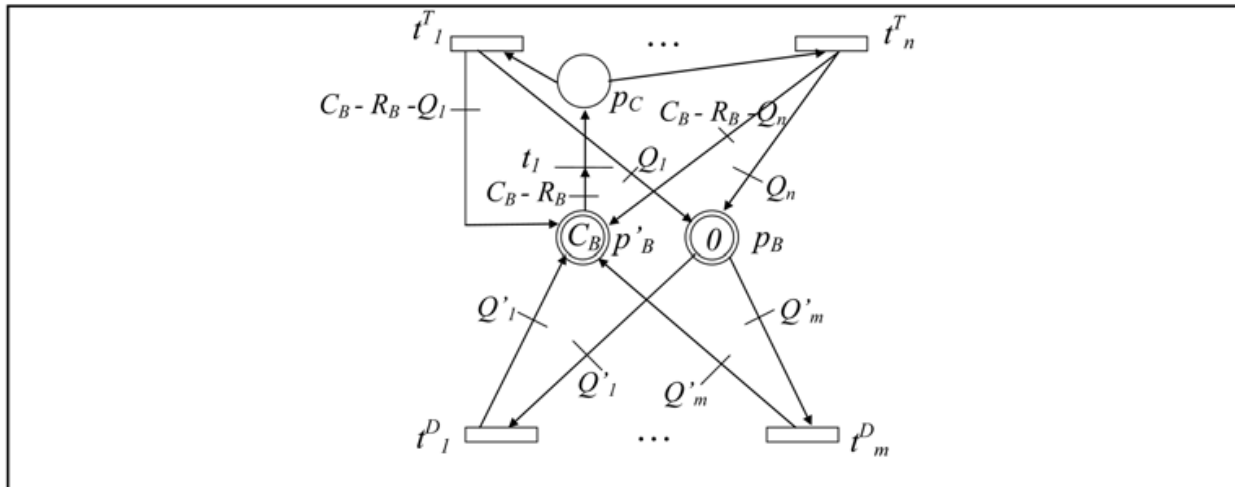

Fig. 6. The FOHPN modeling the transporters.
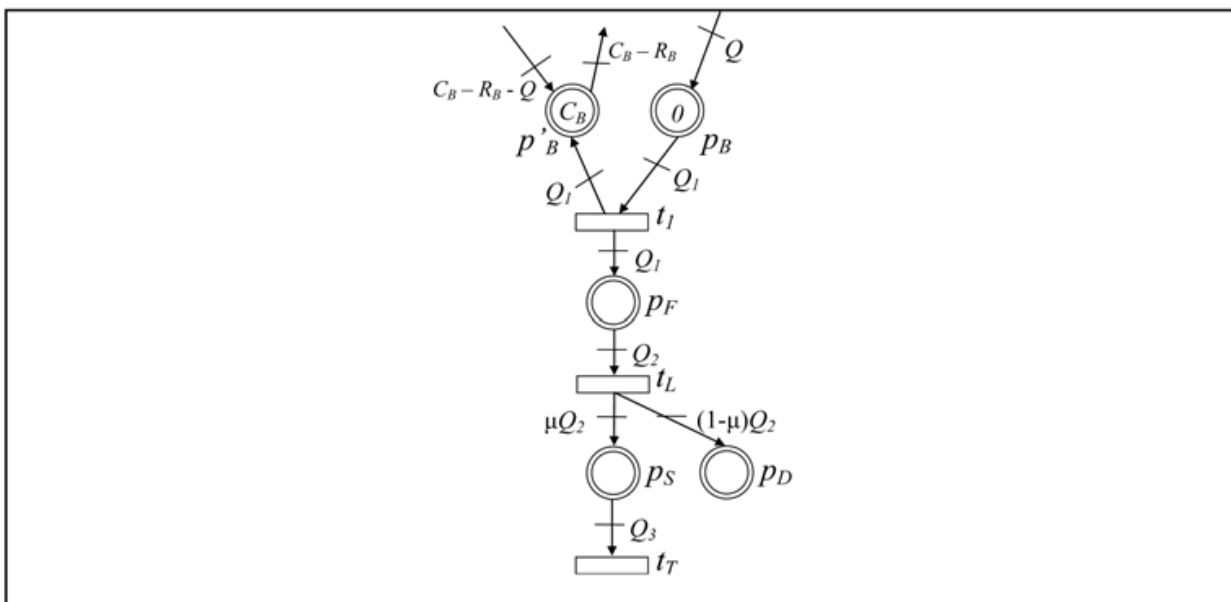
Fig. 7. The FOHPN modeling the distributors.

Fig. 8. The FOHPN modeling the retailers under the MTS strategy.

The distributor model. The model of the distributors is represented by an input buffer managed by the (R,Q) inventory control rule. Hence, the model is similar to the FOHPN represented in Fig. 3, where each downstream continuous transition t^D_i with $i=1,\dots,m$ is substituted by a stochastic timed transition representing a transport operation (see Fig. 7).

The retailer model. The retailer model is different in the two cases of the MTS and MTO management. If the MTS strategy is used, the retailer is represented by an input buffer managed by the (R,Q) policy with a finite lead time and stochastic demand (see Fig. 8). Hence, the model is similar to the FOHPN represented in Fig. 3 where all the downstream continuous transitions are substituted by one or more exponential transition (such as t_1) modeling the stochastic demand of the consumers. Moreover, the continuous place p_F

collects all the products obtained by the retailer, i.e., it corresponds to an infinite capacity buffer. In addition, the discrete transition with triangular distribution t_L models the deterioration of the finished products used by the customer that are stored in the infinite capacity buffers p_S and p_D. In particular, p_S collects the μQ_2 products to be de-manufactured with $\mu \in [0,1]$, and p_D the $(1-\mu)Q_2$ goods to be discarded. In addition, transition t_T represents the transport operation transferring products to the de-manufacturer.

Similarly, under the MTO management the retailer (see Fig. 9) is described by an output buffer place $p_B \in P_c$ and a stochastic transition t_1 modeling the rate with which the consumer withdraws products, that in this case have been previously ordered. The infinite capacity place $p_F \in P_c$ collects all the products obtained by the retailer. In addition, transition t_L models the products deterioration, place p_S denotes the system output and transition t_T represents the transport operation. The customer demand, just as under the MTS policy, is represented by one (or more) exponential transition, such as t_2. On the other hand, differently than the MTS case, when fired transition t_2 stores an order of Q products in the infinite capacity buffer place p_O. Hence, its marking m_o enables transition t_3 modeling the production of the upstream manufacturer. When the Q orders are satisfied, marking m_o becomes zero, t_3 is weakly enabled and its firing speed is equal to zero until the upstream transition t_2 fires again.

The de-manufacturer model. De-manufacturers are modelled by the FOHPN shown by Fig. 10, which is the reverse of the manufacturers model reported in Fig. 5. More precisely, p_{B1} and p'_{B1} model the input buffer and its capacity, respectively, and the continuous places p_{Bi} and p'_{Bi} with $i=2,\ldots n$ describe the output buffers and the corresponding available capacities, respectively. The continuous transition t_1 models the production (or, more properly, the disassembly) rate of the facility.
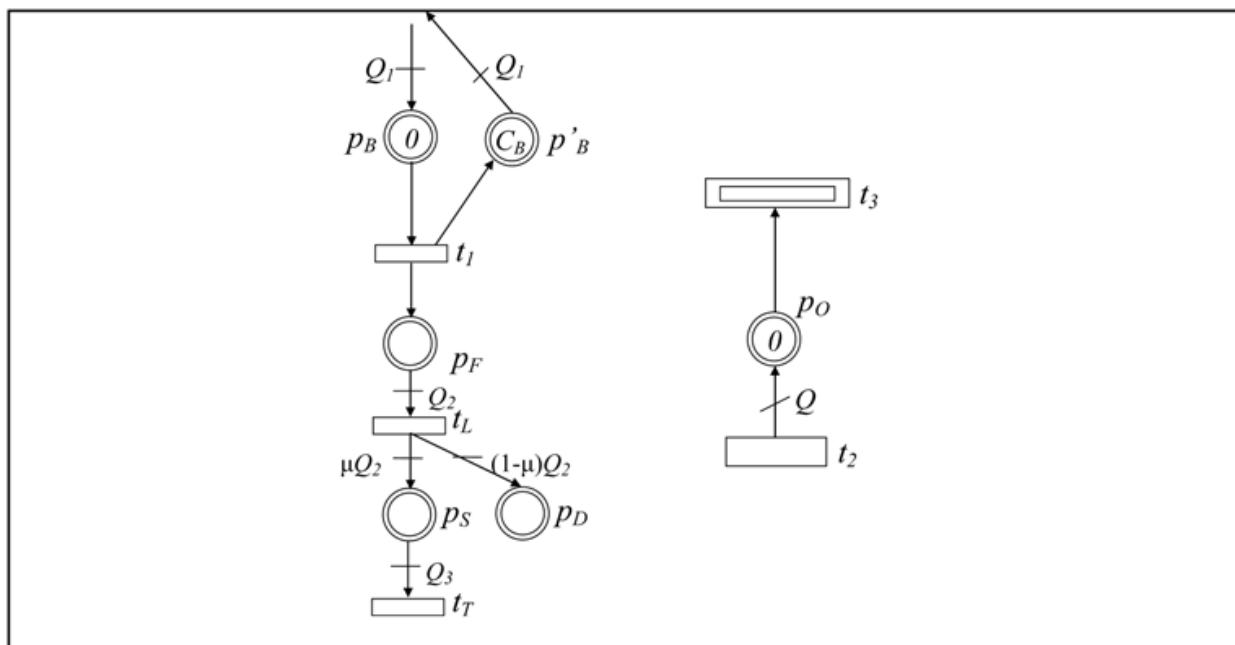

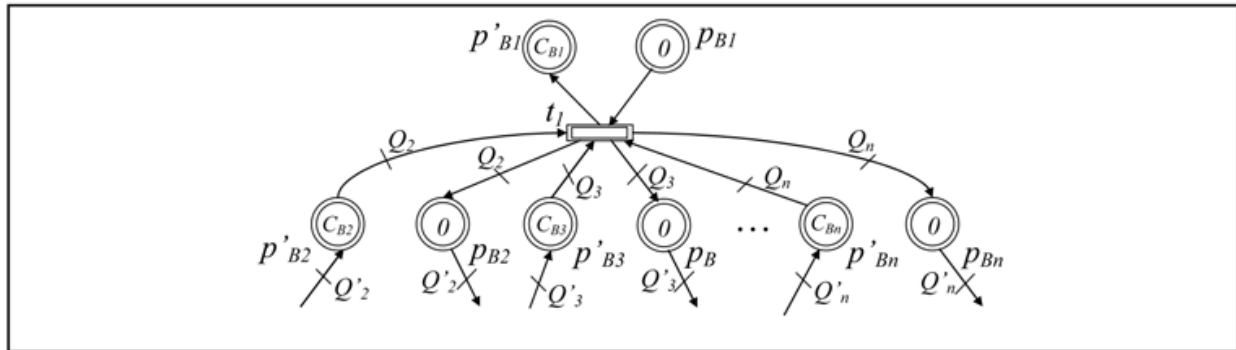Fig. 9. The FOHPN modeling the retailers under the MTO strategy.

Fig. 10. The FOHPN modeling the de-manufacturers.

5. A case study

We consider the SC of Fig. 1 described in Section 2.B. To implement and compare the system managed under the MTS and MTO strategies, we model the whole SC properly merging the elementary modules described in the previous section. Figures 11 and 12 show the FOHPN modeling the SC under MTS and MTO, respectively. The dashed rectangles depict in the two figures the correspondence between each module and the entities of Fig. 1. We remark that for the sake of simplicity in this chapter we consider only single product SCs, since our ultimate aim is to show the effectiveness of a FOHPN formalism for operational management of SCs. However, common multi-product SCs may straightforwardly be considered within the proposed formalism thanks to its simplicity and modularity.

5.1 The SC under the MTS strategy

Figure 11 shows the SC model of the case study managed by the MTS policy. The production is determined by the firing of the continuous transitions t_1, ..., t_7 (modules S1, S2 and S3) that describe the input of the raw materials that can be interrupted by stochastic events only. Consequently, under this control technique, each input buffer is managed by the (R,Q) strategy.

Moreover, if the input buffer of manufacturer M1 (M2) requires a particular product, a request has to be sent to the corresponding transporter. Hence, places p_{60}, p_{63}, p_{66} and p_{67} (modules T1 and T2) (p_{72}, p_{74}, p_{75} and p_{77} (modules T3 and T4)) are introduced to select a particular transporter. For example, if place p_{60} (module T1) is marked then the transport modelled by t_{43} (module T1) is enabled. In addition, transitions t_{56} and t_{58} and place p_{63} (modules T1 and T2) are introduced since the buffer of M1 storing the semi-finished products monitors (denoted by p_{17} and p'_{17}) can require material from S1 by T1 or from S2 by T2. Consequently, place p_{63} with transitions t_{56} and t_{58} model the choice.

Finally, according to the SC scheme of Fig. 1, in the model of Fig 11 the supply of some semifinished products at the manufacturers (i.e., H at place p_{21} in M1 and C at place p_{27} in M2) may be obtained via two different paths, either by a supplier or by the de-manufacturer. The corresponding choice of the replenishment transition to enable (i.e., t_{60} of T2 or t_{82} of T11 for M1 and t_{63} of T4 or t_{83} of T12 for M2) is modelled via a random switch that associates a 100% probability to the less costly supply obtained by the de-manufacturer (i.e., to the enabling of t_{82} and t_{83}): the corresponding transition, however, is enabled via the

respective arc weights Q_{13} and Q_{14} only when the corresponding semi-finished product output buffer of the demanufacturer (i.e., place p_{96} or p_{97} of module DM1) contains sufficient material.
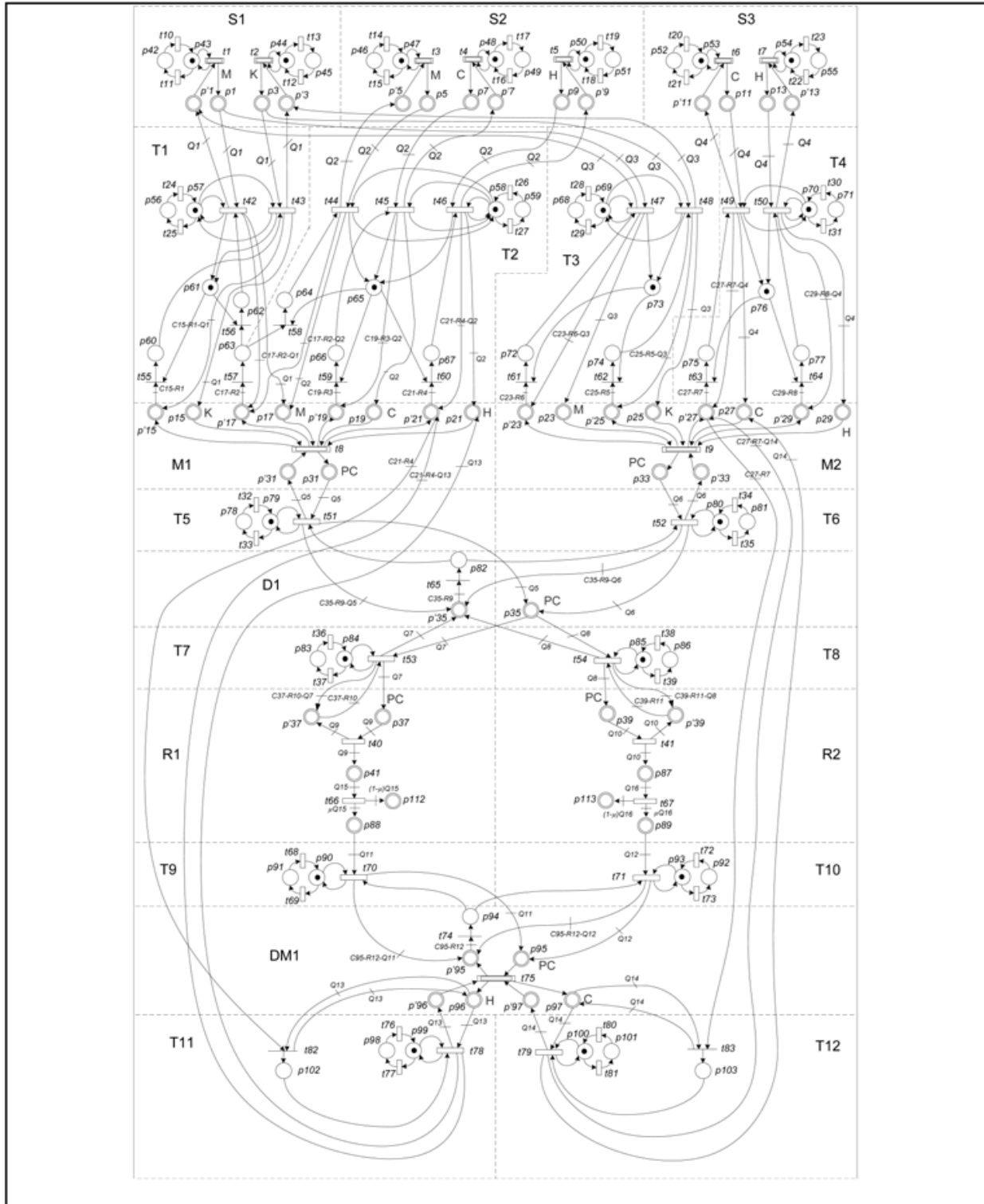

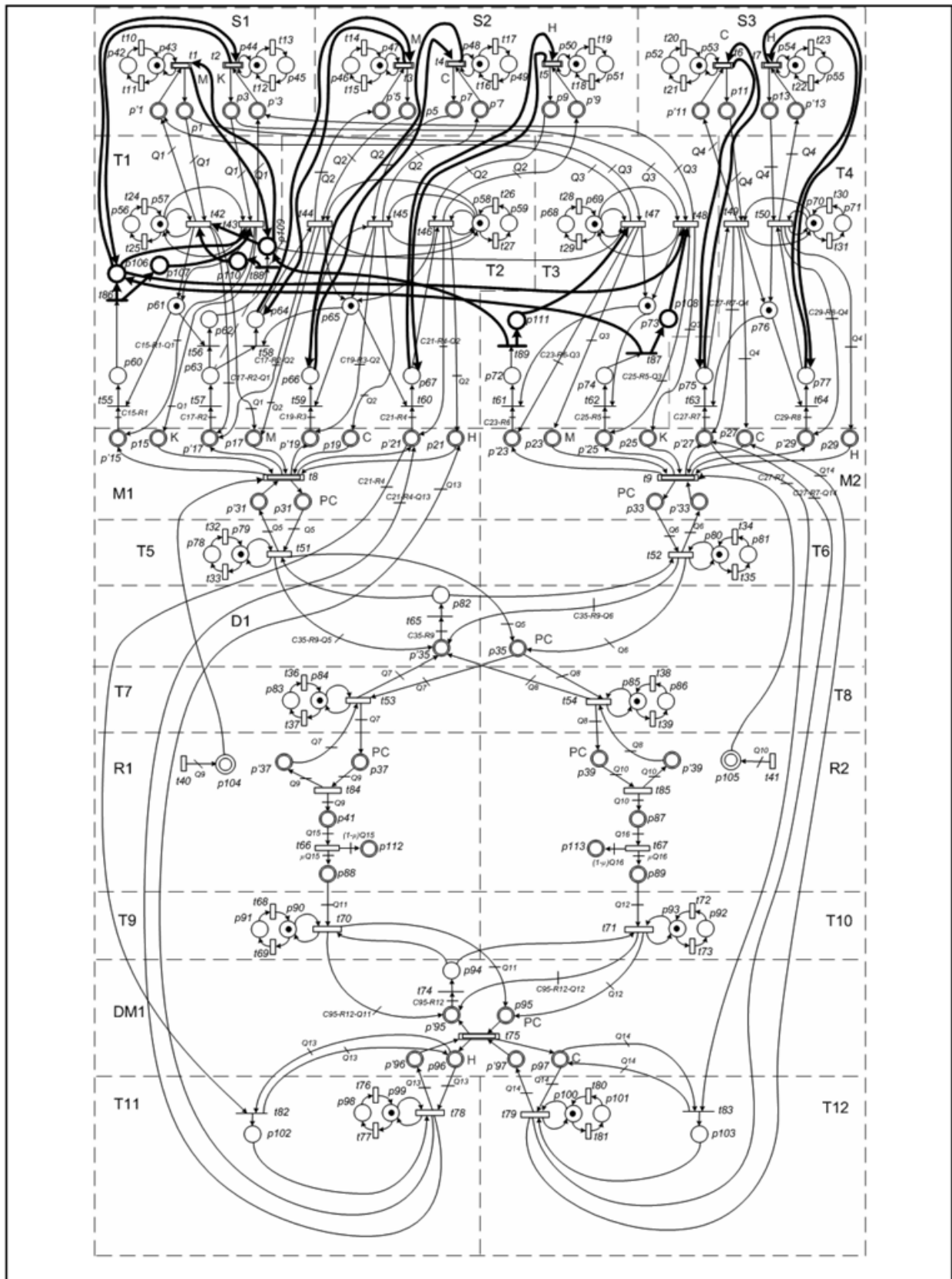Fig. 11. The FOHPN modeling the case study under the MTS policy.

Fig. 12. The FOHPN modeling the case study under the MTO policy.

5.2 The SC under the MTO strategy

Figure 12 shows the SC model of the case study managed by the MTO policy. The model is similar to the model shown in Fig. 11 with two exceptions. First, some additional edges, places and transitions are introduced and drawn in bold. Second, the retailer modules are different than those employed under the MTS strategy, as detailed in Section 5.B. Hence, the actual assembling of a finished product is triggered by the demand of a consumer that in the MTO policy is modelled by the discrete exponential transitions t_{40} and t_{41} (modules R1 and R2, respectively) and by the places p_{104} and p_{105} (modules R1 and R2, respectively). For example, if a request is present for R1 (i.e., $m_{104}>0$), then t_8 (module M1) is enabled and fires. Consequently, the markings of p_{15}, p_{17}, p_{19} and p_{21} (module M1) decrease. In such a condition, if there is material stored in each input buffers, no raw material is requested to the suppliers. Indeed, the markings $m_{106}=m_{109}=m_{64}=m_{66}=m_{67}=0$ (modules T1 and T2) disable transitions t_1, t_2, …, t_5 (modules S1 and S2). On the contrary, if for example the input buffer p_{19} (module M1) is at a low level (i.e., it holds $m_{19}\leq R_3$ and $m'_{19}\geq C_{19}-R_3$), then transition t_{59} (module T2) is enabled. If such an immediate transition fires, it holds $m_{66}=1$ and the continuous transition t_4 (module S2) fires to provide the raw material C in the buffer p_7 (module S2). When it holds $m_7\geq Q_2$, transition t_{45} (module T2) modeling the transport is enabled and fires after $\Delta(t_{45})$ time units on average. After transition t_{45} fires, it holds $m_{66}=0$ so that t_4 is disabled.

We remark that the replenishment requests of a manufacturer input buffer might influence the replenishment of other manufacturer input buffers. Hence, in order to avoid such a condition, we suitably introduce places p_{106}, p_{107}, p_{108}, p_{109}, p_{110} and p_{111} and transitions t_{86}, t_{87}, t_{88} and t_{89} (modules T1, T2 and T3, drawn in bold). As an example, a token in p_{106} (module T1) may be determined by the firing of transition t_{86} (module T1) or t_{87} (module T3). In both cases, the marking $m_{106}=1$ enables transition t_2 (module S1). The subsequent enabling of the transport transition is managed by the added places p_{107} (module T1) and p_{108} (module T3) that respectively enable the transports from supplier S1 to manufacturers M1 and M2 modelled by t_{43} (module T1) and t_{48} (module T3).

Finally, note that the supply of the semi-finished products at the manufacturers from suppliers or de-manufacturers is governed in the same way as in the previously detailed SC model managed by MTS.

5.3 The simulation specification

The SC dynamics under the MTS and MTO management strategies is analyzed via numerical simulation using the data reported in Table 1. This table shows the manufacturer production rates and the average firing delays of discrete stochastic transitions. In addition, Table 2 shows further data necessary to fully describe and simulate the system, namely the buffer capacities for the inventories of each stage and the initial markings of odd continuous places (those of the other continuous places are complementary with respect to the capacities reported in the table and hence omitted). Furthermore, the initial markings of discrete places and the values of the edge weights are depicted in Figs. 11 and 12. Moreover, Table 3 reports the reorder levels and fixed order quantities. Note that the reorder levels are set to zero when the MTO policy is used, since the production is triggered by the customers demand only. In addition, we assign a value of 0.5 to each random switch. Moreover, the fraction of consumed goods to be recycled is set equal to μ=0.5 in both retailers (see modules R1 and R2 of Figs. 11 and 12).

Continuous transitions		Discrete transitions			
	$[V_{min}, V_{max}]$	Exponential	Average firing delay (hours)	Triangular	Average firing delay (hours)
$t_1\ t_5\ t_7$	$[0, 4]$	$t_{22}\ t_{40}\ t_{72}\ t_{84}$	2	t_{53}	1
$t_2\ t_3\ t_4$	$[0, 5]$	$t_{16}\ t_{26}\ t_{34}$	3	$t_{42}\ t_{43}\ t_{70}$	2
t_6	$[0, 6]$	$t_{10}\ t_{14}\ t_{18}\ t_{76}$	4	$t_{47}\ t_{48}\ t_{78}$	2
t_8	$[0,7]$	$t_{24}\ t_{28}\ t_{32}$	4	$t_{52}\ t_{54}\ t_{79}$	2
t_9	$[0,6]$	$t_{36}\ t_{38}\ t_{80}$	4	$t_{44}\ t_{45}\ t_{71}$	3
t_{75}	$[0,7]$	$t_{20}\ t_{30}\ t_{41}\ t_{85}$	5	$t_{46}\ t_{49}$	3
		$t_{12}\ t_{68}$	6	$t_{50}\ t_{51}$	3
		$t_{13}\ t_{69}$	18	t_{66}	60
		$t_{21}\ t_{31}$	19	t_{67}	60
		$t_{11}\ t_{15}\ t_{19}$	20		
		$t_{25}\ t_{29}\ t_{33}\ t_{81}$	20		
		$t_{37}\ t_{39}\ t_{77}$	20		
		$t_{17}\ t_{27}\ t_{35}$	21		
		$t_{23}\ t_{73}$	22		

Table 1. Firing speed and average firing delay of continuous and discrete transitions.

Initial marking	Parts	Capacities	Parts
$m_1(0),\ m_5(0),\ m_{11}(0),\ m_{15}(0)$	20	$C_1,\ C_5,\ C_{11},\ C_{15}$	100
$m_{23}(0),\ m_{25}(0)$	20	$C_{23},\ C_{25}$	100
$m_{31}(0)$	20	C_{31}	150
$m_{37}(0),\ m_{39}(0)$	20	$C_{37},\ C_{39}$	70
$m_3(0),\ m_9(0),\ m_{13}(0)$	15	$C_3,\ C_9,\ C_{13}$	100
$m_7(0),\ m_{27}(0)$	25	$C_7,\ C_{27}$	100
$m_{17}(0),\ m_{19}(0),\ m_{29}(0)$	30	$C_{17},\ C_{19},\ C_{29}$	100
$m_{33}(0)$	30	C_{33}	150
$m_{21}(0)$	35	C_{21}	100
$m_{35}(0)$	0	C_{35}	120
$m_{95}(0),\ m_{96}(0),\ m_{97}(0)$	0	$C_{95},\ C_{96},\ C_{97}$	80
$m_{41}(0),\ m_{87}(0),\ m_{88}(0)$	0	/	/
$m_{89}(0),\ m_{104}(0),\ m_{105}(0)$	0	/	/

Table 2. Initial marking of odd continuous places, capacities and edge weights.

To analyze the SC, the following performance indices are selected (Viswanadham, 1999):
i.   the system throughput T, i.e., the average number of products obtained in a time unit;
ii.   the average system inventory SI, i.e., the average amount of products stored in all the system buffers during the run time TP;
iii. the average lead time LT=SI/T that is a measure of the time spent by the SC to convert the raw material in final products.

Note that in the considered simulation experiments the SI performance index (and, consequently, the LT value) is calculated taking into account only the buffers that are upstream with respect to the retailers.

The FOHPN models of the case study under the MTS and MTO policies are implemented and simulated in the Matlab environment (The Mathworks 2006), which is ideally suited when dealing with modular, numerical matrix-based large-scale systems. All the indices are evaluated by a simulation run of 600 time units with a transient period of 100 time units, so that the run time equals 500 hours if we associate one time unit to one hour. The estimates of

the performance indices are deduced by 100 independent replications with a 95% confidence interval. Besides, we evaluate the percentage value of the confidence interval half width to assess the accuracy of the performance index estimation: the half width of the confidence interval, being 3.3% in the worst case, confirms the sufficient accuracy of the performance indices estimation.

Reorder levels (parts) for the MTS case	Reorder levels (parts) for the MTO case	Fixed order quantities (parts)
$R_1=18$	$R_1=0$	$Q_1=50$
$R_2=25$	$R_2=0$	$Q_2=45$
$R_3=25$	$R_3=0$	$Q_3=55$
$R_4=25$	$R_4=0$	$Q_4=40$
$R_5=15$	$R_5=0$	$Q_5=60$
$R_6=15$	$R_6=0$	$Q_6=50$
$R_7=20$	$R_7=0$	$Q_7=30$
$R_8=20$	$R_8=0$	$Q_8=25$
$R_9=30$	$R_9=0$	$Q_9=2$
$R_{10}=10$	$R_{10}=0$	$Q_{10}=5$
$R_{11}=10$	$R_{11}=0$	$Q_{11}=35$
$R_{12}=10$	$R_{12}=0$	$Q_{12}=45$
		$Q_{13}=40$
		$Q_{14}=40$
		$Q_{15}=25$
		$Q_{16}=35$

Table 3. Reorder levels and fixed order quantities.

The simulation study is performed in three different operative conditions, respectively denoted *OCi* with i=1,2,3, and each operative condition *OCi* corresponds to a different choice of the IFS vector within the set of admissible solutions of (1).

Operative Condition 1 (OC1). At each macro-period the IFS vector *v* is selected so as to maximize the sum of all flow rates:

$$\max \ (\mathbf{1}^{\mathrm{T}} \cdot v)$$
$$\text{s.t. } v \in S(PN, \boldsymbol{m}). \tag{8}$$

Operative Condition 2 (OC2). At each macro-period the IFS vector *v* is selected so as to minimize the sum of all the stored materials:

$$\min \ (\mathbf{c}^{\mathrm{T}} \cdot v)$$
$$\text{s.t. } v \in S(PN, \boldsymbol{m}). \tag{9}$$

with $c_j = \boldsymbol{C}(p, t_j)$ if $t_j \in p^{(c)} \cup^{(c)} p$ and $c_j = 0$ otherwise.

Operative Condition 3 (OC3). At each macro-period the IFS vector *v* is selected so as to maximize the sum of all flow rates as in (8) while setting all the capacities, re-order levels and fixed order quantities of the SC equal to 3/5 of the nominal values reported in Tables 2 and 3.

We remark that the first operative condition allows us to estimate the maximum level of performance of the SC with respect to the production capacity. Obviously, if we wish to maximize the manufacturing of a sub-set of products only, then (8) may be accordingly modified with a suitable objective function. Similarly, in the second operative condition the

stored materials in the SC buffers are minimized and, if we wish to minimize the stocks of inventory of a sub-set of products, then (9) may be accordingly modified by a suitable vector $\mathbf{c}^T$. Moreover, the third operative condition aims at imposing an *almost* constant work-inprocess in each buffer as a fraction of each capacity, similar to the previously described CONWIP management technique (Spearman et al. 1990). In this case the storage costs are a priori limited and at the same time the SC does not evolve at its maximum productivity level.

5.4 The simulation results

Figures 13, 14 and 15 report the selected SC performance indices, i.e., throughput, system inventory and lead time, respectively, obtained under the two management strategies and in the three operative conditions. In particular, Fig. 13 shows that in the two conditions *OC1* and *OC2* the system throughput values obtained under the MTS strategy are always greater than the corresponding values obtained with the MTO policy, since in the latter case the production is triggered by orders only. On the other hand, the reduced productivity imposed to the SC under *OC3* leads to equivalent throughput values in such a condition under MTS and MTO. Moreover, the throughput values obtained under *OC1* for a given management strategy (i.e., MTS or MTO) are greater than the ones corresponding to *OC2* and these are in turn bigger than those obtained in *OC3*. Indeed, solving (8) corresponds to maximizing the flow rates of the SC while the objective of (9) is to minimize the SC inventory and finally the third condition corresponds to a decreased level of the SC overall maximum productivity. In addition, it is noteworthy that the greatest (smallest) average throughput value, i.e. the highest (lowest) productivity level, is obtained when the SC is governed by the MTS (MTS or MTO) policy and in *OC1* (*OC3*).
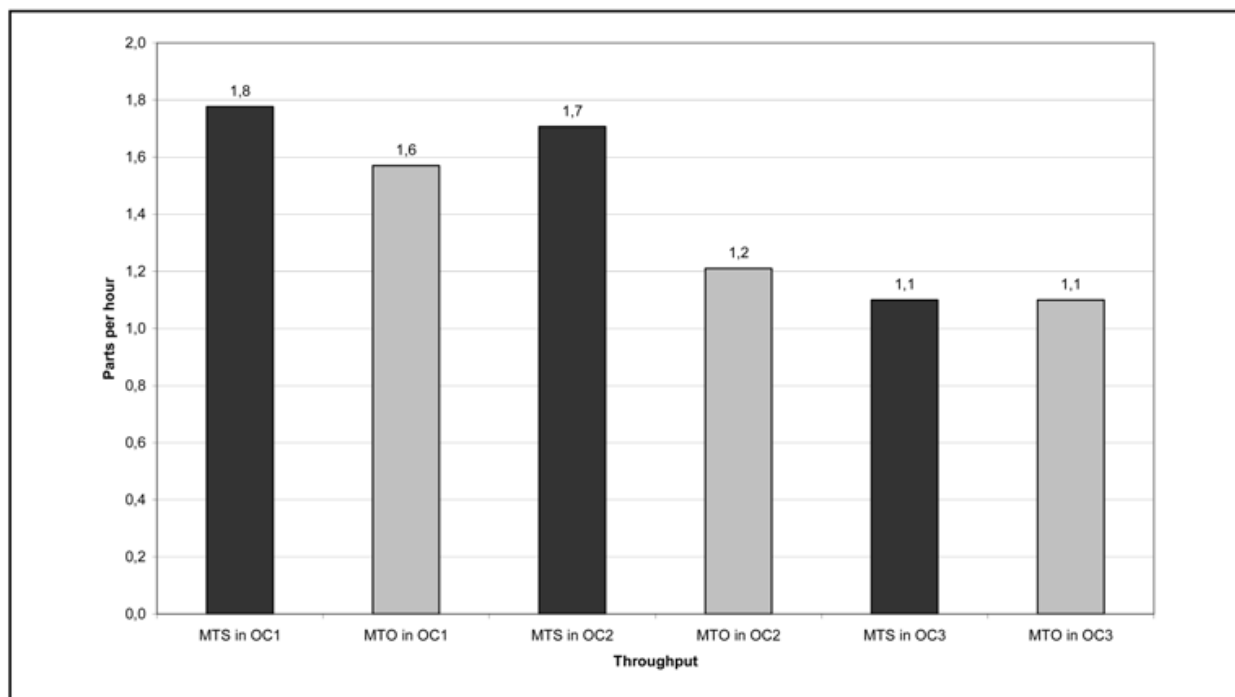

Fig. 13. Throughput (parts per hour) for different operative conditions under MTS and MTO.
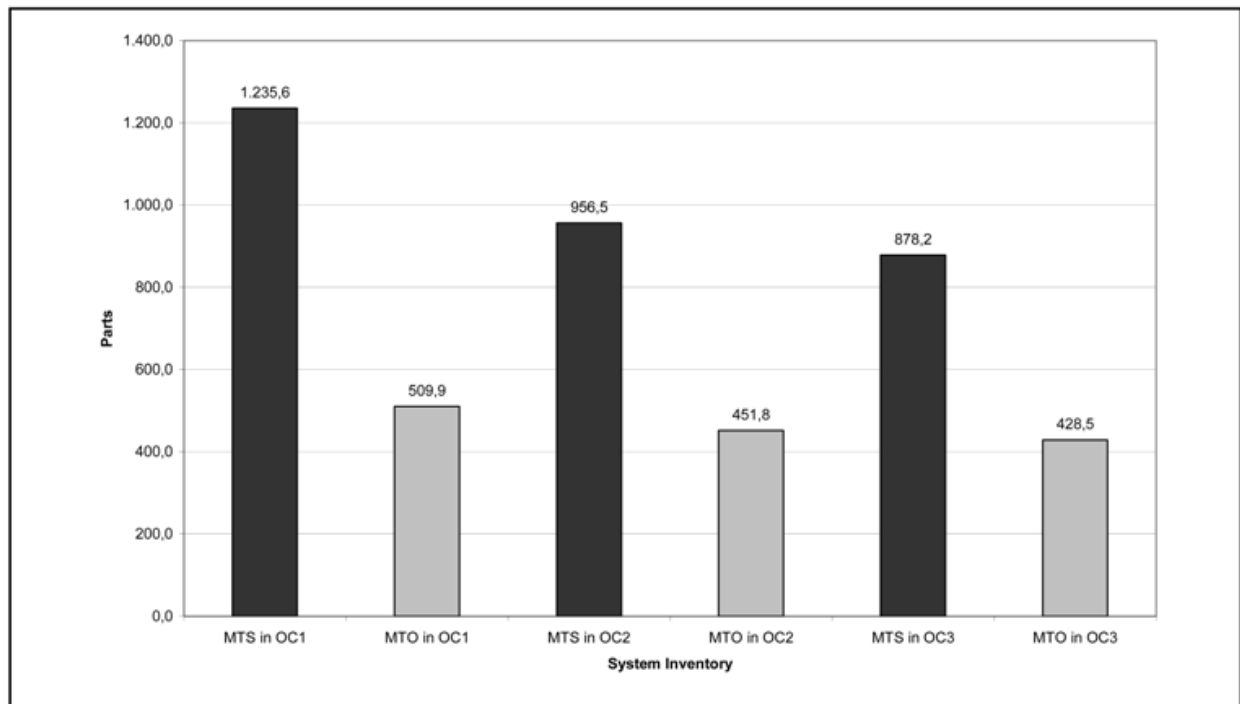
Fig. 14. System inventory (parts) for different operative conditions under MTS and MTO.
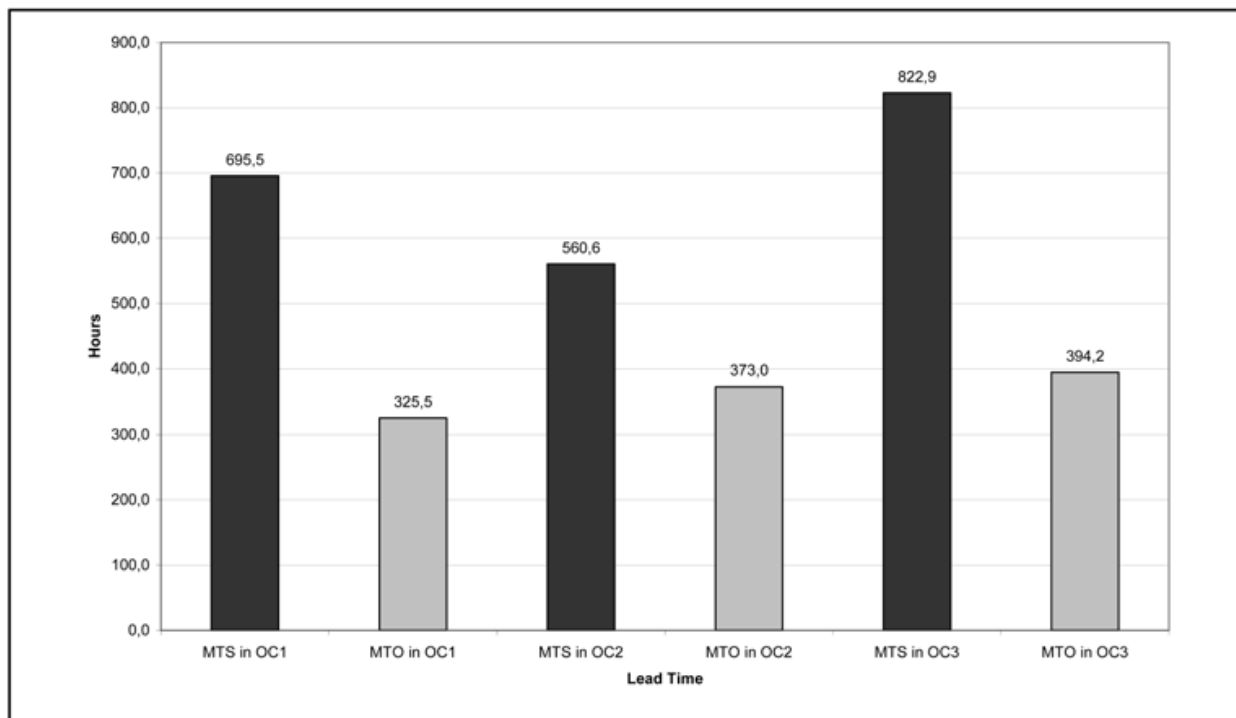

Fig. 15. Lead time (hours) for different operative conditions under MTS and MTO.

On the other hand, Fig. 14 shows that in the three cases the system inventory values evaluated under the MTS policy are always much greater than the corresponding values obtained with the MTO strategy. Moreover, the values of index SI obtained in *OC1* under a given policy (i.e. MTS or MTO) are greater than the ones corresponding to *OC2* for the same

strategy and these are in turn bigger than those obtained in *OC3* with that policy. The greatest (smallest) average system inventory value, i.e. the highest (lowest) storage level, is obtained when the SC is governed by the MTS (MTO) policy and in *OC1* (*OC3*).

Besides, Fig. 15 shows that in any operative condition the lead time values obtained under MTS are always greater than those obtained with the MTO policy, since in the former case the higher production corresponds on average to a longer permanence of materials and products in the system. Furthermore, Fig. 15 shows that under a the MTS operational management policy the LT values obtained in *OC1* are greater than the corresponding ones obtained in *OC2*, since the former case corresponds to a higher productivity. On the other hand, in *OC3* the value of LT under MTS (MTO) is greater than the corresponding index obtained in *OC1* under the same policy, since in the former condition the considerable throughput diminishment counterbalances the less significant system inventory decrease. Conversely, the MTO strategy lead to similar LT values in the three operative conditions.

Summing up, the SC managed under MTS is more productive than the system using MTO. On the other hand, using the latter policy the stocks in the considered SC are reduced and the same applies to the lead time values. Moreover, a different choice of the production rates and inventory management (as in *OC1*, *OC2* and *OC3*) lets the designer further find a tradeoff between different key performance indicators of the SC. For instance, comparing the system under the MTS policy in *OC1* and *OC2*, it is apparent that in these conditions a similar productivity level is attained (compare the corresponding throughput values in Fig. 13) but in the latter case the system inventory is quite smaller (as enlightened by the SI values in Fig. 14), leading to more sustainable storage costs than in the former case, with a lower lead time, as well (see the LT values in Fig. 15). Moreover, *OC3* under the MTS or MTO policy is characterized by the lowest value of average throughput and highest value of lead time compared to the other operative conditions with the same management policy (see Fig. 13 and Fig. 15) while offering the lowest average system inventory (see Fig. 14). On the other hand, under a given management strategy *OC1* is characterized by a lower value of LT with respect to *OC3* (see Fig. 15) but *OC1* involves a higher SI value than *OC3*, while leading to a better (or at most equal) throughput (see Fig. 13 and Fig. 14). With regard to the MTO strategy, the obtained LT values in the three operative conditions are quite similar (see Fig. 15), with *OC3* leading to the lowest SI index (see Fig. 14) at the price of a low value of T (see Fig. 13).

6. Conclusion

The chapter focuses on the problem of managing at the operational level Supply Chains (SCs), new emerging company networks, very complex to describe and manage. The SC system is described by a modular model based on the first order hybrid Petri net formalism, previously proposed by the authors: a fluid approximation of material and products is considered and discrete unpredictable events occurring stochastically (i.e., blocking of suppliers, manufacturers, transporters, etc.) are singled out by the discrete event dynamics. The model can effectively describe the operational management policies and the inventory control rules, and enables the designer to impose an optimal SC dynamics according to given objective functions.

To show the effectiveness and simplicity of the modeling technique, a SC case study is modelled and simulated under the alternative Make-To-Stock and Make-To-Order policies with a reorder point based inventory rule. The simulation results show that the fluid

approximation leads to an effective management policy implementation and to the possibility of choosing some important control parameters of the system, such as the production rates. Perspectives on future research include investigating the optimal decoupling point position, as well as implementing additional inventory control rules.

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Supply Chain the Way to Flat Organisation

Edited by Julio Ponce and Adem Karahoca

With the ever-increasing levels of volatility in demand and more and more turbulent market conditions, there is a growing acceptance that individual businesses can no longer compete as stand-alone entities but rather as supply chains. Supply chain management (SCM) has been both an emergent field of practice and an academic domain to help firms satisfy customer needs more responsively with improved quality, reduction cost and higher flexibility. This book discusses some of the latest development and findings addressing a number of key areas of aspect of supply chain management, including the application and development ICT and the RFID technique in SCM, SCM modeling and control, and number of emerging trends and issues.

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Mariagrazia Dotoli, Maria Pia Fanti and Agostino Marcello Mangini (2009). Operational Management of Supply Chains: A Hybrid Petri Net Approach, Supply Chain the Way to Flat Organisation, Julio Ponce and Adem Karahoca (Ed.), ISBN: 978-953-7619-35-0, InTech, Available from:
http://www.intechopen.com/books/supply_chain_the_way_to_flat_organisation/operational_management_of_supply_chains__a_hybrid_petri_net_approach

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