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# Adaptive Control of Dynamic Systems with Sandwiched Hysteresis Based on Neural Estimator

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## 1. Introduction

The so-called Sandwich system with hysteresis is a class of systems in which a hysteretic subsystem is sandwiched between two smooth dynamic blocks. In engineering, many practical processes can be considered as the sandwich systems with hysteresis. In the following, two typical examples will be presented.

### 1.1 Ultra-precision moving positioning stage

A typical ultra-precision moving positioning stage is often used in ultra-precision manufacturing system for its nanometer displacement and fast linear moving speed. Usually, such platform consists of electric amplifiers, piezoelectric actuators and loads. As hysteresis is inherent in piezoelectric actuator, the amplifier and load can be considered as smooth dynamic subsystems. Therefore, this platform can be considered as a typical sandwich system with hysteresis. Fig.1 shows the architecture of such system.

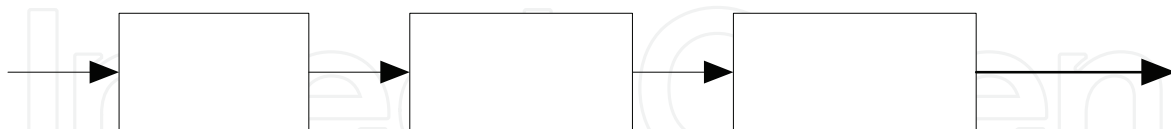


Fig. 1. Architecture of ultra-precision moving stage with piezoelectric actuator

### 1.2 Mechanical Transmission System

Mechanical transmission system often exists in machine tools or many other mechanical systems. A typical mechanical transmission system is shown in Fig.2. In this system, the servomotor is used to drive a gearbox connected with a mechanical work platform through a screw. In this system,  $u$  is the servomotor angle,  $x$  is the angle of the gearbox, and  $y$  is the displacement of the work platform. The servomotor and the work platform can be considered as smooth dynamic subsystems. However, the gearbox and screw in this system is a typical hysteresis nonlinearity due to the tear and wear of the gear teeth. Obviously, this mechanical system can be described by the sandwich system with hysteresis.

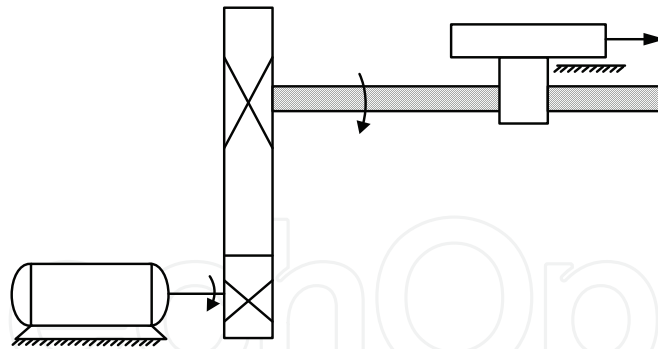


Fig. 2. Mechanical transmission system

Although, sandwich systems with hysteresis often exist in engineering practice, there are only several research reports found on the control of them. Taware & Tao (1999) presented an analysis on the control of such systems with backlash-type hysteresis. Tao & Ma (2001) proposed an optimal control for the systems with sandwiched backlash. In their methods, an optimal control scheme is employed for backlash compensation. Then, the nonlinear feedback control law is used for the control of nonlinear dynamics. Zhao & Tan (2006) proposed a neural adaptive control for sandwich systems with hysteresis. The neural network based hysteresis compensator is developed to compensate for the effect of the hysteresis. Furthermore, Zhao et. al. (2007) presented an adaptive control strategy for sandwich systems with dynamic hysteresis based on Duhem hysteretic operator. Corradini et. al. (2007) proposed a variable structure control of nonlinear uncertain sandwich systems with hysteretic block. Therefore, the control of sandwich systems with hysteresis has become one of the interesting topics in control engineering domain.

It is known that the existence of hysteresis in actuators often leads to oscillation and undesirable inaccuracy. Therefore, the main purpose of design a control scheme for sandwich system with hysteresis is to eliminate the side effect of hysteresis inherent in the system and force the system to track the reference trajectory. Note that hysteresis is a non-differentiable nonlinear system with multi-valued mapping. Moreover the structure of the sandwich system is rather complex. Hence, it is not easy to construct a compensator for the hysteresis in such system. Therefore it is a real challenge to develop a control strategy for the dynamic systems with sandwiched hysteresis.

In this chapter, a mathematical description of the sandwich systems with hysteresis will be described in section 2. Then, in section 3, the control architecture for the sandwich systems with hysteresis will be illustrated. In this architecture, a neural network based inverse model is constructed to cancel the effect of the first dynamic block of sandwich system. Then, the sandwich system can be transformed to a nonlinear system preceded by hysteresis which can be described by a Hammerstein model with hysteresis. In Section 4, a neural network based estimator will be developed in terms of a proposed expanded input space with hysteretic operator. The developed neural hysteretic estimator can be used to compensate for the system residual caused by the effect of hysteresis. Section 5 will present an adaptive control strategy based on pseudo inverse control technique for the obtained Hammerstein system with hysteresis. One of advantages of the controller is that it does not need to construct the hysteresis inverse to cancel hysteretic effect. The neural control strategy and the corresponding adaptive law based on the Lyapunov stability theory will be developed.

Furthermore, Comparison of the simulation results between the proposed method and the PID control strategy will be illustrated in Section 6. Section 7 will present the remarks and conclusions of this Chapter.

### 2. Mathematical Description of Sandwich Systems with Hysteresis

The structure of the sandwich system with hysteresis is shown in Fig.3. Suppose the nonlinear single-input-single-output (SISO) system with sandwiched hysteresis is described by

$$L_i : f_i[v^{(n)}, v^{(n-1)}, \dots, v^{(1)}, v, r^{(m)}, r^{(m-1)}, \dots, r^{(1)}, r] = 0 \tag{1}$$

where  $r$  is the input,  $v$  is the output,  $v^{(n)}$  is the  $n$ -th order derivative of  $v$ ,  $r^{(m)}$  is  $m$ -th order derivative of  $r$ ,  $m$  and  $n$  ( $m \leq n$ ) are the orders of the input and output respectively.

$$H : u = H(v) \tag{2}$$

where  $H$  presents the hysteresis nonlinearity.

$$L_o : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f_o(x) + g_o(x)u \end{cases} \tag{3}$$

and

$$y = x_1 \tag{4}$$

where  $x = [x_1, x_2, \dots, x_n]^T$  is the system state vector,  $u$  is the input,  $y$  is the output,  $v$  is the control input and  $u$  is the actuator output. It is assumed that  $f_o(x)$  and  $g_o(x)$  are sufficiently smooth but unknown functions and satisfy  $\frac{\partial f_o}{\partial u} \neq 0$  and  $\frac{\partial g_o}{\partial u} \neq 0$ . Moreover, assume that  $f_o$  is invertible. Notation  $H[\cdot]$  denotes that the hysteresis nonlinearity is not dependent on an instantaneous value  $v(t)$  but the trajectory,  $v(t) \in C^0[0, t]$ . Assume that all the control and input variables, i.e.  $v^{(n)}, v^{(n-1)}, \dots, v^{(1)}, v, r^{(m)}, r^{(m-1)}, \dots, r^{(1)}, r$  are known.



Fig. 3. The structure of sandwich system with hysteresis

### 3. Control Architecture for Sandwich System with Hysteresis

From Fig. 3, it is known that the architecture of the sandwich system with hysteresis is rather complex. It would be convenient for us to design a control strategy for such system if we could find a method to simplify the structure of the system. In this section, a control architecture for the sandwich system will be discussed. In this architecture, a neural networks (NN) based inverse system  $\hat{L}_i^{-1}$  will be constructed. By connecting the NN based inverse with the system  $L_i$  can form an approximate pseudo-linear unit compensator which leads to  $\hat{L}_i^{-1}L_i \approx 1$ . Then the sandwich system can be transform to a pseudo-linear unit system connected with a nonlinear system preceded with hysteresis nonlinearity which is shown in Fig.4. The obtained the system can be considered as a Hammerstein System with hysteresis.

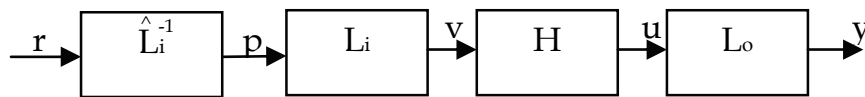


Fig. 4. The sandwich system with a pseudo-linear unit compensation

With the above-mentioned NN based inverse, the effect of  $L_i$  would be cancelled. So we can design the controller for the system  $L_o$  preceded by a hysteresis nonlinearity. Usually, the model uncertainty of the neural network based compensator exists. That implies the NN based compensator cannot completely compensate for the effect of  $L_i$ . Therefore, a model residual should be added to system  $L_o$ . That is  $\hat{L}_i^{-1}L_i = 1 + \xi$ , where  $\xi$  is a bounded modeling error. Hence, the obtained system preceded by a hysteresis can be described as follows:

$$H : u = H(v) , \quad (5)$$

$$L_o : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = f_o(x) + g_o(x)u + \xi \end{cases} , \quad (6)$$

and

$$y = x_1 . \quad (7)$$

The control objective is to design a control law  $v(t)$  to force  $y(t)$ , the plant output, to track a smooth prescribed trajectory  $y_d(t)$  with an acceptable accuracy. The desired state vector is defined as  $x_d(t) = [y_d, \dot{y}_d, \dots, y_d^{(n-1)}]^T$  where  $y_d^{(n-1)}$  is the  $(n-1)$ th order derivative. Moreover, the tracking error vector is defined as  $e = x - x_d$ . It is assumed that the desired states are

bounded, i.e.  $\|x_d\| \leq X_d$ . Moreover,  $\xi$  denotes bounded disturbance caused by NN based inverse, where  $|\xi| \leq \xi_N$ , and  $\xi_N > 0$ .

Define the filtered tracking error as

$$\tau = [\lambda_1, \lambda_2 \cdots \lambda_{n-1}, 1]e = [\Lambda^T, 1]e \tag{8}$$

where  $\Lambda = [\lambda_1, \lambda_2 \cdots \lambda_{n-1}]^T$  is a parameter vector to be designed. Suppose

$s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$  is Hurwitz. Differentiating (8) and using (6), it results in

$$\dot{\tau} = \dot{x}_n - y_d^{(n)} + [0, \Lambda^T]e = f_o(x) + g_o(x)u - y_d^{(n)} + [0, \Lambda^T]e + \xi \tag{9}$$

As  $u$  is the output of hysteresis which is usually unknown, an invertible function  $\hat{f}(x, v)$  is introduced to approximate  $f_o(x) + g_o(x)u$ . Adding and subtracting  $\hat{f}(x, v)$  to and from the right hand side of (9), it yields

$$\begin{aligned} \dot{\tau} &= \delta + f_o(x) + g_o(x)u - \hat{f}(x, v) - y_d^{(n)} + [0, \Lambda^T]e + \xi \\ &= \delta + F(x, u) - \hat{f}(x, v) - y_d^{(n)} + [0, \Lambda^T]e + \xi \\ &= \delta + \tilde{f}(x, v, u) - y_d^{(n)} + [0, \Lambda^T]e + \xi \end{aligned} \tag{10}$$

where  $\delta = \hat{f}(x, v)$  is the so called pseudo-control (Calis & Hovakimyan, 2001) and (Hovakimyan & Nandi, 2002),  $F(x, u) = f_o(x) + g_o(x)u$  and  $\tilde{f}(x, v, u) = F(x, u) - \hat{f}(x, v)$  is the system residual. As  $\hat{f}(x, v)$  is invertible with respect to  $v$  and satisfies (Calis & Hovakimyan, 2001):

$$1. \operatorname{sgn} \frac{\partial F}{\partial u} \frac{\partial u}{\partial v} = \operatorname{sgn} \frac{\partial \hat{f}}{\partial v}, \tag{11}$$

and

$$2. \left| \frac{\partial \hat{f}}{\partial v} \right| > \frac{1}{2} \left| \frac{\partial F}{\partial u} \frac{\partial u}{\partial v} \right| > 0. \tag{12}$$

In order to design the corresponding control strategy, the approximation of the nonlinear residual  $\tilde{f}(x, v, u)$  is required. Neural networks would be one of the recommended alternatives to model this residual. However,  $\tilde{f}(x, v, u)$  involves the characteristic of

hysteresis, the traditional nonlinear identification methods such as neural modeling technique usually cannot be directly applied to the modeling of it since the hysteresis is a non-linearity with multi-valued mapping (Adly & Abd-El-Hafiz, 1998). In Section 4, we will present a method to construct the neural estimator for  $\tilde{f}(x, v, u)$  to compensate for the effect of hysteresis. Moreover, a corresponding adaptive control method based on the control architecture stated-above will be illustrated in Section 5.

#### 4. Neural Estimator for System Residual

In order to approximate the system residual, neural network can be considered as an alternative. However, the system residual contains the characteristic of hysteresis which is a system with multi-valued mapping. In this section, a hysteretic operator is proposed to construct an expanded input space so as to transform the multi-valued mapping of hysteresis into a one-to-one mapping (Zhao & Tan, 2008). Thus, the neural networks can be used for modeling of hysteresis based on the expanded input space with the hysteretic operator. The proposed hysteretic operator is defined as:

$$h(x) = (1 - e^{-|x-x_p|})(x - x_p) + h(x_p), \quad (13)$$

where  $x$  is the current input,  $h(x)$  is the current output,  $x_p$  is the dominant extremum adjacent to the current input  $x$ .  $h(x_p)$  is the output of the operator when the input is  $x_p$ .

*Lemma 1:* Let  $x(t) \in C(R^+)$ , where  $R^+ = \{t | t \geq 0\}$  and  $C(R^+)$  are the sets of continuous functions on  $R^+$ . If there exist two time instants  $t_1, t_2$  and  $t_1 \neq t_2$ , such that  $x(t_1) = x(t_2)$ ,  $x(t_1)$  and  $x(t_2)$  are not the extrema, then  $h[x(t_1)] \neq h[x(t_2)]$ .

*Proof:* For  $x(t)$  decreases or increases monotonically, (13) becomes

$$h(x) = \begin{cases} h_{in}(x) = [1 - e^{-(x-x_p)}](x - x_p) + h(x_p), & \dot{x}(t) > 0 \\ h_{de}(x) = (1 - e^{x-x_p})(x - x_p) + h(x_p), & \dot{x}(t) < 0 \end{cases} \quad (14)$$

$$\begin{aligned} h'_{in}(x) &= e^{-(x-x_p)} \cdot (x - x_p) + [1 - e^{-(x-x_p)}] \\ &= 1 - [1 - (x - x_p)] / e^{x-x_p} \\ &> 1 - 1 / e^{x-x_p} > 0 \end{aligned} \quad (15)$$

Therefore,  $h_{in}(x)$  is monotonic. Similarly one can obtain that  $h_{de}(x)$  is monotonic. It is noted that  $h_{in}(x)$  is obtained from  $h_{in0}(x) = (1 - e^{-x})x$  ( $x \geq 0$ ). That means its origin moves from  $(0,0)$  to  $(x_p, h(x_p))$ . Similarly  $h_{de}(x)$  is obtained from  $h_{de0}(x) = (1 - e^x)x$  ( $x \leq 0$ ). It represents that its origin moves from  $(0,0)$  to  $(x_p, h(x_p))$ . As  $h_{in0}(-x) = -h_{de0}(x)$ , it implies

that  $h_{in}(x)$  and  $h_{de}(x)$  are antisymmetric. Therefore it can be concluded that  $h_{in}(x)$  and  $h_{de}(x)$  intersect only at extremum point  $(x_p, h(x_p))$ . That is, if  $x(t_1)$  and  $x(t_2)$  are not the extrema,  $x(t_1) = x(t_2)$ , then  $h[x(t_1)] \neq h[x(t_2)]$ .

*Remark:* If both  $h(x)$  and  $H[\cdot]$  are fed with the same input  $v(t)$ , the curve of  $h[v(t)]$  exhibits similarity to that of  $H[v(t)]$  such as ascending, turning and descending. Moreover, since  $x(t_1) = x(t_2)$ ,  $x(t_1)$  and  $x(t_2)$  are not the extrema,  $h[x(t_1)] \neq h[x(t_2)]$ , the pair  $(v(t), h[v(t)])$  will uniquely correspond to one of the output values of hysteresis  $H[v(t)]$ .

*Lemma 2:* If there exist two time instants  $t_1, t_2$  and  $t_1 \neq t_2$ , such that  $h[x(t_1)] - h[x(t_2)] \rightarrow 0$ , then  $x(t_1) - x(t_2) \rightarrow 0$ .

*Proof:*

$$\frac{h_{in}[x(t_1)] - h_{in}[x(t_2)]}{x(t_1) - x(t_2)} = k, \quad k \in (0, +\infty), \tag{16}$$

and

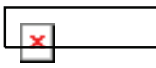
$$x(t_1) - x(t_2) = \frac{h_{in}[x(t_1)] - h_{in}[x(t_2)]}{k}. \tag{17}$$

It is clear that if  $h_{in}[x(t_1)] - h_{in}[x(t_2)] \rightarrow 0$ , then  $x(t_1) - x(t_2) \rightarrow 0$ . Similarly, it is obtained that if  $h_{de}[x(t_1)] - h_{de}[x(t_2)] \rightarrow 0$ , then  $x(t_1) - x(t_2) \rightarrow 0$ . Thus, it leads to the following theorem, i.e.:

*Theorem 1:* For any hysteresis, there exists a continuous one-to-one mapping  $\Gamma : R^2 \rightarrow R$ , such that  $H[v(t)] = \Gamma(v(t), h[v(t)])$ , where  $\{v(t), h[v(t)]\}$  is an expanded input space with hysteresis operator.

*Proof:* The proof can be divided into two cases, i.e.

*Case 1:* If  $v(t)$  is not the extrema. Based on Lemma1, if there exist two time instants  $t_1, t_2$  and  $t_1 \neq t_2$ , then  $(v(t_1), h[v(t_1)]) \neq (v(t_2), h[v(t_2)])$ . Therefore, the pair  $(v(t), h[v(t)])$  uniquely corresponds to an output value of  $H[v(t)]$ .

*Case 2:* If  $v(t)$  is the extrema, then  $(v(t_1), h[v(t_1)]) = (v(t_2), h[v(t_2)])$ . According to the principle of the classical Preisach modeling, i.e.  $H[v(t_1)] = H[v(t_2)]$ , then the pair  uniquely corresponds to an output value of  $H[v(t)]$ .

Combining the above-mentioned two cases, there exists a mapping  $\Gamma : R^2 \rightarrow R$  such that  $H[v(t)] = \Gamma(v(t), h[v(t)])$ .

In theorem 1, the obtained mapping  $\Gamma(\cdot)$  is a continuous function. According to Lemma 2, from  $v(t_1) - v(t_2) \rightarrow 0$ , it leads to  $H[v(t_1)] - H[v(t_2)] \rightarrow 0$ . Also, from  $h[v(t_1)] - h[v(t_2)] \rightarrow 0$ , it



yields  $v(t_1) - v(t_2) \rightarrow 0$ . Then, it results in  $H[v(t_1)] - H[v(t_2)] \rightarrow 0$ . Therefore, it is derived that  $\Gamma$  is a continuous function. Moreover, Theorem 1 indicates that the multi-valued mapping of hysteresis can be transformed to a one-to-one mapping. It can be proved that the obtained mapping is a continuous mapping, i.e.

Let  $T = [t_0, \infty) \in \mathbb{R}$ ,  $V = \{v | T \xrightarrow{v} \mathbb{R}\}$ . Also let  $F = \{h | T \xrightarrow{h} \mathbb{R}\}$  be the input sets. Given  $t_i \in T$  it is obvious that  $v(t_i) < +\infty$  and  $h[v(t_i)] < +\infty$ . So that  $(v(t_i), h[v(t_i)]) \in \mathbb{R}^2$ . Thus, it is obtained that  $\Phi = \{(v(t_i), h[v(t_i)]) | v(t_i) \in V, h[v(t_i)] \in F\}$  is a compact set.

Hence, it provides a premise to apply neural networks to modeling of the behavior of hysteresis. Based on the proposed expanded input space with hysteretic operator, a neural network is used to approximate the system residual, i.e.  $\tilde{f}(x, v, u)$ :

$$\tilde{f}(x, v, u) = W^T \sigma(V^T x_{nn}) + \varepsilon(x_{nn}) \quad (18)$$

where  $\sigma(\cdot)$  is activation function,  $V$  is the first-to-second layer interconnection weights,  $W$  is the second-to-third layer interconnection weights,  $x_{nn} = (x^T, \delta, u)$ ,  $\varepsilon$  is the NN functional reconstruction error,  $\|\varepsilon(x_{nn})\| \leq \varepsilon_N$ , and  $\varepsilon_N > 0$ .

The above-mentioned neural network based on the expanded input space with hysteretic operator can be used to construct the corresponding neural estimator for the system residual  $\tilde{f}(x, v, u)$ . Thus, it can be used for the compensation for the effect of the hysteresis inherent in the sandwich system.

## 5. Adaptive Control Strategy

In section 3, we introduce an architecture of the control strategy for the sandwich system with hysteresis. In the control structure, a neural inverse model is used to compensate for the effect of  $L_i$  in the architecture of the sandwich system with hysteresis. After the compensation, the sandwich system with hysteresis is approximately transformed into a Hammerstein system with hysteresis. In this section, an adaptive control strategy is developed for the obtained Hammerstein system with hysteresis.

*Assumption 1:* If the weight matrices, i.e.  $V$  and  $W$  of the neural estimator are respectively bounded by  $V_p > 0$  and  $W_p > 0$ , i.e.  $\|W\|_F \leq W_p$  and  $\|V\| \leq V_p$ , where  $\|\cdot\|_F$  represents Frobenius norm. Then, the corresponding pseudo-control can be chosen as

$$\delta = y_d^{(n)} - K\tau - [0, \Lambda^T]e - v_{ad} + v_r \quad (19)$$

where  $v_r$  is the term for robust design,  $K$  is a design parameter,  $v_{ad}$  is the output of neural network, i.e.  $v_{ad} = \hat{W}^T \sigma(\hat{V}^T x_{nn})$  where  $\hat{W}$  and  $\hat{V}$  are the estimated values of  $W$  and  $V$ .

From (10) and (19), notice that  $\tilde{f}(x, v, u)$  depends on  $v_{ad}$  through  $\delta$ . However,  $v_{ad}$  has to be designed to cancel the effect of  $\tilde{f}(x, v, u)$ . This should assume that the mapping  $\delta_{ad} \mapsto \tilde{f}$  is a contraction over the entirely interested input domain. It has been proven by Hovakimyan and Nandi (2002) that the assumption is held when (11) and (12) are satisfied. Using (18) and (19), (10) can be written as

$$\dot{\tau} = -K\tau - \hat{W}^T \sigma(\hat{V}^T x_{nn}) + W^T \sigma(V^T x_{nn}) + v_r + \varepsilon + \xi. \tag{20}$$

Define

$$\tilde{V} = V - \hat{V} \text{ and } \tilde{W} = W - \hat{W}. \tag{21}$$

The Taylor series expansion of  $\sigma(Vx_{nn})$  for a given  $x_{nn}$  can be written as

$$\sigma(Vx_{nn}) = \sigma(\hat{V}x_{nn}) + \sigma'(\hat{V}x_{nn})\tilde{V}x_{nn} + o(\tilde{V}x_{nn})^2 \tag{23}$$

where  $\sigma'(\hat{z}) = d\sigma(z)/dz|_{z=\hat{z}}$  and  $o(\tilde{z})^2$  is the term of order two. Denoting  $\sigma = \sigma(V^T x_{nn})$ ,  $\hat{\sigma} = \sigma(\hat{V}^T x_{nn})$ , and  $\hat{\sigma}' = \sigma'(\hat{V}^T x_{nn})$ , with the procedure as Appendix, we have

$$\dot{\tau} = -K\tau + \tilde{W}^T (\hat{\sigma} - \hat{\sigma}' \hat{V}^T x_{nn}) + \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} + v_r + \varepsilon + \xi + w \tag{24}$$

where

$$w = W^T (\sigma - \hat{\sigma}) + W^T \hat{\sigma}' \hat{V}^T x_{nn} - \hat{W}^T \hat{\sigma}' V^T x_{nn}. \tag{25}$$

An upper bound for  $w$  can be presented as:

$$\|w\| \leq \|W\|_1 + \|W\| \|\hat{\sigma}' \hat{V}^T x_{nn}\| + \|V\|_F \|x_{nn} \hat{W}^T \hat{\sigma}'\|_F \tag{26}$$

or

$$\|w\| \leq \rho_w \mathcal{G}_w(\hat{W}, \hat{V}, x_{nn}) \tag{27}$$

where  $\mathcal{G}_w = 1 + \|\hat{\sigma}' \hat{V}^T x_{nn}\| + \|x_{nn} \hat{W}^T \hat{\sigma}'\|_F$  and  $\rho_w = \max(\|W\|_1, \|W\|, \|V\|_F)$ .

*Theorem 2:* Let the desired trajectory be bounded. Consider the system represented by (5), (6) and (7), if the control law and adaptive law are given by

$$v = \hat{f}^{-1}(x, \delta) \quad (28)$$

$$\delta = y_d^{(n)} - K\tau - [0, \Lambda^T]e - v_{ad} + v_r \quad (29)$$

$$\dot{\hat{W}} = F[(\hat{\sigma} - \hat{\sigma}'\hat{V}^T x_m)\tau - k\hat{W}\|\tau\|] \quad (30)$$

$$\dot{\hat{V}} = R[x_m\hat{W}^T\hat{\sigma}'\tau - k\hat{V}\|\tau\|] \quad (31)$$

$$\dot{\hat{\phi}} = \gamma[\|\tau\|(\mathcal{G}_w + 1) - k\|\tau\|\hat{\phi}] \quad (32)$$

and

$$v_r = \begin{cases} -\hat{\phi}(\mathcal{G}_w + 1)\frac{\tau}{\|\tau\|}, & \|\tau\| \neq 0 \\ 0, & \|\tau\| = 0 \end{cases} \quad (33)$$

where  $F = F^T > 0$ ,  $R = R^T > 0$ ,  $\gamma > 0$ ,  $\phi = \max[\rho_w, (\varepsilon_N + \xi_N)]$ , and  $\tilde{\phi} = \phi - \hat{\phi}$ ; then the signals  $e$ ,  $\hat{W}$ ,  $\hat{V}$ , and  $\hat{\phi}$  in the closed-loop system are ultimately bounded.

*Proof:* Consider the following Lyapunov function candidate, i.e.

$$L = \frac{1}{2}\tau^2 + \frac{1}{2}\text{tr}(\tilde{W}^T F^{-1}\tilde{W}) + \frac{1}{2}\text{tr}(\tilde{V}^T R^{-1}\tilde{V}) + \frac{1}{2}\tilde{\phi}^T \gamma^{-1}\tilde{\phi} \quad (34)$$

The derivative of  $L$  will be

$$\dot{L} = \tau\dot{\tau} + \text{tr}(\tilde{W}^T F^{-1}\dot{\tilde{W}}) + \text{tr}(\tilde{V}^T R^{-1}\dot{\tilde{V}}) + \tilde{\phi}^T \gamma^{-1}\dot{\tilde{\phi}} \quad (35)$$

Substituting (20) into (35), it yields

$$\begin{aligned} \dot{L} = & -K\tau^2 + \tau v_r + \tau(w + \varepsilon + \xi) + \tilde{\phi}^T \gamma^{-1}\dot{\tilde{\phi}} + \text{tr}\tilde{W}^T [F^{-1}\dot{\tilde{W}} + (\hat{\sigma} - \hat{\sigma}'\hat{V}^T x_m)\tau] \\ & + \text{tr}\tilde{V}^T (R^{-1}\dot{\tilde{V}} + x_m\tau\hat{W}^T\hat{\sigma}') \end{aligned} \quad (36)$$

Substituting  $\dot{\tilde{W}} = -\hat{W}$  and  $\dot{\tilde{V}} = -\hat{V}$  into (30) and (31), (36) can be rewritten as

$$\dot{L} = -K\tau^2 + \tau v_r + \tau(w + \varepsilon + \xi) + \tilde{\phi}^T \gamma^{-1}\dot{\tilde{\phi}} + k|\tau|[\text{tr}(\tilde{W}^T\hat{W}) + \text{tr}(\tilde{V}^T\hat{V})] \quad (37)$$

Considering (27) and  $\phi = \max[\rho_w, (\varepsilon_N + \xi_N)]$ , we obtain

$$\dot{L} \leq -K\tau^2 + \tau v_r + |\tau| \phi (\mathcal{G}_w + 1) - \tilde{\phi} \gamma^{-1} \hat{\phi} + k |\tau| [tr(\tilde{W}^T \hat{W}) + tr(\tilde{V}^T \hat{V})]. \tag{38}$$

Substituting (32) and (33) into (38), it results in

$$\dot{L} \leq -K\tau^2 + k \|\tau\| [tr(\tilde{W}^T \hat{W}) + tr(\tilde{V}^T \hat{V}) + \tilde{\phi}^T \hat{\phi}]. \tag{39}$$

Defining

$$\tilde{Z} = \begin{bmatrix} \tilde{W} & 0 & 0 \\ 0 & \tilde{V} & 0 \\ 0 & 0 & \tilde{\phi} \end{bmatrix}, \hat{Z} = \begin{bmatrix} \hat{W} & 0 & 0 \\ 0 & \hat{V} & 0 \\ 0 & 0 & \hat{\phi} \end{bmatrix} \text{ and } Z = \begin{bmatrix} W & 0 & 0 \\ 0 & V & 0 \\ 0 & 0 & \phi \end{bmatrix}, \tag{39}$$

can be rewritten as

$$\dot{L} \leq -K\tau^2 + k \|\tau\| tr(\tilde{Z}^T \hat{Z}). \tag{40}$$

As  $tr(\tilde{Z}^T \hat{Z}) \leq \|\tilde{Z}\|_F \|Z\|_F - \|\tilde{Z}\|_F^2$ ,

it leads to

$$\dot{L} \leq -K\tau^2 + k \|\tau\| (\|\tilde{Z}\|_F \|Z\|_F - \|\tilde{Z}\|_F^2). \tag{41}$$

That is

$$\dot{L} \leq -\|\tau\| [K \|\tau\| + k (\|\tilde{Z}\|_F - \frac{\|Z\|_F}{2})^2 - \frac{k \|Z\|_F^2}{4}]. \tag{42}$$

Thus,  $\dot{L}$  is negative as long as either  $\|\tau\| > \frac{k \|Z\|_F^2}{4K}$  or  $\|\tilde{Z}\|_F > \|Z\|_F$ . This demonstrate that  $\tau$ ,  $\tilde{W}$ ,  $\tilde{V}$ , and  $\tilde{\phi}$  are ultimately bounded. According to Assumption 1 and the definition of  $\tau$  and  $\phi$ , we can obtain that the variables  $e$ ,  $\hat{W}$ ,  $\hat{V}$  and  $\hat{\phi}$  in the closed-loop system are ultimately bounded.

### 6. Simulation

In order to illustrate that the proposed approach is applicable to nonlinear system with sandwiched hysteresis, we consider the following nonlinear system:

$$L_i, \dot{v} = -0.2(\sin v - \cos v) - \frac{v}{1+v^2} + (0.4 \sin v \cos v^2 + 0.8)r, v(0) = 0$$

$H$ , The hysteresis is generated by the sum of  $N = 50$  backlash operators, i.e. ,

$$u = H[v(t)] = \sum_{i=1}^N u_i, \text{ and}$$

$$\dot{u}_i = \begin{cases} \dot{v}(t) & \dot{v}(t) > 0, u_i(t) = v(t) - \frac{d_i}{2} \\ \dot{v}(t) & \dot{v}(t) < 0, u_i(t) = v(t) + \frac{d_i}{2} \\ 0 & \text{otherwise} \end{cases}$$

where  $u_i$  and  $d_i$  are respectively the output and the dead-band width of  $i$ -th backlash operator where  $i = 1, 2, \dots, N$  ( $N > 0$  is a positive integer). The values of the dead-band widths are evenly distributed within  $[0.02, 1]$ . All the initial outputs of the operators are set to zero. Fig. 5 shows the response of the hysteresis contained in the system.

$$L_o: \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = (1 - x_1^2)x_2 - x_1 + u \end{cases}$$

and

$$y = x_1.$$

The design procedure of the controller for the sandwich system with hysteresis will be shown in the following.

- 1) *Construction of neural network based  $L_i$  inverse.* An artificial neural network unit inverse, i.e.  $\hat{L}_i^{-1}$  is constructed to cancel the effect of the first dynamic block, i.e.  $L_i$ . The system is excited by the input  $r_i(t) = \sin 2t + \cos t$ . Then, 500 input/output pairs of data  $\{r_i, (v_i, \dot{v}_i)\}$  are obtained. Using these data as learning samples, a neural network based inverse  $\hat{L}_i^{-1}$  is constructed. The architecture of neural network based inverse model consists of 2 input nodes, 10 hidden neurons and 1 output neuron. The sigmoid function and linear function are respectively used as activation function in the hidden layer and the output layer. The conjugate gradient algorithm with Powell-Beale restarted method (Powell, 1977) is used to

train the neural network. The compensation result of the NN based  $\hat{L}_i^{-1}$  is shown in Fig. 6. It is known that there are some larger error happened in the beginning. Then it is gradually reduced as the control proceeded.

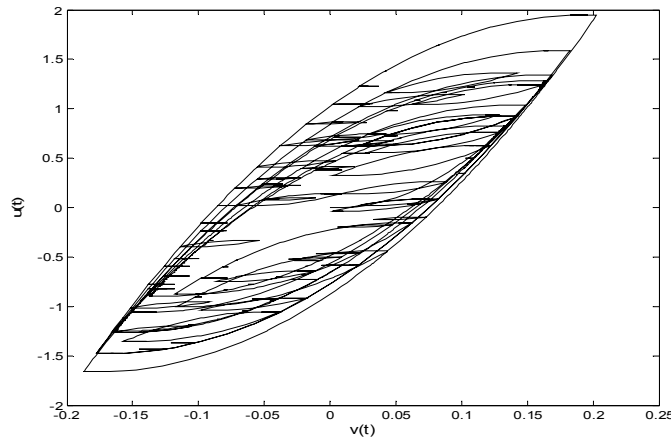


Fig. 5. The hysteresis in the system

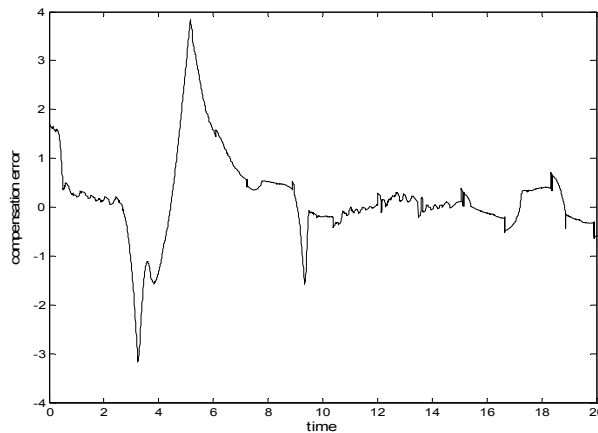


Fig. 6. The compensation error of NN based  $\hat{L}_i^{-1}$

2) *Neural approximator of system residual:* The neural network used to approximate  $\tilde{f}(x, v, u)$  consists of 4 input nodes, 35 hidden neurons and 1 output neuron. The input of the NN is  $x_m = (x^T, \delta, u)$ . The activation function is  $\sigma(x) = \frac{1}{1 + e^{-x}}$ .

3) *The selection of the controller parameters:* The other parameters of the controller are respectively chosen as  $\lambda_1 = 2, K = 11, k = 0.001, \gamma = 0.1, \hat{f}(x, v) = v, F = 8I$ , and  $R = 5I$ , where  $I$  is the unit matrix.

4) *PID control for comparison:* In order to compare the control performance of the proposed control strategy with the PID controller, we choose

$$v(t) = -22e_1 + \int_0^t e_1 dt - 13e_2$$

where  $e_1 = y - y_d$ ,  $e_2 = \dot{y} - \dot{y}_d$ . Moreover, the desired output of the system is  $y_d(t) = 0.1\pi[\sin 2t - \cos t]$ .

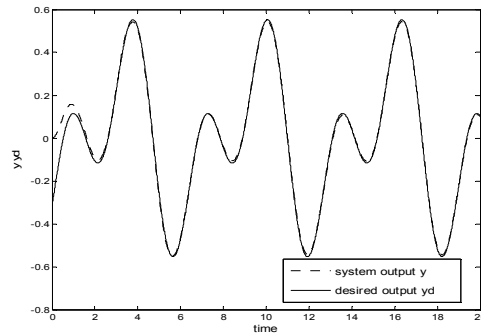


Fig. 7. The control response of the proposed method

From Fig.7, it is known that the control performance of the proposed controller has achieved good control response. Also, Fig.8 illustrates the control performance of the PID controller. It can be seen that the PID control strategy has led to larger control error when the reference signal achieves its local extreme. However, the proposed control strategy obtained better control performance. It can obviously derive more accurate control result.

## 7. Conclusions

An adaptive control strategy for nonlinear dynamic systems with sandwich hysteresis is presented. In the proposed control scheme, a neural network unit inverse is constructed to compensate for the effect of the first smooth dynamic subsystem. Thus, the sandwich system with hysteresis can be transformed to a Hammerstein type nonlinear dynamic system preceded by hysteresis. Considering the modified structure of the sandwich system, an adaptive controller based on the pseudo-control technique is developed. In our method, a neural network is used to approximate the system residual based on the proposed expanded input space with hysteretic operator. The advantage of this method can avoid constructing the hysteresis inverse. Then, the adaptive control law is derived in terms of the Lyapunov stability theorem. It has been proved that the ultimate boundedness of the closed-loop control error is guaranteed. Simulation results have illustrated that the proposed scheme has obtained good control performance.

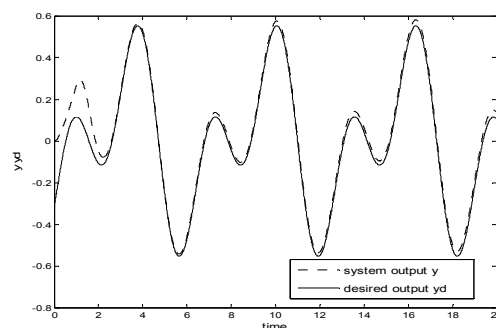


Fig. 8. The control response of the PID control method

## 8. Appendix

From (20), the approximation error can be written as:

$$\begin{aligned} & W^T \sigma - \hat{W}^T \hat{\sigma} \\ &= W^T \sigma - \hat{W}^T \sigma + \hat{W}^T \sigma - \hat{W}^T \hat{\sigma} \\ &= \tilde{W}^T \sigma + \hat{W}^T (\sigma - \hat{\sigma}) \end{aligned} \quad (\text{A1})$$

Substituting (23) into (A1), it yields

$$\begin{aligned} & W^T \sigma - \hat{W}^T \hat{\sigma} \\ &= \tilde{W}^T (\hat{\sigma} + \hat{\sigma}' \tilde{V}^T x_{nn} + o(\tilde{V}^T x_{nn})^2) + \hat{W}^T (\hat{\sigma}' \tilde{V}^T x_{nn} + o(\tilde{V}^T x_{nn})^2) \\ &= \tilde{W}^T \hat{\sigma} + \tilde{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} + \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} + W^T o(\tilde{V}^T x_{nn})^2 \\ &= \tilde{W}^T \hat{\sigma} + \tilde{W}^T \hat{\sigma}' V^T x_{nn} - \tilde{W}^T \hat{\sigma}' \hat{V}^T x_{nn} + \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} + W^T o(\tilde{V}^T x_{nn})^2 \\ &= \tilde{W}^T (\hat{\sigma} - \hat{\sigma}' \hat{V}^T x_{nn}) + \tilde{W}^T \hat{\sigma}' V^T x_{nn} + \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} + W^T o(\tilde{V}^T x_{nn})^2. \end{aligned} \quad (\text{A2})$$

Defining  $w = \tilde{W}^T \hat{\sigma}' V^T x_{nn} + W^T o(\tilde{V}^T x_{nn})^2$ , (A2) becomes

$$\begin{aligned} & W^T \sigma - \hat{W}^T \hat{\sigma} \\ &= \tilde{W}^T (\hat{\sigma} - \hat{\sigma}' \hat{V}^T x_{nn}) + \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} + w. \end{aligned}$$

So that

$$\begin{aligned} w &= W^T \sigma - \hat{W}^T \hat{\sigma} - \tilde{W}^T (\hat{\sigma} - \hat{\sigma}' \hat{V}^T x_{nn}) - \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} \\ &= W^T \sigma - W^T \hat{\sigma} + \tilde{W}^T \hat{\sigma}' \hat{V}^T x_{nn} - \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} \\ &= W^T (\sigma - \hat{\sigma}) + W^T \hat{\sigma}' \hat{V}^T x_{nn} - \hat{W}^T \hat{\sigma}' \hat{V}^T x_{nn} - \hat{W}^T \hat{\sigma}' \tilde{V}^T x_{nn} \\ &= W^T (\sigma - \hat{\sigma}) + W^T \hat{\sigma}' \hat{V}^T x_{nn} - \hat{W}^T \hat{\sigma}' V^T x_{nn} \end{aligned}$$

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