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Adaptive Control for Systems with Randomly Missing Measurements in a Network Environment

Yang Shi[1] and Huazhen Fang[1]
[1]*Department of Mechanical Engineering, University of Saskatchewan*
Canada

1. Introduction

Networked control systems (NCSs) are a type of distributed control systems, where the information of control system components (reference input, plant output, control input, etc.) is exchanged via communication networks. Due to the introduction of networks, NCSs have many attractive advantages, such as reduced system wiring, low weight and space, ease of system diagnosis and maintenance, and increased system agility, which motivated the research in NCSs. The study of NCSs has been an active research area in the past several years, see some recent survey articles (Chow & Tipsuwan, 2001; Hespanha & Naghshtabrizi, 2007; Yang, 2006) and the references therein. On the other hand, the introduction of networks also presents some challenges such as the limited feedback information caused by packet transmission delays and packet loss; both of them are due to the sharing and competition of the transmission medium, and bring difficulties for analysis and design for NCSs. The information transmission delay arises from by the limited capacity of the communication network used in a control system, whereas the packet loss is caused by the unavoidable data losses or transmission errors. Both the information transmission delay and packet loss may result in randomly missing output measurements at the controller node, as shown in Fig. 1. So far different approaches have been used to characterize the limited feedback information. For example, the information transmission delay and packet losses have been modeled as Markov chains (Zhang et al., 2006). The binary Bernoulli distribution is used to model the packet losses in (Sinopoli et al., 2004; Wang et al., 2005 a & 2005 b).
The main challenge of NCS design is the limited feedback information (information transmission delays and packet losses), which can degrade the performance of systems or even cause instability. Various methodologies have been proposed for modeling, stability analysis, and controller design for NCSs in the presence of limited feedback information. A novel feedback stabilization solution of multiple coupled control systems with limited communication is proposed by bringing together communication and control theoretical issues in (Hristu & Morgansen, 1999). Further the control and communication codesign methodology is applied in (Hristu-Varsakelis, 2006; Zhang & Hristu-Varsakelis, 2006) – a method of stabilizing linear NCSs with medium access constraints and transmission delays by designing a delay-compensated feedback controller and an accompanying medium

access policy is presented. In (Zhang et al., 2001), the relationship of sampling time and maximum allowable transfer interval to keep the systems stable is analyzed by using a stability region plot; the stability analysis of NCSs is addressed by using a hybrid system stability analysis technique. In (Walsh et al., 2002), a new NCS protocol, try-once-discard (TOD), which employs dynamic scheduling method, is proposed and the analytic proof of global exponential stability is provided based on Lyapunov's second method. In (Azimi-Sadjadi, 2003), the conditions under which NCSs subject to dropped packets are mean square stable are provided. Output feedback controller that can stabilize the plant in the presence of delay, sampling, and dropout effects in the measurement and actuation channels is developed in (Naghshtabrizi & Hespanha, 2005). In (Yu et al., 2004), the authors model the NCSs with packet dropout and delays as ordinary linear systems with input delays and further design state feedback controllers using Lyapunov-Razumikhin function method for the continuous-time case, and Lyapunov-Krasovskii based method for the discrete-time case, respectively. In (Yue et al., 2004), the time delays and packet dropout are simultaneously considered for state feedback controller design based on a delay-dependent approach; the maximum allowable value of the network-induced delays can be determined by solving a set of linear matrix inequalities (LMIs). Most recently, Gao, et al., for the first time, incorporate simultaneously three types of communication limitation, e.g., measurement quantization, signal transmission delay, and data packet dropout into the NCS design for robust H_∞ state estimation (Gao & Chen, 2007), and passivity based controller design (Gao et al., 2007), respectively. Further, a new delay system approach that consists of multiple successive delay components in the state, is proposed and applied to network-based control in (Gao et al., 2008).

However, the results obtained for NCSs are still limited: Most of the aforementioned results assume that the plant is given and model parameters are available, while few papers address the analysis and synthesis problems for NCSs whose plant parameters are unknown. In fact, while controlling a real plant, the designer rarely knows its parameters accurately (Narendra & Annaswamy, 1989). To the best of our knowledge, adaptive control for systems with unknown parameters and randomly missing outputs in a network environment has not been fully investigated, which is the focus of this paper.

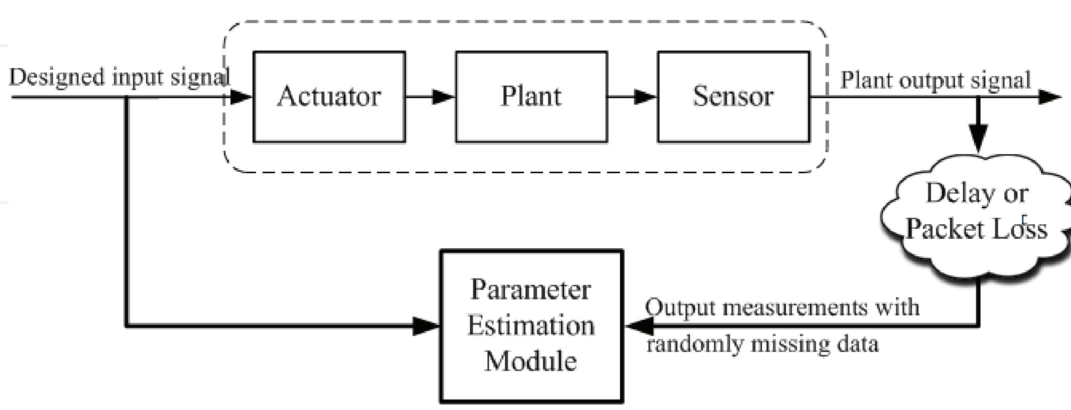

Fig. 1. An NCS with randomly missing outputs.

It is worth noting that systems with regular missing outputs – a special case of those with randomly missing outputs – can also be viewed as multirate systems which have uniform but various input/output sampling rates (Chen & Francis, 1995). Such systems may have regular-output-missing feature. In (Ding & Chen, 2004a), Ding, et al. use an auxiliary model and a modified recursive least squares (RLS) algorithm to realize simultaneous parameter and output estimation of dual-rate systems. Further, a least squares based self-tuning control scheme is studied for dual-rate linear systems (Ding & Chen, 2004b) and nonlinear systems (Ding et al., 2006), respectively. However, network-induced limited feedback information unavoidably results in randomly missing output measurements. To generalize and extend the adaptive control approach for multirate systems (Ding & Chen, 2004b; Ding et al., 2006) to NCSs with randomly missing output measurements and unknown model parameters is another motivation of this work.

In this paper, we first model the availability of output as a Bernoulli process. Then we design an output estimator to online estimate the missing output measurements, and further propose a novel Kalman filter based method for parameter estimation with randomly output missing. Based on the estimated output or the available output, and the estimated model parameters, an adaptive control is proposed to make the output track the desired signal. Convergence of the proposed output estimation and adaptive control algorithms is analyzed.

The rest of this paper is organized as follows. The problem of adaptive control for NCSs with unknown model parameters and randomly missing outputs is formulated in Section 2. In Section 3, the proposed algorithms for output estimation, model parameter estimation, and adaptive control are presented. In Section 4, the convergence properties of the proposed algorithms are analyzed. Section 5 gives several illustrative examples to demonstrate the effectiveness of the proposed algorithms. Finally, concluding remarks are given in Section 6.

Notations: The notations used throughout the paper are fairly standard.' E ' denotes the expectation. The superscript ' T ' stands for matrix transposition; $\lambda_{\max/\min}(X)$ represents the Maximum/minimum eigenvalue of X; $|X| = \det(X)$ is the determinant of a square matrix X; $\|X\|^2 = tr(XX^T)$ stands forthe trace of XX^{T}. If $\exists\, \delta_0 \in \mathrm{R}^+$ and $k_0 \in \mathrm{Z}^+$, $|f(k)| \le \delta_0 g(k)$ for $k \ge k_0$, then $f(k) = O\big(g(k)\big)$; if $f(k)/g(k) \to 0$ for $k \to \infty$, then $f(k) = o\big(g(k)\big)$.

2. Problem Formulation

The problem of interest in this work is to design an adaptive control scheme for networked systems with unknown model parameters and randomly missing outputs. In Fig. 2, the output measurements y_k could be unavailable at the controller node at some time instants because of the network-induced limited feedback information, e.g., transmission delay and/or packet loss. The data transmission protocols like TCP guarantee the delivery of data packets in this way: When one or more packets are lost the transmitter retransmits the lost packets. However, since a retransmitted packet usually has a long delay that is not desirable for control systems, the retransmitted packets are outdated by the time they arrive at the controller (Azimi-Sadjadi, 2003; Hristu-Varsakelis & Levine, 2005). Therefore, in this paper, it is assumed that the output measurements that are delayed in transmission are regarded as missed ones.

The availability of y_k can be viewed as a random variable γ_k. γ_k is assumed to have Bernoulli distribution:

$$\mathrm{E}(\gamma_k\gamma_s) = \mathrm{E}\gamma_k\mathrm{E}\gamma_s \text{ for } k \neq s, \tag{1}$$
$$\mathrm{Prob}(\gamma_k) = \begin{cases} \mu_k, \text{ if } \gamma_k = 1, \\ 1-\mu_k, \text{ else if } \gamma_k = 0, \end{cases}$$

where $0 < \mu_k \leq 1$.
Consider a single-input-single-output (SISO) process (Fig. 2):

$$A_z x_k = B_z u_k, \quad y_k = x_k + v_k \tag{2}$$

where u_k is the system input, y_k the output and v_k the disturbing white noise with variance r_v . A_z and B_z are two backshift polynomials defined as

$$A_z = 1 + a_1 z^{-1} + a_2^{-2} + \cdots + a_{n_a} z^{-n_a},$$
$$B_z = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_{n_b} z^{-n_b}.$$

The polynomial orders n_a and n_b are assumed to be given. Eqn. (2) can be written equivalently as the following linear regression model:

$$y_k = \varphi_{0k}^{\mathrm{T}}\theta + v_k, \tag{3}$$

where

$$\varphi_{0k} = \left[-x_{k-1} - x_{k-2} \cdots - x_{k-n_a} u_k u_{k-1} \cdots u_{k-n_b}\right]^{\mathrm{T}},$$
$$\theta = \left[a_1\ a_2 \cdots a_{n_a}\ b_0\ b_1 \cdots b_{n_b}\right]^{\mathrm{T}}.$$

Vector φ_{0k} represents system's excitation and response information necessary for parameter estimation, while vector θ contains model parameters to be estimated.

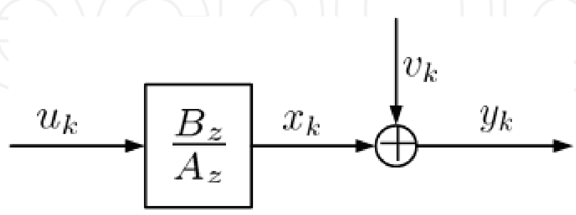

Fig. 2. Output-error (OE) model structure.

For a system with the output-error (OE) model placed in a networked environment subject to randomly missing outputs, the objectives of this paper are:

1. Design an output estimator to online estimate the missing output measurements.
2. Develop a recursive Kalman filter based identification algorithm to estimate unknown model parameters.

3. Propose an adaptive tracking controller to make the system output track a given desired signal.
4. Analyze the convergence properties of the proposed algorithms.

3. Parameter Estimation, Output Estimation, and Adaptive Control Design

There are two main challenges of the adaptive control design for a networked system as depicted in Fig. 1: (1) randomly missing output measurements; (2) unknown system model parameters. Therefore, in this section, we first propose algorithms for missing output estimation and unknown model parameter estimation, and then design the adaptive control scheme.

3.1 Parameter estimation and missing output estimation

Consider the model in (3). It is shown by (Cao & Schwartz, 2003) and (Guo, 1990) that the corresponding Kalman filter can be conveniently used for parameter estimation. In combination with an auxiliary model, the Kalman filter based parameter estimation algorithm for an OE model is given by

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_{a,k}(y_k - \varphi_{a,k}^{\mathrm{T}}\hat{\theta}_{k-1}), \tag{4}$$

$$K_{a,k} = \frac{P_{a,k-1}\varphi_{a,k}}{r_v + \varphi_{a,k}^{\mathrm{T}} P_{a,k-1}\varphi_{a,k}}, \tag{5}$$

$$P_{a,k} = P_{a,k-1} - \frac{P_{a,k-1}\varphi_{a,k}\varphi_{a,k}^{\mathrm{T}}P_{a,k-1}}{r_v + \varphi_{a,k}^{\mathrm{T}} P_{a,k-1}\varphi_{0k}}, \tag{6}$$

$$x_{a,k} = \phi_{a,k}^{\mathrm{T}}\hat{\theta}_k, \tag{7}$$

$$\varphi_{a,k} = \left[-x_{a,k-1} \; -x_{a,k-2} \cdots -x_{a,k-n_a} \; u_k \; u_{k-1} \cdots u_{k-n_b}\right]^{\mathrm{T}}, \tag{8}$$

where $\hat{\theta}_k$ represents the estimated parameter vector at time instant k.

It is worth to note that the above algorithm as shown in (4)-(8) cannot be *directly* applied to the parameter estimation of systems with randomly missing outputs in a network environment, as y_k in (4) may not be available. This motivates us to develop a new algorithm that can simultaneously online estimate the unavailable missing output and estimate system parameters under the network environment. The proposed algorithm consists of two steps.

Step 1: Output estimation

Albertos, et al. propose a simple algorithm that uses the input-output model, replacing the unknown past values by estimates when necessary (Albertos et al., 2006). Inspired by this work, we design the following output estimator:

$$z_k = \gamma_k y_k + (1-\gamma_k)\hat{y}_k, \tag{9}$$

with

$$\hat{y}_k = \varphi_k^{\mathrm{T}}\hat{\theta}_{k-1}.$$

In (9), γ_k is a Bernoulli random variable used to characterize the availability of y_k at time instant k at the controller node, as defined in (1). With the time-stamp technique, the controller node can detect the availability of the output measurements, and thus, the values of γ_k (either 1 or 0) are known. The knowledge of their corresponding probability μ_k is not used in the designed estimator. The structure of the designed output estimator is intuitive and simple yet very effective, which will be seen soon from the simulation examples.

Step 2: Model parameter estimation

Replacing y_k in the algorithm (4)-(8) by z_k, defining a new φ_k, and considering the random variable γ_k, we readily obtain the following algorithm:

$$\hat{\theta}_k = \hat{\theta}_{k-1} + K_k(z_k - \varphi_k^{\mathrm{T}}\hat{\theta}_{k-1}), \tag{10}$$

$$K_k = \frac{P_{k-1}\varphi_k}{r_v + \varphi_k^{\mathrm{T}}P_{k-1}\varphi_k}, \tag{11}$$

$$P_k = P_{k-1} - \gamma_k\frac{P_{k-1}\varphi_k\varphi_k^{\mathrm{T}}P_{k-1}}{r_v + \varphi_k^{\mathrm{T}}P_{k-1}\varphi_k}, \tag{12}$$

$$x_{b,k} = \varphi_k^{\mathrm{T}}\hat{\theta}_k, \tag{13}$$

$$\varphi_k = \left[-x_{b,k-1}\ -x_{b,k-2}\ \cdots -x_{b,k-n_a}\ u_k\ u_{k-1}\cdots u_{k-n_b}\right]^{\mathrm{T}}. \tag{14}$$

Remark 3.1. *Consider two extreme cases. If the availability sequence* $\{\gamma_1,\cdots,\gamma_k\}$ *constantly assumes 1, then no output measurement is lost, and the algorithm above will reduce to the algorithm (4)-(6). On the other hand, if the availability sequence* γ_k *constantly takes 0, then all output measurements are lost, and the parameter estimates just keep the initial values.*

3.2 Adaptive control design

Consider the tracking problem. Let $y_{r,k}$ be a desired output signal, and define the output tracking error

$$\zeta_k := y_k - y_{r,k}.$$

If the control law u_k is appropriately designed such that $y_{r,k} = \varphi_{0k}^{\mathrm{T}}\theta$, then the average tracking error z_k approaches zero finally. Replacing θ by $\hat{\theta}_{k-1}$ and φ_{0k} by φ_k yields

$$\begin{aligned} y_{r,k} &= \varphi_k^{\mathrm{T}}\hat{\theta}_{k-1} = -\sum_{i=1}^{n_a}\hat{\theta}_{i,k-1}x_{k-i} + \sum_{i=0}^{n_b}\hat{\theta}_{n_a+i+1,k-1}u_{k-i} \\ &= -\hat{a}_{1,k-1}x_{b,k-1} - \cdots - \hat{a}_{n_a,k-1}x_{b,k-n_a} + \hat{b}_{0,k-1}u_k + \cdots + \hat{b}_{n_b,k-1}u_{k-n_b}. \end{aligned}$$

Therefore, the control law can be designed as

$$u_k = \frac{1}{\hat{b}_{0,k-1}}\left[y_{r,k} + \sum_{i=1}^{n_a}\hat{a}_{i,k-1}x_{k-i} - \sum_{i=1}^{n_b}\hat{b}_{i,k-1}u_{k-i}\right]. \tag{15}$$

The proposed adaptive control scheme consists of the missing output estimator [Equation (9)], model parameter estimator [Equations (10-14)], and the adaptive control law [Equation (15)]. The overall control diagram is shown in Fig. 3.

4. Convergence Analysis

This section focuses on the analysis of some convergence properties. Some preliminaries are first summarized to facilitate the following convergence analysis of parameter estimation in (10)-(12) and of output estimation in (9). Inspired by the work in (Chen & Guo, 1991; Ding & Chen, 2004a; Ding et al., 2006), the convergence analysis is carried out under the stochastic framework.

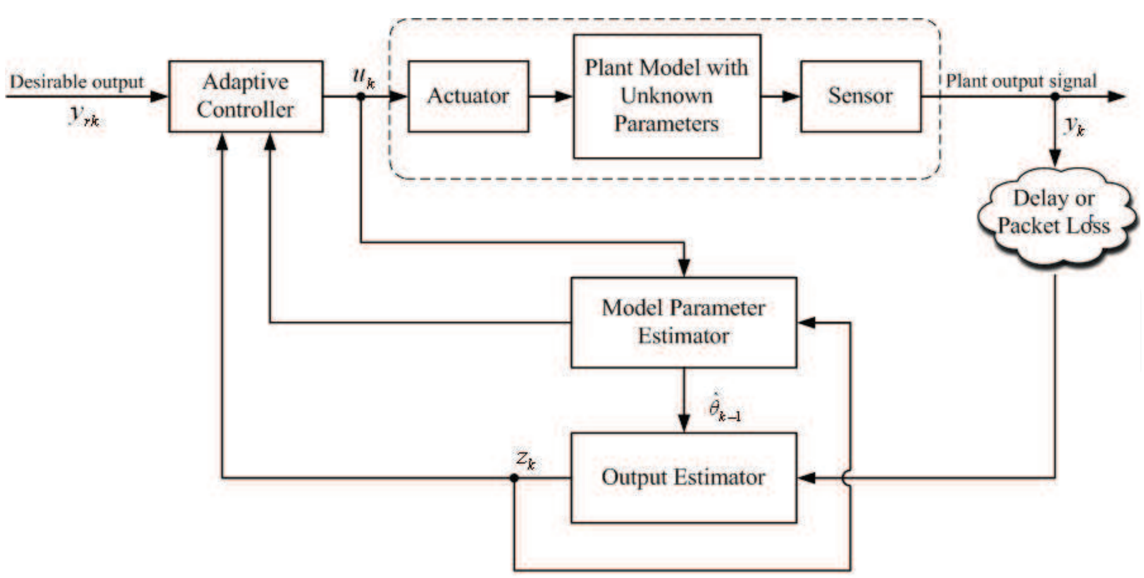

Fig. 3. Adaptive control diagram.

4.1 Preliminaries

To facilitate the convergence analysis, directly applying the matrix inversion formula (Horn

& Johnson, 1991)

$$(A+BCD)^{-1}=A^{-1}-A^{-1}B(C^{-1}+DA^{-1}B)^{-1}DA^{-1},$$

the proposed parameter estimation algorithm in Section 3.1 [(10)-(12] can be equivalently rewritten as:

$$\hat{\theta}_k=\hat{\theta}_{k-1}+r_v^{-1}P_k\varphi_k(z_k-\varphi_k^{\mathrm{T}}\hat{\theta}_{k-1}), \tag{16}$$

$$P_k^{-1}=P_{k-1}^{-1}+r_v^{-1}\gamma_k\varphi_k\varphi_k^{\mathrm{T}}. \tag{17}$$

Suppose that P_k is initialized by p_0I, where p_0 is a positive real value large enough, and define $r_k=\mathrm{tr}(P_k^{-1})$. The relation between r_k and $|P_k^{-1}|$ can be established in the following lemma.

Lemma 4.1. *The following relation holds:*

$$\ln \mathrm{E}|P_k^{-1}|=O\left(\ln \mathrm{E}r_k\right). \tag{18}$$

Proof: Using the formulae

$$\mathrm{tr}(X)=\sum_{i=1}^{n}\lambda_i(X) \ \text{ and } |X|=\prod_{i=1}^{n}\lambda_i(X),$$

where n is the dimension of X, we have

$$\mathrm{E}|P_k^{-1}|\le(\mathrm{E}r_k)^n.$$

This completes the proof.

The next lemma shows the convergence of two infinite series that will be useful later.

Lemma 4.2. *The following inequalities hold:*

$$\sum_{i=1}^{t}\mu_i r_v^{-1}\mathrm{E}\left(\varphi_i^{\mathrm{T}}P_i\varphi_i\right)\le \ln \mathrm{E}|P_k^{-1}|+n_0\ln p_0 \text{ a.s.}, \tag{19}$$

$$\sum_{i=1}^{\infty}\mu_i r_v^{-1}\frac{\mathrm{E}\left(\varphi_i^{\mathrm{T}}P_i\varphi_i\right)}{\left(\ln \mathrm{E}|P_i^{-1}|\right)^c}<\infty \text{ a.s.}, \tag{20}$$

where $c>1$.

Proof: The proof can be done along the similar way as Lemma 2 in (Ding & Chen, 2004b) and is omitted here. □

The following is the well-known martingale convergence theorem that lays the foundation for the convergence analysis of the proposed algorithms.

Theorem 4.1. (Goodwin & Sin, 1984) *Let* $\{X_k\}$ *be a sequence of nonnegative random variables adapted to an increasing* σ*-algebras* $\{F_k\}$. *If*

$$E(X_{k+1} \mid F_k) \le (1+\check{n}_k)X_k - \alpha_k + \beta_k, \text{ a.s.,}$$

where $\alpha_k \ge 0$, $\beta_k \ge 0$, $EX_0 < \infty$, $\sum_{i=0}^{\infty} |\varepsilon_i| < \infty$ *and* $\sum_{i=0}^{\infty} \beta_i < \infty$ *almost surely (a.s.), then* X_k *converges a.s. to a finite random variable and*

$$\lim_{N\to\infty} \sum_{i=0}^{N} \alpha_i < \infty, \text{ a.s.}$$

4.2 Convergence analysis

To carry out the convergence analysis of the proposed algorithms, it is essential to appropriately construct a martingale process satisfying the conditions of Theorem 4.1. Main results on the convergence properties of the proposed algorithm are summarized in the following Theorem.

Theorem 4.2. *For the system considered in (3), assume that*

(A1) $\{v_k, F_k\}$ *is a martingale difference sequence satisfying*

$$E(v_k \mid F_{k-1}) = 0, \text{ a.s.,} \tag{21}$$

$$E(v_k^2 \mid F_{k-1}) = r_v < \infty, \text{ a.s.;} \tag{22}$$

(A2) $\dfrac{1}{A_z} - \dfrac{1}{2}$ *is strictly positive real;*

(A3) B_z *is stable; i.e., zeros o f* B_z *are inside the closed unit disk.*

Suppose the desired output signal is bounded: $|y_{r,k}| < \infty$. *Applying the missing output estimator [Equation (9)], model parameter estimator [Equations (10-14)], and the adaptive control law [Equation (15)], then the output tracking error has the property of minimum variance, i.e.,*

$$(1)\ \lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} (y_{r,i} - y_i + v_i)^2 = 0, \text{ a.s.;}$$

$$(2)\ \lim_{k\to\infty} \frac{1}{k}\sum_{i=1}^{k} \mu_i E\{(z_i - y_{r,i})^2 \mid F_{i-1}\} = r_v < \infty, \text{ a.s.}$$

Proof: As pointed out in (Goodwin & Sin, 1984; Chen & Guo, 1991), from (A2) it follows that

$$\frac{1}{k}\sum_{i=1}^{k} u_i^2 \le O(1) + O\left(\frac{c_1}{k}\sum_{i=1}^{k} y_i^2\right), \text{ a.s.} \tag{23}$$

Here, c_1 is a positive constant. Define the following vectors:

$$e_k = z_k - \varphi^{\mathrm{T}}\hat{\theta}_{k-1},$$

$$\bar{\eta}_k = y_k - x_{b,k},$$

$$\eta_k = \gamma_k \bar{\eta}_k,$$

$$\bar{\tau}_k = y_{r,k} - y_k + v_k,$$

$$\tau_k = \gamma_k \bar{\tau}_k.$$

From (2), (3), (16) and (16), it follows that

$$\eta_k = \gamma_k (x_k - x_{b,k} + v_k), \tag{24}$$

$$\eta_k = (1 + r_v^{-1}\varphi_k^{\mathrm{T}} P_{k-1}\varphi_k)^{-1} e_k, \tag{25}$$

$$e_k = -\tau_k + \gamma_k v_k. \tag{26}$$

Also define the parameter estimation error vector and a Lyapunov-like function as

$$\tilde{\theta}_k = \hat{\theta}_k - \theta,$$
$$V_k = \tilde{\theta}_k^{\mathrm{T}} P_k^{-1} \tilde{\theta}_k.$$

From (9), (16) and (25), we obtain

$$\tilde{\theta}_k = \tilde{\theta}_{k-1} + r_v^{-1} P_k \varphi_k e_k = \tilde{\theta}_{k-1} + r_v^{-1} P_{k-1}\varphi_k \eta_k. \tag{27}$$

With (17) and (27), V_k can be further evaluated as

$$V_k = V_{k-1} + r_v^{-1}\gamma_k (\varphi_k^{\mathrm{T}}\tilde{\theta}_k)^2 + 2r_v^{-1}\varphi_k^{\mathrm{T}}\tilde{\theta}_k \eta_k - r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k) e_k^2.$$

Let us define

$$\tilde{u}_k = -\varphi_k^{\mathrm{T}}\tilde{\theta}_k,$$

$$\tilde{y}_k = \frac{1}{2}\varphi_k^{\mathrm{T}}\tilde{\theta}_k + (\bar{\eta}_k - v_k).$$

Then we have

$$
\begin{aligned}
V_k &= V_{k-1} - 2r_v^{-1}\gamma_k \tilde{u}_k \tilde{y}_k + 2r_v^{-1}\gamma_k \varphi_k^{\mathrm{T}} \tilde{\theta}_k v_k - r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k) e_k^2 \\
&= V_{k-1} - 2r_v^{-1}\gamma_k \tilde{u}_k \tilde{y}_k + 2r_v^{-1}\gamma_k \varphi_k^{\mathrm{T}} \tilde{\theta}_{k-1} v_k + 2r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k \left[(e_k - \gamma_k v_k)^{\mathrm{T}} v_k + \gamma_k v_k^2 \right] \\
&\quad - r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k)\tau_k^2 + 2r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k)\tau_k v_k \\
&\quad - r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k) v_k^2 \\
&\leq V_{k-1} - 2r_v^{-1}\gamma_k \tilde{u}_k \tilde{y}_k + 2r_v^{-1}\gamma_k \varphi_k^{\mathrm{T}} \tilde{\theta}_{k-1} v_k + 2r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k \left[(e_k - \gamma_k v_k)^{\mathrm{T}} v_k + \gamma_k v_k^2 \right] \\
&\quad - r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k)\tau_k^2 + 2r_v^{-2}\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k)\tau_k v_k .
\end{aligned}
\tag{28}
$$

Note that $\varphi_k^{\mathrm{T}} \tilde{\theta}_{k-1}$, $e_k - \gamma_k v_k$, $\varphi_k^{\mathrm{T}} P_k \varphi_k$ and τ_k are uncorrelated with v_k and F_{k-1} -measurable. Thus taking the conditional expectation of both sides of (28) with respect to F_{k-1} gives

$$
\begin{aligned}
\mathrm{E}\left(V_k \mid \mathrm{F}_{k-1}\right) &\leq V_{k-1} - 2r_v^{-1}\mu_k \mathrm{E}\left(\tilde{u}_k \tilde{y}_k\right) + 2r_v^{-1}\mu_k \mathrm{E}\left(\varphi_k^{\mathrm{T}} P_k \varphi_k\right) \\
&\quad - r_v^{-2}\mu_k \mathrm{E}\left[\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k) \right] \bar{\tau}_k^2 .
\end{aligned}
\tag{29}
$$

Consider that

$$
\begin{aligned}
A_z(\bar{\eta}_k - v_k) &= A_z(y_k - x_{b,k}) \\
&= B_z u_k - A_z x_{b,k} \\
&= -\varphi_k^{\mathrm{T}} \tilde{\theta}_k = \tilde{u}_k .
\end{aligned}
$$

Therefore, we have

$$
\tilde{y}_k = \left(\frac{1}{A_z} - \frac{1}{2} \right) \tilde{u}_k .
$$

In (A2), it is assumed that $\left(\frac{1}{A_z} - \frac{1}{2} \right)$ is positive real, which indicates

$$
S_k := 2r_v^{-1} \sum_{i=1}^{k} \mu_k \tilde{u}_k \tilde{y}_k \geq 0, \text{ a.s.} \tag{30}
$$

Adding S_k to both sides of (29) yields

$$
\begin{aligned}
\mathrm{E}\left(V_k + S_k \mid \mathrm{F}_{k-1}\right) &\leq V_{k-1} + S_{k-1} + 2r_v^{-1}\mu_k \mathrm{E}\left(\varphi_k^{\mathrm{T}} P_k \varphi_k\right) \\
&\quad - r_v^{-2}\mu_k \mathrm{E}\left[\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1}\varphi_k^{\mathrm{T}} P_k \varphi_k) \right] \bar{\tau}_k^2 .
\end{aligned}
\tag{31}
$$

Define a new sequence:

$$W_k = \frac{V_k + S_k}{\left(\ln \mathrm{E} \,|\, P_k^{-1} \,|\right)^c}, c > 1. \tag{32}$$

Since $\ln \mathrm{E} \,|\, P_k^{-1} \,|$ is nondecreasing and $\varphi_k^{\mathrm{T}} P_k \varphi_k = o(1)$, there exists a k_0 such that if $k \ge k_0$ we have

$$\begin{aligned} \mathrm{E}\left(W_k \,|\, \mathrm{F}_{k-1}\right) \le & \frac{V_{k-1} + S_{k-1}}{\left(\ln \mathrm{E} \,|\, P_k^{-1} \,|\right)^c} + \frac{2 r_v^{-1} \mu_k \mathrm{E}\left(\varphi_k^{\mathrm{T}} P_k \varphi_k\right)}{\left(\ln \mathrm{E} \,|\, P_k^{-1} \,|\right)^c} \\ & - \frac{r_v^{-2} \mu_k \mathrm{E}\left[\varphi_k^{\mathrm{T}} P_k \varphi_k (1 - r_v^{-1} \varphi_k^{\mathrm{T}} P_k \varphi_k)\right] \overline{\tau}_k^2}{\left(\ln \mathrm{E} \,|\, P_k^{-1} \,|\right)^c} \\ \le & W_{k-1} + \frac{2 r_v^{-1} \mu_k \mathrm{E}\left(\varphi_k^{\mathrm{T}} P_k \varphi_k\right)}{\left(\ln \mathrm{E} \,|\, P_k^{-1} \,|\right)^c} \\ & - \frac{r_v^{-2} \mu_k \mathrm{E}\left(1 - r_v^{-1} \varphi_k^{\mathrm{T}} P_k \varphi_k\right) \overline{\tau}_k^2}{\left(\ln \mathrm{E} \,|\, P_k^{-1} \,|\right)^c}. \end{aligned} \tag{33}$$

From (12) we have

$$\mathrm{E}\left(1 - r_v^{-1} \varphi_k^{\mathrm{T}} P_k \varphi_k\right) > 0.$$

Also note that by Lemma 4.2 the summation of the third term in (33) from 0 to ∞ is finite. Therefore, Theorem 4.1 is applicable, and it gives

$$\sum_{k=1}^{\infty} \frac{r_v^{-2} \mu_k \mathrm{E}\left(1 - r_v^{-1} \varphi_k^{\mathrm{T}} P_k \varphi_k\right) \overline{\tau}_k^2}{\left(\ln E \,|\, P_k^{-1} \,|\right)^c} < \infty \text{ a.s.} \tag{34}$$

Further, Lemma 4.1 indicates

$$\sum_{k=1}^{\infty} \frac{r_v^{-2} \mu_k \mathrm{E}\left(1 - r_v^{-1} \varphi_k^{\mathrm{T}} P_k \varphi_k\right) \overline{\tau}_k^2}{\left(\ln \mathrm{E} r_k\right)^c} < \infty \text{ a.s.} \tag{35}$$

As $\left[1 - r_v^{-1} \mathrm{E}\left(\varphi_k^{\mathrm{T}} P_k \varphi_k\right)\right]$ is positive and nondecreasing, it holds that $1 = O\left[1 - r_v^{-1} \mathrm{E}\left(\varphi_k^{\mathrm{T}} P_k \varphi_k\right)\right]$. Hence,

$$\sum_{i=1}^{\infty} \frac{\overline{\tau}_i^2}{\left(\ln \mathrm{E} r_i\right)^c} < \infty \text{ a.s.} \tag{36}$$

Since $\lim_{k \to \infty} \ln \mathrm{E} r_k = \infty$, then from the Kronecker lemma [15] it follows that

$$\lim_{k\to\infty} \Delta_k = 0, \text{ a.s.,}$$

where

$$\Delta_k := \frac{1}{(\ln \mathrm{E} r_k)^c} \sum_{i=1}^{k} \overline{\tau}_i^2 .$$

With

$$r_k = \frac{n}{p_0} + \sum_{i=1}^{k} r_v^{-1} \gamma_i \varphi_i^{\mathrm{T}} \varphi_i$$

and (23), we obtain

$$\begin{aligned} \frac{1}{k}\sum_{i=1}^{k} \overline{\tau}_i^2 &= \frac{\Delta_k}{k} O\left[(\ln \mathrm{E} r_k)^c \right] \\ &= \frac{\Delta_k}{k} O\left[\mathrm{E}(r_k) \right] \\ &= \frac{\Delta_k}{k} O\left(\frac{n}{p_0} + n_a \sum_{i=1}^{k} \mu_i \mathrm{E}^2(z_i) + n_b \sum_{i=1}^{k} u_i^2 \right) \\ &= \Delta_k O\left(\frac{1}{k} \sum_{i=1}^{k} y_i^2 \right) \end{aligned} \tag{37}$$

By (22) we have

$$\frac{1}{k}\sum_{i=1}^{k} y_i^2 = O(1) + O\left(\frac{1}{k}\sum_{i=1}^{k} \eta_i^2 \right). \tag{38}$$

Substituting (37) into (38) gives

$$\frac{1}{k}\sum_{i=1}^{k} y_i^2 = O(1), \text{ a.s.,}$$

which implies together with (37) that

$$\lim_{k\to\infty} \frac{1}{k} \sum_{i=1}^{k} \overline{\tau}_i^2 = 0, \text{ a.s.,}$$

or equivalently

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}(y_{r,i}-y_i+v_i)^2=0,\ \text{a.s.} \tag{39}$$

Since

$$\begin{aligned}\mathrm{E}\left\{(y_{r,k}-y_k+v_k)^2\mid \mathrm{F}_{k-1}\right\} &= \mathrm{E}[(y_{r,k}-y_k)^2+2y_{r,k}v_k-2y_kv_k+v_k^2\mid \mathrm{F}_{k-1}]\\ &= \mathrm{E}[(y_{r,k}-y_k)^2\mid \mathrm{F}_{k-1}]+0-2r_v+r_v\\ &= \mathrm{E}[(y_{r,k}-y_k)^2\mid \mathrm{F}_{k-1}]-r_v,\ \text{a.s.,}\end{aligned}$$

and $\gamma_k z_k=\gamma_k y_k$, we have

$$\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\mu_i\mathrm{E}\left\{(z_i-y_{r,i})^2\mid \mathrm{F}_{i-1}\right\}=\lim_{k\to\infty}\frac{1}{k}\sum_{i=1}^{k}\mu_i\mathrm{E}\left\{(y_i-y_{r,i})^2\mid \mathrm{F}_{i-1}\right\}=r_v,\ \text{a.s.}$$

This completes the proof. □

5. Illustrative Examples

In this section, we give three examples to illustrate the adaptive control design scheme proposed in the previous sections.
The OE model shown in Fig. 2 in the simulation is chosen as

$$y_k=\frac{b_0+b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}u_k+v_k,$$

which is assumed to be placed in a network environment (Fig. 1) with randomly missing output measurements and unknown model parameters. $\{v_k\}$ is a Gaussian white noise sequence with zero mean and variance $r_v=0.05^2$. The parameter vector $\theta=[a_1\ a_2\ b_0\ b_1\ b_2]^{\mathrm{T}}$ is to be estimated. Here, true values of θ are

$$\theta=[-0.3\ 0.6\ 0.5\ 0.2\ 0.34]^{\mathrm{T}}.$$

For simulation purposes, we assume that: (1) θ is unknown and initialized by ones; (2) the output measurement $\{y_k\}$ is subject to randomly missing when transmitted to the controller node; (3) the availability of the output measurements (y_k) at the controller node is characterized by the probability μ_k; (4) The desired output signal to be tracked is a square wave alternating between -1 and 1 with a period of 1000. Mathematically, it is given by

$$y_{r,(500i+j)}=(-1)^{i+1},\ i=0,1,2,\cdots,\ j=1,2,...500.$$

In the following simulation studies, we carry out experiments for three different scenarios regarding the availability of the output measurements at the controller node and the parameter variation, and examine the control performance, respectively. According to the proposed adaptive control scheme shown in Fig. 3, we apply the algorithms of the missing output estimator, model parameter estimator, and the adaptive control law to the networked control system.

Example 1: $\mu_k = 0.85$. In the first example, 85% of all the measurements are available at the controller node after network transmission from the sensor to the controller. The output response is shown in Fig. 4, from which it is observed that the output tracking performance is satisfactory. In order to take a closer observation on the model parameter estimation and output estimation, we define the relative parameter estimation error as

$$\delta_{par}\% = \frac{\|\hat{\theta}_k - \theta\|}{\|\theta\|} \times 100\%.$$

It is shown in Fig. 5 (solid blue curve) that $\delta_{par}\%$ is becoming smaller with k increasing. Comparison between the estimated outputs and true outputs during the time range $501 \le t \le 550$ is illustrated in Fig. 6: The dashed lines are corresponding to the time instants when data missing occurs, and the small circles on the top of the dashed lines represent the estimated outputs at these time instants. From Fig. 6 it can be found that the missing output estimation also exhibits good performance.

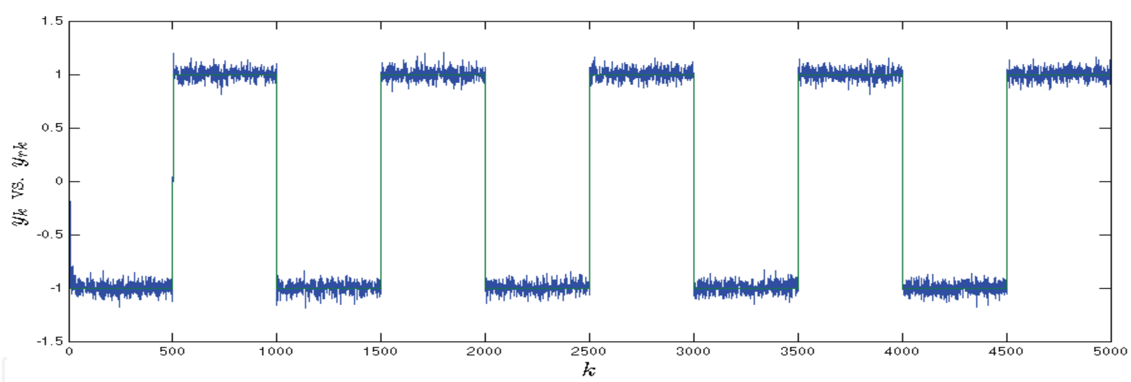

Fig. 4. Example 1: Output response when $\mu_k = 0.85$.

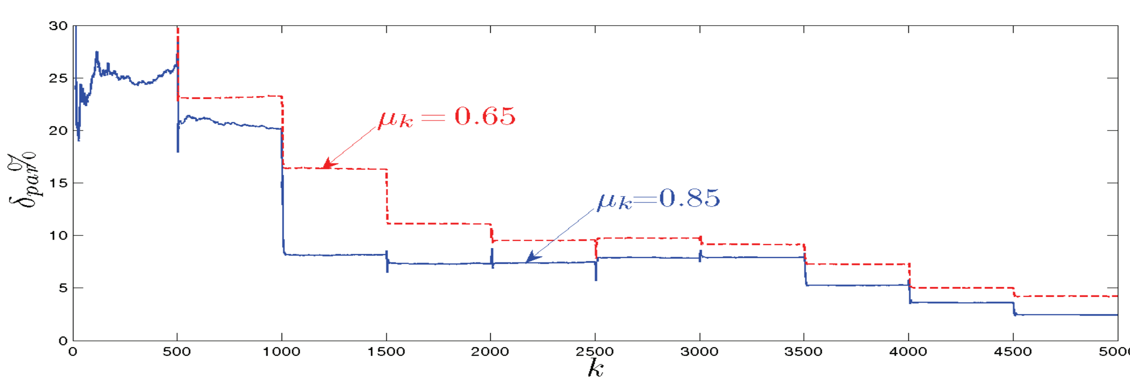

Fig. 5. Comparison of relative Parameter estimation errors for Example 1 and Example 2:

Blue solid line for Example 1; red dotted line for Example 2.

Example 2: $\mu_k = 0.65$. In the second example, a worse case subject to more severe randomly missing outputs is examined: Only 65% of all the measurements are available at the controller node. The output response is shown in Fig. 7. Even though the available output measurements are more scarce than those in Example 1, it is still observed that the output is tracking the desired signal with satisfactory performance. The relative parameter estimation error, $\delta_{par}\%$, is shown in Fig. 5 (dashed red curve). Clearly, it is decreasing when k is increasing. The estimated outputs and the true outputs are illustrated in Fig. 8, from which we can see good output estimation performance.

For the comparison purpose, the relative parameter estimation errors of these two examples are shown in Figure 5. We can see that the parameter estimation performance when $\mu_k = 0.85$ is better than that when $\mu_k = 0.65$ .It is no doubt that the estimation performance largely depends on data completeness that is characterized by μ_k .

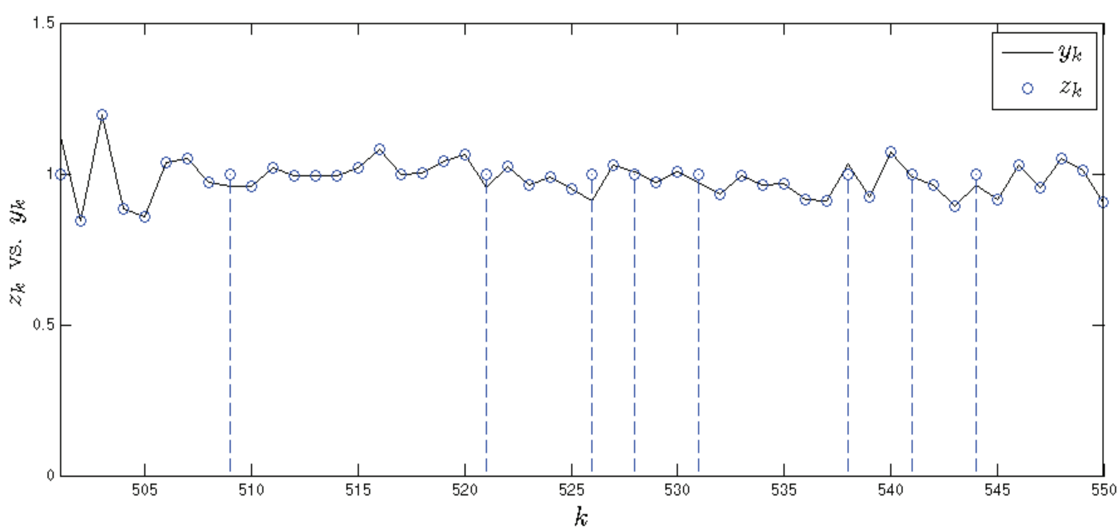

Fig. 6. Example 1: Comparison between estimated and true outputs when $\mu_k = 0.85$ (The dashed line represents output missing).

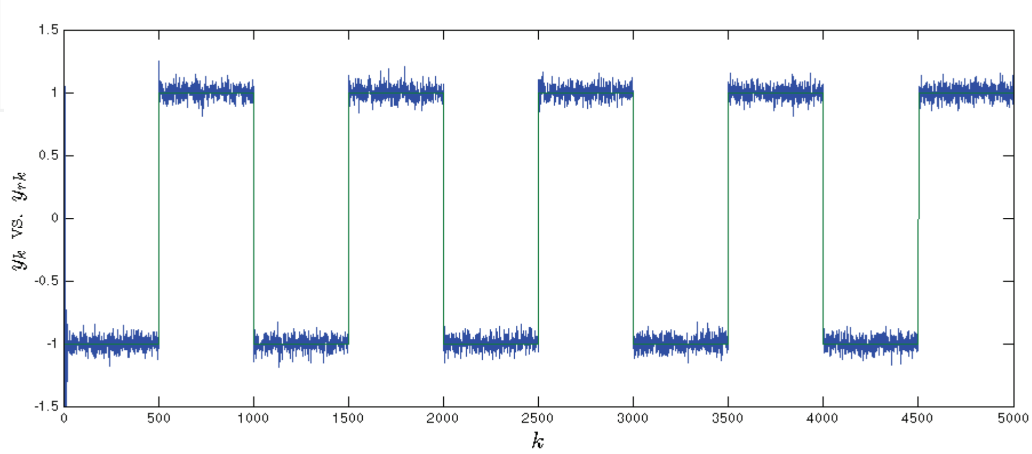

Fig. 7. Example 2: Output response when $\mu_k = 0.65$.

Example 3: Output tracking performance subject to parameter variation. In practice, the model parameters may vary during the course of operation due to the change of load, external disturbance, noise, and so on. Hence, it is also paramount to explore the robustness of the designed controller against he influence of parameter variation. In this example, we assume that at $k = 2500$, model parameters are all increased by 50%. The output response is shown in Fig. 9. It can be seen that: At $k = 2500$, the output response has a big overshoot because of the parameter variation; however, the adaptive control scheme quickly forces the system output to track the desired signal again.

Observing Figs. 4, 7, and 9 in three examples, we notice that the tracking error and oscillation still exist. This is mainly due to (1) the missing output measurements, and, (2) the relatively high noise-signal ratio (around 25%). On the other hand, it is desirable to develop new control schemes to further improve the control performance for networked systems subject to limited feedback information, which is worth to do extensive research.

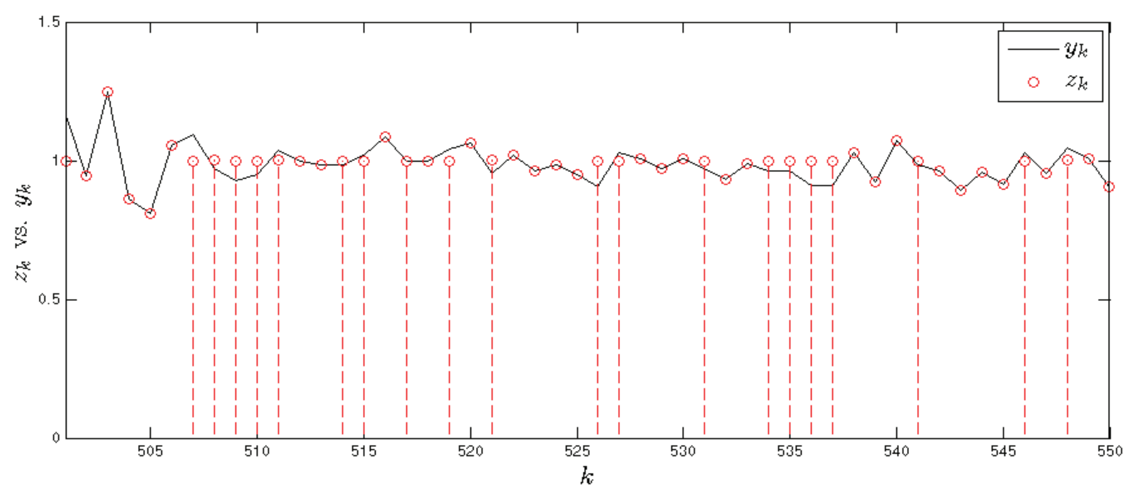

Fig. 8. Example 2: Comparison between estimated and true outputs when $\mu_k = 0.65$ (The dashed line represents output missing).

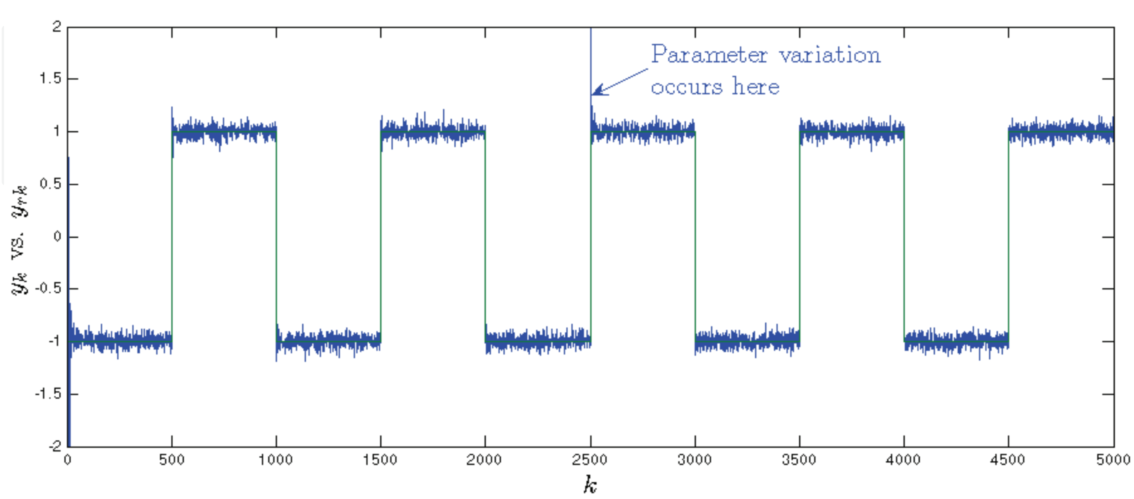

Fig. 9. Example 3: Output response subject to parameter variation: At time instant $k = 2500$, all parameters are increased by 50%.

6. Conclusion

This paper has investigated the problem of adaptive control for systems with SISO OE models placed in a network environment subject to unknown model parameters and randomly missing output measurements. The missing output estimator, Kalman filter based model parameter estimator, and adaptive controller have been designed to achieve output tracking. Convergence performance of the proposed algorithms is analyzed under the stochastic framework. Simulation examples verify the proposed methods. It is worth mentioning that the proposed scheme is developed for SISO systems in this work, and the extension to multi-input-multi-output (MIMO) systems is a subject worth further researching

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Adaptive control has been a remarkable field for industrial and academic research since 1950s. Since more and more adaptive algorithms are applied in various control applications, it is becoming very important for practical implementation. As it can be confirmed from the increasing number of conferences and journals on adaptive control topics, it is certain that the adaptive control is a significant guidance for technology development.The authors the chapters in this book are professionals in their areas and their recent research results are presented in this book which will also provide new ideas for improved performance of various control application problems.