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# Discrete Model Matching Adaptive Control for Potentially Inversely Non-Stable Continuous-Time Plants by Using Multirate Sampling

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## 1. Introduction

Adaptive control theory has been widely applied for stabilizing linear time invariant plants of unknown parameters (Goodwin & Sin, 1984). One of the more used methods for such a purpose is based on the model reference adaptive control (MRAC) problem (Aström & Wittenmark, 1997). Such a method requires some assumptions relative to the plant to be controlled in order to carry out the synthesis of a stable controller (Narendra & Annaswamy, 1989). One of them is that the plant has to be inversely stable, what means that its zeros have to be located within the stability domain. However, this information is not always available to the designer when the system under control contains unknown parameters. There are several alternatives to circumvent this drawback and carry out the stable adaptive control design. Some of them consist on relaxing the control performance from the model matching to that achievable from the closed-loop pole placement (Alonso-Quesada & De la Sen, 2004 and Arvanitis, 1999). In this way, the stabilization of the closed-loop system can be ensured although its transient behaviour cannot be fixed to a predefined one.

On one hand, the work (Alonso-Quesada & De la Sen, 2004) includes an estimates modification in the estimation algorithm to ensure the controllability of the estimated plant model without assuming any knowledge about the parameters of the plant to be controlled. This controllability property is crucial to avoid pole-zero cancellations between the estimated plant and the controller, which are both time-varying. In this context, a projection of the estimated plant parameters into a region in the parameter space where the closed-loop system is free of pole-zero cancellations for all time can be alternatively used provided that the true plant is controllable and the knowledge of a region where the true plant parameters belong to (Goodwin & Mayne, 1987).

On the other hand, the research (Arvanitis, 1999) proposes an adaptive pole-placement control for linear systems using generalized sampled-data hold functions. Following such a technique, gain controllers essentially need to be designed. Concretely, a periodic piecewise constant gain controller is added in the feedback chain. In the non-adaptive case, such constant gain values are those required so that the discretized closed-loop model under a fundamental sampling period and a zero-order hold (ZOH) be stabilized. For such a

purpose, each sampling period is divided in a certain finite number of uniform subintervals and the controller gain takes a different value within each of them in order to locate the discretized poles at the stable desired locations. In other words, the controller consists of a constant vector of gains. In this sense, the controller works with a sampling rate faster than that used to discretize the plant to be controlled. In the adaptive case, an estimated model of the discretized plant is on-line updated by means of an estimation algorithm. Such a model is used to parameterize the controller gains vector which becomes time-varying and converges asymptotically to a constant one.

Another alternative, which does not relax the MRAC objective, to overcome the drawback of the unstable zeros in a continuous-time plant is the design of discrete-time controllers which are synthesized from the discretization of the continuous-time plant by means of a holder device combined with a multirate with fast input sampling rate (De la Sen & Alonso-Quesada, 2007 and Liang & Ishitobi, 2004). The main motivation of this method is that an inversely stable discretized model of the plant can be obtained with an appropriate choice of the multirate gains. In this way, an adaptive controller can be designed to match a discrete-time reference model since all the discretized plant zeros may be cancelled if suited.

In this context, a *fractional-order hold (FROH) with a multirate input* is used in this paper to obtain an inversely stable discretized plant model from a possible non-inversely stable and unstable time invariant continuous-time plant. Then, a control design for matching a discrete-time reference model is developed for both non-adaptive and adaptive cases. Note that a FROH includes as particular cases the ZOH and the FOH (first-order hold). In this way, the stabilization of the continuous-time plant is guaranteed without any assumption about the stability of its zeros and without requiring estimates modification in contrast with previous works on the subject. In this sense, this paper is an extension of the work (De La Sen & Alonso-Quesada, 2007) where the same problem is addressed. *The main contribution is related to the method used to build the continuous-time plant input from the discrete-time controller output.* In the present paper, the FROH acts on the fundamental sampling period used to discretize the plant (plant output sampling) while in the aforementioned paper the FROH acted on the sampling period used to define the multirate device at the plant input. This later sampling period is an integer fraction of the plant output one, i.e. an integer number of input samples takes place within each output sampling period. Such an integer has to be suitably chosen for disposing of the enough freedom degrees being necessary to place the discretized plant zeros at desired locations, namely within the unity circle in order to guarantee the inverse stability of the discretized plant model.

The assumptions about the plant to guarantee the closed-loop stability of the adaptive control system are the following: (1) the stabilizability of the plant and (2) the knowledge of the continuous-time plant order. The motivation for using a multirate sampling input instead of the most conventional single rate one resides in the fact that the former, with the appropriate multirate gains, provides an inversely stable discretized plant model without requirements on either the stability of the continuous-time plant zeros or the size of the sampling period. In this sense, a single rate input can only provide an inversely stable discretized plant from an inversely stable continuous-time plant and, moreover, the fundamental sampling period to discretized the plant has to be sufficiently small (Blachuta, 1999). Finally, the use of a FROH, instead of the most conventional ZOH, allows to accommodate better some discrete adaptive techniques to the transient response of discrete-time controlled continuous-time plants (Bárcena et al., 2000 and Liang et al., 2003).

The paper is organized as follows. Section 2 formulates a discrete state-space description under fast input sampling to then obtain an input-output discrete transfer function for the running slow sampling rate, namely, that acting on the output signal. The selection of the scalar gains that generate the fast sampled input so that the discrete plant zeros are stable is focused on depending on the continuous-time plant parametrization. Section 3 discusses the synthesis of a model-matching based controller with a possible potential free design of all the zeros of the reference model. The case of known plant parameters and the adaptive case for not fully known plant parameters are both considered. Two alternatives are proposed to update on-line the time-varying multirate gains in the adaptive case. The first one updates the multirate gains for all sampling instants in order to maintain the zeros of the estimated discretized plant fixed within the stability domain. On the contrary, the other one updates the multirate gains only when the change of gains is crucial to guarantee the stability of the estimated discretized plant zeros. In this way, the multirate gains are not updated for all sampling instants and then they became piecewise constant. As a result, the zeros of the estimated discretized plant become time-varying within the stability domain. Section 4 deals with the stability analysis of the adaptive control system. Simulated examples which highlight the proposed design philosophy are provided in Section 5. A comparison of the results obtained with the two different methods for updating the multirate gains is presented. Finally, conclusions end the paper in Section 6.

## 2. Discretized Plant Representation

Consider a linear time-invariant, single-input single-output and strictly proper continuous-time plant described by the following state space equations:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad ; \quad y(t) = Cx(t) \tag{1}$$

where  $u(t)$  and  $y(t)$  are, respectively, the input and output signals,  $x(t) \in \mathbb{R}^n$  denotes the state vector and  $A$ ,  $B$  and  $C$  are constant matrices of suitable dimensions. A FROH and a multirate sampling on the fast input sampling will be used in order to obtain an inversely stable discretized plant model. The signal generated by such a device is given by,

$$u(t) = \alpha_j \left\{ u(k) + \beta \frac{u(k) - u(k-1)}{T} (t - kT) \right\} \tag{2}$$

for  $t \in [kT + (j-1)T', kT + jT']$ ,  $j \in \{1, 2, \dots, N\}$  and FROH correcting gain  $\beta \in [-1, 0) \cup (0, 1]$ , where  $T$  is the sampling period (slow sampling) which is divided in  $N$  equal subperiods of length  $T' = \frac{T}{N}$  (fast sampling) to generate the multirate plant input,  $u(k)$  denotes the value of a discrete-time controller output signal at the instant  $kT$ , for all non-negative integer  $k$ , and  $\alpha_j$ 's are real constants. Note that the FROH device operates on the sequence  $\{u(k)\}$  defined at the slow sampling instants  $kT$  and then the input  $u(t)$  is generated over each subperiod with the corresponding gain  $\alpha_j$ , for  $j \in \{1, 2, \dots, N\}$ , via (2). By substituting (2) into (1) and sampling the plant output  $y(t)$  over the sampling period  $T$ , the following

state space representation is obtained which corresponds to the discrete-time plant model that relates the sequences  $\{u(k)\}$  and  $\{y(k)\}$  :

$$x(k+1) = F x(k) + G_1 u(k) + G_2 u(k-1) \quad ; \quad y(k) = C x(k) \quad (3)$$

where  $F = \psi^N = \phi(T) = e^{AT}$  is the continuous-time state transition matrix computed for a slow sampling period and,

$$G_1 = \left( \sum_{\ell=1}^N \alpha_\ell \psi^{N-\ell} \right) \left( \Gamma + \frac{\beta}{T} \Gamma' \right) \in \mathfrak{R}^{n \times 1} \quad ; \quad G_2 = -\frac{\beta}{T} \left( \sum_{\ell=1}^N \alpha_\ell \psi^{N-\ell} \right) \Gamma' \in \mathfrak{R}^{n \times 1} \quad (4)$$

with,

$$\Gamma = \int_0^T \phi(T-s) B ds \in \mathfrak{R}^{n \times 1} \quad ; \quad \Gamma' = \int_0^T \phi(T-s) B s ds \in \mathfrak{R}^{n \times 1} \quad (5)$$

The transfer function of this discrete-time plant model is,

$$H(z) = C(zI_n - \psi^N)^{-1} \left( G_1 + \frac{1}{z} G_2 \right) = \frac{B(z)}{A(z)} \quad (6)$$

where,

$$B(z) = C \text{Adj}(zI_n - \psi^N) C_\Delta(z) g = \text{Det} \begin{bmatrix} zI_n - \psi^N & C_\Delta(z) g \\ -C & 0 \end{bmatrix} = \sum_{i=1}^{n+1} b_i z^{n-i+1} \quad (7)$$

$$A(z) = z \text{Det}(zI_n - \psi^N) = z^{n+1} + \sum_{i=1}^n a_i z^{n-i+1}$$

with  $\text{Adj}(\cdot)$  and  $\text{Det}(\cdot)$  denoting, respectively, the adjoint matrix and the determinant of the square matrix ( $\cdot$ ),  $I_n$  denoting the  $n$ -th order identity matrix, and

$$g = [\alpha_1 \cdots \alpha_n]^T \in \mathfrak{R}^{n+1}; C_\Delta(z) \cdots [\psi^{N-1} \Delta(z) \cdots \psi \Delta(z) \quad \Delta(z)] \in \mathfrak{R}^{n \times N} \quad (8)$$

$$\Delta(z) = z \left( \Gamma + \frac{\beta}{T} \Gamma' \right) - \frac{\beta}{T} \Gamma'$$

Note that the coefficients  $b_i$ , for  $i \in \{1, 2, \dots, n+1\}$ , of the polynomial  $B(z)$  in (7) depend on the values  $\alpha_j$ , for  $j \in \{1, 2, \dots, N\}$ , which define the multirate device. This fact lets to allocate the zeros of the discretized plant model at desired locations if a suitable number of multirate gains is provided. In this sense, the multirate gains  $\alpha_j$ , being the components of the vector  $g$ , are calculated to guarantee that such zeros are maintained within the stability domain, i.e., the unity circle. In particular, the coefficients  $b_i$  can be

expressed as:

$$b_i = \sum_{j=1}^N b_{i,j} \alpha_j \Leftrightarrow v = M g \tag{9}$$

where  $M = [b_{i,j}] \in \mathfrak{R}^{(n+1) \times N}$  and  $v = [b_1 \ b_2 \ \dots \ b_{n+1}]^T$ . The coefficients  $b_{i,j}$  depend on the parameters of the continuous-time plant, the sampling period  $T$  and the correcting gain  $\beta$  of the FROH considered in the discretization process.

**Assumptions 1.**

- (i) The plant is stabilizable, i.e. its transfer function does not possess unstable pole-zero cancellations,
- (ii) the plant order  $n$  is known, and
- (iii) the correcting gain  $\beta$  of the FROH device and the sampling period  $T$  are chosen such that  $M$  is a full rank matrix. \*\*\*

**Remark 1.** The multirate gains vector  $g$  required to place the zeros of the discretized plant transfer function (6) at desired locations may be calculated from (9) provided Assumptions 1 and that  $N \geq n + 1$ . In this sense, such locations are prefixed via a suitable choice of the vector  $v$  composed by the coefficients of the desired polynomial for the transfer function numerator. If  $N > n + 1$ , different solutions can be obtained for  $g$ . Otherwise, i.e. if  $N = n + 1$ , there is a unique solution for the multirate gains vector from the linear algebraic system (9) which places the discretized zeros at desired locations. \*\*\*

The discretized model (6) can be described by the following difference equation:

$$\begin{aligned}
 y(k) &= -\sum_{i=1}^n a_i y(k-i) + \sum_{i=1}^{n+1} b_i u(k-i) = -\sum_{i=1}^n a_i y(k-i) + \sum_{i=1}^{n+1} \sum_{j=1}^N b_{i,j} \alpha_j u(k-i) \\
 &= -\sum_{i=1}^n a_i y(k-i) + \sum_{i=1}^{n+1} \sum_{j=1}^N b_{i,j} \bar{u}_j(k-i) = \theta^T \varphi(k-1)
 \end{aligned} \tag{10}$$

where,

$$\begin{aligned}
 \theta &= [\theta_a^T \ \theta_{b,1}^T \ \theta_{b,2}^T \ \dots \ \theta_{b,n+1}^T]^T ; \ \varphi(k-1) = [\varphi_y^T(k-1) \ \varphi_u^T(k-1) \ \varphi_u^T(k-2) \ \dots \ \varphi_u^T(k-n-1)]^T \\
 \theta_a &= [-a_1 \ -a_2 \ \dots \ -a_n]^T ; \ \varphi_y(k-1) = [y(k-1) \ y(k-2) \ \dots \ y(k-n)]^T \\
 \theta_{b,i} &= [b_{i,1} \ b_{i,2} \ \dots \ b_{i,N}]^T ; \ \varphi_u(k-i) = [\bar{u}_1(k-i) \ \bar{u}_2(k-i) \ \dots \ \bar{u}_N(k-i)]^T \\
 \bar{u}_j(k-i) &= \alpha_j u(k-i)
 \end{aligned} \tag{11}$$

for all  $i \in \{1, 2, \dots, n+1\}$  and all  $j \in \{1, 2, \dots, N\}$ . In the rest of the paper, the case  $N = n + 1$  will be considered for simplicity purposes.

### 3. Control Design

The control objective in the case of known plant parameters is that the discretized plant model matches a stable discrete-time reference model  $H_m(z) = \frac{B_m(z)}{A_m(z)}$  whose zeros can be

freely chosen, where  $z$  is the Z-transform argument. Such an objective is achievable if the discretization process uses the multirate sampling input with the appropriate multirate gains, what guarantees the inverse stability of the discretized plant. Then, all the discretized plant zeros may be cancelled by controller poles. In this way, the continuous-time plant output tracks the reference model output at the sampling instants. The tracking-error between such signals is zero at all sampling instants in the case of known plant parameters while it is maintained bounded for all time while it converges asymptotically to zero as time tends to infinity in the adaptive case considered when the plant parameters are fully or partially unknown. A self-tuning regulator scheme is used to meet the control objective in both non-adaptive and adaptive cases.

#### 3.1 Known Plant

The proposed control law is obtained from the difference equation:

$$R(q) u(k) = T(q) c(k) - S(q) y(k) \quad (12)$$

for all non-negative integer  $k$ , where  $\{c(k)\}$  is the input reference sequence and  $q$  is the running sample rate advance operator being formally equivalent to the Z-argument used in discrete transfer functions. The reconstruction of the continuous-time plant input  $u(t)$  is made by using (2), with the control sequence  $\{u(k)\}$  obtained from (12), with the appropriate multirate gains  $\alpha_j$ , for  $j \in \{1, 2, \dots, N\}$ , to guarantee the stability of the discretized plant zeros.

The discrete-time transfer function of the closed-loop system obtained from the application of the control law (12) to the discretized plant (6) is given by:

$$\frac{Y(z)}{C(z)} = \frac{B(z)T(z)}{A(z)R(z) + B(z)S(z)} = \frac{T(z)}{A(z) + S(z)} \quad (13)$$

where the second equality is fulfilled if the control polynomial  $R(z) = B(z)$ . In this way, the polynomial  $B(z)$ , which is stable, is cancelled. Then, the polynomials  $T(z)$ ,  $R(z)$  and  $S(z)$

of the controller (12) so that  $\frac{Y(z)}{C(z)} = \frac{B_m(z)}{A_m(z)}$  are obtained from:

$$T(z) = B_m(z)A_s(z) \quad ; \quad R(z) = B(z) \quad ; \quad S(z) = A_m(z)A_s(z) - A(z) \quad (14)$$

where  $A_s(z)$  is a stable monic polynomial of zero-pole cancellations of the closed-loop system. The following degree constraints are satisfied in the synthesis of the controller:

$$\begin{cases} \text{Deg}[A_m(z)] + \text{Deg}[A_s(z)] = \text{Deg}[A(z)] = n + 1 \\ \text{Deg}[S(z)] = \text{Deg}[A(z)] - 1 = n \Rightarrow S(z) = \sum_{i=0}^n s_{i+1} z^{n-i} \\ \text{Deg}[T(z)] = \text{Deg}[B_m(z)] + \text{Deg}[A_s(z)] \leq n \end{cases} \quad (15)$$

### 3.2 Unknown Plant

If the continuous-time plant parameters are unknown then the vector  $\theta$  in (11) composed of the discretized plant model parameters is also unknown. However, all the above control design in the previous subsection remains valid if such a parameter vector is estimated by an estimation algorithm. In this way, the controller parameterization can be obtained from  $R(z,k) = \hat{B}(z,k)$ , with  $\hat{B}(z,k)$  denoting the estimated of  $B(z)$  at the current slow sampling instant  $kT$ , and equations similar to (14) by replacing the discretized plant polynomial  $A(z)$  by its corresponding estimated one  $\hat{A}(z,k)$  (Alonso-Quesada & De la Sen, 2004). Note that  $T(z)$  in (14) has to be calculated once for all since  $B_m(z)$  and  $A_s(z)$  are time-invariant while  $S(z)$  is updated at each running sampling time since the polynomial  $\hat{A}(z,k)$  is time-varying. The coefficients of the unknown polynomial  $B(z)$  depend, via (9), on the multirate input gains  $\alpha_j$ , for  $j \in \{1, 2, \dots, N\}$ , being applicable to calculate the input within the inter-sample slow period. However, the estimation algorithm provides an adaptation of each parameter  $b_{i,j}$ , namely  $\hat{b}_{i,j}(k)$ , for  $i, j \in \{1, 2, \dots, N\}$  and all non negative integer  $k$ . Then, the  $\alpha_j$ -gains have to be also updated in order to ensure the stability of the zeros of the estimated discretized plant, i.e. the roots of  $\hat{B}(z,k)$  be stable. Then, the gains  $\alpha_j$  become time-varying, namely  $\hat{\alpha}_j(k)$ . The estimation algorithm for updating the parameters vector  $\hat{\theta}(k)$ , which denotes the estimated of  $\theta$ , and two different design alternatives for the adaptation of the multirate gains are presented below. Also, the main boundedness and convergence properties derived from the use of such algorithms are established.

#### 3.2.1. Estimation algorithm

An 'a priori' estimated parameters vector is obtained at each slow sampling instant by using a recursive least-squares algorithm (Goodwin & Sin, 1984) defined by:

$$\begin{aligned} P(k) &= P(k-1) - \frac{P(k-1) \varphi(k-1) \varphi^T(k-1) P(k-1)}{1 + \varphi^T(k-1) P(k-1) \varphi(k-1)} \\ \hat{\theta}^0(k) &= \hat{\theta}^0(k-1) + \frac{P(k-1) \varphi(k-1) e^0(k)}{1 + \varphi^T(k-1) P(k-1) \varphi(k-1)} \end{aligned} \quad (16)$$

for all integer  $k > 0$  where  $e^0(k) = (\theta - \hat{\theta}^0(k-1))^T \varphi(k-1) = \tilde{\theta}^{0T}(k-1) \varphi(k-1)$  denotes the 'a priori' estimation error and  $P(k)$  is the covariance matrix initialized as  $P(0) = P^T(0) > 0$ .



Such an algorithm provides an estimation  $\hat{\theta}^0(k)$  of the parameters vector by using the regressor  $\varphi(k-1)$ , defined in (11), built with the output and input measurements with the multirate gains  $\hat{\alpha}_j(k-1)$  obtained at the previous slow sampling instant, i.e.  $\bar{u}_j(k-i) = \hat{\alpha}_j(k-1) u(k-i)$  for all  $i \in \{1, 2, \dots, n+1\}$ . Then, an 'a posteriori' estimates vector is obtained in the following way:

**Modification algorithm.**

This algorithm consists of three steps:

Step 1: Built the matrix  $\hat{M}^0(k) = [\hat{b}_{i,j}^0(k)] \in \mathfrak{R}^{N \times N}$ , for  $i, j \in \{1, 2, \dots, N\}$ , from the 'a priori' estimates  $\hat{\theta}_{b,i}^0(k)$ , included in  $\hat{\theta}^0(k)$ , of the corresponding  $\theta_{b,i}$  defined in (11).

Step 2:  $\hat{M}(k) = \hat{M}^0(k)$

If  $\text{Det}[\hat{M}(k)] \geq \delta_0$  then  $\hat{\theta}_{b,i}(k) = \hat{\theta}_{b,i}^0(k)$   
 else while  $\text{Det}[\hat{M}(k)] < \delta_0$   
      $\hat{M}(k) = \hat{M}(k) + \delta I_N$   
   end;  
   for  $i = 1$  to  $N$   
      $\hat{\theta}_{b,i}(k) = \hat{M}_i(k)$   
   end

end.

Step 3:  $\hat{\theta}(k) = [\hat{\theta}_a^T(k) \quad \hat{\theta}_{b,1}^T(k) \quad \hat{\theta}_{b,2}^T(k) \quad \dots \quad \hat{\theta}_{b,N}^T(k)]^T$ ,

for some real positive constants  $\delta \ll 1$  and  $\delta_0 \ll 1$ , and where  $\hat{M}_i(k)$  denotes the  $i$ -th row of  $\hat{M}(k)$ . \*\*\*

**Remark 2.** Note that the estimate  $\hat{\theta}_a^0(k)$  corresponding to the parameters of  $\theta_a$  is not affected by the modification algorithm. Also, note that the *while* instruction part of the second step is doing a finite number of times since there exists a finite integer number  $\ell$  such that  $\text{Det}[\hat{M}(k)] = \text{Det}[\hat{M}^0(k) + \ell \delta I_N] = (\ell \delta)^N + f(\delta, \hat{\theta}_{b,1}^0(k), \hat{\theta}_{b,2}^0(k), \dots, \hat{\theta}_{b,N}^0(k)) \geq \delta_0$ . \*\*\*

### 3.2.2. Updating of the time-varying multirate gains

Once the estimated parameters vector is obtained at each slow sampling instant the multirate input gains have to be updated. Two alternative algorithms are considered to carry out such an operation.

**Algorithm 1.**

A vector of multirate gains is updated at all slow sampling instants in order to maintain the zeros of the estimated discretized plant fixed at desired locations within the stability domain  $|z| < 1$ . Such desired zeros are the roots of a predefined polynomial  $B'(z)$ . For such a

purpose, the required vector  $\hat{g}(k)$  is obtained from the resolution of the following matrix equation:

$$\hat{M}(k) \hat{g}(k) = v \tag{17}$$

at each slow sampling instant, where  $v = [b'_1 \ b'_2 \ \dots \ b'_N]^T$  is composed by the coefficients of  $B'(z)$ ,  $\hat{M}(k) = [\hat{b}_{i,j}(k)] \in \mathfrak{R}^{N \times N}$ , with  $\hat{b}_{i,j}(k)$  denoting each of the 'a posteriori' estimated parameters corresponding to the components of the vectors  $\theta_{b,i}$  defined in (11), and  $\hat{g}(k) = [\hat{\alpha}_1(k) \ \hat{\alpha}_2(k) \ \dots \ \hat{\alpha}_N(k)]^T$ . In this way,  $\hat{g}(k)$  is composed by the multirate gains which make the numerator of the estimated discretized plant model be equal to the desired polynomial  $B'(z)$ . Note that the matrix equation (17) can be solved at all slow sampling instants since the parameters modification added to the estimation algorithm ensures the non-singularity of the matrix  $\hat{M}(k)$ .

**Algorithm 2.**

It consists of solving the equation (17) only when it is necessary to modify the previous values of the multirate gains in order to guarantee the stability of the zeros of the estimated discretized plant model. i.e., the multirate gains remain equal to those of the preceding slow sampling instant if the zeros of the estimated discretized plant obtained with the current estimated parameters vector,  $\hat{\theta}(k)$ , and the previous multirate gains,  $\hat{\alpha}_i(k-1)$ , are within the discrete-time stability domain. Otherwise, the multirate gains are updated by the resolution of the equation (17), which can be solved whenever it is necessary since the matrix  $\hat{M}(k)$  is invertible at all slow sampling instant due to the modification included in the estimation algorithm. In this way, the multirate gains are piecewise constant, the estimated discretized plant zeros are time-varying and the computational burden associated with the updating of the multirate gains is reduced with respect to that of Algorithm 1.

**3.2.3. Properties of the estimated models**

The parameter estimation algorithm, together with any of the considered adaptation algorithms for the multirate gains, possesses the properties given in the following lemma, whose proof is presented in Appendix A.

**Lemma 1. Main properties of the estimation and multirate gains adaptation algorithms**

- (i)  $P(k)$  is uniformly bounded for all non-negative integer  $k$ , and it asymptotically converges to a finite, at least semidefinite positive, limit as  $k \rightarrow \infty$ .
- (ii)  $\hat{\theta}^0(k)$  and  $\hat{\theta}(k)$  are uniformly bounded and they asymptotically converge to a finite limit as  $k \rightarrow \infty$ .
- (iii) The vector  $\hat{g}(k)$  of multirate gains is bounded and converges to a finite limit as  $k \rightarrow \infty$ .
- (iv)  $\frac{(e^0(k))^2}{1 + \varphi^T(k-1) P(k-1) \varphi(k-1)}$  is uniformly bounded and it asymptotically converges to

zero as  $k \rightarrow \infty$ .

(v)  $e^0(k)$  asymptotically converges to zero as  $k \rightarrow \infty$ .

(vi) Assuming that the external input  $c(k)$  is sufficiently rich such that  $\varphi(k-1)$  in (11) is persistently exciting,  $\hat{\theta}^0(k)$  tends to the true parameters vector  $\theta$  as  $k \rightarrow \infty$ . Then,  $\hat{\theta}(k)$  tends to  $\hat{\theta}^0(k)$  and  $e(k) = (\theta - \hat{\theta}(k-1))^T \varphi(k-1)$  tends to zero as  $k \rightarrow \infty$ . \*\*\*

**Remark 3.** The convergence of the estimated parameters to their true values in  $\theta$  requires that  $\varphi(k-1)$  is persistently exciting. In this context,  $\varphi(k-1)$  is persistently exciting if there exists an integer  $\ell$  such that  $\rho_1 I_m > \sum_{k=k_0}^{k_0+\ell} \varphi(k-1)\varphi^T(k-1) > \rho_2 I_m$  where  $\rho_1 > 0$ ,  $\rho_2 > 0$  and  $m = n + N^2 = n^2 + 3n + 1$  is the number of components of the regressor  $\varphi(k-1)$ . Such a condition may be ensured by choosing an external input sufficiently rich of order  $m$ , i.e. it consists of at least  $m/2$  frequencies in the frequency domain (Ioannou & Sun, 1996). \*\*\*

#### 4. Stability Analysis

The plant discretized model can be written as follows,

$$y(k) = \hat{y}(k) + e(k) = \hat{\theta}^T(k-1)\varphi(k-1) + e(k) = -\sum_{i=1}^n \hat{a}_i(k-1)y(k-i) + \sum_{i=1}^{n+1} \hat{b}_i(k-1)u(k-i) + e(k) \quad (18)$$

and the adaptive control law as,

$$u(k) = \frac{1}{\hat{b}_1(k)} \left\{ \sum_{i=1}^n (\hat{s}_1(k-1)\hat{a}_i(k-1) - \hat{s}_{i+1}(k-1))y(k-i) - \sum_{i=1}^n (\hat{s}_1(k-1)\hat{b}_i(k-1) + \hat{b}_{i+1}(k-1))u(k-i) \right. \\ \left. - \hat{s}_1(k-1)\hat{b}_{n+1}(k-1)u(k-n-1) + \sum_{i=1}^{n+1} b_{mi}c(k-i+1) \right\} - \frac{\hat{s}_1(k)}{\hat{b}_1(k)}e(k) + \delta(k) \quad (19)$$

where (12) has been used with  $R(q)$  and  $S(q)$  substituted, respectively, by time-varying polynomials  $\hat{R}(z,k) = \hat{B}(z,k)$  and  $\hat{S}(z,k)$ , which is the solution of the equation (14) for the adaptive case, and,

$$\delta(k) = \frac{1}{\hat{b}_1(k)} \left\{ \sum_{i=1}^n [(\hat{s}_1(k) - \hat{s}_1(k-1))\hat{a}_i(k-1) - (\hat{s}_{i+1}(k) - \hat{s}_{i+1}(k-1))]y(k-i) \right. \\ \left. - \sum_{i=1}^n [(\hat{s}_1(k) - \hat{s}_1(k-1))\hat{b}_i(k-1) + (\hat{b}_{i+1}(k) - \hat{b}_{i+1}(k-1))]u(k-i) \right. \\ \left. - (\hat{s}_1(k) - \hat{s}_1(k-1))\hat{b}_{n+1}(k-1)u(k-n-1) \right\} \quad (20)$$

By combining (18) and (19), the discrete-time closed-loop system can be written as:

$$x(k) = \Lambda(k-1)x(k-1) + \Psi_1 e(k) + \Psi_2 \vartheta(k) \tag{21}$$

where  $\vartheta(k) = \frac{1}{\hat{b}_1(k)} \left( \sum_{i=1}^{n+1} b_{m_i} c(k-i+1) - \hat{s}_1(k) e(k) \right) + \delta(k)$  and,

$$x(k-1) = [y(k-1)y(k-2)\dots y(k-n)u(k-1)u(k-2)\dots u(k-n-1)]$$

$$\psi_1 = [1\ 0 \dots 0]^T \in \mathfrak{R}^{2(n+1) \times 1}; \psi_2 = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 0 \end{bmatrix}^T \in \mathfrak{R}^{2(n+1) \times 1}$$

$$\Lambda(k-1) = \begin{bmatrix} -\hat{a}_1(k-1) & -\hat{a}_2(k-1) & \dots & -\hat{a}_{n-1}(k-1) & -\hat{a}_n(k-1) & \hat{b}_1(k-1) & \hat{b}_2(k-1) & \dots & \hat{b}_n(k-1) & \hat{b}_{n+1}(k-1) \\ 1 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & \vdots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & 0 & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ \frac{\hat{f}_1(k-1)}{\hat{b}_1(k)} & \frac{\hat{f}_2(k-1)}{\hat{b}_2(k)} & \dots & \frac{\hat{f}_{n-1}(k-1)}{\hat{b}_{n-1}(k)} & \frac{\hat{f}_n(k-1)}{\hat{b}_n(k)} & \frac{\hat{h}_1(k-1)}{\hat{b}_1(k)} & \frac{\hat{h}_2(k-1)}{\hat{b}_2(k)} & \dots & \frac{\hat{h}_n(k-1)}{\hat{b}_n(k)} & \frac{\hat{h}_{n+1}(k-1)}{\hat{b}_{n+1}(k)} \\ 0 & 0 & \dots & 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 1 & 0 \end{bmatrix} \tag{22}$$

with  $\hat{f}_i(k-1) = \hat{s}_1(k-1)\hat{a}_i(k-1) - \hat{s}_{i+1}(k-1)$ ,  $\hat{h}_i(k-1) = -(\hat{s}_i(k-1)\hat{b}_i(k-1) + \hat{b}_{i+1}(k-1))$ , for  $i \in \{1, 2, \dots, n\}$ , and  $\hat{h}_{n+1}(k-1) = -\hat{s}_1(k-1)\hat{b}_{n+1}(k-1)$ .

Note that  $\hat{a}_i(k-1)$  and  $\hat{b}_i(k-1) = \sum_{j=1}^N \hat{b}_{i,j}(k-1)\hat{\alpha}_j(k-1)$  are uniformly bounded from

Lemma 1 (properties ii and iii). Also,  $\hat{b}_1(k) \neq 0$  since the adaptation of the multirate gains makes such a parameter fixed to a prefixed one which is suitably chosen and  $\hat{s}_1(k-1)$  is uniformly bounded from the resolution of a equation being similar to that of (14) replacing polynomials  $A(z)$  and  $S(z)$  by time-varying polynomials  $\hat{A}(z, k-1)$  and  $\hat{S}(z, k-1)$ , respectively.

The following theorem, whose proof is presented in Appendix B, establishes the main stability result of the adaptive control system.

**Theorem 1. Main stability result.**

- (i) The adaptive control law stabilizes the discrete-time plant model (6) in the sense that  $\{u(k)\}$  and  $\{y(k)\}$  are bounded for all finite initial states and any uniformly bounded reference input sequence  $\{c(k)\}$  subject to Assumptions 1,
- (ii)  $\{y(k)\}$  converges to  $\{y_m(k)\}$  as  $k$  tends to infinity, and
- (iii) the continuous plant input and output signals,  $u(t)$  and  $y(t)$ , are bounded for all  $t$ . \*\*\*

## 5. Simulations Results

Some simulation results which illustrate the effectiveness of the proposed method are shown in the current section. A continuous-time unstable plant of transfer function

$$G(s) = \frac{s-2}{(s-1)(s+3)}$$

with an unstable zero, and whose internal representation is defined by

the matrices  $A = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $B = [1 \ 1]^T$  and  $C = [1.25 \ -0.25]$ , is considered. A suitable

multirate scheme with fast input sampling through a FROH device is used to place the zeros of the discretized plant within the stability region and a discrete-time controller is synthesized so that the discrete-time closed-loop system matches a reference model. The results for the case of known plant parameters are presented in a first example and then two more examples with the described adaptive control strategies are considered. The difference among such adaptive control strategies relies on the way of updating the multirate gains for ensuring the stability of the estimated discretized plant zeros.

### 5.1. Known Plant Parameters

The discretization of the continuous-time plant with a multirate,  $N = 3$ , and a FROH device with  $\beta = 0.7$  for a slow sampling time  $T = 0.3$  is performed leading to the discrete transfer

$$\text{function } H(z) = \frac{B(z)}{A(z)} = \frac{b_1(g)z^2 + b_2(g)z + b_3(g)}{z(z^2 - 1.7564z + 0.5488)}$$

where  $b_1(g) = 0.0307\alpha_1 + 0.0693\alpha_2 + 0.13\alpha_3$ ,

$b_2(g) = -(0.0788\alpha_1 + 0.1488\alpha_2 + 0.2631\alpha_3)$  and  $b_3(g) = 0.0083\alpha_1 + 0.0343\alpha_2 + 0.0797\alpha_3$  are the coefficients of the transfer function numerator of the discretized model. Such coefficients depend on the multirate gains  $\alpha_i$ , for  $i \in \{1, 2, 3\}$ , included as components in the vector  $g$ .

The zeros of such a discretized plant can be fixed within the stability domain via a suitable choice of the multirate gains. In this example such gains are  $\alpha_1 = -621.8706$ ,  $\alpha_2 = 848.4241$

and  $\alpha_3 = -297.4867$  so that  $B(z) = B'(z) = z^2 + z + 0.25$  and then both zeros are placed at  $z_0 = -0.5$ . The control objective is the matching of the reference model defined by the

$$\text{transfer function } G_m(z) = \frac{z^2 + z - 0.272}{(z + 0.2)^3}$$

. For such a purpose, the controller has to cancel the

discretized plant zeros, which are stable, and add those of the reference model to the discrete-time closed-loop system. The values of the control parameters to meet such an objective are  $s_1 = 2.3564$ ,  $s_2 = -0.4288$  and  $s_3 = 0.008$ . A unitary step is considered as

external input signal. Figure 1 displays the time evolution of the closed-loop system output, its values at the slow sampling instants and the sequence of the discrete-time reference model output. Figure 2 shows the plant input signal. Note that perfect model matching is achieved, at the slow sampling instants, without any constraints in the choice of the zeros of the reference model  $G_m(z)$ , in spite of the continuous-time plant possesses an unstable zero.

Furthermore, the continuous-time output and input signals are maintained bounded for all time.

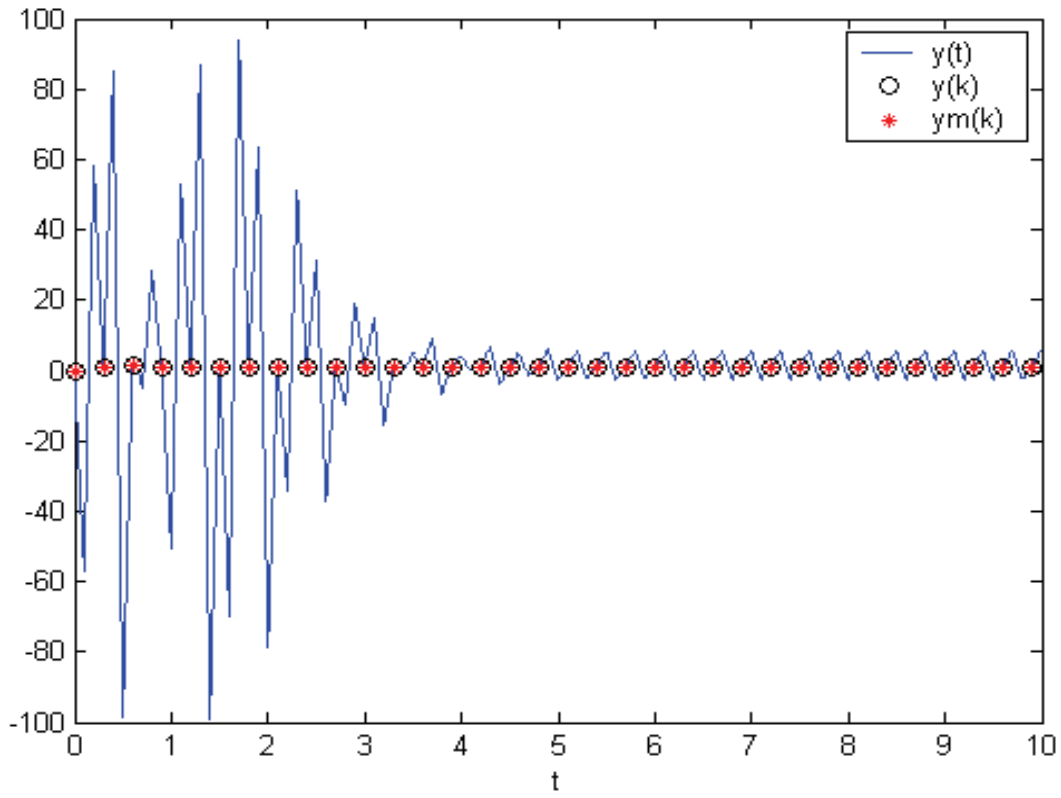


Fig. 1. Plant and reference model output signals

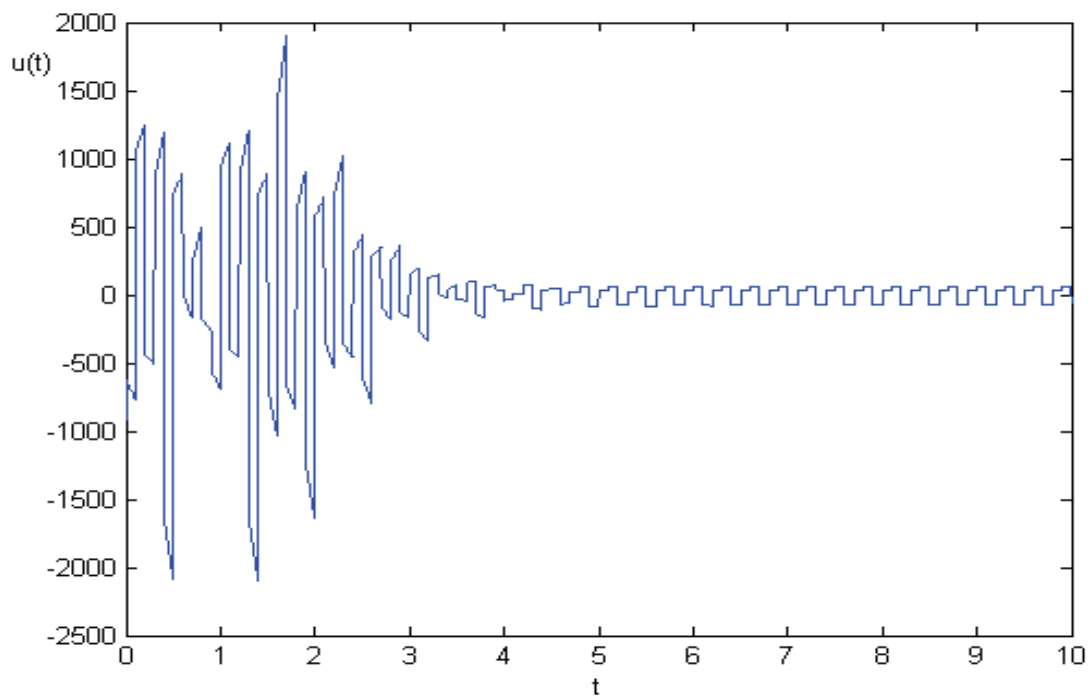


Fig. 2. Plant input signal

## 5.2. Unknown Plant Parameters

An adaptive version of the discrete-time controller designed in the previous example is considered with the parameters estimation algorithm being initialized with  $\hat{\theta}^0(0) = 10^{-2} \times [263.46 \quad -82.32 \quad 4.61 \quad 10.39 \quad 19.51 \quad -11.82 \quad -22.33 \quad -39.46 \quad 1.25 \quad 5.15 \quad 11.95]^T$  and  $P(0) = 1000 \cdot I_{11}$ . Furthermore, the values  $\delta = \delta_0 = 10^{-6}$  are chosen for the modification algorithm included in such an estimation process. Two different methods are considered to update the multirate gains. The first one consists of updating such gains at all the slow sampling instants so that the discretized zeros are maintained constant within the stability domain (Algorithm 1). The second one consists of changing the value of the multirate gains only when at least one of the discretized zeros, which are time-varying, is going out of the stability domain. Otherwise, the values for the multirate gains are maintained equal to those of the previous slow sampling instant (Algorithm 2).

### 5.2.1. Algorithm 1: Discretized plant zeros are maintained constant

Figure 3 displays the time evolution of the closed-loop adaptive control system output, its values at the slow sampling instants and the sequence of the discrete-time reference model output under a unitary step as external input signal. Note that the discrete-time model matching is reached after a transient time interval. Figures 4 and 5 show, respectively, the plant output signal and the input signal generated from the multirate with the FROH applied to the control sequence  $\{u(k)\}$ . It can be observed that both signals are bounded for all time. Finally, Figures 6 and 7 display, respectively, the time evolution of the multirate gains and the adaptive controller parameters. Note that the multirate gains and the adaptive controller parameters are time-varying until they converge to constant values.

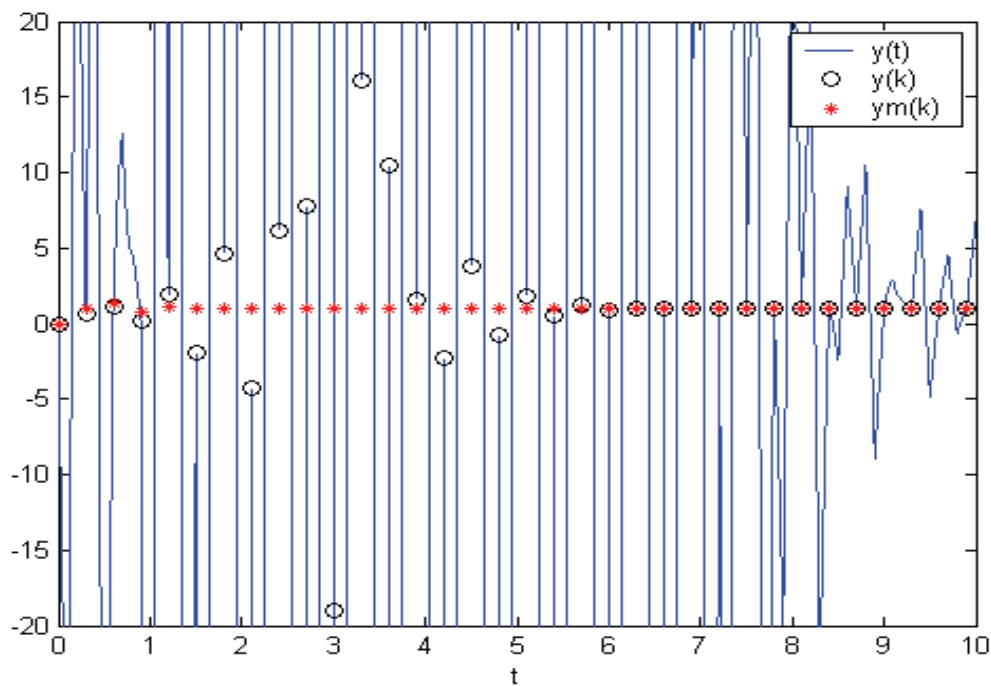


Fig. 3. Plant and reference model output signals

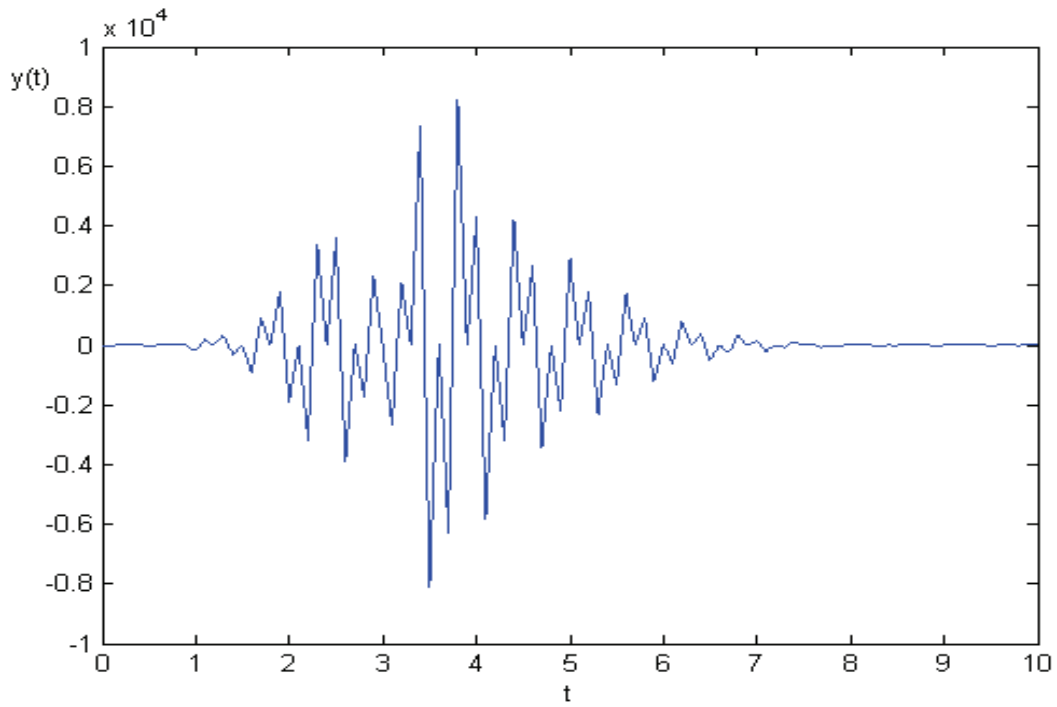


Fig. 4. Plant output signal

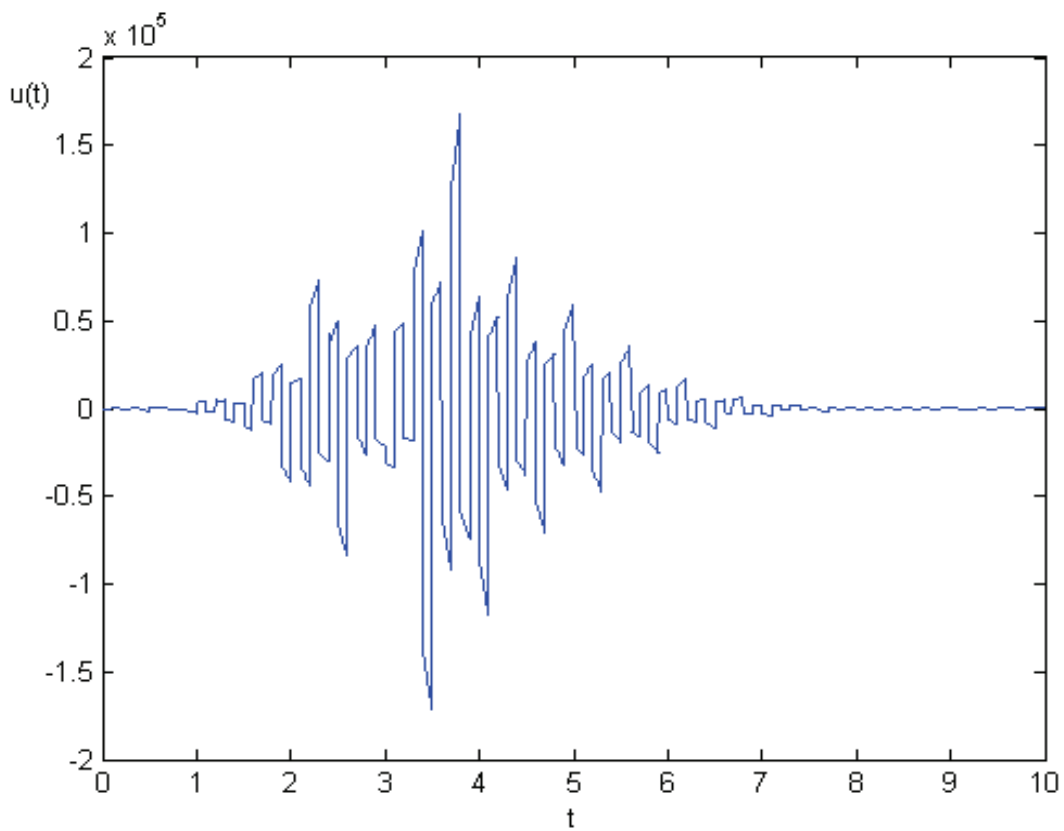


Fig. 5. Plant input signal



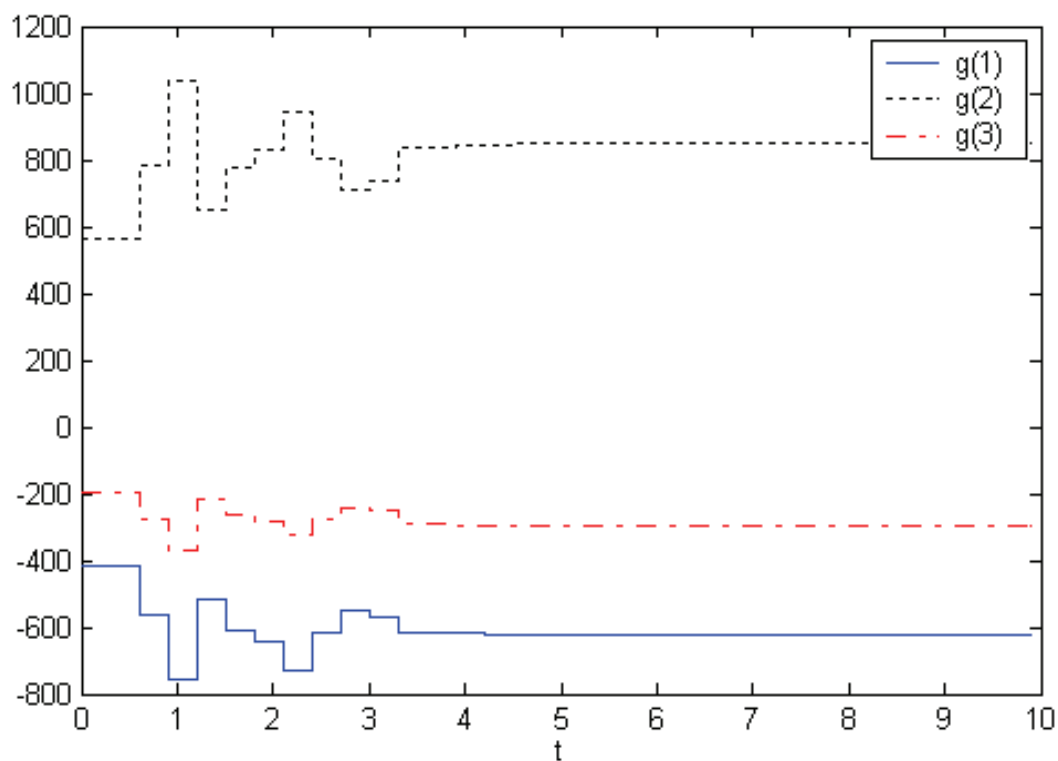


Fig. 6. Multirate gains

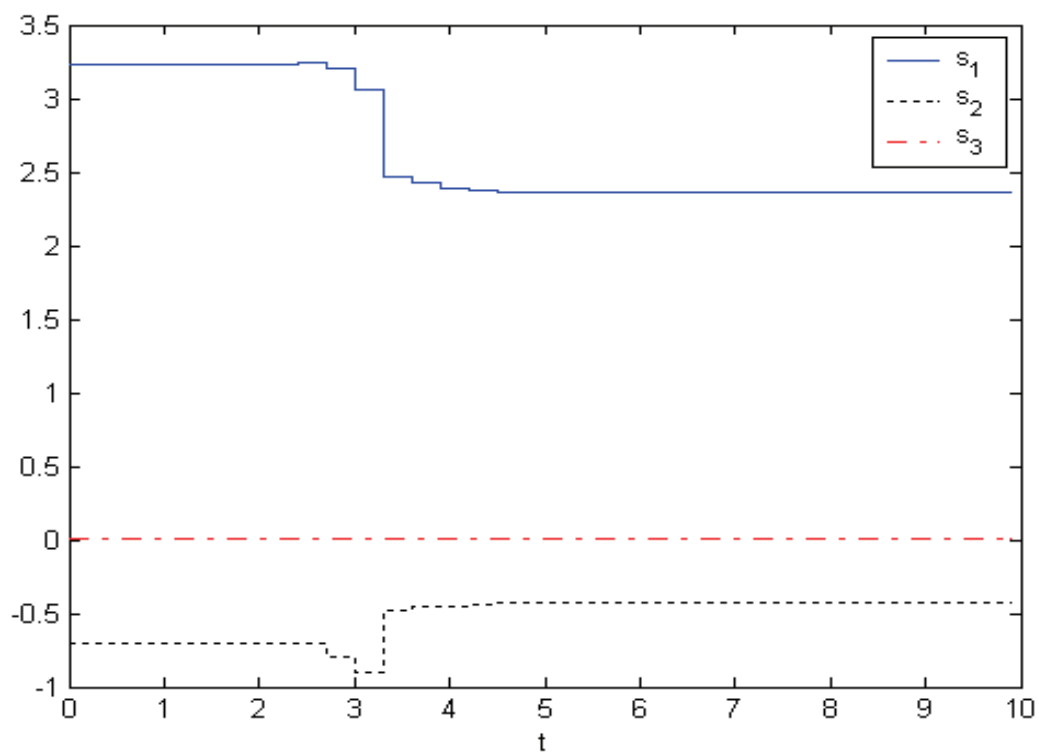


Fig. 7. Adaptive control parameters

**5.2.2. Algorithm 2: Discretized plant zeros are time-varying**

The multirate gains are maintained constant to their values at the previous slow sampling instant until at least one of the discretized plant zeros is going out of the stability domain. In this sense, note that the discretized zeros vary when the values of the multirate gains are maintained constant and eventually they can go out of the stability domain. When this happens such gains are again calculated to place both discretized zeros at  $z_0 = -0.5$ . The discrete-time model matching is reached after a transient time interval and the continuous-time plant output and input signals are bounded for all time as it can be observed from Figures 8, 9 and 10 where the response to a unitary step is shown. The maximum values reached by both continuous-time output and input signals are larger than those obtained with the previous method (Algorithm 1) for updating the multirate gains. Figures 11 and 12 display, respectively, the evolution of the multirate gains and the controller parameters. The adaptive control parameters are time-varying until they converge to constant values while the multirate gains are piecewise constant and also they converge to constant values. Note that this second method ensures a small number of changes in the values of the multirate gains compared with the first method since such gains only vary when it is necessary to maintain the zeros within the stability domain. This fact gives place to a less computational effort to generate the control law than that required with the first method. However, the behaviour of the continuous-time plant output and input signals is worse with the use of this second alternative in this particular example. Finally, the evolution of the modules of the discretized plant zeros and the coefficients of the time-varying numerator of such an estimated model are, respectively, shown in Figures 13 and 14.

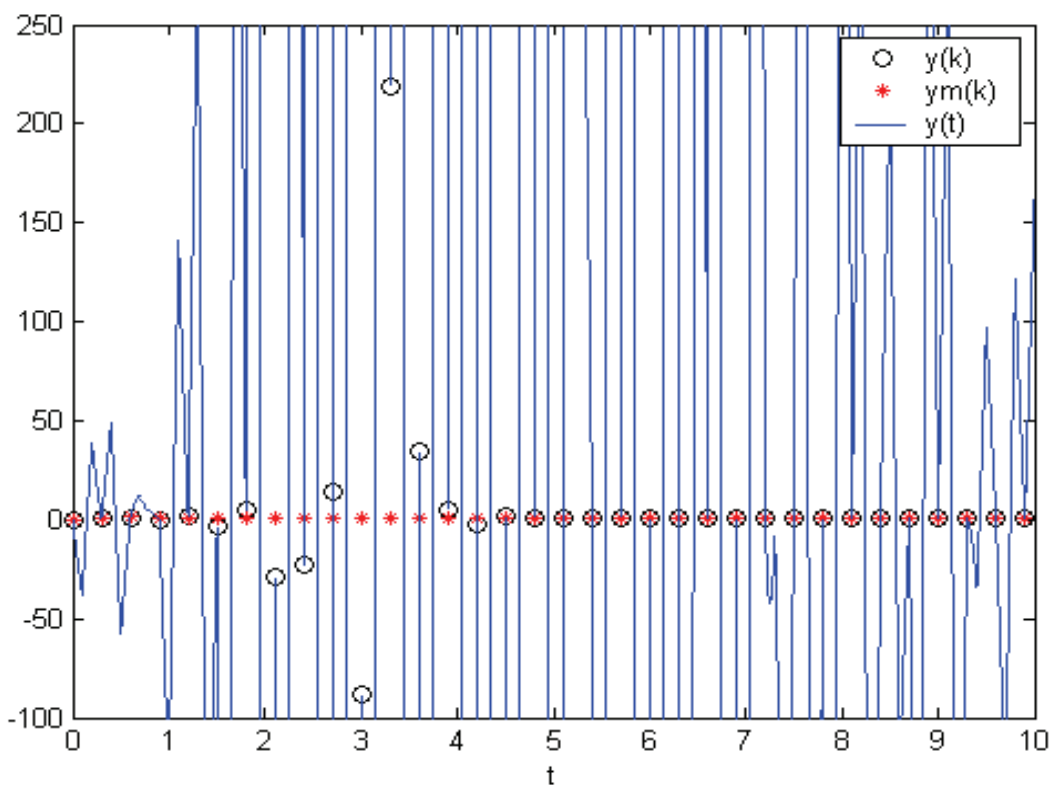


Fig. 8. Plant and reference model output signals

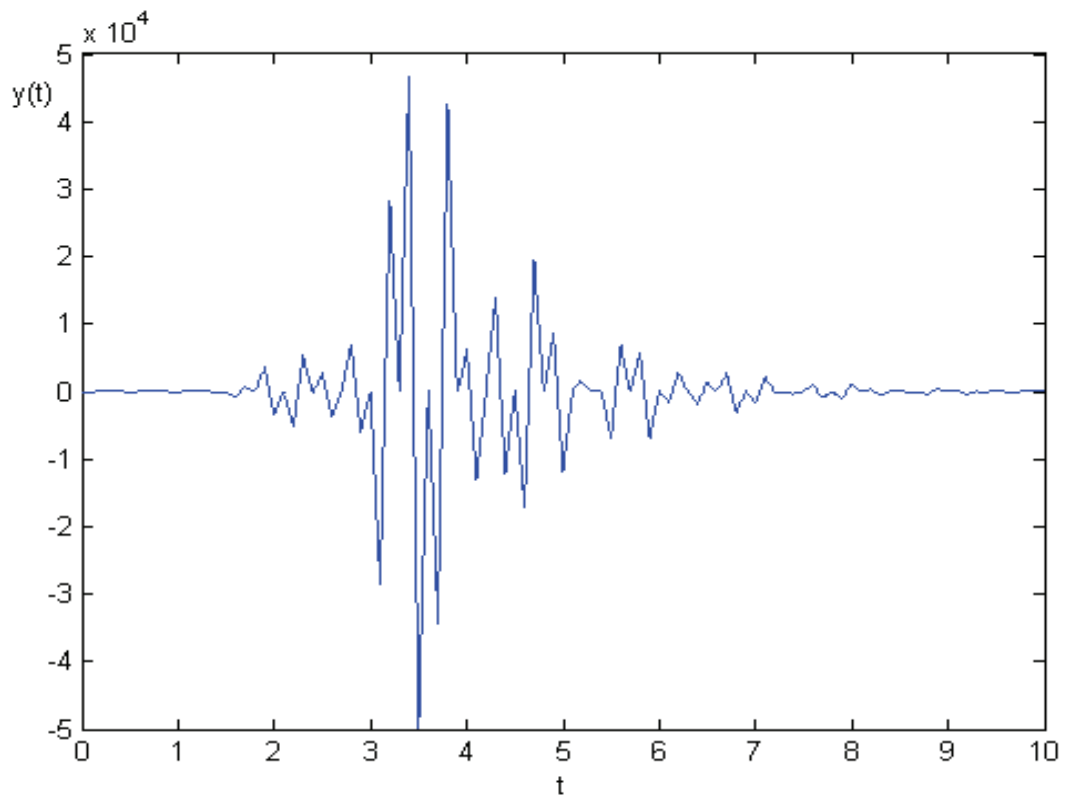


Fig. 9. Plant output signal

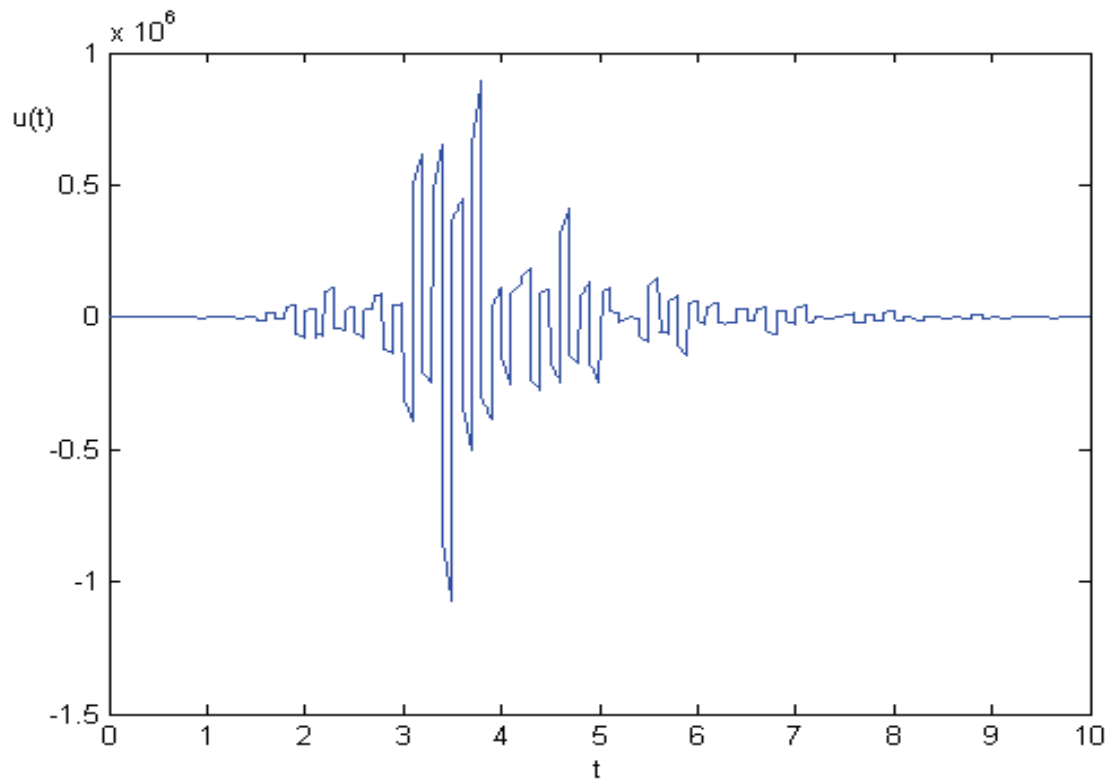


Fig. 10. Plant input signal

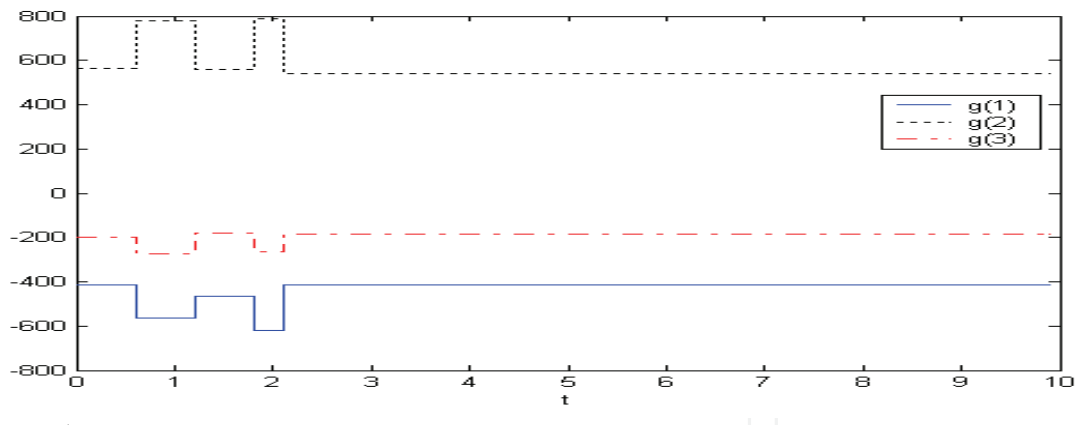


Fig. 11. Multirate gains

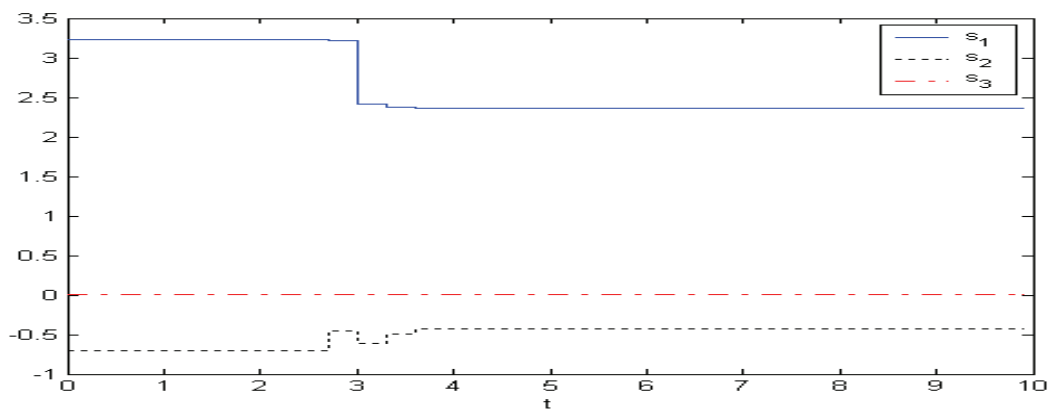


Fig. 12. Adaptive control parameters

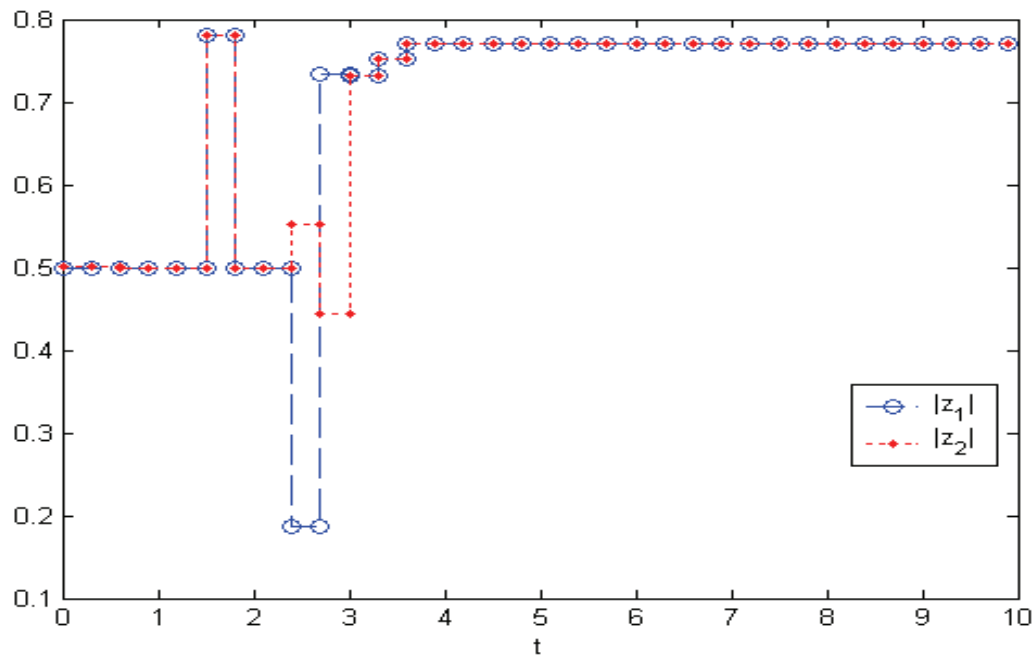


Fig. 13. Modules of the estimated discretized plant zeros

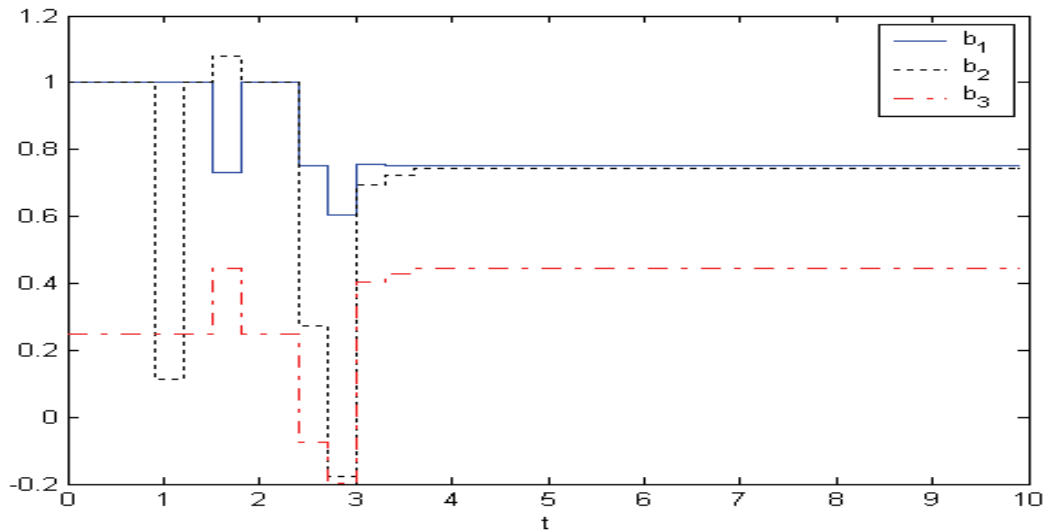


Fig. 14. Coefficients of the estimated discretized plant numerator

## 6. Conclusion

This paper deals with the stabilization of an unstable and possibly non-inversely stable continuous-time plant. The mechanism used to fulfill the stabilization objective consists of two steps. The first one is the discretization of the continuous-time plant by using a FROH device combined with a multirate input in order to obtain an inversely stable discretized model of the plant. Then, a discrete-time controller is designed to match a discrete-time reference model by such a discretized plant. There is not any restriction in the choice of the reference model since the zeros of the discretized plant model are guaranteed to be stable by the fast sampled input generated by the multirate sampling device.

An adaptive version of such a controller constitutes the main contribution of the present manuscript. The model matching between the discretized plant and the discrete-time reference model is asymptotically reached in the adaptive case of unknown plant. Also, the boundedness of the continuous-time plant input and output signals are ensured, as it is illustrated by means of some simulation examples. In this context, the behaviour of the designed adaptive control system in the inter-samples period may be improved. In this sense, an improvement in such a behaviour has been already reached with a multi-estimation scheme where several discretization/estimation processes, each one with its proper FROH and multirate device, are working in parallel providing different discretized plant estimated models (Alonso-Quesada & De la Sen, 2007). Such a scheme is completed with a supervisory system which activates one of the discretization/estimation processes. Such a process optimizes a performance index related with the inter-sample behaviour. In this sense, each of the discretization/estimation processes gives a measure of its quality by means of such an index which may measure the size of the tracking-error and/or the size of the plant input for the inter-sample period. The supervisor switches on-line from the current process to a new one when the last is better than the former, i.e. the performance index of the new process is smaller than that of the current one. Moreover, the supervisor has to guarantee a minimum residence time between two consecutive switches in order to ensure the stability of the adaptive control system.

### 7. Appendix A. Proof of Lemma 1

- (i)  $P(k)$  is a monotonic non-increasing matrix sequence since  $P(k) - P(k-1) \leq 0$  for all integer  $k > 0$  from (16). Moreover, if  $P(k_1) = 0$  for any integer  $k_1 > 0$  then  $P(k_1 + 1) - P(k_1) = 0$  from (16) and then  $P(k) = 0$  for all integer  $k \geq k_1$ . Thus,  $0 \leq P(k) \leq P(0)$  and  $P(k)$  asymptotically converges to a finite limit as  $k \rightarrow \infty$ .
- (ii) By considering the non-negative sequence  $V(k) = \tilde{\theta}^{0T}(k)P^{-1}(k)\tilde{\theta}^0(k)$  and applying the matrix inversion lemma (Goodwin & Sin, 1984) to (16) it follows that,

$$V(k) - V(k-1) = -\frac{(e^0(k))^2}{1 + \varphi^T(k-1)P(k-1)\varphi(k-1)} \leq 0 \tag{23}$$

where (16) and the definition of the estimation error have been used. Then,  $V(k) \leq V(0)$  and  $\|\tilde{\theta}^0(k)\| \leq \frac{\lambda_{\max}\{P(0)\}}{\lambda_{\min}\{P(0)\}}\|\tilde{\theta}^0(0)\| < \infty$  where  $\lambda_{\max}\{P(0)\}$  and  $\lambda_{\min}\{P(0)\}$  denote the maximum and the minimum eigenvalues of  $P(0)$ , respectively. It implies that  $\tilde{\theta}^0(k)$ , and then also  $\hat{\theta}^0(k)$ , is uniformly bounded. Then,  $\hat{\theta}(k)$  is also bounded since the modification algorithm guarantees the boundedness of  $\hat{M}(k)$  provided that  $\hat{\theta}^0(k)$  is bounded. On other hand,  $V(k)$  asymptotically converges to a finite limit as  $k \rightarrow \infty$  from its definition and the fact that such a sequence is non-negative and monotonic non-increasing. Then,  $\tilde{\theta}^0(k)$ , and also  $\hat{\theta}^0(k)$ , converges to a finite limit as  $k \rightarrow \infty$  since  $P(k)$  also converges as it has been proved in (i). Then,  $\hat{M}(k)$  and  $\hat{\theta}(k)$  also converge to finite limits as  $k \rightarrow \infty$ .

- (iii) The boundedness and convergence of the estimation model parameters vector together with the non-singularity of the matrix  $\hat{M}(k)$ , guaranteed by the modification algorithm, implies the boundedness and convergence of the vector  $\hat{g}(k)$  obtained by resolution of equation (17).

- (iv) It follows that  $\sum_{i=1}^k \frac{(e^0(i))^2}{1 + \varphi^T(i-1)P(i-1)\varphi(i-1)} = V(0) - V(k) \leq V(0) < \infty$  from (23), then  $\frac{(e^0(k))^2}{1 + \varphi^T(k-1)P(k-1)\varphi(k-1)}$  is uniformly bounded and it converges to zero as  $k \rightarrow \infty$ .

- (v) It follows that  $\lim_{k \rightarrow \infty} \{e^0(k)\} = 0$  irrespective of the boundedness of  $\varphi(k-1)$  from the fact

that  $\lim_{k \rightarrow \infty} \left\{ \frac{(e^0(k))^2}{1 + \varphi^T(k-1)P(k-1)\varphi(k-1)} \right\} = 0$ . On one hand, if  $\varphi(k-1)$  is bounded then

$\lim_{k \rightarrow \infty} \left\{ \frac{(e^0(k))^2}{1 + \varphi^T(k-1)P(k-1)\varphi(k-1)} \right\} = 0$  implies directly that  $\lim_{k \rightarrow \infty} \{e^0(k)\} = 0$ . On the other

hand, if  $\varphi(k-1)$  is unbounded then  $\lim_{k \rightarrow \infty} \left\{ \frac{(e^0(k))^2}{1 + \varphi^T(k-1)P(k-1)\varphi(k-1)} \right\} = 0$  implies that

$\lim_{k \rightarrow \infty} \left\{ \frac{\|\tilde{\theta}^0(k-1)\|^2}{\|P(k-1)\|^2} \right\} = 0$  since  $e^0(k) = \tilde{\theta}^{0T}(k-1)\varphi(k-1)$  and then  $\lim_{k \rightarrow \infty} \left\{ \|\tilde{\theta}^0(k-1)\|^2 \right\} = 0$  from

the fact that  $P(k)$  is uniformly bounded. Thus,  $\lim_{k \rightarrow \infty} \{e^0(k)\} = 0$ .

- (vi) Provided that the external input sequence  $\{c(k)\}$  is sufficiently rich such that  $\varphi(k-1)$  is persistently exciting,  $\hat{\theta}^0(k)$  tends to the true parameters vector  $\theta$  as  $k \rightarrow \infty$  (Goodwin & Sin, 1984). Then,  $\hat{M}(k)$  tends to  $\hat{M}^0(k)$  from the modification algorithm and, consequently,  $\hat{\theta}(k)$  tends to  $\hat{\theta}^0(k)$  and  $e(k)$  tends to zero as  $k \rightarrow \infty$ . \*\*\*

## 8. Appendix B. Proof of Theorem 1

- (i)  $\Lambda(k)$  is bounded since the estimation model parameters  $\hat{a}_i(k)$  and  $\hat{b}_j(k)$ , and the controller parameters  $\hat{s}_j(k)$ , for  $i \in \{1, 2, \dots, n\}$  and  $j \in \{1, 2, \dots, n+1\}$ , are bounded thanks to the estimated parameters vector  $\hat{\theta}(k)$  and the multirate gains vector  $\hat{g}(k)$  are bounded for all integer  $k \geq 0$ . The eigenvalues of  $\Lambda(k)$  are in  $|z| < 1$  since they are the roots of  $A_m(z)$  and  $A_s(z)$ , due to the designed control law, and the roots of  $\hat{B}(z, k)$  which are within the unit circle due to the suitable adaptation of the multirate gains. Besides,

$$\sum_{k'=k_0+1}^k \|\Lambda(k') - \Lambda(k'-1)\|^2 \leq \gamma_0 + \gamma_1(k - k_0) \quad (24)$$

for all integers  $k$  and  $k_0$  such that  $k > k_0 \geq 0$ , and some sufficiently small positive real constants  $\gamma_0$  and  $\gamma_1$  (Bilbao-Guillerna et al., 2005). Note that (24) is fulfilled with a slow enough estimation rate via a suitable choice of  $P(0)$  in (16) so that  $\gamma_1$  is sufficiently small. Thus, the time-varying homogeneous system  $x(k) = \Lambda(k-1)x(k-1)$  is

exponentially stable and its transition matrix  $\phi(k, k') = \prod_{j=k'}^{k-1} \Lambda(j)$  satisfies  $\|\phi(k, k')\| \leq \rho_1 \sigma_0^{k-k'}$

for all  $k \geq k'$  where  $\sigma_0 \in (0, 1)$  and  $\rho_1$  is a non-dependent constant (Alonso-Quesada & De la Sen, 2004). It follows from (21) that:

$$x(k) = \phi(k, k_0) x(k_0) + \sum_{k'=k_0}^k \phi(k, k') (\Psi_1 e(k') + \Psi_2 \vartheta(k')) \quad (25)$$

for all integer  $k \geq k_0 \geq 0$ . Then,

$$\|x(k)\| = \rho_1 \sigma_0^{k-k_0} \|x(k_0)\| + \sum_{k'=k_0}^k \rho_1 \sigma_0^{k-k'} (\rho_2 + \rho_3 |e(k')| + \rho_4 |\delta(k')|) \quad (26)$$

for some positive real constants  $\rho_2$ ,  $\rho_3$  and  $\rho_4$ , provided that the input reference sequence  $\{c(k)\}$  is bounded. It follows that  $\lim_{k \rightarrow \infty} |\hat{a}_i(k) - \hat{a}_i(k-1)| = 0$  and  $\lim_{k \rightarrow \infty} |\hat{b}_j(k) - \hat{b}_j(k-1)| = 0$  for all  $i \in \{1, 2, \dots, n\}$  and  $j \in \{1, 2, \dots, n+1\}$  from the convergence property of the estimation algorithm. Then,  $\lim_{k \rightarrow \infty} |\hat{s}_i(k) - \hat{s}_i(k-1)| = 0$  as it follows from the adaptive control resolution. Consequently,  $\lim_{k \rightarrow \infty} |\delta(k)| = 0$ . Besides,  $\lim_{k \rightarrow \infty} |e(k)| = 0$  from the estimation algorithm. Then,  $x(k)$  is bounded from (26), which implies that sequences  $\{u(k)\}$  and  $\{y(k)\}$  are also bounded.

- (ii) On one hand, the adaptive control law ensures that the estimated sequence  $\{\hat{y}(k)\}$  matches the reference model one  $\{y_m(k)\}$  for all integer  $k \geq 0$ . On the other hand, the estimation algorithm guarantees the asymptotic convergence of the estimation error  $e(k)$  to zero. Then, the output sequence  $\{y(k)\}$  tends to  $\{y_m(k)\}$  asymptotically as  $k \rightarrow \infty$ .
- (iii) The adaptive control algorithm ensures that there is no finite escapes. Then, the boundedness of the sequences  $\{u(k)\}$  and  $\{y(k)\}$  implies that the plant input and output continuous-time signals  $u(t)$  and  $y(t)$  are bounded for all  $t$ .

## 9. Acknowledgment

The authors are very grateful to MCYT by its partial support through grants DPI 2003-0164 and DPI2006-00714.

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Edited by Kwanho You

ISBN 978-953-7619-47-3

Hard cover, 372 pages

**Publisher** InTech

**Published online** 01, January, 2009

**Published in print edition** January, 2009

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S. Alonso-Quesada and M. De La Sen (2009). Discrete Model Matching Adaptive Control for Potentially Inversely Non-Stable Continuous-Time Plants by Using Multirate Sampling, Adaptive Control, Kwanho You (Ed.), ISBN: 978-953-7619-47-3, InTech, Available from:  
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