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Adaptive output regulation of unknown linear systems with unknown exosystems

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1. Introduction

The problems of the output regulations and/or disturbance reductions have attracted a lot of interest and have been actively researched in the consideration of the control problem for systems which are required to have servomechanism and for vibration attenuation in mechanical systems. It is well known that such problems are solvable using the Internal Model Principle in cases where the system to be controlled and the exosystem which generates the output reference signal and external disturbances are known. In the case where the controlled system is unknown and/or the exosystem is unknown, adaptive control strategies have played active roles in solving such problems for systems with uncertainties. For known systems with unknown exosystems, solutions with adaptive internal models have been provided in (Feg & Palaniswami, 1991), (Nikiforov, 1996) and (Marino & Tomei, 2001). In (Marino & Tomei, 2001), an output regulation system with an adaptive internal model is proposed for known non-minimum phase systems with unknown exosystems. Adaptive regulation problems have also been presented for time varying systems and nonlinear systems (Marino & Tomei, 2000; Ding, 2001; Serrani et al., 2001). Most of these methods, however, assumed that either the controlled system or the exosystem was known. Only few adaptive regulation methods for unknown systems with unknown exosystems have been provided (Nikiforov, 1997a; Nikiforov, 1997b). The method in (Nikiforov, 1997a) is an adaptive servo controller design based on the MRAC strategy, so that it was essentially assumed that the order of the controlled system was known. The method in (Nikiforov, 1997b) is one based on an adaptive backstepping strategy. In this method, it was necessary to design an adaptive observer that had to estimate all of the unknown system parameters depending on the order of the controlled system. Further, the controller design based on the backstepping strategy essentially depends on the order of the relative degree of the controlled system. As a result, the controller's structure was quite complex in both methods for higher order systems with higher order relative degrees.

In this paper, the adaptive regulation problem for unknown controlled systems is dealt with and an adaptive output feedback controller with an adaptive internal model is proposed for single input/single output linear minimum phase unknown systems with unknown exosystems. The proposed method is based on the adaptive output feedback control utilizing the almost strictly positive real-ness (ASPR-ness) of the controlled system and the controller is designed based on an expanded backstepping strategy with a parallel feedforward compensator (PFC) (Mizumoto et al., 2005). It is shown that, under certain assumptions, without a priori knowledge of the order of the controlled system and without state variables, one can design an adaptive controller with a single step backstepping strategy even when the system to be controlled has an unknown order and a higher order relative degree. Using the proposed method, one can not attain perfect output regulation, however, the obtained controller structure is relatively simple even if the system has a higher order and a higher order relative degree.

2. Problem Statement

Consider the following single input/single output LTI system.

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + \mathbf{b}u(t) + C_d\mathbf{w}(t)\\ y(t) &= \mathbf{c}^T\mathbf{x}(t) + \mathbf{d}^T\mathbf{w}(t),\end{aligned} \tag{1}$$

where $\mathbf{x} = [x_1, \cdots, x_n]^T \in R^n$ is the state vector and $u, y \in R$ are the input and the output, respectively. Further $\mathbf{w}(t) \in R^m$ is an unknown vector disturbance.

We assume that the disturbances and the reference signal which the output y is required to track are generated by the following unknown exosystem:

$$\begin{aligned}\dot{\mathbf{w}}(t) &= A_d\mathbf{w}(t)\\ y_m(t) &= \mathbf{c}_m^T\mathbf{w}(t),\end{aligned} \tag{2}$$

where $A_d \in R^{m\times m}$ is a stable matrix with all its eigenvalues on the imaginary axis. It is also assumed that the characteristic polynomial of A_d is expressed by

$$\det(\lambda\mathbf{I} - A_d) = \lambda^m + \alpha_{m-1}\lambda^{m-1} + \cdots + \alpha_1\lambda + \alpha_0. \tag{3}$$

The objective is to design an adaptive controller that has the output y(t) track the reference signal $y_m(t)$ generated by an unknown exosystem given in (2) for unknown systems with unknown disturbances generated by the unknown exosystem in (2) using only the output signal under the following assumptions.

Assumption 1 The system (1) is minimum-phase.

Assumption 2 The system (1) has a relative degree of r.

Assumption 3 $\mathbf{c}^T A^{r-1}\mathbf{b} > 0$, i.e. the high frequency gain of the system (1) is positive.

Assumption 4 The output y(t) and the reference signal $y_m(t)$ are available for measurement.

3. System Representation

From Assumption 2, since the system (1) has a relative degree of r, there exists a smooth nonsingular variable transformation: $\left[\mathbf{z}^T, \boldsymbol{\eta}^T\right]^T = \Phi\mathbf{x}$ such that the system (1) can be transformed into the form (Isidori, 1995):

$$\begin{aligned}\dot{\mathbf{z}}(t) &= A_z\mathbf{z}(t) + \mathbf{b}_z u(t) + \begin{bmatrix}\mathbf{0}\\ \mathbf{c}_z^T\end{bmatrix}\boldsymbol{\eta}(t) + D_d\mathbf{w}(t)\\ \dot{\boldsymbol{\eta}}(t) &= Q_\eta\boldsymbol{\eta}(t) + \begin{bmatrix}\mathbf{0}\\ 1\end{bmatrix} z_1(t) + F_d\mathbf{w}(t)\\ y &= [1,0,\cdots,0]z_1(t) + \mathbf{d}^T\mathbf{w}(t),\end{aligned} \tag{4}$$

where

$$\begin{aligned}A_z &= \begin{bmatrix}\mathbf{0} & \mathbf{I}_{r-1\times r-1}\\ -a_0 \cdots & -a_{r-1}\end{bmatrix},\\ \mathbf{b}_z &= [0,\cdots,b_z],\ b_z = \mathbf{c}^T A^{r-1}\mathbf{b},\end{aligned}$$

and $\mathbf{c}_z \in R^{n-r}$ is an appropriate constant vector. From assumption 1, Q_η is a stable matrix because $\dot{\boldsymbol{\eta}}(t) = Q_\eta\boldsymbol{\eta}(t)$ denotes the zero dynamics of system (1).

3.1 Virtual controlled system

We shall introduce the following (r-1)th order stable virtual filter $1/f(s)$ with a state space representation:

$$\begin{aligned}\dot{\mathbf{z}}_f(t) &= A_{u_f}\mathbf{z}_f(t) + \mathbf{b}_{u_f}u(t)\\ u_{f_1}(t) &= \mathbf{c}_{u_f}^T\mathbf{z}_f(t),\end{aligned} \tag{5}$$

where $\mathbf{z}_f = \left[z_{f_1},\cdots,z_{f_{r-1}}\right]^T$ and

$$\begin{aligned}A_{u_f} &= \begin{bmatrix}\mathbf{0} & \mathbf{I}_{r-2\times r-2}\\ -\beta_0 \cdots & -\beta_{r-2}\end{bmatrix},\\ \mathbf{b}_{u_f}^T &= [0,\cdots,1],\ \mathbf{c}_{u_f}^T = [1,0,\cdots,0].\end{aligned}$$

With the following variable transformation using the filtered signal z_{f_i} given in (5):

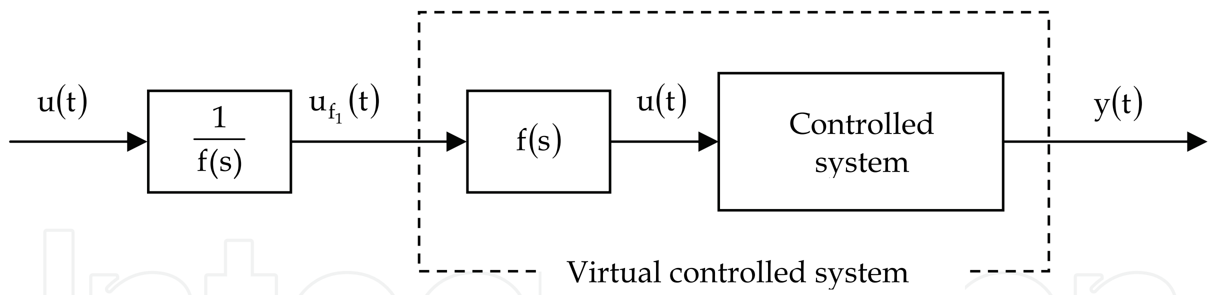


Fig. 1. Virtual controlled system with a virtual filter

$$\begin{aligned}\xi_1(t) &= z_1(t)\\ \xi_i(t) &= -b_z u_{f_{i-1}}(t) + z_i(t) + \sum_{j=1}^{i-1} c_{\xi_j} z_{i-j}(t),\end{aligned} \tag{6}$$

where

$$\begin{aligned}c_{\xi_i} &= \theta_i - a_{r-i}, \quad (1 \le i \le r-1)\\ c_{\xi_r} &= -a_0 + \sum_{j=1}^{r-1} \beta_{j-1} c_{\xi_j}\\ \theta_1 &= \beta_{r-2}\\ \theta_i &= \beta_{r-i-1} + \sum_{j=1}^{i-1} \beta_{r-i+j-1} c_{\xi_j},\end{aligned}$$

the system (1) can be transformed into the following virtual system which has u_{f_1} given from a virtual input filter as the control input (Michino et al., 2004) (see Fig.1):

$$\begin{aligned}\dot{\xi}_1(t) &= \alpha_z \xi(t) + \mathbf{c}_1^T \boldsymbol{\eta}_y(t) + b_z u_{f_1}(t) + \bar{\mathbf{c}}_{d_1}^T \mathbf{w}(t)\\ \dot{\boldsymbol{\eta}}_y(t) &= A_\eta \boldsymbol{\eta}_y(t) + \mathbf{c}_\eta \xi_1(t) + \overline{C}_{d_\eta} \mathbf{w}(t)\\ y(t) &= \xi_1(t) + \mathbf{d}^T \mathbf{w}(t),\end{aligned} \tag{7}$$

where $\boldsymbol{\eta}_y = \left[\boldsymbol{\xi}^T, \boldsymbol{\eta}^T\right]^T$, $\boldsymbol{\xi} = [\xi_2, \xi_3, \cdots, \xi_r]^T$ and $\mathbf{c}_1^T = [1, 0, \cdots, 0]$, $\mathbf{c}_\eta = \left[\mathbf{c}_\xi^T, 0, \cdots, 0, 1\right]^T$. $\bar{\mathbf{c}}_{d_1}$ and $\overline{C}_{d_\eta}$ are a vector and a matrix with appropriate dimensions, respectively. Further, A_η is given by the form of

$$A_\eta = \left[\begin{array}{c|c} A_{u_f} & \begin{matrix}\mathbf{0}\\ \mathbf{c}_z^T\end{matrix} \\ \hline \mathbf{0} & Q_\eta \end{array}\right].$$

Since A_{u_f} and Q_η are stable matrices, A_η is a stable matrix.

3.2 Virtual error system

Now, consider a stable filter of the form:

$$\begin{aligned}\dot{\mathbf{z}}_{c_f}(t) &= A_{c_f}\mathbf{z}_{c_f}(t) + \mathbf{c}_{c_f}u_{f_1}(t)\\ u_f(t) &= \boldsymbol{\theta}^T\mathbf{z}_{c_f}(t) + u_{f_1}(t),\end{aligned} \tag{8}$$

where $\mathbf{c}_{c_f} = [0,\cdots,0,1]^T$ and

$$A_{c_f} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{m-1\times m-1} \\ -\beta_{c_0},\cdots,-\beta_{c_{m-1}} & \end{bmatrix}$$
$$\boldsymbol{\theta}^T = [\alpha_0 - \beta_{c_0},\cdots,\alpha_{m-1} - \beta_{c_{m-1}}].$$

$\beta_{c_0},\beta_{c_1},\cdots,\beta_{c_{m-1}}$ are chosen such that A_{c_f} is stable.

Let's consider transforming the system (7) into a one with u_f given in (8) as the input. Define new variables X_1 and $\mathbf{X}_2$ as follows:

$$\begin{aligned}X_1 &= \xi_1^{(m)} + \alpha_{m-1}\xi_1^{(m-1)} + \cdots + \alpha_1\dot{\xi}_1 + \alpha_0\xi_1\\ \mathbf{X}_2 &= \boldsymbol{\eta}_y^{(m)} + \alpha_{m-1}\boldsymbol{\eta}_y^{(m-1)} + \cdots + \alpha_1\dot{\boldsymbol{\eta}}_y + \alpha_0\boldsymbol{\eta}_y.\end{aligned} \tag{9}$$

Since it follows from the Cayley-Hamilton theorem that

$$A_m^m + \alpha_{m-1}A_m^{m-1} + \cdots + \alpha_1 A_m + \alpha_0 I = 0, \tag{10}$$

we have from (2) and (7) that

$$\begin{aligned}\dot{X}_1(t) &= \alpha_z X_1(t) + \mathbf{c}_1^T\mathbf{X}_2(t) + b_z\overline{u}_f(t)\\ \dot{\mathbf{X}}_2(t) &= A_\eta\mathbf{X}_2(t) + \mathbf{c}_\eta X_1(t),\end{aligned} \tag{11}$$

where

$$\overline{u}_f = u_{f_1}^{(m)} + \alpha_{m-1}u_{f_1}^{(m-1)} + \cdots + \alpha_1\dot{u}_{f_1} + \alpha_0 u_{f_1} \tag{12}$$

Further we have from (10) that

$$e^{(m)} + \alpha_{m-1}e^{(m-1)} + \cdots + \alpha_1\dot{e} + \alpha_0 e = X_1. \tag{13}$$

Therefore defining $\mathbf{E}=\left[e,\dot{e},\cdots,e^{(m-1)}\right]^T$, the following error system is obtained:

$$\begin{aligned}\dot{\mathbf{E}}(t)&=A_E\mathbf{E}(t)+X_1(t)\\ \dot{X}_1(t)&=\alpha_z X_1(t)+\mathbf{c}_1^T\mathbf{X}_2(t)+b_z\overline{u}_f(t)\\ \dot{\mathbf{X}}_2(t)&=A_\eta\mathbf{X}_2(t)+\mathbf{c}_\eta X_1(t)\\ e(t)&=[1,0,\cdots,0]\mathbf{E}(t).\end{aligned} \tag{14}$$

Obviously this error system with the input $\overline{u}_f$ and the output e has a relative degree of $m+1$ and a stable zero dynamics (because A_η is stable).
Furthermore, there exists an appropriate variable transformation such that the error system (14) can be represented by the following form (Isidori, 1995):

$$\begin{aligned}\dot{\mathbf{z}}_e(t)&=A_{z_e}\mathbf{z}_e(t)+\begin{bmatrix}\mathbf{0}\\ b_{z_e}\end{bmatrix}\overline{u}_f(t)+\begin{bmatrix}\mathbf{0}\\ \mathbf{c}_{z_e}^T\end{bmatrix}\boldsymbol{\eta}_{z_e}(t)\\ \dot{\boldsymbol{\eta}}_{z_e}(t)&=\mathbf{Q}_{z_e}\boldsymbol{\eta}_{z_e}(t)+\begin{bmatrix}\mathbf{0}\\ 1\end{bmatrix}z_{e_1}(t)\\ e(t)&=z_{e_1}(t),\end{aligned} \tag{15}$$

where $\mathbf{z}_e=\left[z_{e_1},\cdots,z_{e_{m+1}}\right]^T$ and $\boldsymbol{\eta}_{z_e}\in R^{n-1}$. Since the error system (14) has stable zero dynamics, $\mathbf{Q}_{z_e}$ is a stable matrix.

Recall the stable filter given in (8). Since we have from (8) that

$$\begin{aligned}&u_f^{(m)}+\beta_{c_{m-1}}u_f^{(m-1)}+\cdots+\beta_{c_1}\dot{u}_f+\beta_{c_0}u_f\\ &=u_{f_1}^{(m)}+\alpha_{m-1}u_{f_1}^{(m-1)}+\cdots+\alpha_1\dot{u}_{f_1}+\alpha_0 u_{f_1}=\overline{u}_f,\end{aligned} \tag{16}$$

the filter's output signal u_f can also be obtained from

$$\begin{aligned}\dot{\overline{\mathbf{z}}}_{c_f}(t)&=A_{c_f}\overline{\mathbf{z}}_{c_f}(t)+\begin{bmatrix}\mathbf{0}\\ 1\end{bmatrix}\overline{u}_f(t)\\ u_f(t)&=[1,0,\cdots,0]\overline{\mathbf{z}}_{c_f}(t)\end{aligned}$$

by defining $\overline{\mathbf{z}}_{c_f}=\left[u_f,\dot{u}_f,\cdots,u_f^{(m-1)}\right]^T$ .Using this virtual filter signal in the variable transformation given in (6), the error system (15) can be transformed into the following form, the same way as the virtual system (7) was derived, with u_f as the input.

$$\begin{aligned}\dot{e}(t)&=\alpha_e e(t)+b_e u_f(t)+\mathbf{c}_e^T\boldsymbol{\eta}_e(t)\\ \dot{\boldsymbol{\eta}}_e(t)&=Q_e\boldsymbol{\eta}_e(t)+\mathbf{b}_\eta e(t),\end{aligned} \tag{17}$$

where

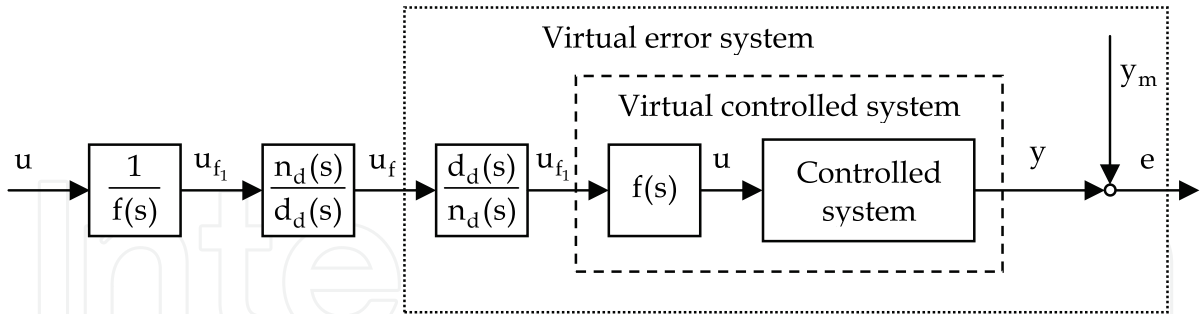


Fig. 2. Virtual error system with an virtual internal model

$$Q_e = \left[\begin{array}{c|c} A_{c_f} & \begin{matrix}\mathbf{0} \\ \mathbf{c}_{z_e}^T\end{matrix} \\ \hline \mathbf{0} & Q_{z_e} \end{array}\right].$$

Since A_{c_f} and Q_{z_e} are stable matrices, Q_e is a stable matrix. Thus the obtained virtual error system (17) is ASPR from the input u_f to the output e.

The overall configuration of the virtual error system is shown in Fig.2.

4. Adaptive Controller Design

Since the virtual error system (17) is ASPR, there exists an ideal feedback gain k^* such that the control objective is achieved with the control input: $u_f(t) = -k^* e(t)$ (Kaufman et al., 1998; Iwai & Mizumoto, 1994). That is, from (8), if the filter signal u_{f_1} can be obtained by

$$u_{f_1}(t) = -k^* e(t) - \boldsymbol{\theta}^T \mathbf{z}_{c_f}(t), \tag{18}$$

one can attain the goal. Unfortunately one can not design u_{f_1} directly by (18), because u_{f_1} is a filter signal given in (8) and the controlled system is assumed to be unknown. In such cases, the use of the backstepping strategy on the filter (5) can be considered as a countermeasure. However, since the controller structure depends on the relative degree of the system, i.e. the order of the filter (5), it will become very complex in cases where the controlled system has higher order relative degrees. Here we adopt a novel design strategy using a parallel feedforward compensator (PFC) that allows us to design the controller through a backstepping of only one step (Mizumoto et al., 2005; Michino et al., 2004).

4.1 Augmented virtual filter

For the virtual input filter (5), consider the following stable and minimum-phase PFC with an appropriate order n_f:

$$\begin{aligned}\dot{y}_f(t) &= -a_{f_1} y_f(t) + \mathbf{a}_{f_2}^T \boldsymbol{\eta}_f(t) + b_a u(t) \\ \dot{\boldsymbol{\eta}}_f(t) &= A_f \boldsymbol{\eta}_f(t) + \mathbf{b}_f y_f(t),\end{aligned} \tag{19}$$

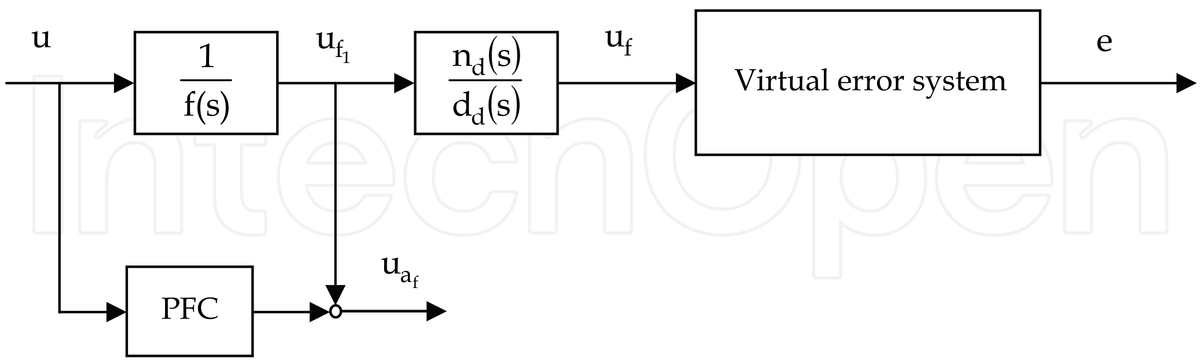


Fig. 3. Virtual error system with an augmented filter

where $y_f \in R$ is the output of the PFC. Since the PFC is minimum-phase A_f is a stable matrix.
The augmented filter obtained from the filter (5) by introducing the PFC (19) can then be represented by

$$\begin{aligned}\dot{\mathbf{z}}_{u_f}(t) &= A_{z_f} \mathbf{z}_{u_f}(t) + \mathbf{b}_{z_f} u(t) \\ u_{a_f}(t) &= \mathbf{c}_{z_f}^T \mathbf{z}_{u_f}(t) = u_{f_1}(t) + y_f(t),\end{aligned} \tag{20}$$

where $\mathbf{z}_{u_f} = \left[\mathbf{z}_f^T, y_f, \boldsymbol{\eta}_f^T\right]^T$ and

$$A_{z_f} = \begin{bmatrix} A_{u_f} & 0 & \mathbf{0} \\ \mathbf{0} & -a_{f_1} & \mathbf{a}_{f_2}^T \\ \mathbf{0} & \mathbf{b}_f & A_f \end{bmatrix}, \mathbf{b}_{z_f} = \begin{bmatrix} \mathbf{b}_{u_f} \\ b_a \\ \mathbf{0} \end{bmatrix},$$
$$\mathbf{c}_{z_f}^T = \left[\mathbf{c}_{u_f}, 1, 0, \cdots, 0\right]$$

Here we assume that the PFC (19) is designed so that the augmented filter is ASPR, i.e. minimum-phase and a relative degree of one. In this case, there exists an appropriate variable transformation such that the augmented filter can be transformed into the following form (Isidori, 1995):

$$\begin{aligned}\dot{u}_{a_f}(t) &= a_{a_1} u_{a_f}(t) + \mathbf{a}_{a_2}^T \boldsymbol{\eta}_a(t) + b_a u(t) \\ \dot{\boldsymbol{\eta}}_a(t) &= A_a \boldsymbol{\eta}_a(t) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} u_{a_f}(t),\end{aligned}$$

where A_a is a stable matrix because the augmented filter is minimum-phase.

Using the augmented filter's output u_{a_f}, the virtual error system is rewritten as follows (see Fig.3):

$$\begin{aligned}\dot{e}(t) &= \alpha_e e(t) + b_e\left(u_{a_f}(t) + \boldsymbol{\theta}^T \mathbf{z}_{c_f}(t) - y_f(t)\right) + \mathbf{c}_e^T \boldsymbol{\eta}_e(t)\\ \dot{\boldsymbol{\eta}}_e(t) &= Q_e \boldsymbol{\eta}_e(t) + \mathbf{b}_\eta e(t).\end{aligned} \tag{21}$$

4.2 Controller design by single step backstepping

[Pre-step] We first design the virtual input α_1 for the augmented filter output u_{a_f} in (21) as follows:

$$\alpha_1(t) = -k(t)e(t) - \hat{\boldsymbol{\theta}}(t)^T \mathbf{z}_{c_f}(t) + \Psi_0(t), \tag{22}$$

where $k(t)$ is an adaptive feedback gain and $\hat{\boldsymbol{\theta}}(t)$ is an estimated value of $\boldsymbol{\theta}$, these are adaptively adjusted by

$$\begin{aligned}\dot{k}(t) &= \gamma_k e^2(t) - \sigma_k k(t), \quad \gamma_k > 0, \sigma_k > 0\\ \dot{\hat{\boldsymbol{\theta}}}(t) &= \Gamma_\theta \mathbf{z}_{c_f}(t)e(t) - \sigma_\theta \hat{\boldsymbol{\theta}}(t), \quad \Gamma_\theta^T = \Gamma_\theta > 0, \sigma_\theta > 0.\end{aligned} \tag{23}$$

Further, $\Psi_0(t)$ is given as follows:

$$\begin{aligned}\dot{\Psi}_0(t) &= D(y_f)\left(-a_{f_1}\Psi_0(t) + b_a u(t)\right)\\ D(y_f) &= \begin{cases} 0, & \text{if } |y_f| \le \delta_{y_f} \\ 1, & \text{if } |y_f| > \delta_{y_f} \end{cases}\end{aligned} \tag{24}$$

where δ_{y_f} is any positive constant.

Now consider the following positive definite function:

$$V_0 = \frac{1}{2b_e}e^2 + \frac{1}{2\gamma_k}\Delta k^2 + \frac{1}{2}\Delta\boldsymbol{\theta}^T \Gamma_\theta^{-1}\Delta\boldsymbol{\theta} + \boldsymbol{\eta}_e^T P_e \boldsymbol{\eta}_e, \tag{25}$$

where

$$\Delta k = k(t) - k^*, \ \Delta\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}(t) - \boldsymbol{\theta},$$

k^* is an ideal feedback gain to be determined later and P_e is a positive definite matrix that satisfies the following Lyapunov equation for any positive definite matrix R_e.

$$P_e Q_e + Q_e^T P_e = -R_e < 0.$$

Since Q_e is a stable matrix, there exists such P_e.
The time derivative of V_0 can be evaluated by

$$\begin{aligned}\dot{V}_0 \le& -\left(k^* - v_0\right)e^2 - \left(\lambda_{\min}\left[R_e\right] - \rho_1\right)\|\boldsymbol{\eta}_e\|^2 + \omega_1 e \\ &- \left\{y_f - \Psi_0\right\}e - \left(\frac{\sigma_k}{\gamma_k} - \rho_2\right)\Delta k^2 \\ &- \left(\sigma_\theta \lambda_{\min}\left[\Gamma_\theta^{-1}\right] - \rho_3\right)\Delta\boldsymbol{\theta}^2 + R_0\end{aligned} \tag{26}$$

with any positive constant ρ_1 to ρ_3. Where $\omega_1 = u_{a_f} - \alpha_1$ and

$$\begin{aligned}v_0 &= \frac{\alpha_e}{b_e} + \frac{\left(\|\mathbf{c}_e\| + 2\|P_e\|\|\mathbf{b}_\eta\|b_e\right)^2}{4b_e^2\rho_1} \\ R_0 &= \frac{\sigma_k^2 k^{*2}}{4\rho_2\gamma_k^2} + \frac{\sigma_\theta^2\left(\lambda_{\min}\left[\Gamma_\theta^{-1}\right]\right)^2}{4\rho_3}\|\boldsymbol{\theta}\|^2.\end{aligned} \tag{27}$$

[Step 1] Consider the error system, ω_1-system, between u_{a_f} and α_1. The ω_1-system is given from (21) by

$$\begin{aligned}\dot{\omega}_1 &= \dot{u}_{a_f} - \dot{\alpha}_1 \\ &= a_{a_1}u_{a_f} + \boldsymbol{a}_{a_2}^T\boldsymbol{\eta}_a + b_a u - \dot{\alpha}_1.\end{aligned} \tag{28}$$

The time derivative of α_1 is obtained as follows:

$$\begin{aligned}\dot{\alpha}_1 =& \frac{\partial\alpha_1}{\partial e}\alpha_e e + \frac{\partial\alpha_1}{\partial e}\boldsymbol{\theta}_1^T\mathbf{z}_{c_f} + \frac{\partial\alpha_1}{\partial e}b_e u_{f_1} + \frac{\partial\alpha_1}{\partial e}\mathbf{c}_e^T\boldsymbol{\eta}_e \\ &+ \frac{\partial\alpha_1}{\partial k(t)}\dot{k} + \frac{\partial\alpha_1}{\partial\mathbf{z}_{c_f}}\dot{\mathbf{z}}_{c_f} + \frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}}\dot{\hat{\boldsymbol{\theta}}} \\ &+ D\left(y_f\right)\left(-a_{f_1}\Psi_0 + b_a u\right),\end{aligned} \tag{29}$$

where $\boldsymbol{\theta}_1^T = b_e\boldsymbol{\theta}^T$. Taking (28) and (29) into consideration, the actual control input is designed as follows:

$$u = \begin{cases} -\dfrac{1}{b_a}\left[c_1\omega_1 + \varepsilon_0\left(\left|u_{a_f}\right|^2 + \|\boldsymbol{\eta}_a\|^2\right)\omega_1 + \varepsilon_1\Psi_1\omega_1 - \Psi_2\right], \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if} \quad \left|y_f\right| \le \delta_{y_f} \\ -\dfrac{\omega_1}{b_a y_f}\left[c_1\omega_1 + \varepsilon_0\left(\left|u_{a_f}\right|^2 + \|\boldsymbol{\eta}_a\|^2\right)\omega_1 + \varepsilon_1\Psi_1\omega_1 - \Psi_2\right] \\ \quad -\dfrac{1}{b_a}\left[\gamma_f y_f + \varepsilon_2\|\boldsymbol{\eta}_f\|^2 y_f\right] - \dfrac{\varepsilon_3}{b_a y_f}\Psi_0^2 \quad , \quad \text{if} \quad \left|y_f\right| > \delta_{y_f} \end{cases} \tag{30}$$

where ε_0 to ε_3 and γ_f are any positive constants, and Ψ_1 and Ψ_2 are given by

$$\Psi_1 = \left|\frac{\partial\alpha_1}{\partial k}\right|^2 |\dot{k}|^2 + \left\|\frac{\partial\alpha_1}{\partial\hat{\boldsymbol{\theta}}}\right\|^2 \left\|\dot{\hat{\boldsymbol{\theta}}}\right\|^2 + \left\|\frac{\partial\alpha_1}{\partial \mathbf{z}_{c_f}}\right\|^2 \left\|\dot{\mathbf{z}}_{c_f}\right\|^2 + l$$

$$\Psi_2 = -\frac{\partial\alpha_1}{\partial e}\hat{\alpha}_e e - \frac{\partial\alpha_1}{\partial e}\hat{\boldsymbol{\theta}}_1^T \mathbf{z}_{c_f} - \frac{\partial\alpha_1}{\partial e}\hat{b}_e u_{f_1} + \hat{\beta}_1\left(\frac{\partial\alpha_1}{\partial e}\right)^2 \omega_1,$$

where l is any positive constant and $\hat{\alpha}_e, \hat{b}_e, \hat{\boldsymbol{\theta}}_1, \hat{\beta}_1$ are estimated values of $\alpha_e, b_e, \boldsymbol{\theta}_1, \beta_1$, respectively, and adaptively adjusted by the following parameter adjusting laws.

$$\begin{aligned}
\dot{\hat{\alpha}}_e(t) &= -\gamma_\alpha \omega_1(t)\frac{\partial\alpha_1}{\partial e} e(t) - \sigma_\alpha \hat{\alpha}_e(t) \\
\dot{\hat{b}}_e(t) &= -\gamma_b \omega_1(t)\frac{\partial\alpha_1}{\partial e} u_{f_1}(t) - \sigma_b \hat{b}_e(t) \\
\dot{\hat{\boldsymbol{\theta}}}_1(t) &= -\Gamma_{\theta_1} \mathbf{z}_{c_f}(t)\frac{\partial\alpha_1}{\partial e}\omega_1(t) - \sigma_{\theta_1}\hat{\boldsymbol{\theta}}_1(t) \\
\dot{\hat{\beta}}_1(t) &= \gamma_{\beta_1}\omega_1(t)^2\left(\frac{\partial\alpha_1}{\partial e}\right)^2 - \sigma_{\beta_1}\hat{\beta}_1(t)
\end{aligned} \tag{31}$$

where $\gamma_\alpha, \gamma_b, \gamma_{\beta_1}, \sigma_\alpha, \sigma_b, \sigma_{\theta_1}, \sigma_{\beta_1}$ are any positive constants and $\Gamma_{\theta_1} = \Gamma_{\theta_1}^T > 0$.

4.3 Boundedness analysis

For the designed control system with control input (30), we have the following theorem concerning the boundedness of all the signals in the control system.

Theorem 1 Under assumptions 1 to 3 on the controlled system (1), all the signals in the resulting closed loop system with the controller (30) are uniformly bounded.

Proof: Consider the following positive and continuous function V_1.

$$V_1 = \begin{cases} V_0 + \frac{1}{2}\omega_1^2 + \frac{1}{2}\Delta\boldsymbol{\theta}_1^T\Gamma_{\theta_1}^{-1}\Delta\boldsymbol{\theta}_1 + \frac{1}{2\gamma_\alpha}\Delta\alpha_e^2 \\ \quad + \frac{1}{2\gamma_b}\Delta b_e^2 + \frac{1}{2\gamma_{\beta_1}}\Delta\beta_1^2 + \frac{1}{2}\delta_{y_f}^2, \quad \text{if } |y_f| \le \delta_{y_f} \\ V_0 + \frac{1}{2}\omega_1^2 + \frac{1}{2}\Delta\boldsymbol{\theta}_1^T\Gamma_{\theta_1}^{-1}\Delta\boldsymbol{\theta}_1 + \frac{1}{2\gamma_\alpha}\Delta\alpha_e^2 \\ \quad + \frac{1}{2\gamma_b}\Delta b_e^2 + \frac{1}{2\gamma_{\beta_1}}\Delta\beta_1^2 + \frac{1}{2}y_f^2, \quad \text{if } |y_f| > \delta_{y_f}, \end{cases} \tag{32}$$

where

$$\Delta\alpha_e = \hat{\alpha}_e(t) - \alpha_e, \ \ \Delta b_e = \hat{b}_e(t) - b_e$$
$$\Delta\boldsymbol{\theta}_1 = \hat{\boldsymbol{\theta}}_1(t) - \boldsymbol{\theta}_1, \ \ \Delta\beta_1 = \hat{\beta}_1(t) - \beta_1,$$

and δ_{y_f} is any positive constant.

From (26) and (32), the time derivative of V_1 for $|y_f| \le \delta_{y_f}$ can be evaluated by

$$\begin{aligned}\dot{V}_1 \le & -\left(k^* - v_0 - \frac{1}{4\varepsilon l}\right)e^2 - \left(\lambda_{\min}[R_e] - \rho_1 - \mu_0\right)\|\boldsymbol{\eta}_e\|^2 \\ & -\left(\frac{\sigma_k}{\gamma_k} - \rho_2\right)\Delta k^2 - \left(\sigma_\theta \lambda_{\min}\left[\Gamma_\theta^{-1}\right] - \rho_3\right)\Delta\boldsymbol{\theta}^2 \\ & - c_1\omega_1^2 - \left(\sigma_{\theta_1}\lambda_{\min}\left[\Gamma_{\theta_1}^{-1}\right] - \mu_1\right)\Delta\boldsymbol{\theta}_1^2 \\ & -\left(\frac{\sigma_\alpha}{\gamma_\alpha} - \mu_2\right)\Delta\alpha_e^2 - \left(\frac{\sigma_b}{\gamma_b} - \mu_3\right)\Delta b_e^2 \\ & -\left(\frac{\sigma_{\beta_1}}{\gamma_{\beta_1}} - \mu_4\right)\Delta\beta_1^2 - \left(y_f - \Psi_0(y_f)\right)e + R_1\end{aligned} \tag{33}$$

with any positive constants μ_0 to μ_4. Where

$$R_1 = R_0 + \frac{3}{4\varepsilon_1} + \frac{\sigma_{\theta_1}^2\left(\lambda_{\min}\left[\Gamma_{\theta_1}^{-1}\right]\right)^2}{4\mu_1}\|\boldsymbol{\theta}_1\|^2 + \frac{\sigma_\alpha^2\alpha_e^2}{4\mu_2\gamma_\alpha} + \frac{\sigma_b^2 b_e^2}{4\mu_3\gamma_b} + \frac{\sigma_\beta^2\beta^2}{4\mu_4\gamma_\beta}.$$

Here we have

$$-(y_f - \Psi_0)e = -\mu_5\left\{e - \frac{(y_f + \Psi_0)}{2\mu_5}\right\}^2 + \frac{(y_f - \Psi_0)^2}{4\mu_5} + \mu_5 e^2 \tag{34}$$

with any positive constant μ_5. Furthermore, for $|y_f| \le \delta_{y_f}$, since $\dot{\Psi}_0(t) = 0$ is held, there exists a positive constant Ψ_M such that $|y_f(t) - \Psi_0(t)| \le \Psi_M$.

Therefore the time derivative of V_1 can be evaluated by

$$\dot{V}_1 \le -\alpha_a V_1 + \overline{R}_1 \tag{35}$$

for $|y_f| \le \delta_{y_f}$, where

$$\alpha_a = \min\left[2b_e\left(k^* - v_0 - \frac{1}{4\varepsilon l} - \mu_5\right), s_a, 2\right]$$

$$s_a = \min\left[\frac{\lambda_{\min}[R_e] - \rho_1 - \mu_0}{\lambda_{\max}[P_e]}, 2\gamma_k\left(\frac{\sigma_k}{\gamma_k} - \rho_2\right), 2\frac{\left(\sigma_\theta\lambda_{\min}\left[\Gamma_\theta^{-1}\right] - \rho_3\right)}{\lambda_{\mathrm{mac}}\left[\Gamma_\theta^{-1}\right]},\right.$$
$$2c_1, 2\frac{\left(\sigma_{\theta_1}\lambda_{\min}\left[\Gamma_{\theta_1}^{-1}\right] - \mu_1\right)}{\lambda_{\max}\left[\Gamma_{\theta_1}^{-1}\right]}, 2\gamma_\alpha\left(\frac{\sigma_\alpha}{\gamma_\alpha} - \mu_2\right),$$
$$\left. 2\gamma_b\left(\frac{\sigma_b}{\gamma_b} - \mu_3\right), 2\gamma_\beta\left(\frac{\sigma_{\beta_1}}{\gamma_{\beta_1}} - \mu_4\right)\right]$$

$$\overline{R}_1 = R_1 + \frac{\Psi_M^2}{4\mu_5} + \delta_{y_f}^2.$$

For $|y_f| > \delta_{y_f}$, the time derivative of V_1 is evaluated as

$$\begin{aligned}\dot{V}_1 \le &-\left(k^* - v_0 - \frac{1}{4\varepsilon l}\right)e^2 - \left(\lambda_{\min}[R_e] - \rho_1 - \mu_0\right)\|\boldsymbol{\eta}_e\|^2 \\ &-\left(\frac{\sigma_k}{\gamma_k} - \rho_2\right)\Delta k^2 - \left(\sigma_\theta\lambda_{\min}\left[\Gamma_\theta^{-1}\right] - \rho_3\right)\Delta\boldsymbol{\theta}^2 - c_1\omega_1^2 \\ &-\left(\sigma_{\theta_1}\lambda_{\min}\left[\Gamma_{\theta_1}^{-1}\right] - \mu_1\right)\Delta\boldsymbol{\theta}_1^2 - \left(\frac{\sigma_\alpha}{\gamma_\alpha} - \mu_2\right)\Delta\alpha_e^2 \\ &-\left(\frac{\sigma_b}{\gamma_b} - \mu_3\right)\Delta b_e^2 - \left(\frac{\sigma_{\beta_1}}{\gamma_{\beta_1}} - \mu_4\right)\Delta\beta_1^2 + R_1 \\ &- a_{f_1}y_f^2 + \mathbf{a}_{f_2}\boldsymbol{\eta}_f y_f - \gamma_f y_f^2 - \varepsilon_2\|\boldsymbol{\eta}_f\|^2 y_f^2 - \varepsilon_3\Psi_0^2 \\ &+ \Psi_0 e - y_f e,\end{aligned} \tag{36}$$

and thus we have for $|y_f| > \delta_{y_f}$ that

$$\dot{V}_1 \le -\alpha_b V_1 + R_2, \tag{37}$$

where

$$\begin{aligned}\alpha_b &= \min\left[2b_e\left(k^* - v_0 - \frac{1}{4\varepsilon l} - \frac{1}{a_{f_1}} - \frac{1}{4\varepsilon_3}\right), s_a, 2\gamma_f\right] \\ R_2 &= R_1 + \frac{\|\mathbf{a}_{f_2}\|^2}{4\varepsilon_2}.\end{aligned} \tag{38}$$

Finally, for an ideal feedback gain k^* which satisfies

$$k^* > v_0 + \frac{1}{4\varepsilon l} + v_1, \quad v_1 = \max\left[\mu_5, \frac{1}{a_{f_1}} - \frac{1}{4\varepsilon_3}\right],$$

the time derivative of V_1 can be evaluated by

$$\dot{V}_1 \le -\alpha V_1 + R, \tag{39}$$

where $\alpha = \min[\alpha_a, \alpha_b] > 0$, $R = \max[\overline{R}_1, R_2]$. Consequently it follows that V_1 is uniformly bounded and thus the signals $e(t), \omega_1(t), \boldsymbol{\eta}_e(t), y_f(t), \boldsymbol{\eta}_f(t)$ and adjusted parameters $k(t), \hat{\boldsymbol{\theta}}(t)$, $\hat{\alpha}_e(t), \hat{\boldsymbol{\theta}}_1(t), \hat{b}_e(t), \hat{\beta}_1(t)$ are also uniformly bounded.

Next, we show that the filter signal $\mathbf{z}_{c_f}$ and the control input u are uniformly bounded. Define new variable z_{ξ_1} as follows:

$$z_{\xi_1}^{(m)} + \beta_{c_{m-1}} z_{\xi_1}^{(m-1)} + \cdots + \beta_{c_0} z_{\xi_1} = \xi_1 \tag{40}$$

$$\dot{\xi}_1 = \alpha_z \xi_1 + \mathbf{c}_\xi^T \boldsymbol{\eta}_y + b_z u_{f_1} + \overline{\mathbf{c}}_{d_1}^T \mathbf{w} \tag{41}$$

$$\dot{\boldsymbol{\eta}}_y = A_\eta \boldsymbol{\eta}_y + \mathbf{b}_\eta \xi_1 + \overline{C}_{d_\eta} \mathbf{w}, \tag{42}$$

where ξ_1 and $\boldsymbol{\eta}_y$ have been given in (7). Further define z_{β_1} by

$$\dot{z}_{\beta_1} = \alpha_z z_{\beta_1} + b_z z_{c_f} + \eta_{\beta_1} \tag{43}$$

$$\eta_{\beta_1}^{(m)} + \beta_{c_{m-1}} \eta_{\beta_1}^{(m-1)} + \cdots + \beta_{c_0} \eta_{\beta_1} = \mathbf{c}_\xi^T \boldsymbol{\eta}_y + \overline{\mathbf{c}}_{d_1}^T \mathbf{w}, \tag{44}$$

where $z_{c_{f1}} = [1, 0, \cdots, 0]\mathbf{z}_{c_f}$ and we set $z_{\beta_1}^{(k)}(0) = z_{\xi_1}^{(k)}(0), k = 0, \cdots, m$ .We have from (40) and (41) that

$$\begin{aligned} \dot{\xi}_1 - \alpha_z \xi_1 = z_{\xi_1}^{(m+1)} &+ \left(\beta_{c_{m-1}} - \alpha_z\right) z_{\xi_1}^{(m)} \\ &+ \left(\beta_{c_{m-2}} - \alpha_z \beta_{c_{m-1}}\right) z_{\xi_1}^{(m-1)} + \cdots \\ &+ \left(\beta_{c_0} - \alpha_z \beta_{c_1}\right) \dot{z}_{\xi_1} - \alpha_z z_{\xi_1} \\ = b_z u_{f_1} &+ \mathbf{c}_\xi^T \boldsymbol{\eta}_y + \overline{\mathbf{c}}_{d_1}^T \mathbf{w}. \end{aligned} \tag{45}$$

Further, we have from (43), (44) and (8) that

$$\begin{aligned} &z_{\beta_1}^{(m+1)} + \left(\beta_{c_{m-1}} - \alpha_z\right) z_{\beta_1}^{(m)} + \left(\beta_{c_{m-2}} - \alpha_z \beta_{c_{m-1}}\right) z_{\beta_1}^{(m-1)} \\ &\quad + \cdots + \left(\beta_{c_0} - \alpha_z \beta_{c_1}\right) \dot{z}_{\beta_1} - \alpha_z z_{\beta_1} \\ &\quad = b_z u_{f_1} + \mathbf{c}_\xi^T \boldsymbol{\eta}_y + \bar{\mathbf{c}}_{d_1}^T \mathbf{w}. \end{aligned} \tag{46}$$

It follows from (45) and (46) that $z_{\xi_1}^{(k)} = z_{\beta_1}^{(k)},\ k = 0, \cdots, m$.

Define $\mathbf{z}_\xi = \left[z_{\xi_1}, \dot{z}_{\xi_1}, \cdots, z_{\xi_1}^{(m-1)}\right]^T$ and $\mathbf{z}_\beta = \left[z_{\beta_1}, \dot{z}_{\beta_1}, \cdots, z_{\beta_1}^{(m-1)}\right]^T$. Since $s^m + \beta_{c_{m-1}} s^{m-1} + \cdots + \beta_{c_0}$ is a stable polynomial, we obtain from (40) that

$$\|\mathbf{z}_\beta\| = \|\mathbf{z}_\xi\| \le l_1 |\xi_1| + l_2, \tag{47}$$

with appropriate positive constants l_1, l_2. From the boundedness of $\mathbf{w}(t)$ and e(t), we have $\xi_1(t)$ is bounded and thus $\mathbf{z}_\beta$ is also bounded.

Furthermore defining $\boldsymbol{\eta}_\beta = \left[\eta_{\beta_1}, \dot{\eta}_{\beta_1}, \cdots, \eta_{\beta_1}^{(m-1)}\right]^T$, we have from (44) that

$$\dot{\boldsymbol{\eta}}_\beta(t) = A_{c_f} \boldsymbol{\eta}_\beta(t) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \left(\mathbf{c}_\xi^T \boldsymbol{\eta}_y(t) + \bar{\mathbf{c}}_{d_1}^T \mathbf{w}(t)\right). \tag{48}$$

From (8) and (48), we obtain

$$\begin{aligned} &b_z \dot{\mathbf{z}}_{c_f}(t) + \dot{\boldsymbol{\eta}}_\beta(t) \\ &= A_{c_f}\left(b_z \mathbf{z}_{c_f}(t) + \boldsymbol{\eta}_\beta(t)\right) + b_z \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} u_{f_1}(t) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \left(\mathbf{c}_\xi^T \boldsymbol{\eta}_y(t) + \bar{\mathbf{c}}_{d_1}^T \mathbf{w}(t)\right). \end{aligned} \tag{49}$$

Therefore $b_z \dot{\mathbf{z}}_{c_f}(t) + \dot{\boldsymbol{\eta}}_\beta(t)$ can be evaluated from (48) and the fact that $u_{f_1} = \omega_1 + \alpha_1 - y_f$ by

$$\begin{aligned} \left\|b_z \dot{\mathbf{z}}_{c_f}(t) + \dot{\boldsymbol{\eta}}_\beta(t)\right\| &\le \left\|A_{c_f}\right\| \left\|b_z \mathbf{z}_{c_f}(t) + \boldsymbol{\eta}_\beta(t)\right\| \\ &\quad + b_z\left|\alpha_1(t) - y_f(t)\right| + b_z\left|\omega_1(t)\right| \\ &\quad + \left\|\mathbf{c}_\xi\right\| \left\|\boldsymbol{\eta}_y(t)\right\| + \left\|\bar{\mathbf{c}}_{d_1}\right\| \left\|\mathbf{w}(t)\right\|. \end{aligned} \tag{50}$$

Here, we have from (22) that

$$\alpha_1(t) - y_f(t) = -k(t)e(t) - \frac{\hat{\boldsymbol{\theta}}^T(t)}{b_z}\left\{b_z \mathbf{z}_{c_f}(t) + \boldsymbol{\eta}_\beta(t)\right\} + \frac{\hat{\boldsymbol{\theta}}^T(t)}{b_z} \boldsymbol{\eta}_\beta(t) + \Psi_0(y_f) - y_f(t). \tag{51}$$

Since it follows from (19) and (24) that

$$\dot{y}_f(t)-\dot{\Psi}_0(t)=-a_{f_1}\left(y_f(t)-\Psi_0(t)\right)+\mathbf{a}_{f_2}^T\boldsymbol{\eta}_f(t) \tag{52}$$

for $|y_f|>\delta_{y_f}$ and from the boundedness of $\boldsymbol{\eta}_f(t)$, there exists a positive constant such that $|y_f-\Psi_0(t)|\le\overline{\Psi}_M$. Further, from the boundedness of $\mathbf{w}(t)$ and e(t) i.e. $\xi_1(t)$, we can confirm that $\boldsymbol{\eta}_y(t)$ and $\boldsymbol{\eta}_\beta(t)$ are also bounded from (7) and (48). Finally, taking the boundedness of the signals e(t), $\omega_1(t), k(t), \hat{\boldsymbol{\theta}}(t)$ and $\boldsymbol{\eta}_y(t), \boldsymbol{\eta}_\beta(t)$ into consideration, from (50) $b_z\dot{\mathbf{z}}_{c_f}(t)+\dot{\boldsymbol{\eta}}_\beta(t)$ can be evaluated by

$$\left\|b_z\dot{\mathbf{z}}_{c_f}(t)+\dot{\boldsymbol{\eta}}_\beta(t)\right\|\le l_{z_1}\left\|b_z\mathbf{z}_{c_f}(t)+\boldsymbol{\eta}_\beta(t)\right\|+l_{z_2} \tag{53}$$

with appropriate positive constants l_{z_1} and l_{z_2}. Consequently, considering the system:

$$\dot{\mathbf{z}}_\beta(t)=\alpha_z\mathbf{z}_\beta+b_z\mathbf{z}_{c_f}+\boldsymbol{\eta}_\beta(t) \tag{54}$$

from (44) with $b_z\mathbf{z}_{c_f}+\boldsymbol{\eta}_\beta(t)$ as the input and $\mathbf{z}_\beta$ as the output, since this system is minimum-phase and the inequality (53) is held, we have from the Output/Input L_p Stability Lemma (Sastry & Bodson, 1989) that the input $b_z\mathbf{z}_{c_f}+\boldsymbol{\eta}_\beta(t)$ in the system (54) can be evaluated by

$$\left\|b_z\mathbf{z}_{c_f}(t)+\boldsymbol{\eta}_\beta(t)\right\|\le\bar{l}_{z_1}\left\|\mathbf{z}_\beta(t)\right\|+\bar{l}_{z_2} \tag{55}$$

with appropriate positive constants $\bar{l}_{z_1}$ and $\bar{l}_{z_2}$. From the boundedness of $\mathbf{z}_\beta(t)$ and $\boldsymbol{\eta}_\beta(t)$, we can conclude that $\mathbf{z}_{c_f}(t)$ is uniformly bounded and then the control input u(t) is also uniformly bounded. Thus all the signals in the resulting closed loop system with the controller (30) are uniformly bounded.

5. Simulation Results

The effectiveness of the proposed method is confirmed through numerical simulation for a 3rd order SISO system with a relative degree of 3, which is given by

$$\begin{aligned}\dot{\mathbf{z}}&=\begin{bmatrix}-1&-0.5&0.5\\1.5&-2.5&-0.5\\-2.5&0.5&1\end{bmatrix}\mathbf{z}+\begin{bmatrix}0\\0\\1\end{bmatrix}u+\begin{bmatrix}1&0.1&0.1&0.1\\0.1&0.1&1&0.1\\0.1&0.1&0.1&1\end{bmatrix}\mathbf{w}\\y&=z_1+\begin{bmatrix}0.1&0.1&1&0.1\end{bmatrix}\mathbf{w},\end{aligned} \tag{56}$$

where $\mathbf{w}$ is an unknown disturbance which has the following form:

$$\mathbf{w} = \begin{bmatrix} \sin(2t) \\ 2\cos(2t) \\ 0.5\sin(5t) \\ 2.5\cos(5t) \end{bmatrix} \tag{57}$$

Before designing a controller, we first introduce the following pre-filter:

$$\frac{b}{s+a} \tag{58}$$

in order to reduce the chattering phenomenon to be expected by switching the controller given in (30). Therefore, the considered controlled system has a relative degree of 4.

Since the relative degree of the controlled system is 4, we consider a 3rd order input virtual filter in (5). Further we consider a stable internal model filter (8) of the order of 4.

For the input virtual filter, in this simulation, we consider a first order PFC:

$$\dot{y}_f = -a_{f_1} y_f + b_a u$$

in order to make an ASPR augmented filter.

The design parameters for the pre-filter (58), the input virtual filter (5) and the internal model filter (8) are set as follows:

$$\begin{gathered} a = b = 1000 \\ \beta_0 = 15, \beta_1 = 75, \beta_2 = 125 \\ \beta_{c_0} = 20, \beta_{c_1} = 150, \beta_{c_2} = 500, \beta_{c_3} = 625 \end{gathered}$$

and the PFC parameters are set by

$$a_{f_1} = 10, b_a = 0.01.$$

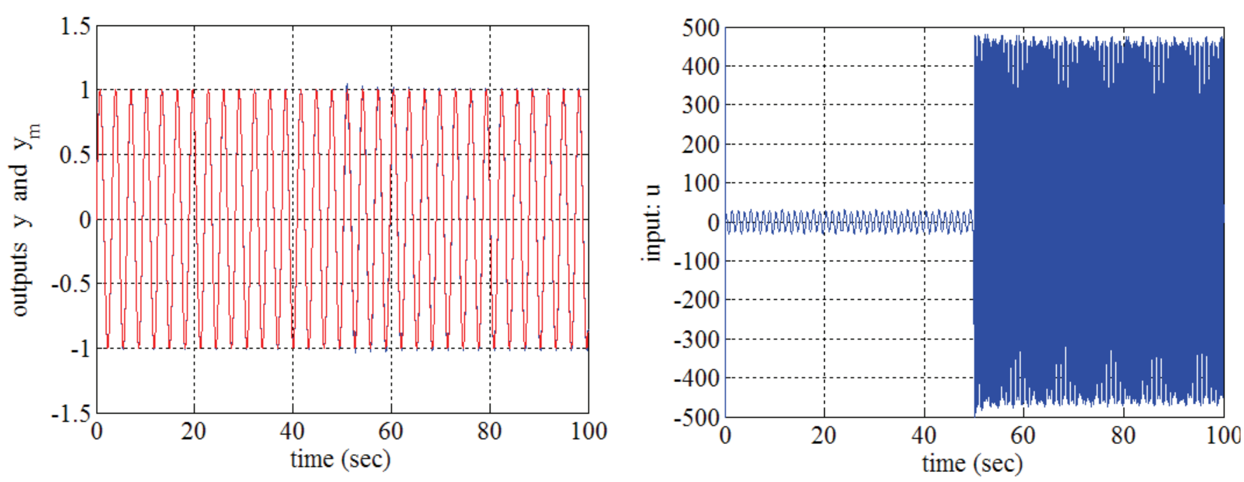
Further design parameters in the controller given in (23), (24), (30) and (31) are designed by

$$\begin{gathered} \gamma_k = 500, \sigma_k = 0.01, \delta_{y_f} = 10 \\ l = 0.5, \sigma_\theta = 0.05, \sigma_{\theta_1} = \sigma_a = \sigma_b = \sigma_{\beta_1} = 0.1 \\ \Gamma_\theta = \Gamma_{\theta_1} = 5000 I_4, \gamma_a = \gamma_b = \gamma_\beta = 100 \\ c_1 = 1000, \varepsilon_0 = \varepsilon_1 = \varepsilon_2 = 0.01, \varepsilon_3 = \gamma_f = 100. \end{gathered}$$

Figure 4 shows the simulation results with the proposed controller. In this simulation, the disturbance **w** is changed at 50 [sec]:

$$\mathbf{w} = \begin{bmatrix} \sin(2t) \\ 2\cos(2t) \\ 0.5\sin(5t) \\ 2.5\cos(5t) \end{bmatrix} \Rightarrow \mathbf{w} = \begin{bmatrix} 2\sin(4t) \\ 4\cos(4t) \\ 0.5\sin(20t) \\ 2.5\cos(20t) \end{bmatrix}.$$

Figure 5 is the tracking error and Fig.6 shows the adaptively adjusted parameters in the controller.



output input

Fig. 4. Simulation results with the proposed controller

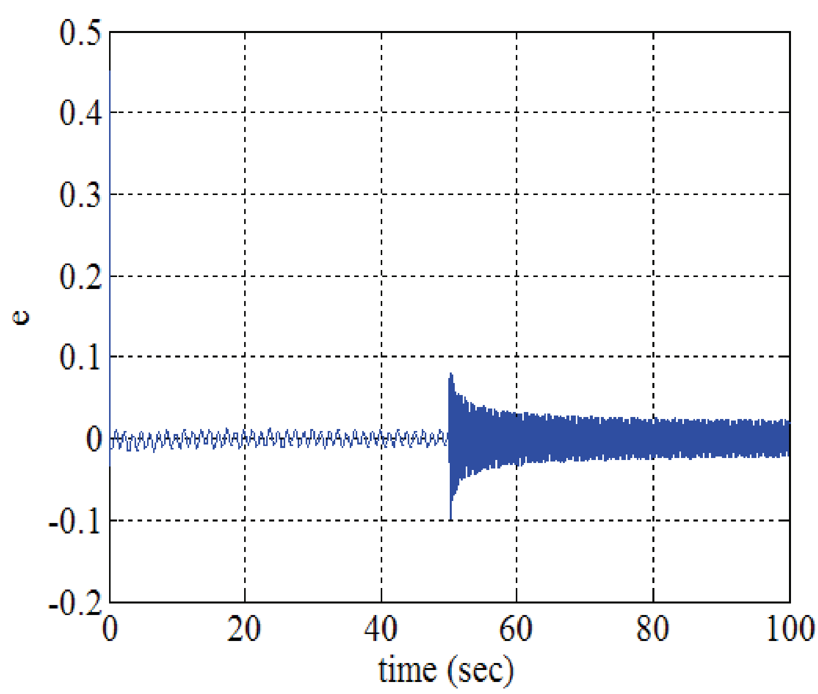


Fig. 5. Tracking error with the proposed controller

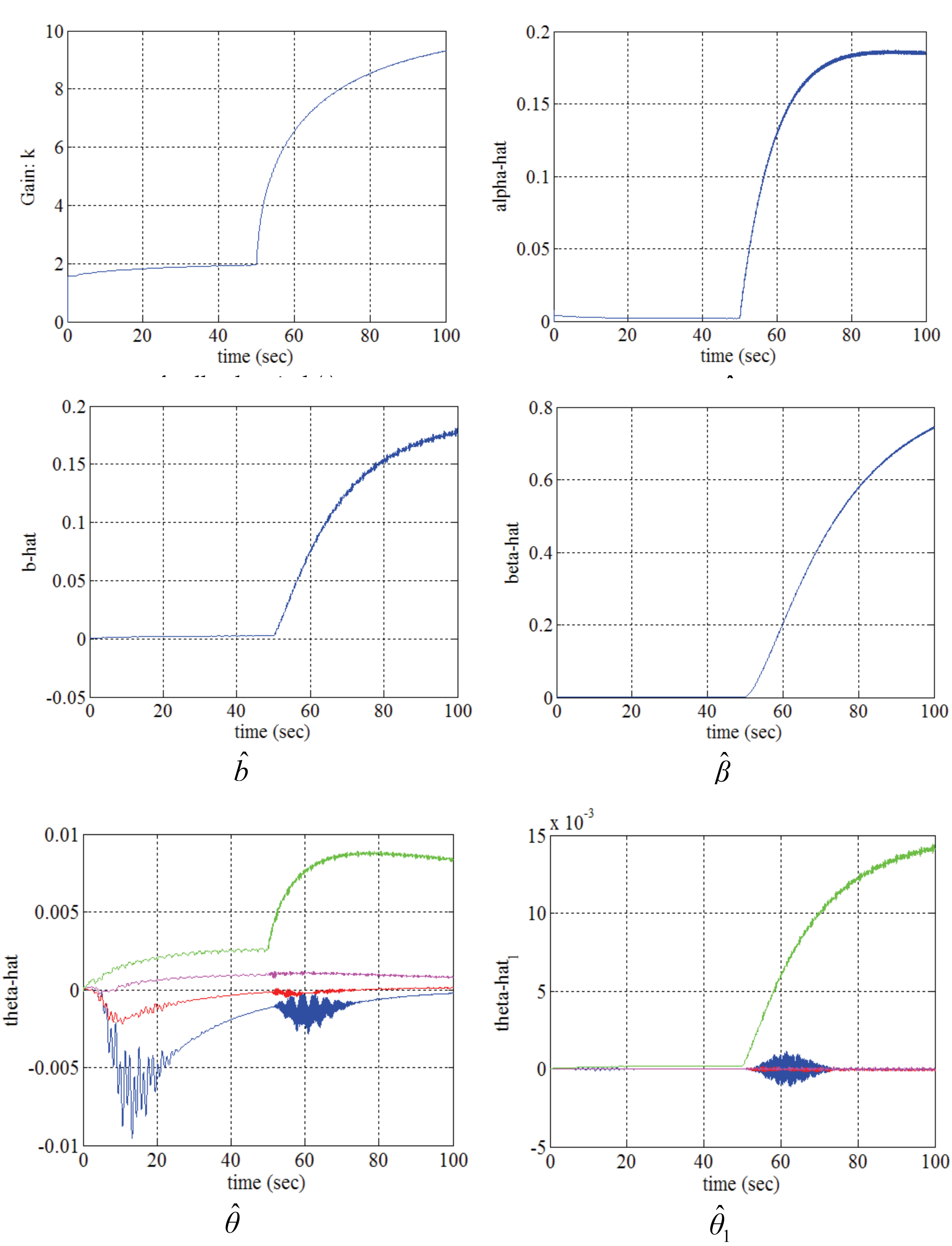
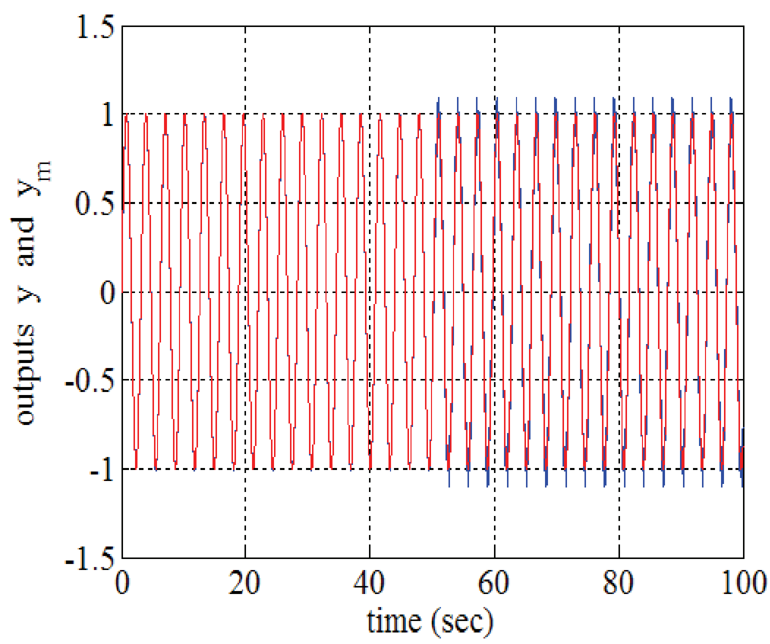


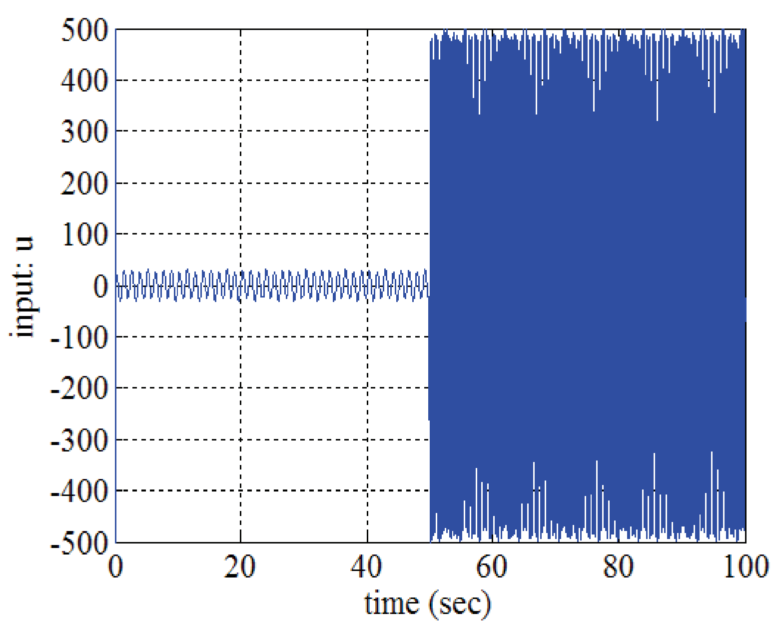
Fig. 6. Adaptively adjusted parameters

A very good control result was obtained and we can see that a good control performance is maintained even as the frequencies of the disturbances were changed at 50 [sec].

Figures 7 and 8 show the simulation results in which the adaptively adjusted parameters in the controller were kept constant after 40 [sec]. After the disturbances were changed, the control performance deteriorated.



output



input

Fig. 7. Simulation results without adaptation after 40 [sec].

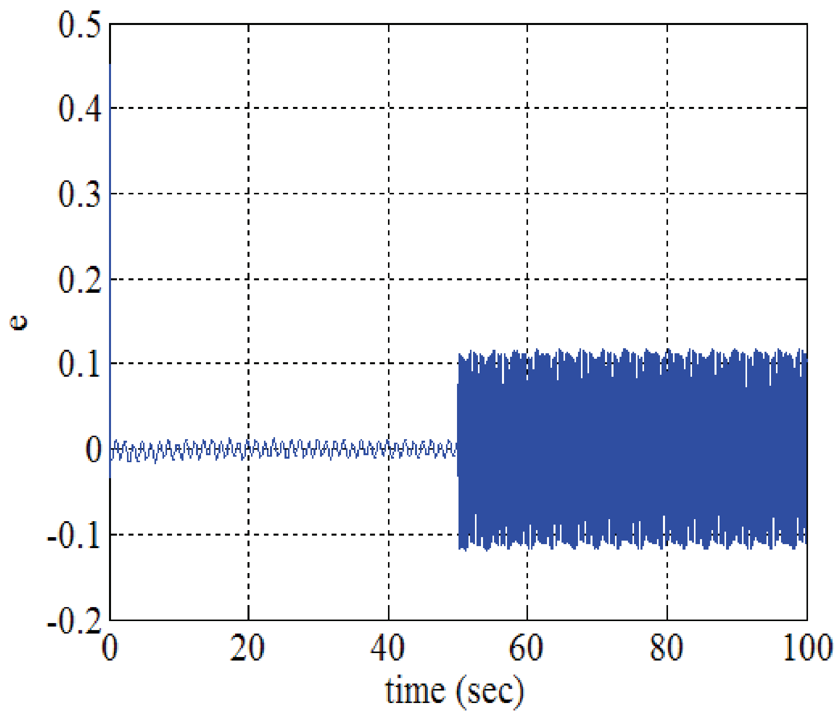


Fig. 8. Tracking error without adaptation

6. Conclusions

In this paper, the adaptive regulation problem for unknown controlled systems with unknown exosystems was considered. An adaptive output feedback controller with an adaptive internal model was proposed for single input/single output linear minimum phase systems. In the proposed method, a controller with an adaptive internal model was designed through an expanded backstepping strategy of only one step with a parallel feedforward compensator (PFC).

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Adaptive control has been a remarkable field for industrial and academic research since 1950s. Since more and more adaptive algorithms are applied in various control applications, it is becoming very important for practical implementation. As it can be confirmed from the increasing number of conferences and journals on adaptive control topics, it is certain that the adaptive control is a significant guidance for technology development.The authors the chapters in this book are professionals in their areas and their recent research results are presented in this book which will also provide new ideas for improved performance of various control application problems.

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