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# Adaptive Control Design for Uncertain and Constrained Vehicle Yaw Dynamics

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## 1. Introduction

Nonlinear models describing vehicle yaw dynamics are considered in inevitably simplified forms using certain assumptions to serve relevant control design purposes. The corresponding modeling errors, however, might have adverse effects on the lateral performance of ground vehicles operating under conditions where these simplifying assumptions are no longer valid. The variations in operating conditions are seldom trivial to monitor and likely to result in significant compromises in the overall performance of the vehicle if the uncertain model parameters are not properly taken into account during the control design phase. In particular, vehicle yaw dynamics might demonstrate unexpected behavior in the presence of unusual external conditions, different side friction coefficients, and steering steps necessary to avoid obstacles. (Canale et al., 2007) Mastering vehicle yaw motions becomes a challenging task while driving on icy road or running on a flat tire. (Ackermann, 1994) Yaw dynamics control problem is additionally complicated in the presence of control input saturation constraints which are in most cases physically inherent acting to limit the magnitude or the rate of change of the effective control signal. In this work we consider a simplified model for vehicle yaw dynamics with steering angle constraints. A nominal control design is developed for the yaw rate tracking performance of the vehicle in Section 2. In order to account for potential uncertainties in the lateral dynamics an adaptive control design is proposed and presented in detail in Section 3. The performance of our yaw rate control strategy is examined through simulations where the road adhesion factor, the vehicle velocity and the vehicle mass are unknown. Our simulation results for several scenarios are demonstrated in Section 4. Finally, our conclusions appear in Section 5.

## 2. Vehicle Dynamics and Nominal Control Design

We consider linear vehicle yaw dynamics and impose magnitude saturation nonlinearities on the steering angle which is introduced as the control input. One can also handle vehicle yaw dynamics with control inputs subject to rate constraints using an extension of our design if a rate-limited actuator is modeled as a first-order lag and a symmetric rate-limiting nonlinearity. (Kahveci & Ioannou, 2008) We investigate several variations in the environmental conditions and unknown changes in the vehicle mass and velocity as

parametric uncertainties which can be shown to be efficiently addressed by our adaptive control design approach. We begin our design using the simplified vehicle dynamics:

$$\dot{x} = Ax + B \text{sat}(\delta_f) \quad (1)$$

$$z = C_p x \quad (2)$$

where  $x = [\beta \ r]^T$  is the measurable state vector,  $\beta$  is the side-slip angle,  $r$  is the yaw rate,  $z = r$  is the performance output,  $\delta_f$  is the steering angle on which magnitude constraints are imposed through a scalar input saturation function defined as:

$$\text{sat}(\delta_f) = \text{sign}(\delta_f) \min(|\delta_f|, \bar{\delta}_f), \quad \bar{\delta}_f \in R, \quad \bar{\delta}_f > 0 \quad (3)$$

with  $\bar{\delta}_f$  and  $-\bar{\delta}_f$  representing the upper and lower saturation limits respectively. We consider the following system matrices and the performance output matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad C_p = [0 \ 1] \quad (4)$$

and define the system parameters accordingly as discussed in (Ackermann & Sienel, 1993; Ackermann et al., 1995; Mammar, 1996) :

$$a_{11} = -(c_r + c_f) / (\tilde{m}_v v) \quad (5)$$

$$a_{12} = -1 + (c_r l_r - c_f l_f) / (\tilde{m}_v v^2) \quad (6)$$

$$a_{21} = (c_r l_r - c_f l_f) / \tilde{J} \quad (7)$$

$$a_{22} = -(c_r l_r^2 + c_f l_f^2) / (\tilde{J} v) \quad (8)$$

$$b_1 = c_f / (\tilde{m}_v v) \quad (9)$$

$$b_2 = c_f l_f / \tilde{J} \quad (10)$$

where  $c_r$  and  $c_f$  are the rear and front cornering stiffness coefficients,  $v$  is the magnitude of the velocity vector,  $l_f$  and  $l_r$  are the distances between the center of gravity and the front and rear axles respectively. Using the distances,  $l_f$  and  $l_r$ , and the total vehicle mass,  $m_v$ , we formulate the vehicle's moment of inertia,  $J$  as:

$$J = m_v l_r l_f \quad (11)$$

One can also normalize the moment of inertia of the vehicle into:

$$\tilde{J} = J / \mu \quad (12)$$

and the normalized mass of the vehicle can be represented by  $\tilde{m}_v$ :

$$\tilde{m}_v = m_v / \mu \quad (13)$$

where  $\mu$  is the common road adhesion factor equal to 1 for dry and 0.5 for wet road. We use the data for the city bus O 305 which is provided in (Ackermann et al., 1995) with  $l_f = 3.67$  m,  $l_r = 1.93$  m,  $c_f = 198000$  N/rad,  $c_r = 470000$  N/rad,  $J = 10.85 m_v$  kgm<sup>2</sup>. The steering angle limits are  $\pm \pi / 8$  rad. The uncertainties in the yaw dynamics are mainly due to:

$$v \in [1, 20] \text{ m/s} \quad (14)$$

$$m_v \in [9950, 16000] \text{ kg} \quad (15)$$

$$\mu \in [0.5, 1] \quad (16)$$

which represent the ranges for the vehicle velocity, the vehicle mass, and the road adhesion factor. The tools of stability analysis have been recently used to investigate the control design with anti-windup augmentation in the adaptive context, and upon combining the control structure with an adaptive law, the closed-loop system stability has been established. (Kahveci & Ioannou, 2007) The design has been employed in aircraft control applications with unknown parameters. (Kahveci et al., 2008) We follow the corresponding control design method and evaluate compatible states,  $x_r$  for desired yaw rate. The state tracking error is hence defined as:

$$e = x - x_r \quad (17)$$

and can be regulated by first augmenting the state vector in the form:

$$x_{aug} = [\dot{e}^T \quad C_p e]^T \quad (18)$$

Using controllability and observability assumptions we consider the following Algebraic Riccati Equation (ARE):

$$A_{aug}^T P + P A_{aug} + Q_z - P B_{aug} R_z^{-1} B_{aug}^T P = 0 \quad (19)$$

$$A_{aug} = \begin{bmatrix} A & 0 \\ C_p & 0 \end{bmatrix}, \quad B_{aug} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad (20)$$

$$Q_z = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} \times Q, \quad Q > 0, \quad R_z > 0 \quad (21)$$

The solution of the above ARE can then be used to obtain a PI controller as:

$$u = -K_1 e - K_2 \int_0^t e(\tau) d\tau \quad (22)$$

$$[K_1 \quad K_2] = R_z^{-1} B_{aug}^T P \quad (23)$$

Using  $A_c = 0$ ,  $B_c = I$ ,  $C_c = K_2$ ,  $D_c = K_1$  we represent the controller in state space form:

$$\dot{x}_c = A_c x_c + B_c (r - x) \quad (24)$$

$$u = C_c x_c + D_c (r - x) \quad (25)$$

Given  $N(s)M^{-1}(s)$  as a full-order right coprime factorization of  $G(s) = (sI - A)^{-1}B$ , the anti-windup compensator can be described by its transfer function matrix:

$$K_{aw}(s) = \begin{bmatrix} M(s) - I \\ N(s) \end{bmatrix} \quad (26)$$

whereas it can also be represented in its state space form through the respective system matrices,  $(A_{aw}, B_{aw}, C_{aw}, D_{aw})$  which can be defined as:

$$A_{aw} = A + BLQ^{-1} \quad (27)$$

$$B_{aw} = B \quad (28)$$

$$C_{aw} = [(LQ^{-1})^T \quad I]^T \quad (29)$$

$$D_{aw} = 0 \quad (30)$$

One needs to generate the term,  $LQ^{-1}$  to implement the two anti-windup compensation matrices,  $A_{aw}$  and  $C_{aw}$  such that  $A + BLQ^{-1}$  is Hurwitz. This term can possibly be evaluated by solving the following set of Linear Matrix Inequalities (LMIs):

$$\begin{bmatrix} QA^T + AQ + L^T B^T + BL & BU - L^T & 0 & Q & L^T \\ U^T B^T - L & -2U & I & 0 & U \\ 0 & I & -\mu I & 0 & -I \\ Q^T & 0 & 0 & -W_p^{-1} & 0 \\ L & U^T & -I & 0 & -W_r^{-1} \end{bmatrix} < 0 \quad (31)$$

$$Q > 0, \quad U > 0, \quad \mu > 0 \quad (32)$$

The selection of weighting matrices,  $W_p > 0$  and  $W_r > 0$  is discussed in (Turner et al., 2004) for system performance and robustness. As a result, the anti-windup is augmented as:

$$\dot{x}_{aw} = A_{aw} x_{aw} + B_{aw} (\delta_f - \text{sat}(\delta_f)) \quad (33)$$

$$y_{aw} = C_{aw} x_{aw} + D_{aw} (\delta_f - \text{sat}(\delta_f)) \quad (34)$$

and the term,  $y_{aw} = [y_{aw1} \quad y_{aw2}]^T$ ,  $y_{aw2} \in R^2$ ,  $y_{aw1} \in R$  modifies the controller into:

$$\dot{x}_{cm} = A_c x_{cm} + B_c (x_r - x) - B_c y_{aw2} \quad (35)$$

$$\delta_f = C_c x_{cm} + D_c (x_r - x) - D_c y_{aw2} - y_{aw1} \quad (36)$$

### 3. Adaptive Control Design

In order to avoid high frequency sensor noise amplification by the derivative term we employ a prefilter,  $1/(s + \lambda)$ ,  $\lambda > 0$ , and for any set of fixed plant parameters we obtain:

$$\frac{s}{s + \lambda} = A \frac{1}{s + \lambda} x + B \frac{1}{s + \lambda} \text{sat}(\delta_f) \quad (37)$$

At any particular time instant,  $t$  we estimate the vectors,  $\theta_1^*(t)$  and  $\theta_2^*(t)$  which are defined as:

$$\theta_1^*(t) = [a_{11} \quad a_{12} \quad b_1]^T, \quad \theta_2^*(t) = [a_{21} \quad a_{22} \quad b_2]^T \quad (38)$$

and denote these estimates by  $\theta_1(t)$  and  $\theta_2(t)$ . The estimation model consists of:

$$\hat{z}_1 = \theta_1^T \phi, \quad \hat{z}_2 = \theta_2^T \phi \quad (39)$$

$$\phi = \frac{1}{s + \lambda} [\beta \quad r \quad \text{sat}(\delta_f)]^T \quad (40)$$

As a next step we construct the normalized estimation errors,  $\varepsilon_1$  and  $\varepsilon_2$  :

$$\varepsilon_1 = (z_1 - \hat{z}_1) / m^2 \quad (41)$$

$$\varepsilon_2 = (z_2 - \hat{z}_2) / m^2 \quad (42)$$

$$m^2 = 1 + \phi^T \phi \quad (43)$$

We use the discrete version of the Least-Squares Algorithm (LSA) given in (Ioannou & Sun, 1996) along with a relevant orthogonal term-by-term projection as described in (Kahveci et al., 2008). Based on the Certainty Equivalence Principle we can implement the control input as:

$$\delta_f = -\hat{K}_2 \frac{1}{s} (e + y_{aw2}) - \hat{K}_1 (e + y_{aw2}) - y_{aw1} \quad (44)$$

where  $[\hat{K}_1 \quad \hat{K}_2]$  is evaluated through the ARE and  $\hat{L}\hat{Q}^{-1}$  is calculated through the LMI solution using the parameter estimates. The modification terms can be explicitly written as:

$$y_{aw1} = \hat{L}\hat{Q}^{-1} (sI - \hat{A} - \hat{B}\hat{L}\hat{Q}^{-1})^{-1} \hat{B}(\delta_f - \text{sat}(\delta_f)) \quad (45)$$

$$y_{aw2} = (sI - \hat{A} - \hat{B}\hat{L}\hat{Q}^{-1})^{-1} \hat{B}(\delta_f - \text{sat}(\delta_f)) \quad (46)$$

The overall adaptive control scheme is summarized in Figure 1.

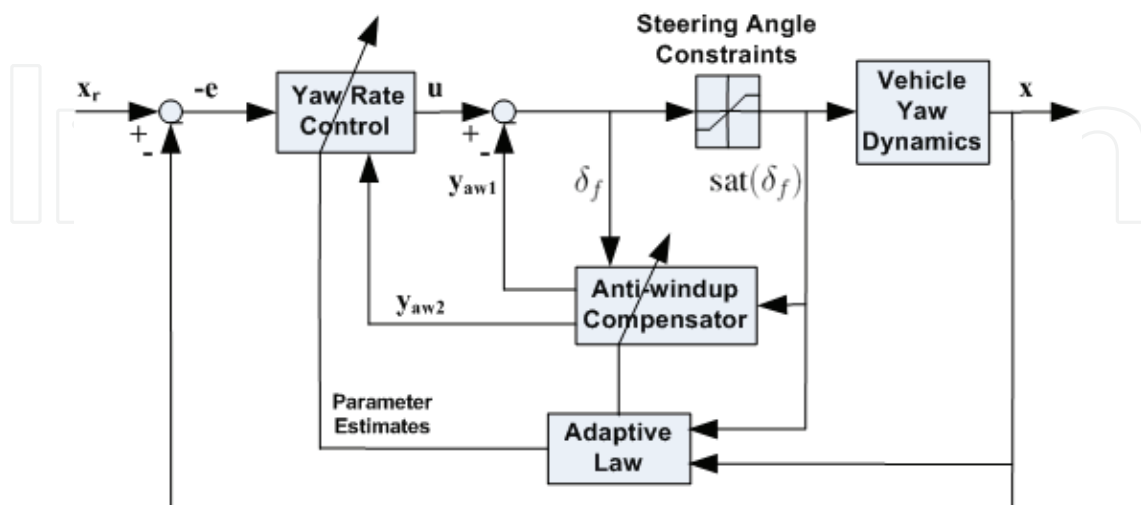


Figure 1. Adaptive control design for constrained vehicle yaw dynamics

#### 4. Simulations

The first set of simulations is conducted using the adaptive control design with no anti-windup compensation. The approximate ranges for the road adhesion factor, the vehicle mass, and the vehicle velocity are provided although the specific values of these parameters are unknown. The commanded input signal is demonstrated in Figure 2. The reference signal for the system response and the observed vehicle yaw rate are shown in Figure 3. The adaptive control design with anti-windup compensation is implemented in the second set of simulations where the target for the yaw rate of the vehicle is the same as before. The steering angle, the system response, and the anti-windup modification terms are presented in Figure 4, Figure 5, and Figure 6. When the adaptive anti-windup compensator design is included in the overall system, the overshoots in the system response are observed to be eliminated despite the parametric uncertainties in vehicle dynamics and unknown variations in external driving conditions.

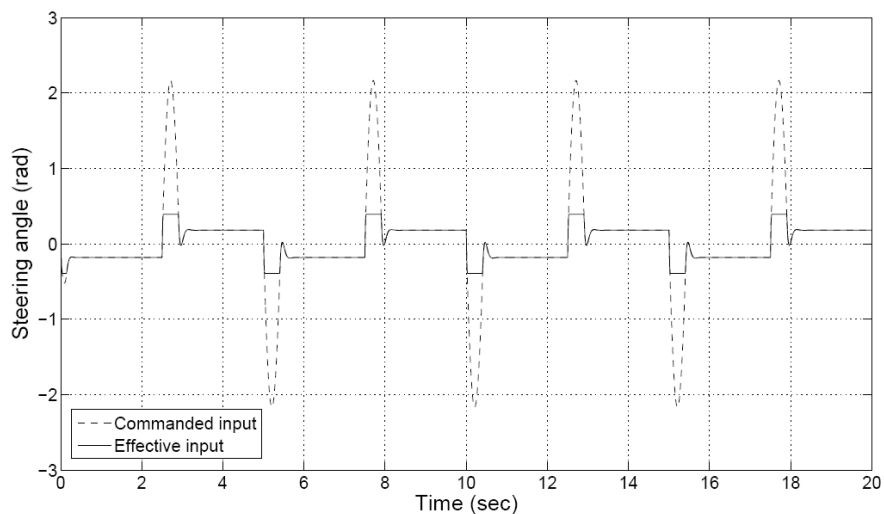


Figure 2. The commanded steering angle and the effective control signal within limits

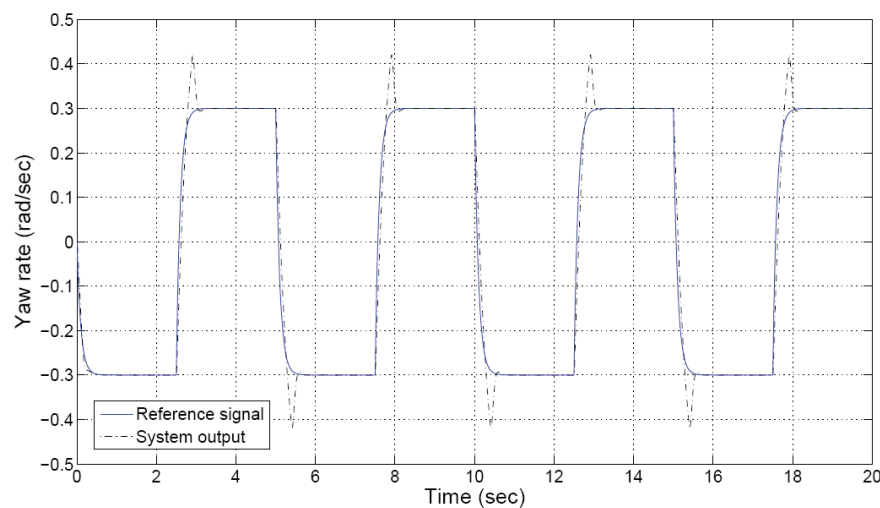


Figure 3. The system response compared with the desired yaw rate



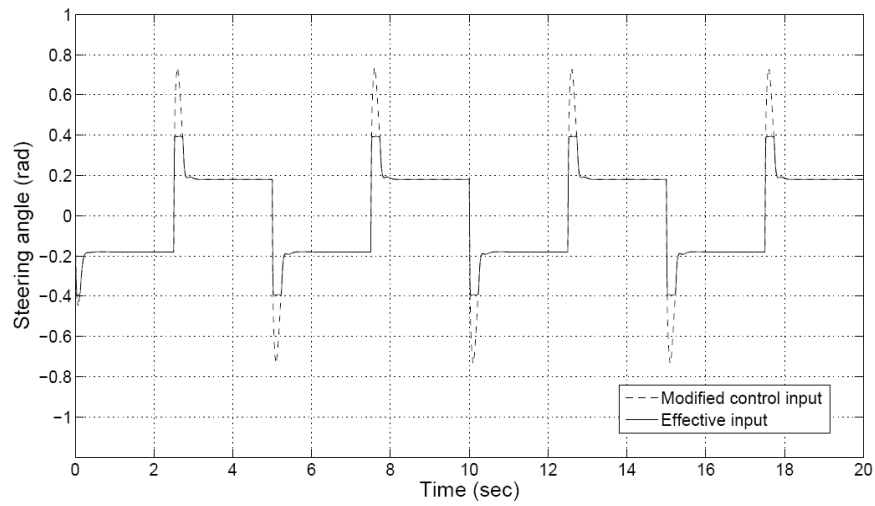


Figure 4. The modified control input subject to saturation and the effective control signal

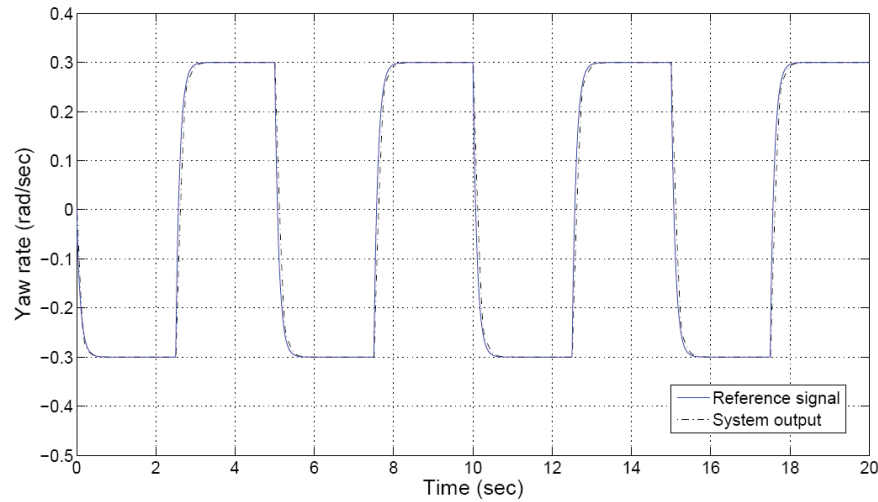


Figure 5. The system response tracking the desired reference signal

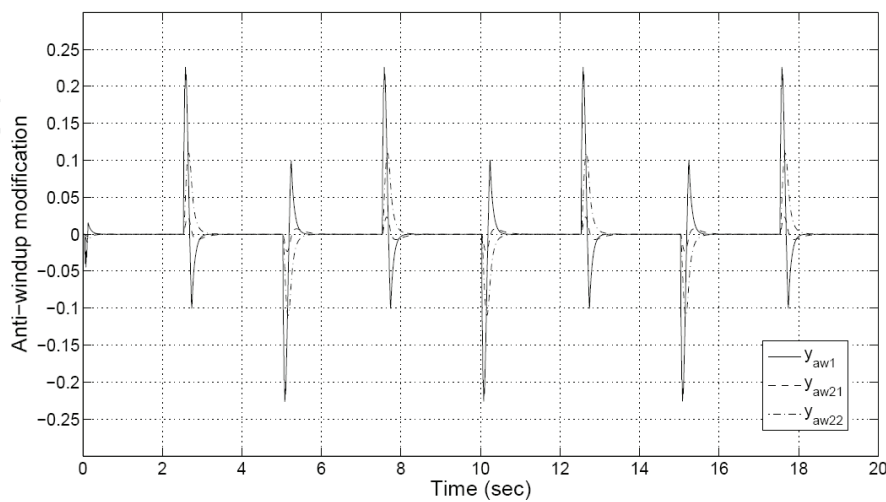


Figure 6. The adaptive anti-windup modification terms

Interested reader might also refer to (Kahveci, 2008) for extensive simulations with an adaptive steering controller under various road conditions, crosswind effects acting on the vehicle dynamics as lateral disturbances, and actuator failure scenarios in addition to more restrictive magnitude saturation constraints imposed on the steering angle of the vehicle.

## 5. Conclusion

We consider yaw dynamics of a vehicle operating under uncertain road conditions with unknown road adhesion factor, vehicle velocity and mass. We develop an adaptive control design technique motivated by the demand for a system capable of adjusting to deviations in vehicle parameters with almost negligible performance compromises despite the changes in environmental conditions. Our simulations with the adaptive control scheme display significant enhancements in the performance of the vehicle in case the steering angle experiences saturation. The adaptive control design methodology can hence be used to address modeling uncertainties in vehicle yaw dynamics with steering angle constraints.

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