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Lipschitzian Parameterization-Based Approach for Adaptive Controls of Nonlinear Dynamic Systems with Nonlinearly Parameterized Uncertainties: A Theoretical Framework and Its Applications

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1. Introduction

The original and popular adaptive control theory usually deals with linear parameterizations (LP) of uncertainties, that is, it is assumed that uncertain quantities in dynamic systems are expressed linearly with respect to unknown parameters. Actually, most developed approaches such as gradient-based ones or recursive least squares [1, 2] rely heavily on this assumption and effective techniques have been proposed in this context [2].
However, LP is impossible in practical applications whose dynamic parameters are highly coupled with system states. Stribeck effect of frictional forces at joints of the manipulators [3] or nonlinear dynamics of space-robot in inertia space are typical examples [4].
Unfortunately, there were very few results in the literature addressing the adaptive control problem for NP in a general and direct manner. Recently, adaptation schemes for NP have been proposed [5, 6] with the assumption on the convexity/concavity and smoothness of the nonlinear functions in unknown parameters. In this approach, the controllers search a known compact set bounding the unknown parameters (i.e. the unknown parameter must belong to a prescribed closed and bounded set) for min-max parameter estimation. Also, the resulting controllers posse a complex structure and need delicate switching due to change of adaptation mechanism up to the convexity/concavity of the nonlinear functions. Such tasks may be hard to be implemented in a real-time manner.
In this chapter, we propose novel adaptive control technique, which is applicable to any NP systems under Lipschitzian structure. Such structure is exploited to design linear-in-parameter upper bounds for the nonlinear functions. This idea enables the design of adaptive controllers, which can compensate effectively for NP uncertainties in the sense that it can guarantee global boundedness of the closed-loop system signals and tracking error within any prescribed accuracies. The structures of the resulting controllers are simple since they are designed based on the nonlinear functions' upper bound, which depends only on

the system variables. Therefore, an important feature of the proposed technique is that the compactness of uncertain parametric sets is not required. Another interesting feature of the technique is that regardless of parametric dimension, even 1-dimension estimator-based control is available. This is an important feature from practical implementation viewpoint. This result is of course new even for traditional LP systems. As a result, the designed adaptive controls can gain a great amount of computation reduced. Also, a very broad class of nonlinearly parameterized adaptive control problems such as Lipschitzian parameterization (including convex/concave, smooth parameterizations as a particular case), multiplicative parameterization, fractional parameterization or their combinations can be solved by the proposed framework.

The chapter is organized as follows. In Section 2, we formulate the control problem of nonlinear dynamic system with NP uncertainties. Adaptive control is designed for uncertainties, which satisfy Lipschitz condition (Lipschitzian parameterization). Our formulating Lipschitzian parameterization plays a central role to convert the NP adaptive control problem to a handleable form. Adaptation laws are designed for both nonnegative unknown parameters and unknown parameters with unknown sign. With the ability to design 1-dimension-observer for unknown parameter, we also redesign the traditional adaptive control of LP uncertain plants. Next is a design of adaptive control for a difficult but popular form of uncertainties, the multiplicative parameterizations. Examples of a control design of the proposed approach is illustrated at the end of the section. Section 3 remarks our results extended to the adaptive controls in systems with indirect control inputs. In this section, we describe the control problems of the backstepping design method to control complex dynamic structures whose their control input can not directly compensate for the effect of unknown parameters (un-matching system). Section 4 is devoted to the incorporation of proposed techniques to a practical application: adaptive controller design applied to path tracking of robot manipulators in the presence of NP. A general framework of adaptive control for NP in the system is developed first. Then, adaptive control for friction compensation in tracking problem of a 2DOF planar robot is introduced together with comparative simulations and experiments. Conclusions and discussions are given in Section 5.

2. Lipschitzian parameterization-based techniques for Adaptive Control of NP

2.1 Problem formulation

We consider adaptive systems admitting a nonlinear parameterization in the form

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}(\mathbf{f}(\mathbf{x}, \boldsymbol{\theta}) + \mathbf{u}), \tag{1}$$

where $\mathbf{u}$ is the control input, $\mathbf{e}$ is the state vector, $\mathbf{x}$ is the system variable, $\boldsymbol{\theta}$ is an unknown time-invariant parameter. Both the state $\mathbf{e}$ and variable $\mathbf{x}$ are available for online measurement. The function $\mathbf{f}(\mathbf{x}, \boldsymbol{\theta})$ is nonlinear in both the system variable $\mathbf{x}$ and unknown parameter $\boldsymbol{\theta}$. The problem is to design a control signal $\mathbf{u}(\mathbf{t})$ enforcing asymptotic convergence of the state, that is, $\mathbf{e}(\mathbf{t}) \to 0$ as $t \to \infty$.

Note that any general adaptive control problem where the state $\mathbf{x_m}$ of uncertain plants (satisfying the model matching condition) is required to track the state of a reference model $\mathbf{x_p}$ can actually be reduced to the above described problem.

For the simplicity of description and without loss of generality, the following standard assumption is used throughout the chapter.

Assumption 1. $e \in R, u \in R$ *(i.e. the state and control are scalars) and* $A = -I$, *whereas* $x(t) \in R$ *is bounded.*

From this assumption, it is clear that we can set $B = 1$ without loss of generality, so from now on, we are considering the system

$$\dot{e} = -e + f(x, \boldsymbol{\theta}) + u. \tag{2}$$

Note that from [5, Lemma 3], it is known that indeed the vector case of the state can be easily transformed to the scalar case. Clearly, under the model matching condition, the methodology for scalar control can be easily and naturally extended to multidimensional controls.

Let us also recall that a function $f : R_+^p \to R$ *is increasing (decreasing,* resp.) if and only if $f(\boldsymbol{\theta}) \le f(\bar{\boldsymbol{\theta}})$ $(f(\boldsymbol{\theta}) \ge f(\bar{\boldsymbol{\theta}})$, resp.) whenever $\boldsymbol{\theta} \le \bar{\boldsymbol{\theta}}$ (i.e. $\theta_j \le \theta_j, j = 1, 2, ..., p$).

We shall use the absolute value of a vector, which is defined as

$$|\mathbf{w}| = [\,|w_1| \quad |w_2| \quad ... \quad |w_p|\,]^T, \quad \forall\, \mathbf{w} \in \mathbf{R^P}.$$

2.2 Lipschitzian parameterization

We consider the case where $f(x, \boldsymbol{\theta})$ in (1) is Lipschitzian in $\boldsymbol{\theta}$. It suffices to say that any convex or concave or smooth function is Lipschitzian in their effective domain [7] . As we discuss later on, the Lipschitzian parameterization-based method allows us to solve the adaptive control problems in a very efficient and direct manner. The Lipschitz condition is recalled first.

Assumption 2. *The function* $f(x, .)$ *is Lipschitzian in* $\boldsymbol{\theta}$, *i.e. there are continuous functions* $0 \le L_j(x) < +\infty$, *j = 1, 2, ...,p such that*

$$|f(x, \bar{\boldsymbol{\theta}}) - f(x, \boldsymbol{\theta})| \le \sum_{j=1}^{p} L_j(x) |\bar{\theta}_j - \theta_j| \tag{3}$$

Note that in literature, the Lipschitz condition is often described by

$$|f(x, \bar{\boldsymbol{\theta}}) - f(x, \boldsymbol{\theta})| \le \mathbf{L}(x) ||\bar{\boldsymbol{\theta}} - \boldsymbol{\theta}||$$

which can be shown to be equivalent to (3). In what follows, we shall set

$$\mathbf{L}(x) = [\,L_1(x) \quad L_2(x) \quad ... \quad L_p(x)\,]. \tag{4}$$

2.3 Adaptation techniques for unknown parameters

To make our theory easier to follows, let's first assume that

$$\boldsymbol{\theta} \in R_+^p, \text{ i.e. } \theta_j \ge 0, \; j = 1, 2, ..., p. \tag{5}$$

The following lemma plays a key role in the subsequent developments.

Lemma 1 *The function* $f(x, \boldsymbol{\theta}) - \mathbf{L}(x)\boldsymbol{\theta}$ *is decreasing in* $\boldsymbol{\theta}$ *whereas the function* $f(x, \boldsymbol{\theta}) + \mathbf{L}(x)\boldsymbol{\theta}$ *is increasing in* $\boldsymbol{\theta}$.

Proof. By (3), for every $\boldsymbol{\theta} \ge \bar{\boldsymbol{\theta}}$,

$$\max\{f(x,\boldsymbol{\theta}) - f(x,\bar{\boldsymbol{\theta}}), f(x,\bar{\boldsymbol{\theta}}) - f(x,\boldsymbol{\theta})\} \leq \mathbf{L}(x)(\boldsymbol{\theta} - \bar{\boldsymbol{\theta}})$$

which implies

$$\begin{aligned} f(x,\boldsymbol{\theta}) - \mathbf{L}(x)\boldsymbol{\theta} &\leq f(x,\bar{\boldsymbol{\theta}}) - \mathbf{L}(x)\bar{\boldsymbol{\theta}}, \\ f(x,\boldsymbol{\theta}) + \mathbf{L}(x)\boldsymbol{\theta} &\geq f(x,\bar{\boldsymbol{\theta}}) + \mathbf{L}(x)\bar{\boldsymbol{\theta}}, \end{aligned}$$

i.e. function $f(x,\boldsymbol{\theta}) - \mathbf{L}(x)\boldsymbol{\theta}$ is decreasing in $\boldsymbol{\theta}$, while $f(x,\boldsymbol{\theta}) + \mathbf{L}(x)\boldsymbol{\theta}$ in increasing in $\boldsymbol{\theta}$. Now, take the following Lyapunov function for studying the stabilization of system (2)

$$V(e,\hat{\boldsymbol{\theta}}) = \frac{1}{2}[e^2 + ||\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}||^2], \tag{6}$$

where $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}(t)$ is an "observer" of $\boldsymbol{\theta}$ to be designed with the controller u. Then

$$\begin{aligned} \dot{V} &= -e(t)^2 + e(t)[f(x(t),\boldsymbol{\theta}) + u] - \dot{\hat{\boldsymbol{\theta}}}^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \\ &= -e(t)^2 + e(t)[f(x(t),\boldsymbol{\theta}) - \text{sgn}(e(t))\mathbf{L}(x(t))\boldsymbol{\theta}] \\ &\quad + e(t)[\text{sgn}(e(t))\mathbf{L}(x(t))\boldsymbol{\theta} + u] - \dot{\hat{\boldsymbol{\theta}}}^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}). \end{aligned} \tag{7}$$

As a consequence of lemma 1 , we have
Lemma 2 *The function*

$$e(t)[f(x(t),\boldsymbol{\theta}) - \text{sgn}(e(t))\mathbf{L}(x(t))\boldsymbol{\theta}]$$

is decreasing in $\boldsymbol{\theta}$.
Proof. It suffices to show that function

$$k(e,x,\boldsymbol{\theta}) := e[f(x,\boldsymbol{\theta}) - \text{sgn}(e)\mathbf{L}(x)\boldsymbol{\theta}]$$

is decreasing in $\boldsymbol{\theta}$.
When $e > 0$,

$$k(e,x,\boldsymbol{\theta}) = e[f(x,\boldsymbol{\theta}) - \mathbf{L}(x)\boldsymbol{\theta}]$$

and thus $k(e,x,\boldsymbol{\theta})$ is decreasing because $f(x,\boldsymbol{\theta}) - \mathbf{L}(x)\boldsymbol{\theta}$ is decreasing (by lemma 1) and $e > 0$. On the other hand, when $e < 0$,

$$k(e,x,\boldsymbol{\theta}) = e[f(x,\boldsymbol{\theta}) + \mathbf{L}(x)\boldsymbol{\theta}]$$

and again $k(e,x,\boldsymbol{\theta})$ is decreasing because $f(x,\boldsymbol{\theta}) + \mathbf{L}(x)\boldsymbol{\theta}$ is increasing (by lemma 1) and e < 0. Finally, $k(e,x,\boldsymbol{\theta})$ is obviously decreasing (constant) when $e = 0$, completing the proof of lemma 2.
From (7) and lemma 2, we have

$$\dot{V} \leq -e(t)^2 + e(t)[f(x(t),\mathbf{0}) + \text{sgn}(e(t))\mathbf{L}(x(t))\boldsymbol{\theta} + u] - \dot{\hat{\boldsymbol{\theta}}}^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}). \tag{8}$$

Therefore, we design the following controller u

$$u \;=\; -f(x,\mathbf{0}) - \text{sgn}(e)\mathbf{L}(x)\hat{\boldsymbol{\theta}} \tag{9}$$

in tandem with the adaptive rule

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}} &= e\text{sgn}(e)\mathbf{L}(x)^T \\ \Leftrightarrow \dot{\hat{\theta}}_j &= |e|L_j(x),\ j = 1, 2, ..., p, \end{aligned} \tag{10}$$

which together lead to

$$\dot{V} \leq -e(t)^2\,. \tag{11}$$

The last inequality implies that V is decreasing as a function of time, and thus is bounded by $y(0)$. Therefore, by definition (6), $e(t)$ and $\hat{\boldsymbol{\theta}}(t)$ must be bounded from which we infer the boundness of $\dot{e}(t)$ as well. Also, (11) also gives $\int_0^T e(t)^2 dt \leq V(0),\ \forall\ T > 0$, i.e. $e(.)$ is in L_2. Therefore, by a consequence of Barbalat's lemma [1, p. 205],

$$\lim_{t \to +\infty} e(t) = 0.$$

Finally, let us mention that equation (10) guarantees $\hat{\boldsymbol{\theta}}(t) \in R_+^p,\ \forall t > 0$ provided that $\hat{\boldsymbol{\theta}}(0) \in R_+^p$. The following theorem summarizes the results obtained so far.

Theorem 1 *Under the assumption 2, the control u and observer* $\hat{\boldsymbol{\theta}}$ *defined by (9) and (10) stabilizes system (2).*

The control law determined by (9) and (10) is discontinuous at $e(t) = 0$. According to a suggested technique in [5], we can modify the control (9) and (10) to get a continuous one as follows

$$u = -f(x, \mathbf{0}) - \text{sat}(e/\epsilon)\mathbf{L}(x)\hat{\boldsymbol{\theta}}, \tag{12}$$

$$\dot{\hat{\boldsymbol{\theta}}}(t) = |e_\epsilon(t)|\mathbf{L}(x(t))^T, \tag{13}$$

where $\epsilon > 0$ and

$$\text{sat}(e/\epsilon) = \begin{cases} e/\epsilon & \text{when} \quad -\epsilon \leq e \leq \epsilon \\ 1 & \text{when} \quad e > \epsilon \\ -1 & \text{when} \quad e < -\epsilon \end{cases} \tag{14}$$

$$e_\epsilon = e - \epsilon\text{sat}(e/\epsilon). \tag{15}$$

Note that whenever $|e| > \epsilon$,

$$e_\epsilon^2 \leq e_\epsilon e \ \& \quad \text{sat}(e/\epsilon) = \text{sgn}(e_\epsilon).$$

Then, instead of Lyapunov function (8), take the function

$$V(e, \hat{\boldsymbol{\theta}}) = \frac{1}{2}[e_\epsilon^2 + ||\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}||^2]$$

and thus

$$\dot{V} = 0 \quad \text{when } |e(t)| \leq \epsilon \tag{17}$$

$$\begin{aligned}\dot{V} &= -e_\epsilon e + e_\epsilon[f(x,\boldsymbol{\theta}) - f(x,\mathbf{0}) - \mathrm{sat}(e/\epsilon)\mathbf{L}(x)\hat{\boldsymbol{\theta}}] - \dot{\hat{\boldsymbol{\theta}}}^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \\ &\le -e_\epsilon^2 + e_\epsilon[f(x,\boldsymbol{\theta}) - f(x,\mathbf{0}) - \mathrm{sgn}(e_\epsilon)\mathbf{L}(x)\hat{\boldsymbol{\theta}}] - \dot{\hat{\boldsymbol{\theta}}}^T(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}) \\ &\le -e_\epsilon^2 \quad \text{when } |e(t)| > \epsilon. \end{aligned} \tag{18}$$

The last inequality follows from the fact that function

$$e_\epsilon[f(x,\boldsymbol{\theta}) - \mathrm{sgn}(e_\epsilon)\mathbf{L}(x)\boldsymbol{\theta}]$$

is decreasing in $\boldsymbol{\theta}$.
Therefore, it can be proved that the control (12)-(13) guarantees that $e(t)$ asymptotically tracks 0 within a precision of ϵ.

2.4 1-dimension estimator for unknown parameters

In controls defined by (9)-(10) and (12)-(13), the dimension of the observer $\hat{\boldsymbol{\theta}}$ is the same as that of the unknown parameter $\boldsymbol{\theta}$. We now reveal that we can design a control with new observer $\hat{\boldsymbol{\theta}}$ of even dimension 1 (!) which does not depend on the dimension of the unknown parameter $\boldsymbol{\theta}$. For that, instead of $L(x)$ defined by (4), take

$$L(x) := \max_{j=1,2,\ldots,s} L_j(x) \tag{19}$$

then, by (3), it is obvious that

$$|f(x,\bar{\boldsymbol{\theta}}) - f(x,\boldsymbol{\theta})| \le L(x)\sum_{j=1}^{p}|\bar{\theta}_j - \theta_j| \tag{20}$$

and by an analogous argument as that used in the proof of lemmas 1, 2, it can be shown that

Lemma 3 *The function* $f(x,\boldsymbol{\theta}) - L(x)\sum_{j=1}^{p}\theta_j$ *is decreasing in* $\boldsymbol{\theta}$ *whereas the function* $f(x,\boldsymbol{\theta}) + L(x)\sum_{j=1}^{p}\theta_j$ *is increasing in 9.*

Consequently, the function $e(t)[f(x(t),\boldsymbol{\theta}) - \mathrm{sgn}(e(t))\mathbf{L}(x(t))\sum_{j=1}^{p}\theta_j]$ *is decreasing in* $\boldsymbol{\theta}$.

Based on the result of this lemma, instead of the Lyapunov function defined by (6) and the estimator defined by (10), (13), taking

$$V(e,\hat{\theta}) = \frac{1}{2}[e^2 + (\sum_{j=1}^{p}\theta_j - \hat{\theta})^2], \tag{21}$$

$$\dot{\hat{\theta}}(t) = |e(t)|L(x), \tag{22}$$

$$\dot{\hat{\theta}}(t) = |e_\epsilon(t)|L(x), \tag{23}$$

it can be readily shown that

Theorem 2 *With function $sat(e/\epsilon), e_\epsilon$ defined by (14), (15) and the scalar estimator $\hat{\theta}(t)$ obeys either differential equation (22) or differential equation (23), the control (9) still stabilizes the system (2) whereas the control (12) guarantees that e(t) asymptotically tracks* 0 *within a precision of* ϵ.

2.5 Estimator for unknown parameters with unknown sign

First, every $\boldsymbol{\theta} \in R^p$ can be trivially expressed as

$$\boldsymbol{\theta} = \frac{\boldsymbol{\theta}^{(1)} - \boldsymbol{\theta}^{(2)}}{2}, \ \boldsymbol{\theta}^{(1)} = |\boldsymbol{\theta}| + \boldsymbol{\theta} \in R_+^p, \ \boldsymbol{\theta}^{(2)} = |\boldsymbol{\theta}| - \boldsymbol{\theta} \in R_+^p. \tag{24}$$

For the new function $\tilde{f}$ defined by $\tilde{f}(x, \boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}) : R \times R_+^{2p} \to R$ by

$$\tilde{f}(x, \boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}) = f(x, (\boldsymbol{\theta}^{(1)} - \boldsymbol{\theta}^{(2)})/2). \tag{25}$$

it is immediate to check that the Lipschitz condition (3) implies

$$|\tilde{f}(x, \bar{\boldsymbol{\theta}}^{(1)}, \bar{\boldsymbol{\theta}}^{(2)}) - \tilde{f}(x, \boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)})| \le \frac{1}{2}\sum_{j=1}^{p} L_j(x)[|\bar{\theta}_j^{(1)} - \theta_j^{(1)}| + |\bar{\theta}_j^{(2)} - \theta_j^{(2)}|]. \tag{26}$$

Then according to lemma 2, the function

$$e(t)[\tilde{f}(x(t), \boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}) - \frac{1}{2}\mathrm{sgn}(e(t))\mathbf{L}(x)(\boldsymbol{\theta}^{(1)} + \boldsymbol{\theta}^{(2)})]$$

is decreasing in $(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)})$.

Note that for $\boldsymbol{\theta}^{(1)}$, $\boldsymbol{\theta}^{(2)}$ defined by (24),

$$\frac{1}{2}(\boldsymbol{\theta}^{(1)} + \boldsymbol{\theta}^{(2)}) = |\boldsymbol{\theta}|. \tag{27}$$

Therefore, using the Lyapunov functions defined by

$$V(e, \hat{\boldsymbol{\theta}}) = \frac{1}{2}[e^2 + |||\boldsymbol{\theta}| - \hat{\boldsymbol{\theta}}||^2], \tag{28}$$

$$V(e, \hat{\theta}) = \frac{1}{2}[e^2 + (\sum_{j=1}^{p} |\theta_j| - \hat{\theta}^2], \tag{29}$$

analogously to Theorems 1 and 2, it can be shown that

Theorem 3 *All statements of Theorems 1 and 2 remain valid with the assumption in (5) removed. Namely,*

i. *With the Lyapunov function (28) used for checking the stability and* $\mathbf{L}(\mathbf{x})$ *defined by (4), the control (9), (10) still stabilizes system (2) while the control (12), (13) guarantees that e(t) asymptotically tracks* 0 *with a precision of* ϵ.

ii. *(ii) With the Lyapunov function (29) used for checking stability and* $\boldsymbol{L}(x)$ *defined by (19), the control (9), (22) still stabilizes system (2) while the control (12), (23) still guarantees that e(t) asymptotically tracks* 0 *within a precision of* ϵ.

2.6 New nonlinear control for linearly parameterized uncertain plants

It is clear that when applied to linearly parameterized uncertain plants, Theorem 3 provides a new result on 1-dimension estimator for uncertain parameters as well. Let's describe this application in some details. A typical adaptive control problem for linearly parameterized uncertain plants can be formulated as follows [1, 8]. For uncertain system

$$\dot{\mathbf{X}}_{\mathbf{p}} = \mathbf{A}_{\mathbf{m}}\mathbf{X}_{\mathbf{p}} + \mathbf{B}(\boldsymbol{\alpha}^{\boldsymbol{T}}\mathbf{X}_{\mathbf{p}} + u),\ \mathbf{A}_{\mathbf{m}} \in R^{n\times n},\ \mathbf{B} \in R^n,\ \boldsymbol{\alpha} \in R^n,\ u \in R \quad (30)$$

with unknown parameter $\boldsymbol{\alpha}$, design a control to makes the state $\mathbf{X}_{\mathbf{p}}(t)$ track a reference trajectory $\mathbf{X}_{\mathbf{m}}$ described by the equation

$$\dot{\mathbf{X}}_{\mathbf{m}} = \mathbf{A}_{\mathbf{m}}\mathbf{X}_{\mathbf{m}} + \mathbf{B}r, \quad (31)$$

where $\mathbf{A}_{\mathbf{m}}$ is asymptotically stable with one negative real eigenvalue $-k$.
The problem is thus to design the control u such that the state $\mathbf{E} := \mathbf{X}_{\mathbf{p}} - \mathbf{X}_{\mathbf{m}}$ of the error equation

$$\dot{\mathbf{E}} = \mathbf{A}_{\mathbf{m}}\mathbf{E} + \mathbf{B}[\boldsymbol{\alpha}^{\boldsymbol{T}}\mathbf{X}_{\mathbf{p}} + u - r] \quad (32)$$

is asymptotically stable. Taking $\mathbf{h} \in R^n$ such that $\mathbf{h}^{\mathbf{T}}(s\mathbf{I} - \mathbf{A}_{\mathbf{m}})^{-1}\mathbf{B} = 1/(s+k)$ and defining $e = \mathbf{h}^{\mathbf{T}}\mathbf{E}$, then

$$\dot{e}(t) = -ke(t) + [\boldsymbol{\alpha}^{\boldsymbol{T}}\boldsymbol{X}_{\boldsymbol{p}} + u - r] \quad (33)$$

and it is known [5] that $\mathbf{E}(t) \to \mathbf{0}$ if and only if $e(t) \to 0$. It is obvious that the function $\boldsymbol{\alpha}^{\boldsymbol{T}}\boldsymbol{X}_{\boldsymbol{p}}$ satisfies the Lipschitz condition

$$|\boldsymbol{\alpha}^{\boldsymbol{T}}\boldsymbol{X}_{\boldsymbol{p}} - \bar{\boldsymbol{\alpha}}^{\boldsymbol{T}}\boldsymbol{X}_{\boldsymbol{p}}| \le \max_{j=1,..,n} |X_{pj}| \sum_{j=1}^{n} |\alpha_j - \bar{\alpha}_j|$$

and applying Theorem 3, we have the following result showing that 1-dimension estimator can be used for update law, instead of full n-dimension estimator in previously developed results of linear adaptive control.

Theorem 4 *The nonlinear control*

$$\begin{aligned} u &= r - \operatorname{sgn}(e) \max_{j=1,..,n} |X_{pj}|\hat{\alpha} \\ \dot{\hat{\alpha}}(t) &= |e(t)| \max_{j=1,..,n} |X_{pj}| \end{aligned} \quad (34)$$

makes $\mathbf{X}_{\mathbf{p}}$ *track;* $\mathbf{X}_{\mathbf{m}}$ *asymptotically, while the nonlinear control*

$$\begin{aligned} u &= r - sat(e/\epsilon) \max_{j=1,..,n} |X_{pj}|\hat{\alpha} \\ \dot{\hat{\alpha}}(t) &= |e(t) - \epsilon sat(e(t)/\epsilon)| \max_{j=1,..,n} |X_{pj} \end{aligned} \quad (35)$$

guarantees $\mathbf{X}_{\mathbf{p}}$ *tracking* $\mathbf{X}_{\mathbf{m}}$ *asymptotically with a precision of* ϵ.

2.7 Case of Multiplicative parameterizations

It is assumed in this section that

$$f(x,\boldsymbol{\theta}) = \mathbf{g}(x,\boldsymbol{\theta})\mathbf{h}(x,\boldsymbol{\theta}), \tag{36}$$

where the assumption below is made.
Assumption 3. *The functions* $\mathbf{g}(x,\boldsymbol{\theta}) : R \times R^p \to (R^m)^T$ *and* $\mathbf{h}(x,\boldsymbol{\theta}) : R \times R^p \to R^m$ *are Lipschitzian in* $\boldsymbol{\theta}$*, i.e. there are continuous functions* $L_j(x) \geq 0, \ell_j(x) \geq 0$ *such that*

$$\begin{aligned} ||\mathbf{g}(x,\boldsymbol{\theta}) - \mathbf{g}(x,\bar{\boldsymbol{\theta}})|| &\leq \sum_{j=1}^{p} L_j(x)|\theta_j - \bar{\theta}_j|, \qquad \forall(x,\boldsymbol{\theta},\bar{\boldsymbol{\theta}}) \\ ||\mathbf{h}(x,\boldsymbol{\theta}) - \mathbf{h}(x,\bar{\boldsymbol{\theta}})|| &\leq \sum_{j=1}^{p} \ell_j(x)|\theta_j - \bar{\theta}_j|, \qquad \forall(x,\boldsymbol{\theta},\bar{\boldsymbol{\theta}}) \end{aligned} \tag{37}$$

true.
Let $L(x)$ be defined by (19) and

$$\ell(x) := \max_{j=1,2,\ldots,p} \ell_j(x).$$

Then, whenever $\boldsymbol{\theta} \geq \bar{\boldsymbol{\theta}}$,

$$\begin{aligned} &\max\{\mathbf{g}(x,\boldsymbol{\theta})\mathbf{h}(x,\boldsymbol{\theta}) - \mathbf{g}(x,\bar{\boldsymbol{\theta}})\mathbf{h}(x,\bar{\boldsymbol{\theta}}), \mathbf{g}(x,\bar{\boldsymbol{\theta}})\mathbf{h}(x,\bar{\boldsymbol{\theta}}) - \mathbf{g}(x,\boldsymbol{\theta})\mathbf{h}(x,\boldsymbol{\theta})\} \leq \\ &||\mathbf{g}(x,\boldsymbol{\theta}) - \mathbf{g}(x,\bar{\boldsymbol{\theta}})||||\mathbf{h}(x,\boldsymbol{\theta})|| + ||\mathbf{g}(x,\bar{\boldsymbol{\theta}})||||\mathbf{h}(x,\boldsymbol{\theta}) - \mathbf{h}(x,\bar{\boldsymbol{\theta}})|| \leq \\ &[L(x)\sum_{j=1}^{p}(\theta_j - \bar{\theta}_j)][\ell(x)\sum_{j=1}^{p}\theta_j + ||\mathbf{h}(x,\mathbf{0})||]+ \\ &[L(x)\sum_{j=1}^{p}\bar{\theta}_j + ||\mathbf{g}(x,\mathbf{0})||]\ell(x)\sum_{j=1}^{p}(\theta_j - \bar{\theta}_j) = \\ &[L(x)\ell(x)(\sum_{j=1}^{p}\theta_j)^2 + (||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x))\sum_{j=1}^{p}\theta_j] - \\ &[L(x)\ell(x)(\sum_{j=1}^{p}\bar{\theta}_j)^2 + (||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x))\sum_{j=1}^{p}\bar{\theta}_j]. \end{aligned}$$

As in the proof of lemma 2, the last inequality is enough to conclude:
Lemma 4 *On* R_+^p*, the function*

$$\begin{aligned} k(e,x,\boldsymbol{\theta}) \;:=\; & e[\mathbf{g}(x,\boldsymbol{\theta})\mathbf{h}(x,\boldsymbol{\theta}) - \mathrm{sgn}(e)(L(x)\ell(x)(\sum_{j=1}^{p}\theta_j)^2 + \\ & (||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x))\sum_{j=1}^{p}\theta_j)] \end{aligned} \tag{38}$$

is decreasing in $\boldsymbol{\theta}$.

Therefore, similarly to Sub-section 2.5, using the Lyapunov function

$$V(e,\hat{\theta},\hat{\alpha}) = \frac{e^2}{2} + \frac{1}{2}[(\hat{\theta} - \sum_{j=1}^{p} |\theta_j|)^2 + (\hat{\alpha} - (\sum_{j=1}^{p} |\theta_j|)^2)^2] \tag{39}$$

we can prove the following theorem

Theorem 5 *The following discontinuous control guarantees* $e(t) \to 0$

$$u = -\mathbf{g}(x,\mathbf{0})\mathbf{h}(x,\mathbf{0}) - \text{sgn}(e)[L(x)\ell(x)\hat{\alpha} + (||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x))\hat{\theta}], \tag{40}$$

$$\dot{\hat{\alpha}}(t) = |e|L(x)\ell(x), \tag{41}$$

$$\dot{\hat{\theta}}(t) = |e|(||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x)), \tag{42}$$

while the following continuous control with $sat(e/\epsilon)$ and e_ϵ defined by (12), (13) guarantees the tracking of *e*(*t*) to 0 with any prescribed precision ϵ,

$$\begin{aligned} u = & -\mathbf{g}(x,\mathbf{0})\mathbf{h}(x,\mathbf{0}) - sat(e/\epsilon)[L(x)\ell(x)\hat{\alpha} + \\ & (||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x))\hat{\theta}], \end{aligned} \tag{43}$$

$$\dot{\hat{\alpha}}(t) = |e_\epsilon|L(x)\ell(x), \tag{44}$$

$$\dot{\hat{\theta}}(t) = |e_\epsilon|(||\mathbf{h}(x,\mathbf{0})||L(x) + ||\mathbf{g}(x,\mathbf{0})||\ell(x)). \tag{45}$$

Remark 1 By reseting $h_j(x,\boldsymbol{\theta}) \leftarrow -h_j(x,\boldsymbol{\theta})$ if necessarily, we can also assume without loss of generality that $g_j(x,0) \geq 0$. Then, using the inequality

$$||\mathbf{v}|| \leq \sum_{j=1}^{p} |v_j| \quad \forall v \in R^p$$

it can be shown that the function

$$\begin{aligned} k(e,x,\boldsymbol{\theta}) := & \; e[\mathbf{g}(x,\boldsymbol{\theta})\mathbf{h}(x,\boldsymbol{\theta}) - \text{sgn}(e)(L(x)\ell(x)(\sum_{j=1}^{p}\theta_j)^2 + \\ & \sum_{k=1}^{p}(|h_j(x,0)|L(x) + g_j(x,0)\ell(x))\sum_{j=1}^{p}\theta_j)] \end{aligned}$$

is still decreasing in $\boldsymbol{\theta} \in R^p_+$. Thus, the statement of Theorem 5 remains valid with $||\mathbf{h}(x,\mathbf{0})||$ and $||\mathbf{g}(x,\mathbf{0})||$ in (40)-(45) replaced by $\sum_{j=1}^{p} |h_j(x,0)|$ and $\sum_{j=1}^{p} g_j(x,0)$, respectively.

Remark 2 Clearly, the statements of lemma 4 and Theorem 5 remain valid by replacing $||\mathbf{h}(x,\mathbf{0})||$ and $||\mathbf{g}(x,\mathbf{0})||$ in (38) and (40)-(42), (43)-(45) by any continuous functions $\bar{\mathbf{h}}(x) \geq ||\mathbf{h}(x,\mathbf{0})||$ and $\bar{\mathbf{g}}(x) \geq ||\mathbf{g}(x,\mathbf{0})||$, respectively.

2.8 Example of controller design

We examine some problems of adaptive friction compensation and show that they belong to the classes considered in Sections 2.2-2.7 and thus the results there can be directly applied to solve these problems.

The model of a process with friction is given as

$$\ddot{x}(t) = u - F \tag{46}$$

where u is the control force, x is the motor shaft angular position, and F is the frictional force that can be described in different ways depending on model types. In this discussion, we consider the Armstrong-Helouvry model [3]

$$F = F_C \text{sgn}(\dot{x})[1 - e^{-\dot{x}^2/v_S^2}] + F_S \text{sgn}(\dot{x}) e^{-\dot{x}^2/v_S^2} + F_v \dot{x}, \tag{47}$$

where F_C, F_S, F_v are coefficients characterizing the Coulomb friction, static friction and viscous friction, respectively, and v_S is the Stribeck parameter. The unknown static parameters are F_C, F_S, F_v, v_S.

To facilitate the developed results, we introduce the new variable

$$e = x + \dot{x} \tag{48}$$

which according to (46) obeys the equation

$$\dot{e} = \dot{x} + u - F \tag{49}$$

For (47), set $\boldsymbol{\theta} = (F_C, F_S, F_v, 1/v_S^2)$. First consider F defined by (47),

$$F = f(\dot{x}, \boldsymbol{\theta}) + \theta_3 \dot{x} \tag{50}$$

where f has the form (36) with

$$\mathbf{g}(\dot{x}, \boldsymbol{\theta}) = [\theta_1 \quad \theta_2], \ \mathbf{h}(\dot{x}, \boldsymbol{\theta}) = \text{sgn}(\dot{x}) \begin{bmatrix} (1 - e^{-\dot{x}^2\theta_4}) \\ e^{-\dot{x}^2\theta_4} \end{bmatrix}. \tag{51}$$

Clearly, $||\mathbf{h}(x, \mathbf{0})|| \leq 1$ and function $\mathbf{h}(\dot{x}, \boldsymbol{\theta})$ is Lipschitzian in $\boldsymbol{\theta}$:

$$|h_j(\dot{x}, \boldsymbol{\theta}) - h_j(\dot{x}, \bar{\boldsymbol{\theta}})| \leq \dot{x}^2 |\theta_4 - \bar{\theta}_4|, j = 1, 2,$$

Applying Theorem 5 to system (49) and taking the Remark 2 in Section 2.7 into account, the following controls are proposed for stabilizing system (46), (47),

$$\begin{gathered} u = -\dot{x} - e + \hat{F}_v \dot{x} - \text{sgn}(e)[\dot{x}^2 \hat{\alpha} + \hat{\theta}], \\ \dot{\hat{F}}_v = -e\dot{x}, \ \dot{\hat{\alpha}} = |e|\dot{x}^2, \ \dot{\hat{\theta}} = |e| \end{gathered} \tag{52}$$

and

$$\begin{gathered} u = -\dot{x} - e + \hat{F}_v \dot{x} - \text{sat}(e/\epsilon)[\dot{x}^2 \hat{\alpha} + \hat{\theta}], \\ \dot{\hat{F}}_v = -\dot{x}(e - \epsilon \text{sat}(e/\epsilon)), \ \dot{\hat{\alpha}} = |e - \epsilon \text{sat}(e/\epsilon)|\dot{x}^2, \ \dot{\hat{\theta}} = |e - \epsilon \text{sat}(e/\epsilon)| \end{gathered} \tag{53}$$

On the other hand, (47) can be rewritten alternatively as

$$F = \mathrm{sgn}(\dot{x})F_c + F_v\dot{x} + (F_s - F_c)\mathrm{sgn}(\dot{x})e^{-\dot{x}^2/v_S^2} \tag{54}$$

with known parameters $(F_c, F_v, F_s - F_c, 1/v_S^2)$. Again, by Theorem 5, the following controller is proposed

$$\begin{gathered} u = -\dot{x} - e + \mathrm{sgn}(\dot{x})\hat{F}_c + \dot{x}\hat{F}_v - \mathrm{sat}(e/\epsilon)(\dot{x}^2\hat{\alpha} + \hat{\theta}) \\ \begin{bmatrix} \dot{\hat{F}}_c \\ \dot{\hat{F}}_v \end{bmatrix} = -\begin{bmatrix} \mathrm{sgn}(\dot{x}) \\ \dot{x} \end{bmatrix} (e - \epsilon\mathrm{sat}(e/\epsilon)),\ \dot{\hat{\alpha}} = |e - \epsilon\mathrm{sat}(e/\epsilon)|\dot{x}^2,\ \dot{\hat{\theta}} = |e - \epsilon\mathrm{sat}(e/\epsilon)|. \end{gathered} \tag{55}$$

One may guess that the term $\mathrm{sgn}(\dot{x})$ in (55) causes its nonsmooth behavior. Considering the term $\mathrm{sgn}(\dot{x})F_c$ in (54) as a Lipschitz function in F_c with Lipschitz constant 1 and applying Theorem 2 to handle this term, an alternative continuous control to (55) is derived as

$$\begin{gathered} u = -\dot{x} - e + \mathrm{sat}(e/\epsilon)\hat{F}_c + \dot{x}\hat{F}_v - \mathrm{sat}(e/\epsilon)(\dot{x}^2\hat{\alpha} + \hat{\theta}) \\ \begin{bmatrix} \dot{\hat{F}}_c \\ \dot{\hat{F}}_v \end{bmatrix} = \begin{bmatrix} |e - \epsilon\mathrm{sat}(e/\epsilon)| \\ -\dot{x}(e - \epsilon\mathrm{sat}(e/\epsilon)) \end{bmatrix},\ \dot{\hat{\alpha}} = |e - \epsilon\mathrm{sat}(e/\epsilon)|\dot{x}^2,\ \dot{\hat{\theta}} = |e - \epsilon\mathrm{sat}(e/\epsilon) \end{gathered} \tag{56}$$

3. Extension to the adaptive controls in systems with indirect control inputs

3.1 Control problems of the generalized matching system with second-order

Without loss of generality, we describe the control problems of the generalized matching system with second-order, i.e.

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi(x_1, \boldsymbol{\theta}), \\ \dot{x}_2 &= u, \end{aligned} \tag{57}$$

where $u \in R$ is the control input, $x = [x_1, x_2]^T$ is the system state. Function $\varphi(x_1, \boldsymbol{\theta})$ is nonlinear in both the variable x_1 and the unknown parameter $\boldsymbol{\theta} \in R^p$. The problem is to design a stabilizing state-feedback control u such that the state $x_1(t)$ converges to 0.

A useful methodology for designing controllers of this class is the adaptive backstepping method [9], under the assumption of a linear parameterization (LP) in the unknown parameter $\boldsymbol{\theta}$, i.e. the function $\varphi(x_1, \boldsymbol{\theta})$ in (57) is assumed linear. The basic idea of backstepping is to design a "stabilizing function", which prescribes a desired behavior for x_2 so that $x_1(t)$ is stabilized. Then, an effective control $u(t)$ is synthesized to regulate x_2 to track this stabilizing function. Very few results, however, are available in the literature that address adaptive backstepping for NP systems of the general form (57) [10]. The difficulty here is attributed to two main factors inherent in the adaptive backstepping. The first one is how to construct the stabilizing function for xi in the presence of nonlinear parameterizations [5, 11, 12]. The second one arises from the fact that as the actual control $u(t)$ involves derivatives of the stabilizing function, the later must be constructed in such a way that it does not lead to multiple parameter estimates (or overparameterization) [13].

3.2 Remarks on adaptive back-stepping design incorporated with Lipschitzian parameterization-based techniques

The proposed approach in Sectionl has been extended to address the adaptive backstepping for the above general matching system. Our approach enables the design of the stabilizing function containing estimates of the unknown parameter $\boldsymbol{\theta}$ without overparameterization. The compactness of parametric sets is not required. The proposed approach is naturally applicable to smooth nonlinearities but also to the broader class of Lipschitzian functions. Interested reader can refer [14, 15, 16] for the results in details.

4. Adaptive controller design applied to path tracking of robot manipulators in the presence of NP.

4.1 Robot manipulators with NP uncertainties

Nonlinear frictions such as Stribeck effect are very common in practical robot manipulators. However, adaptive controls for robot manipulators (see [17, 18] for a survey) cannot successfully compensate for NP frictions since they are based on the LP structure of unknown parameters. Also, most of adaptive friction compensation schemes in the literature of motion control only deal with either frictions with LP structure [19] or linearized models at the nominal values of the Stribeck friction parameters [20]. Recently, a Lyapunov-based adaptive control has been designed to compensate for the Stribeck effect under set-point control [21].

In this section, a general framework of adaptive control for NP in the system is developed. An application of adaptive control for friction compensation in tracking problem of a 2DOF planar robot is introduced together with comparative simulations and experiments.

4.2 Problem formulation

The dynamic model of a robot manipulator can be described by the following equation

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}) = \boldsymbol{\tau}(t), \tag{58}$$

where $\mathbf{q}(t) \in R^n$ is the joint coordinates of the manipulator, $\boldsymbol{\tau} \in R^n$ is the torque applied to the joints, $\mathbf{H}(q) \in R^{n\times n}$ is the symmetric positive definite inertia matrix of the links, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) \in R^{n\times n}$ is a matrix representing Coriolis and centrifugal effects, $\mathbf{g}(\mathbf{q}) \in R^n$ is the gravitational torques, $\mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}) \in R^n$ represents dynamics whose constant or slowly-varying uncertain parameter $\boldsymbol{\theta}$

appears nonlinearly in the system. Note that $\mathbf{x}$ can be any component of the system state, for instance $\mathbf{x} = [\mathbf{q}^T, \dot{\mathbf{q}}^T]^T$.

We focus on the case where the uncertainties admit a general multiplicative form, i.e.,

$$\begin{aligned} \mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}) &= [f_{N1}(\mathbf{x},\boldsymbol{\theta}_1), \quad \dots \quad, f_{Nn}(\mathbf{x},\boldsymbol{\theta}_n)]^T \\ f_{Ni}(\mathbf{x},\boldsymbol{\theta}_i) &= \mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i)\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i), \quad i = 1,...,n. \end{aligned} \tag{59}$$

Here i stands for the *i-ih* joint of the manipulator and functions $\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i), \mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i)$ are assumed nonlinear and Lipschitzian in $\boldsymbol{\theta}_i$, $\boldsymbol{\theta}_i = [\theta_{i1}, \quad \dots \quad, \theta_{ip_i}]^T \in R^{p_i}$. As it will be discussed later, a typical example of uncertainty admitting this form is the Stribeck effect of frictional forces in joints of robot manipulators [3].

Property 2.1 The inertia matrix $\mathbf{H}(\mathbf{q})$ is positive definite and satisfies $\lambda_{\min}\mathbf{I} \leq \mathbf{H}(\mathbf{q}) \leq \lambda_{\max}\mathbf{I}$, with $0 < \lambda_{\min} < \lambda_{\max} < \infty$, $\lambda_{\min}$, where $\lambda_{\max}$ are minimal and maximal eigenvalues of $\mathbf{H}(\mathbf{q})$.
Property 2.2 The matrix $\dot{\mathbf{H}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ is skew-symmetric.
Property 2.3 The sum of the first three terms in the LHS of equation (58) are expressed linearly with respect to a suitable set of constant dynamic parameters:

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\mathbf{a}, \tag{60}$$

where $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \in R^{n\times\omega}$ matrix function and $\mathbf{a} \in R^{\omega}$ is a vector of unknown dynamic parameters.
The following lemma will be frequently used in subsequent developments
Lemma 5 *Given Lipschitzian functions* $\mathbf{g}_i(\mathbf{x}, \boldsymbol{\theta}_i)$, $\mathbf{h}_i(\mathbf{x}, \boldsymbol{\theta}_i)$, *let* $L_i(\mathbf{x})$ *and* $\ell_i(\mathbf{x})$ *be defined as*

$$\begin{aligned} L_i(\mathbf{x}) &:= \max\nolimits_{j=1,2,\ldots,p} L_{ij}(\mathbf{x}), \\ \ell_i(\mathbf{x}) &:= \max\nolimits_{j=1,2,\ldots,p} \ell_{ij}(\mathbf{x}), \end{aligned} \tag{61}$$

then, for $\boldsymbol{\theta}_i \in R_+^{p_i}$ *the following inequalities*

$$\begin{aligned} e(t)\mathbf{g}_i(\mathbf{x}, \boldsymbol{\theta}_i)\mathbf{h}_i(\mathbf{x}, \boldsymbol{\theta}_i) \leq \ & e(t)\mathbf{g}_i(\mathbf{x}, 0)\mathbf{h}_i(\mathbf{x}, 0) + |e(t)|\{L_i(\mathbf{x})\ell_i(\mathbf{x})(\sum_{j=1}^{p_i}\theta_{ij})^2 \\ & + [||\mathbf{h}_i(\mathbf{x}, 0)||\, L_i(\mathbf{x}) + ||\mathbf{g}_i(\mathbf{x}, 0)||\, \ell_i(\mathbf{x})]\sum_{j=1}^{p_i}\theta_{ij}\}, \end{aligned} \tag{62}$$

hold true for any $e(t) \in R$.
Proof. Since,

$$e(t)\left[\mathbf{g}(\mathbf{x}, \boldsymbol{\theta})\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{g}(\mathbf{x}, 0)\mathbf{h}(\mathbf{x}, 0)\right] \leq |e(t)||\mathbf{g}(\mathbf{x}, \boldsymbol{\theta})\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{g}(\mathbf{x}, 0)\mathbf{h}(\mathbf{x}, 0)|,$$

it is sufficient to prove that

$$|\mathbf{g}(\mathbf{x}, \boldsymbol{\theta})\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{g}(\mathbf{x}, 0)\mathbf{h}(\mathbf{x}, 0)| \leq \{L(\mathbf{x})\ell(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right)^2 + [||\mathbf{h}(\mathbf{x}, 0)||\, L(\mathbf{x}) + ||\mathbf{g}(\mathbf{x}, 0)||\, \ell(\mathbf{x})]\sum_{j=1}^{p}\theta_j\}, \tag{63}$$

where $L(\mathbf{x}), \ell(\mathbf{x})$ are defined in (61) and note that the subscripts *i* is neglected lor simplicity.
Actually,

$$\begin{aligned} |\mathbf{g}(\mathbf{x}, \boldsymbol{\theta})\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{g}(\mathbf{x}, 0)\mathbf{h}(\mathbf{x}, 0)| &= |[\mathbf{g}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{g}(\mathbf{x}, 0)]\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) + \mathbf{g}(\mathbf{x}, 0)[\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{h}(\mathbf{x}, 0)]| \\ &\leq ||\mathbf{g}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{g}(\mathbf{x}, 0)||\,||\mathbf{h}(\mathbf{x}, \boldsymbol{\theta})|| + ||\mathbf{g}(\mathbf{x}, 0)||\,||\mathbf{h}(\mathbf{x}, \boldsymbol{\theta}) - \mathbf{h}(\mathbf{x}, 0)|| \\ &\leq \left(\sum_{j=1}^{p} L_j(\mathbf{x})\theta_j\right)||\mathbf{h}(\mathbf{x}, \boldsymbol{\theta})|| + ||\mathbf{g}(\mathbf{x}, 0)||\left(\sum_{j=1}^{p} l_j(x)\theta_j\right) \\ &\leq L(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right)||\mathbf{h}(\mathbf{x}, \boldsymbol{\theta})|| + ||\mathbf{g}(\mathbf{x}, 0)||l(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right) \end{aligned} \tag{64}$$

and

$$\begin{aligned} ||\mathbf{h}(\mathbf{x},\boldsymbol{\theta})|| &\le ||\mathbf{h}(\mathbf{x},\boldsymbol{\theta})-\mathbf{h}(\mathbf{x},0)||+||\mathbf{h}(\mathbf{x},0)|| \\ &\le l(\mathbf{x})\left(\sum_{j=1}^{p}\theta_j\right)+||\mathbf{h}(\mathbf{x},0)||. \end{aligned} \tag{65}$$

leads to (63).

Our goal is to control the rigid manipulator to track a given trajectory $\mathbf{q}_d(t)$ by designing a nonlinear adaptive control to compensate for all uncertainties which are either LP uncertain dynamics according to Property 2.3 or NP as defined by (59), in system (58). For simplicity of the derivations throughout the paper, it is assumed that $\boldsymbol{\theta}_i \in R_+^{p_i}$, i.e. $\theta_{ij} \ge 0$, j = 1, 2, 3, ...,p_i. At the end of Section 4.3.2, we will see that the general case $\boldsymbol{\theta}_i \in R^{p_i}$ can be easily retrieved from our results. While traditional adaptive controls can be effectively applied only in the context of LP [2], lemma 5 reveals an ability to approximate the NP by its certain part plus a part of LP. We will use the key property (62) to design a novel nonlinear adaptive control for the system.

4.3 A framework for adaptive control design

Define vector $\mathbf{s}(t) \in R^n$ as a "velocity error" term

$$\mathbf{s}(t)=\dot{\tilde{\mathbf{q}}}(t)+\boldsymbol{\Lambda}\tilde{\mathbf{q}}(t)=\dot{\mathbf{q}}(t)-\dot{\mathbf{q}}_r(t), \tag{66}$$

where $\boldsymbol{\Lambda}=\mathrm{diag}\,[\lambda_1,\lambda_2,\ldots,\lambda_n] \in R^{n\times n}$ is an arbitrary positive definite matrix, $\tilde{\mathbf{q}}(t)=\mathbf{q}(t)-\mathbf{q}_d(t)$ is the position tracking error, and $\dot{\mathbf{q}}_r(t)=\dot{\mathbf{q}}_d(t)-\boldsymbol{\Lambda}\tilde{\mathbf{q}}(t)$, called the "reference velocity". According to Property 2.3, the dynamics of the system (58) can be rewritten in terms of the "velocity error" $s(t)$ as

$$\mathbf{H}(\mathbf{q})\dot{\mathbf{s}}(t)+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s}(t)=\boldsymbol{\tau}(t)-\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)\mathbf{a}-\mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}), \tag{67}$$

with the identity $\mathbf{Y}(\mathbf{q},\dot{\mathbf{q}},\dot{\mathbf{q}}_r,\ddot{\mathbf{q}}_r)\mathbf{a}=\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}}_r+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}_r+\mathbf{g}(\mathbf{q})$ used.

By definition (66), the tracking error $\tilde{q}_i(t)$ obtained from *Si(t)* through the above designed first-order low pass filter is

$$\tilde{q}_i(t)=\tilde{q}_i(t_0)e^{-\lambda_i(t-t_0)}+\int_{t_0}^{t}s_i(\zeta)e^{\lambda_i(\zeta-t)}d\zeta,$$

where $\tilde{q}_i(t_0)$ is the tracking error of joint i^{th} of the robot manipulator at the time to. If $|s_i(t)|\le\rho, \forall t\ge t_0$ then

$$\begin{aligned} |\tilde{q}_i(t)| &\le |\tilde{q}_i(t_0)|e^{-\lambda_i(t-t_0)}+\int_{t_0}^{t}|s_i(\zeta)|e^{\lambda_i(\zeta-t)}d\zeta \\ &\le \left(|\tilde{q}_i(t_0)|-\frac{\rho}{\lambda_i}\right)e^{-\lambda_i(t-t_0)}+\frac{\rho}{\lambda_i}. \end{aligned} \tag{68}$$

The relation (68) means that $\lim_{t\to\infty}|\tilde{q}_i(t)|\le\dfrac{\rho}{\lambda_i}$ whenever $\lim_{t\to\infty}|s_i(t)|\le\rho$. Therefore, in the next development, the model (67) is used for designing a control input $\tau(t)$ which guarantees the velocity error $s(t) \to 0$ under LP uncertainty a and NP uncertainty θ. As shown above, such performance of $s(t)$ ensures the convergence to 0 of tracking error $\tilde{q}(t)$ when $t \to \infty$.

4.3.1 Discontinuous adaptive control design

Consider a quadratic Lyapunov function candidate

$$V_1(t) := \frac{1}{2}\mathbf{s}^T(t)\mathbf{H}(\mathbf{q})\mathbf{s}(t).$$

By Property 2.2, its time derivative can be written as

$$\dot{V}_1(t) = \mathbf{s}^T\left(\boldsymbol{\tau} - \mathbf{Y}\mathbf{a} - \mathbf{f}_N(\mathbf{x},\boldsymbol{\theta})\right) = \mathbf{s}^T(\boldsymbol{\tau} - \mathbf{Y}\mathbf{a}) - \sum_{i=1}^{n} s_i f_{Ni}(\mathbf{x},\boldsymbol{\theta}_i).$$

where the notations on $t, \mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r$ are neglected for simplicity. In view of relation (62), it follows that

$$\begin{aligned} \dot{V}_1(t) \leq\; & \mathbf{s}^T(\boldsymbol{\tau} - \mathbf{Y}\mathbf{a}) + \left(\sum_{i=1}^{n} s_i \mathbf{g}_i(\mathbf{x},0)\mathbf{h}_i(\mathbf{x},0)\right) \\ & + \sum_{i=1}^{n}|s_i|\left\{L_i(\mathbf{x})\ell_i(\mathbf{x})(\sum_{j=1}^{p_i}\theta_{ij})^2 + [||\mathbf{h}_i(\mathbf{x},0)||L_i(\mathbf{x}) + ||\mathbf{g}_i(\mathbf{x},0)||\ell_i(\mathbf{x})]\sum_{j=1}^{p_i}\theta_{ij}\right\}. \end{aligned} \tag{69}$$

With the definitions

$$\begin{aligned} \mathbf{W}(\mathbf{x}) &:= \operatorname{diag}[\mathbf{w}_1(\mathbf{x}), \mathbf{w}_2(\mathbf{x}), \ldots, \mathbf{w}_n(\mathbf{x})] \in R^{n\times 2n}, \\ \boldsymbol{\Phi}(\mathbf{s},\mathbf{x}) &:= \operatorname{diag}[\operatorname{sgn}(s_1)\mathbf{w}_1(\mathbf{x}), \ldots, \operatorname{sgn}(s_n)\mathbf{w}_n(\mathbf{x})] \in R^{n\times 2n}, \\ \boldsymbol{\beta} &:= [\boldsymbol{\beta}_1^T \quad \boldsymbol{\beta}_2^T \quad \ldots \quad \boldsymbol{\beta}_n^T]^T \in R^{2n}, \\ \mathbf{w}_i(\mathbf{x}) &= [w_{i1} \quad w_{i2}] \\ &:= [L_i(\mathbf{x})\ell_i(\mathbf{x}) \quad ||\mathbf{h}_i(\mathbf{x},0)||L_i(\mathbf{x}) + ||\mathbf{g}_i(\mathbf{x},0)||\ell_i(\mathbf{x})], \\ \boldsymbol{\beta}_i &= [\beta_{i1} \quad \beta_{i2}]^T \\ &:= \left[(\sum_{j=1}^{p_i}\theta_{ij})^2 \quad \sum_{j=1}^{p_i}\theta_{ij}\right]^T, \end{aligned} \tag{70}$$

the inequality (69) can be rewritten as

$$\dot{V}_1(t) \leq \mathbf{s}^T(\boldsymbol{\tau} - \mathbf{Y}\mathbf{a}) + \mathbf{s}^T\mathbf{f}_N(\mathbf{x},0) + \mathbf{s}^T\boldsymbol{\Phi}(\mathbf{s},\mathbf{x})\boldsymbol{\beta}, \tag{71}$$

Therefore, the control input

$$\boldsymbol{\tau} = -\mathbf{K}_D\mathbf{s} + \mathbf{Y}\hat{\mathbf{a}} - \mathbf{f}_N(\mathbf{x},0) - \boldsymbol{\Phi}(\mathbf{s},\mathbf{x})\hat{\boldsymbol{\beta}}, \tag{72}$$

results in

$$\dot{V}_1(t) \leq -\mathbf{s}^T\mathbf{K}_D\mathbf{s} + \mathbf{s}^T[\mathbf{Y}\tilde{\mathbf{a}} - \boldsymbol{\Phi}(\mathbf{s},\mathbf{x})]\tilde{\boldsymbol{\beta}}, \tag{73}$$

where $\tilde{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{a}$ and $\tilde{\boldsymbol{\beta}} = \hat{\boldsymbol{\beta}} - \boldsymbol{\beta}$ are parameter errors and $\mathbf{K}_D \in R^{n\times n}$ is an arbitrary positive definite matrix.

To derive update laws for the parameter estimates, we employ the following Lyapunov function

$$V(t) = V_1(t) + \frac{1}{2}(\tilde{\mathbf{a}}^T \boldsymbol{\Gamma}_a^{-1} \tilde{\mathbf{a}} + \tilde{\boldsymbol{\beta}}^T \boldsymbol{\Gamma}_\beta^{-1} \tilde{\boldsymbol{\beta}}), \tag{74}$$

where $\boldsymbol{\Gamma}_a, \boldsymbol{\Gamma}_\beta$ are arbitrary positive definite matrices. It follows form (73) that

$$\dot{V}(t) \quad \leq \quad -\mathbf{s}^T \mathbf{K}_D \mathbf{s} + \mathbf{s}^T[\mathbf{Y}\tilde{\mathbf{a}} - \boldsymbol{\Phi}(\mathbf{s},\mathbf{x})]\tilde{\boldsymbol{\beta}} + \dot{\hat{\mathbf{a}}}^T \boldsymbol{\Gamma}_a^{-1} \tilde{\mathbf{a}} + \dot{\hat{\boldsymbol{\beta}}}^T \boldsymbol{\Gamma}_\beta^{-1} \tilde{\boldsymbol{\beta}}. \tag{75}$$

Therefore, the following update laws

$$\dot{\hat{\mathbf{a}}} = -\boldsymbol{\Gamma}_a \mathbf{Y}^{\mathbf{T}} \mathbf{s}, \dot{\hat{\boldsymbol{\beta}}} = \boldsymbol{\Gamma}_\beta \mathbf{W}^T(\mathbf{x})|\mathbf{s}|, \quad |\mathbf{s}| = [\,|s_1| \quad |s_2| \quad \ldots \quad |s_n|\,]^T \tag{76}$$

yield

$$\dot{V}(t) \leq -\mathbf{s}^T \mathbf{K}_D \mathbf{s}. \tag{77}$$

The last inequality implies that $V(t)$ is decreasing, and thus is bounded by $V(0)$. Consequently, $\mathbf{s}(t)$ and $\tilde{\mathbf{a}}(t), \tilde{\boldsymbol{\theta}}(t)$ must be bounded quantities by virtue of definition (74). Given the boundedness of the reference trajectory $\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d$, one has $\dot{\mathbf{s}}(t) \in L_\infty$ from the system dynamics (67). Also, relation (77) gives $\lambda_{\min}(\mathbf{K}_D) \int_0^T ||\mathbf{s}(t)||^2 dt \leq V(0), \forall T > 0$, i.e. $\mathbf{s}(t) \in L_2$, where $\lambda_{\min}(\mathbf{K}_D)$ denotes the minimum eigenvalue of $\mathbf{K}_D$. Applying Barbalat's lemma [2] yields $\lim_{t\to\infty} \mathbf{s}(t) = 0$. However, the control (72) is still discontinuous at $\mathbf{s}(t) = 0$, and thus is not readily implemented. As a next stage, we make the control action continuous by a standard modification technique which leads to a practically implementable control law.

4.3.2 Continuous adaptive control design

A continuous control action can be derived by modifying the velocity error $\mathbf{s}(t)$. First, introduce a new variable $\mathbf{s}_\varepsilon(t)$ by setting

$$\mathbf{s}_\varepsilon = \mathbf{s} - \frac{1}{\sqrt{3}}\mathbf{c}(\mathbf{s}), \tag{78}$$

where

$$\mathbf{c}(\mathbf{s}) \quad = \quad [c_1(s_1) \quad \ldots \quad c_n(s_n)]^T,$$

$$c_i(s_i) = \begin{cases} b_i + \sqrt{r_i^2 - (s_i - \varepsilon_i)^2}, & \frac{\sqrt{3}-1}{2}\varepsilon_i \leq s_i \leq \varepsilon_i \\ \sqrt{3}s_i, & |s_i| \leq \frac{\sqrt{3}-1}{2}\varepsilon_i \\ -b_i - \sqrt{r_i^2 - (s_i + \varepsilon_i)^2}, & -\varepsilon_i \leq s_i \leq -\frac{\sqrt{3}-1}{2}\varepsilon_i \\ \varepsilon_i \mathrm{sgn}(s_i), & |s_i| > \varepsilon_i \end{cases} \tag{79}$$

$r_i = (\sqrt{3}-1)\varepsilon_i, b_i = (2-\sqrt{3})\varepsilon_i$, for $\varepsilon_i > 0, \forall i = 1, \ldots, n$.

It is standard to show that such $\mathbf{s}_\varepsilon(t)$ is continuously differentiable in time t (see also Figure 1). Using Property 2.3, the dynamics of system (58) in terms of the modified "velocity error" $\mathbf{s}_\varepsilon(t)$ is expressed by

$$\mathbf{H}(\mathbf{q})\dot{\mathbf{s}}_\varepsilon + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{s}_\varepsilon = \boldsymbol{\tau} - \mathbf{Y}_\varepsilon\mathbf{a} - \mathbf{f}_N(\mathbf{x},\boldsymbol{\theta}), \tag{80}$$

where $\mathbf{Y}_\varepsilon = \mathbf{H}(\mathbf{q})(\ddot{\mathbf{q}}_r + \frac{1}{\sqrt{3}}\dot{\mathbf{c}}(\mathbf{s})) + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})(\dot{\mathbf{q}}_r + \frac{1}{\sqrt{3}}\mathbf{c}(\mathbf{s})) + \mathbf{g}(\mathbf{q})$.

Now, take the following Lyapunov function

$$V_\varepsilon(t) = \frac{1}{2}\mathbf{s}_\varepsilon^T\mathbf{H}(\mathbf{q})\mathbf{s}_\varepsilon + \frac{1}{2}(\tilde{\mathbf{a}}^T\boldsymbol{\Gamma}_a^{-1}\tilde{\mathbf{a}} + \tilde{\boldsymbol{\beta}}^T\boldsymbol{\Gamma}_\beta^{-1}\tilde{\boldsymbol{\beta}}). \tag{81}$$

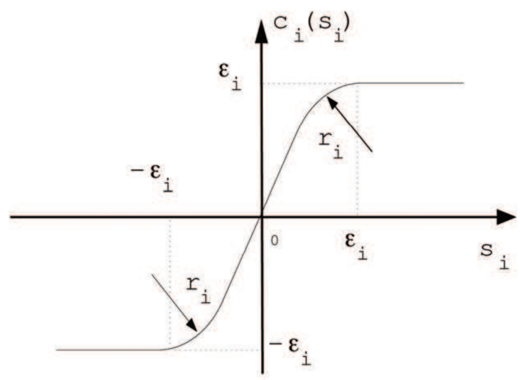

Figure 1. Smooth function $c_i(s_i)$

Like (71), it is clear that

$$\dot{V}_\epsilon(t) \leq \mathbf{s}_\varepsilon^T(\boldsymbol{\tau} - \mathbf{Y}_\varepsilon\mathbf{a}) + \mathbf{s}_\varepsilon^T\mathbf{f}_N(\mathbf{x},0) + \mathbf{s}_\varepsilon^T\boldsymbol{\Phi}(\mathbf{s}_\varepsilon,\mathbf{x})\boldsymbol{\beta} + \dot{\hat{\mathbf{a}}}^T\boldsymbol{\Gamma}_a^{-1}\tilde{\mathbf{a}} + \dot{\hat{\boldsymbol{\beta}}}^T\boldsymbol{\Gamma}_\beta^{-1}\tilde{\beta} \tag{82}$$

where let us recall that $\boldsymbol{\Phi}$ is already defined by formula (70).

Note that whenever $|s_i| \leq \frac{\sqrt{3}-1}{2}\varepsilon_i$, one has

$$s_{i\varepsilon} = 0,$$

and for $|s_i| > \frac{\sqrt{3}-1}{2}\varepsilon_i$

$$\begin{aligned} s_{i\varepsilon}^2 &\leq s_{i\varepsilon}s_i, \\ \mathrm{sgn}(s_i) &= \mathrm{sgn}(s_{i\varepsilon}). \end{aligned} \tag{83}$$

Hence, introducing the saturated function

$$\mathrm{sat}_{\varepsilon_i}(s_i) = \begin{cases} \dfrac{s_i}{\frac{\sqrt{3}-1}{2}\varepsilon_i} & \text{when} \quad |s_i| \leq \frac{\sqrt{3}-1}{2}\varepsilon_i \\ \mathrm{sgn}(s_i) & \text{when} \quad |s_i| > \frac{\sqrt{3}-1}{2}\varepsilon_i, \end{cases} \tag{84}$$

and taking (82), (83) into account, the following continuous control input

$$\boldsymbol{\tau} = -\mathbf{K}_D\mathbf{s} + \mathbf{Y}_\varepsilon\hat{\mathbf{a}} - \mathbf{f}_N(\mathbf{x},0) - \boldsymbol{\Phi}_\varepsilon(\mathbf{s},\mathbf{x})\hat{\boldsymbol{\beta}}, \tag{85}$$

with

$$\boldsymbol{\Phi}_\varepsilon(\mathbf{s},\mathbf{x}) := \quad \mathrm{diag}\left[\mathrm{sat}_{\varepsilon_1}(s_1)\mathbf{w}_1(x), \ldots, \mathrm{sat}_{\varepsilon_n}(s_n)\mathbf{w}_n(x)\right] \in R^{n\times 2n},$$

together with the update laws

$$\dot{\hat{\mathbf{a}}} = -\Gamma_a\mathbf{Y}^T\mathbf{s}_\varepsilon, \dot{\hat{\boldsymbol{\beta}}} = \Gamma_\beta\mathbf{W}^T(\mathbf{x})|\mathbf{s}_\varepsilon|, \quad |\mathbf{s}_\epsilon| = [\,|s_{1\epsilon}| \quad |s_{2\epsilon}| \quad \ldots \quad |s_{n\epsilon}|\,]^T \tag{86}$$

yield

$$\dot{V}_\varepsilon(t) \leq -\mathbf{s}_\varepsilon^T\mathbf{K}_D\mathbf{s}_\varepsilon.$$

Finally, by a similar analysis as done in Section 4.3.1, the error $\mathbf{s}_\varepsilon(t)$ of the system converges to 0, or equivalently $\lim_{t\to\infty}|s_i| \leq \frac{\sqrt{3}-1}{2}\varepsilon_i,\ i = 1,...,n$. From relation (68), the tracking error $\tilde{q}_i(t)$ converges to $\dfrac{\sqrt{3}-1}{2\lambda_i}\varepsilon_i$ as $t \to \infty$. We are now in a position to sum up our results.

Theorem 6 *The adaptive controller defined by equations (78),(79),(84)-(86) enables system to asymptotically track a desired trajectory* $\mathbf{q}_d(t)$ *within a precision of* $\dfrac{\sqrt{3}-1}{2\lambda_i}\varepsilon_i, i = 1,...,n$.

Remark 1 In the general case where $\boldsymbol{\theta}_i \in R^{p_i}$, it follows in a straightforward manner from lemma 5 that

$$\begin{aligned} e(t)\mathbf{g}_i(\mathbf{x},\boldsymbol{\theta}_i)\mathbf{h}_i(\mathbf{x},\boldsymbol{\theta}_i) \quad &\leq \quad e(t)\mathbf{g}_i(\mathbf{x},0)\mathbf{h}_i(\mathbf{x},0) + |e(t)|\{L_i(\mathbf{x})\ell_i(\mathbf{x})(\sum_{j=1}^{p_i}|\theta_{ij}|)^2 \\ &+ [||\mathbf{h}_i(\mathbf{x},0)||L_i(\mathbf{x}) + ||\mathbf{g}_i(\mathbf{x},0)||\ell_i(\mathbf{x})]\sum_{j=1}^{p_i}|\theta_{ij}|\}. \end{aligned}$$

Therefore, with a Lyapunov function defined in (81) where

$$\boldsymbol{\beta}_i = [\beta_{i1}, \quad \beta_{i2}]^T = \left[(\sum_{j=1}^{p}|\theta_{ij}|)^2, \quad \sum_{j=1}^{p}|\theta_{ij}|\right]^T,$$

Theorem 6 remains valid for $\boldsymbol{\theta}_i \in R_i^p$.

Remark 2 The new variable (78) and the function (79) are properly designed to make the stabilizing control (72) continuous. Of course, there are other appropriate choices other than the variable (78) and the function (79), which also make the stabilizing control (72) continuous, too.

4.3.3 1-dimension estimator

In the design of sections 4.3.1 and 4.3.2, the dimensions of estimators are equal to the number of unknown parameters in the system, i.e. $\hat{\mathbf{a}} \in R^\omega, \hat{\boldsymbol{\beta}} \in R^{2n}$. Thus, increasing the

number of links may result in estimators of excessively large dimension. Tuning updating gains $\mathbf{\Gamma}_a, \mathbf{\Gamma}_\beta$ for those estimators then becomes a very laborious task. In this section, we show that it is possible to design an adaptive controller for system (58) with simple 1-dimension estimators $\hat{a}, \hat{\beta}$ independently of the dimensions of the unknown parameters $\mathbf{a}$ and $\boldsymbol{\beta}$.
For that purpose, first consider the term $\mathbf{Ya}$ in (69) where $\mathbf{Y} \in R^{n\times\omega}$,$\mathbf{a} \in R^{\omega}$. It is clear that

$$\sum_{j=1}^{\omega} Y_{ij}a_j \leq \left(\max_{j=1,\ldots,\omega} |Y_{ij}|\right) \sum_{j=1}^{\omega} |a_j|, \;\; i = 1, ..., n.$$

Also note from (70) that

$$\mathbf{w}_i(\mathbf{x})\boldsymbol{\beta}_i \leq \max_{j=1,2} |w_{ij}(\mathbf{x})|(|\beta_{i1}| + |\beta_{i2}|), \;\; i = 1, ..., n.$$

As a result, the inequality (71) can be rewritten as follows

$$\dot{V}_1(t) \;\; \leq \;\; \mathbf{s}^T\boldsymbol{\tau} + \mathbf{s}^T\mathbf{f}_N(\mathbf{x}, 0) + |\mathbf{s}^T|(\mathbf{y}_{\max} \sum_{j=1}^{\omega} |a_j| + \mathbf{w}_{\max}(\mathbf{x}) \sum_{i=1}^{n} (|\beta_{i1}| + |\beta_{i2}|)),$$

where

$$\begin{aligned} \mathbf{y}_{\max} &:= \left[\max_{j=1,\ldots,\omega} |Y_{1j}|, \ldots, \max_{j=1,\ldots,\omega} |Y_{nj}|\right]^T \in R^n, \\ \mathbf{w}_{\max}(\mathbf{x}) &:= \left[\max_{j=1,2} |w_{1j}(\mathbf{x})|, \ldots, \max_{j=1,2} |w_{nj}(\mathbf{x})|\right]^T \in R^n. \end{aligned}$$

Note that $\mathbf{y}_{\max}$ is the function whose notations on variables $\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r$ are neglected for simplicity. Therefore, with the definitions

$$\begin{aligned} \boldsymbol{\psi}_{\max}(\mathbf{s}) &:= \mathrm{diag}\,[\mathrm{sgn}(s_1), \ldots, \mathrm{sgn}(s_n)]\,\mathbf{y}_{\max} \in R^n, \\ \boldsymbol{\phi}_{\max}(\mathbf{s}, \mathbf{x}) &:= \mathrm{diag}\,[\mathrm{sgn}(s_1), \ldots, \mathrm{sgn}(s_n)]\,\mathbf{w}_{\max}(\mathbf{x}) \in R^n, \end{aligned}$$

the following control input

$$\begin{aligned} \boldsymbol{\tau} &= -\mathbf{K}_D\mathbf{s} - \mathbf{f}_N(\mathbf{x}, 0) - \left(\boldsymbol{\psi}_{\max}(\mathbf{s})\hat{a} + \boldsymbol{\phi}_{\max}(\mathbf{s}, \mathbf{x})\hat{\beta}\right), \\ \dot{\hat{a}} &= \gamma_a \mathbf{y}_{\max}^T|\mathbf{s}|, \dot{\hat{\beta}} = \gamma_\beta \mathbf{w}_{\max}^T(\mathbf{x})|\mathbf{s}|, \end{aligned} \tag{87}$$

where γ_a and γ_a are arbitrary positive scalars, together with the following Lyapunov function

$$V_{1D}(t) \;\; = \;\; V_1(t) + \frac{1}{2}\gamma_a^{-1}(\sum_{j=1}^{\omega} |a_j| - \hat{a})^2 + \frac{1}{2}\gamma_\beta^{-1}(\sum_{i=1}^{n} (|\beta_{i1}| + |\beta_{i2}|) - \hat{\beta})^2, \tag{88}$$

yield

$$\dot{V}(t)_{1D} \leq -\mathbf{s}^T\mathbf{K}_D\mathbf{s}.$$

Therefore, the discontinuous control (87) results in the convergence to 0 of velocity error $\mathbf{s}(t)$, which ensures the convergence to 0 of tracking error $\tilde{\mathbf{q}}(t)$ when $t \to \infty$. As in section 4.3.2, we can alter the discontinuous control (87) into a continuous one as follows

$$\begin{aligned} \boldsymbol{\tau} &= -\mathbf{K}_D\mathbf{s} - \mathbf{f}_N(\mathbf{x},0) - (\boldsymbol{\psi}_{\max\varepsilon}(\mathbf{s})\hat{a} + \boldsymbol{\phi}_{\max\varepsilon}(\mathbf{s},\mathbf{x})\hat{\beta}), \\ \dot{\hat{a}} &= \gamma_a \mathbf{y}_{\max}^T|\mathbf{s}_\varepsilon|, \ \dot{\hat{\beta}} = \gamma_\beta \mathbf{w}_{\max}^T(\mathbf{x})|\mathbf{s}_\varepsilon|, \end{aligned} \tag{89}$$

where

$$\begin{aligned} \boldsymbol{\psi}_{\max\varepsilon}(\mathbf{s}) &:= \operatorname{diag}[\operatorname{sat}_{\varepsilon_1}(s_1),\ldots,\operatorname{sat}_{\varepsilon_n}(s_n)]\,\mathbf{y}_{\max} \in R^n, \\ \boldsymbol{\phi}_{\max\varepsilon}(\mathbf{s},\mathbf{x}) &:= \operatorname{diag}[\operatorname{sat}_{\varepsilon_1}(s_1),\ldots,\operatorname{sat}_{\varepsilon_n}(s_n)]\,\mathbf{w}_{\max}(\mathbf{x}) \in R^n. \end{aligned}$$

Then the continuous control (89) ensures the convergence to $\frac{\sqrt{3}-1}{2\lambda_i}\varepsilon_i$, i = 1, ..., n of the tracking error $\tilde{\mathbf{q}}(t)$ when $t \to \infty$.

4.4 Example of nonlinear friction compensation

In this section, we examine how effectively our designed adaptive controllers can compensate for the frictional forces in joints of robot manipulators.

4.4.1 Friction model and friction compensators

Frictional forces in system (58) can be described in different ways. Here, we consider the well-known Amstrong-Helouvry model [3]. For joint *i*, the frictional force is described as

$$\begin{aligned} f_i &= F_{ci}\operatorname{sgn}(\dot{q}_i)[1 - \exp(-\frac{\dot{q}_i^2}{v_{si}^2})] \\ &\quad + F_{si}\operatorname{sgn}(\dot{q}_i)\exp(-\frac{\dot{q}_i^2}{v_{si}^2}) + F_{vi}\dot{q}_i, \end{aligned} \tag{90}$$

where F_{ci}, F_{si}, F_{vi} are coefficients characterizing the Coulomb friction, static friction and viscous friction, respectively, and v_{si} is the Stribeck parameter. Note that the friction term (90) can be decomposed into a linear part f_{Li} and a nonlinear part f_{Ni} as

$$f_i = f_{Li} + f_{Ni}, \tag{91}$$

where

$$f_{Li} = F_{ci}\operatorname{sgn}(\dot{q}_i) + F_{vi}\dot{q}_i = \mathbf{z}_i\boldsymbol{\alpha}_i, \tag{92}$$

with $\boldsymbol{\alpha_i} = [F_{ci} \quad F_{vi}]^T$, $\mathbf{z}_i = [\operatorname{sgn}(\dot{q}_i) \quad \dot{q}_i]$, and

$$f_{Ni} = (F_{si} - F_{ci})\operatorname{sgn}(\dot{q}_i)\exp(-\frac{\dot{q}_i^2}{v_{si}^2}). \tag{93}$$

Practically, the frictional coefficients are not exactly known. In such case, the frictional force f_{Li} can be compensated by a traditional adaptive control for LP. However, the situation becomes non trivial when there are unknown parameters appearing nonlinearly in the model of f_{Ni}.
The NP friction term of joint i, f_{Ni}, can be expressed in the form (59) with

$$f_{Ni} = g_i(\dot{q}_i, \boldsymbol{\theta}_i) h_i(\dot{q}_i, \boldsymbol{\theta}_i), \tag{94}$$

where

$$\begin{aligned} \boldsymbol{\theta}_i &= \left[(F_{si} - F_{ci}) \quad \frac{1}{v_{si}^2} \right]^T = [\,\theta_{i1} \quad \theta_{i2}\,]^T, \\ g_i(\dot{q}_i, \theta_i) &= [\,1 \quad 0\,]\,\theta_i, h_i(\dot{q}_i, \theta_i) = \operatorname{sgn}(\dot{q}_i) \exp\left(-\dot{q}_i^2 \theta_{i2}\right). \end{aligned}$$

Clearly, g_i and h_i are Lipschitzian in θ_i with Lipschitzian coefficients $l_i(\dot{q}) = \dot{q}_i^2, L_i(\dot{q}) = 1$. Also, we have $g_i(\dot{q}_i, 0) = 0, h_i(\dot{q}_i, 0) = 1$. Therefore, by Theorem 6, the following adaptive controller enables the system (58), (90), (94) to asymptotically track a desired trajectory $q_{di}(t)$ within a precision of $\frac{\sqrt{3}-1}{2\lambda_i}\varepsilon_i$, i=1,...,n.

$$\begin{aligned} &\boldsymbol{\tau} = -\mathbf{K}_D \mathbf{s} + \mathbf{Y}_\varepsilon \hat{\mathbf{a}} + \mathbf{Z}\hat{\boldsymbol{\alpha}} - \boldsymbol{\Phi}_\varepsilon(\mathbf{s}, \mathbf{x})\hat{\boldsymbol{\beta}} \\ &\dot{\hat{\mathbf{a}}} = -\boldsymbol{\Gamma}_a \mathbf{Y}^T \mathbf{s}_\varepsilon, \dot{\hat{\boldsymbol{\alpha}}} = -\boldsymbol{\Gamma}_\alpha \mathbf{Z}^T \mathbf{s}_\varepsilon, \dot{\hat{\boldsymbol{\beta}}} = \boldsymbol{\Gamma}_\beta \mathbf{W}^T(\mathbf{x})|\mathbf{s}_\varepsilon|, \end{aligned} \tag{95}$$

where

$$\begin{aligned} \mathbf{Z} = &\quad \operatorname{diag}[\,\mathbf{z}_1(q_1), \ldots, \mathbf{z}_n(q_n)\,] \in R^{n \times 2n}, \\ \mathbf{w}_i(\dot{\mathbf{q}}) = &\quad [\,\dot{q}_i^2, \quad 1\,]. \end{aligned} \tag{96}$$

Note that with the control (95), the term $\mathbf{Z}\hat{\boldsymbol{\alpha}}$ compensates for the LP frictions f_{Li}.

4.4.2 Simulations

A prototype of a planar 2DOF robot manipulator is built to assess the validity of the proposed methods (Figure 2) . The dynamic model of the manipulator and its linearized dynamics parameter are given in Section 6 (Appendix).
The manipulator model is characterized by a real parameter a, which is identified by a standard technique (See Table 3 in Section 6). The parameters of friction model (90) are chosen such that the effect of the NP frictions f_{Ni} are significant, i.e.

$$F_{ci} = 0.49, Fsi = 3.5, F_{vi} = 0.15, v_{si} = 0.189, \forall i = 1, 2.$$

In order to focus on the compensation of nonlinearly parameterized frictions, we have selected the objective of low-velocity tracking. The manipulator must track the desired trajectory $q_{d1}(t) = \frac{\pi}{6}(1 - \cos(t)), q_{d2}(t) = \frac{\pi}{4}(1 - \cos(t))$. Clearly, the selected trajectory contains various zero velocity crossings.
For comparison, we use 2 different controllers to accomplish the tracking task.

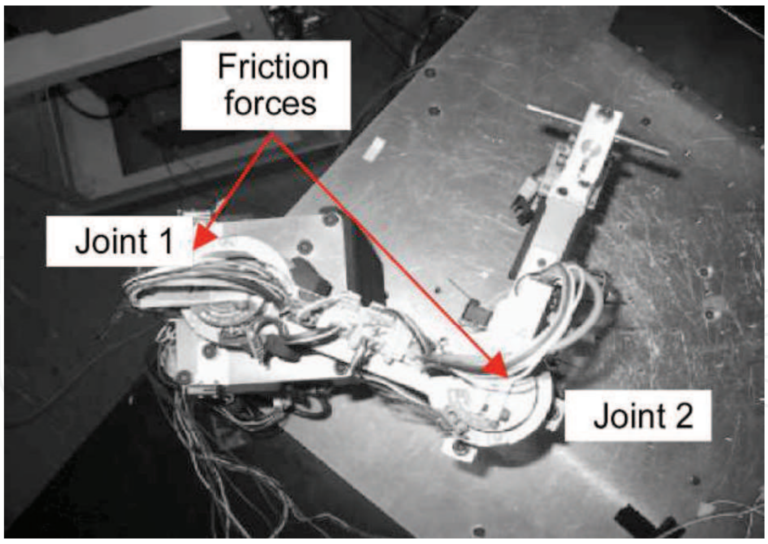

Figure 2. Prototype of robot manipulator

$\mathbf{K}_D$	$\mathbf{\Lambda}$	$\mathbf{\Gamma}_a$	$\mathbf{\Gamma}_\alpha$
$10\mathbf{I}(2,2)$	$5\mathbf{I}(2,2)$	diag(5,5,5)	diag(3,3,3,3)

Table 1. Parameters of the controllers for simulations

- A traditional adaptive control based on the LP structure to compensate for uncertainty in dynamic parameter a of the manipulator links and the linearly parameterized frictions f_{Li} (92) in joints of motors.

$$\begin{aligned}\boldsymbol{\tau} &= -\mathbf{K}_D\mathbf{s} + \mathbf{Y}\hat{\mathbf{a}} + \mathbf{Z}\hat{\boldsymbol{\alpha}} \\ \dot{\hat{\mathbf{a}}} &= -\mathbf{\Gamma}_a\mathbf{Y}^T\mathbf{s}, \dot{\hat{\boldsymbol{\alpha}}} = -\mathbf{\Gamma}_\alpha\mathbf{Z}^T\mathbf{s},\end{aligned} \tag{97}$$

  The gains of the controller are chosen as in Table 1, $\hat{\mathbf{a}} \in R^3, \hat{\boldsymbol{\alpha}} \in R^4$.
- Our proposed controller (95) with the same control parameters for LP uncertainties. Additionally, $\mathbf{\Gamma}_\beta$ = diag(50, 50, 50, 50), ϵ = 0.05 for NP friction compensation, $\hat{\boldsymbol{\beta}} \in R^4$.

Both controllers start without any prior information of dynamic and frictional parameters, i.e. $\hat{\mathbf{a}}(0) = 0, \hat{\boldsymbol{\alpha}}(0) = 0, \hat{\boldsymbol{\beta}}(0) = 0$.

Tradition LP adaptive control vs. proposed control

It can be seen that the position error is much smaller with the proposed control (Figure 3), especially at points where manipulator velocities cross the value of zero. Indeed, the position error of joint 1 decreases about 20 times. The position tracking of joint 2 is improved in the sense that our proposed control obtains a same level of position error as the one of LP, but the bound of control input is reduced about 3 times. This means that the nonlinearly parameterized frictions are effectively compensated by our method.

1-dimension estimators

The performances of the controller with 1-dimension estimators (89) is shown in Figure 4. One estimate is designed for the manipulator dynamics a $\mathbf{a} \in R^3$, one is for the LP friction parameters a $\boldsymbol{\alpha} \in R^4$, and one is for the NP friction parameters $\boldsymbol{\beta} \in R^4$. Thus, by using 1-dimension estimators, the estimates dimension reduces from 11 to 3. The resulting controller benefits not only from a simpler tuning scheme, but also from a minimum amount of on-line calculation since the regressor matrices $\mathbf{Y}, \mathbf{W}$ reduce to the vectors y_{max},w_{max} in this case.

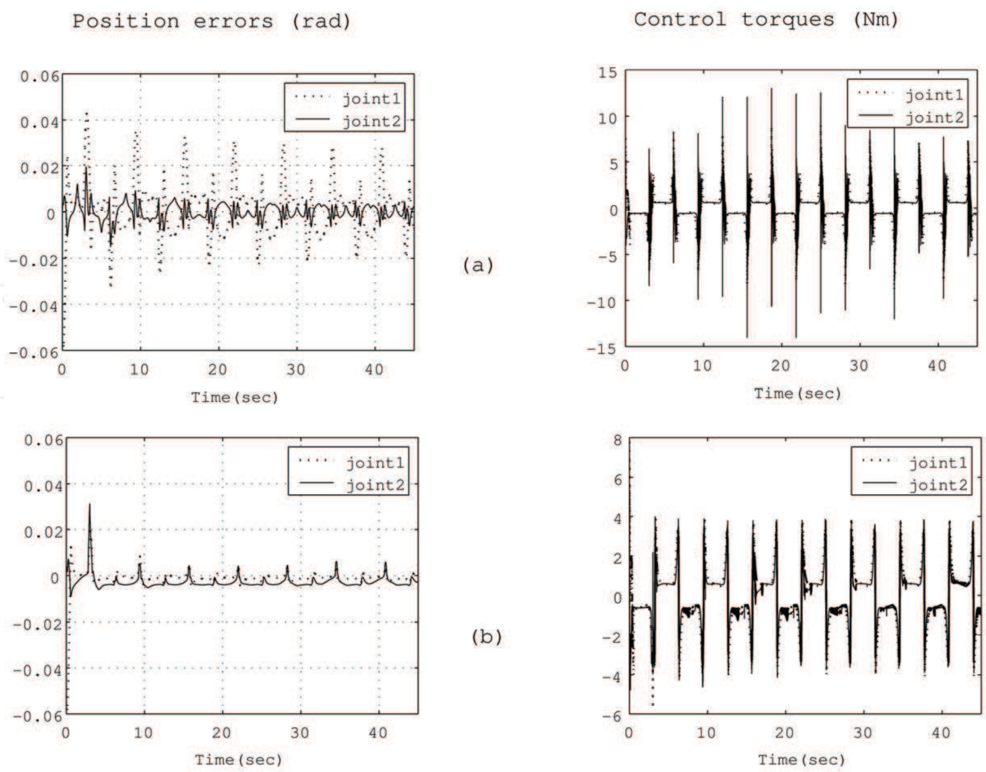

Figure 3. Simulation results: Tracking errors of joints (left) and characteristics of control inputs (right), (a): Traditional LP adaptive controller (97), (b): proposed controller (95)

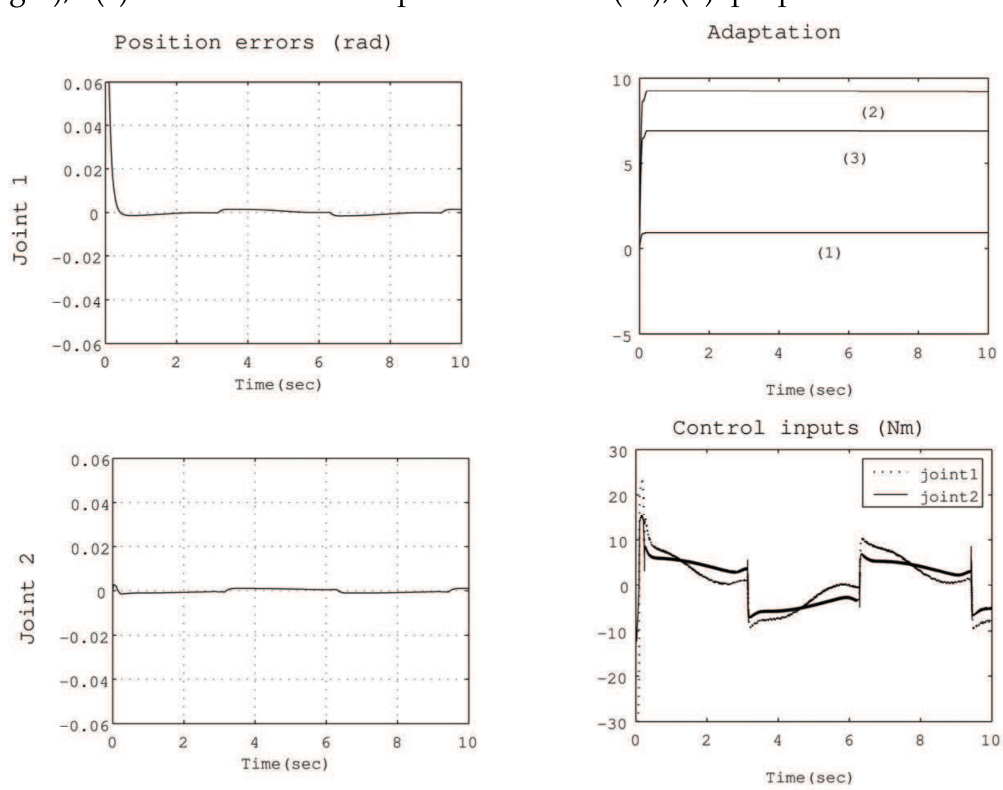

Figure 4. Simulation results for proposed 1-dimension estimators (89): Tracking errors of joints (left), the adaptation of the estimates and characteristics of control inputs (right) .
$(1) - \hat{\mathbf{a}}$, $(2)\text{-}\hat{\boldsymbol{\alpha}}$, $(3)\text{-}\hat{\boldsymbol{\beta}}$

$\mathbf{K}_D$	$\mathbf{\Lambda}$	$\mathbf{\Gamma}_a$	$\mathbf{\Gamma}_\alpha$
$3\mathbf{I}(2,2)$	$3\mathbf{I}(2,2)$	diag(.05,.05,.05)	diag(2.5,5,2.5,5)

Table 2. Parameters of the controllers for experiments

Indeed, under the current simulation environment (WindowsXP/Matlab Simulink), controller (89) requires a computation load 0.7 time less than the one of controller (95) and only 1.2 time bigger than the one of tradition LP adaptive control (97). Also, it can be seen in Figure 4 that these advantages result in a faster convergence (just few instants after the initial time) of the tracking errors to the designed value (0.0035 (rad) in this simulation). Note that the estimates converge to constant values since the adaptation mechanism in controller (89) becomes standstill whenever the tracking errors become less than the design value. However, it is worth noting that the maximum value of control inputs of controller (89), which is required only at the adaptation process of the estimates, is about 6 times bigger than the one of controller (95). It can be learnt from the simulation result that controller (89) can effectively compensates the NP uncertainties in the system provided that there is no limitation to the control inputs. Therefore, controller (95) can be a good choice for practical applications whose the power of actuators are limited.

4.4.3 Experiments

All joints of the manipulator are driven by YASKAWA DC motors UGRMEM-02SA2. The range of motor power is $[-5,5]$ (Nm). The joint angles are detected by potentiometers (350°, ±0.5). Control input signals are sent to each DC motor via a METRONIX amplifier (±35V, ±*3A).* The joint velocities are also calculated from the derivation of joint positions with low-pass niters. Designed controller is implemented on ADSP324-OOA, 32bit DSP board with SOMhz CPU clock. I/O interface is ADSP32X-03/53, 12bit A/D, D/A card. The DSP and the interface card are mounted on Windows98-based PC. The sampling time is 2ms.

Here again, the performances of controller (97) and the proposed control (95) are compared. The gains of the controllers are chosen as in Table 2. The additional control parameters for NP friction compensation with (95) are $\mathbf{\Gamma}_\beta$ = diag(l, 1,1,1), ϵ = .1.

Figure 5 depicts the performances of LP adaptive controller (97). The fact that the trajectory tracking error of joint 2 become about twice smaller as shown by Figure 6 highlights how effectively the NP frictions are compensated by the proposed controller. The estimates of unknown parameters with adaptation mechanisms in LP adaptive controller (97) and proposed controller (95) are shown by Figure 7 and Figure 8, respectively. Since the adaptation mechanism of LP adaptive controller (97) can not compensate for the NP friction terms, its estimates can not converge to any values able to make the trajectory tracking errors converge to 0. For the proposed controller, a better convergence of the estimates can be observed. That the motion of the manipulator has lower frequencies in case of the proposed control (see Figure 9) shows its more robustness in face of noisy inputs. These results can be obtained because the NP frictions are compensated effectively.

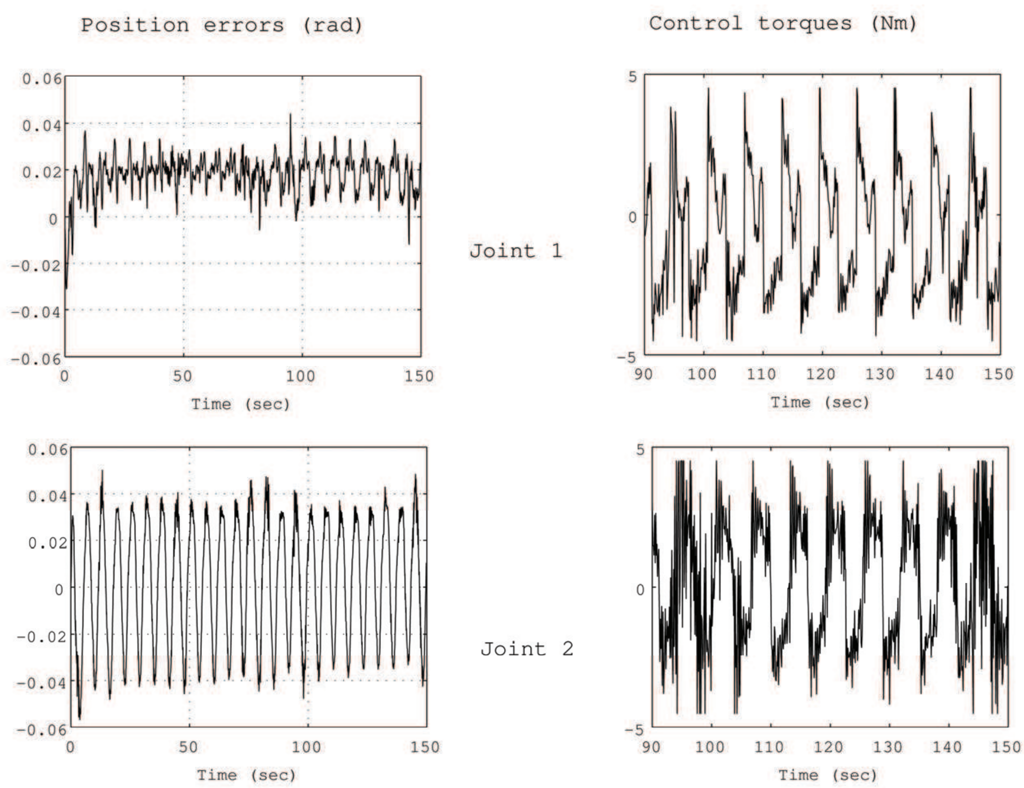

Figure 5. Experimental results for traditional LP adaptive controller (97): Tracking errors of joints (left) and characteristics of control inputs (right)

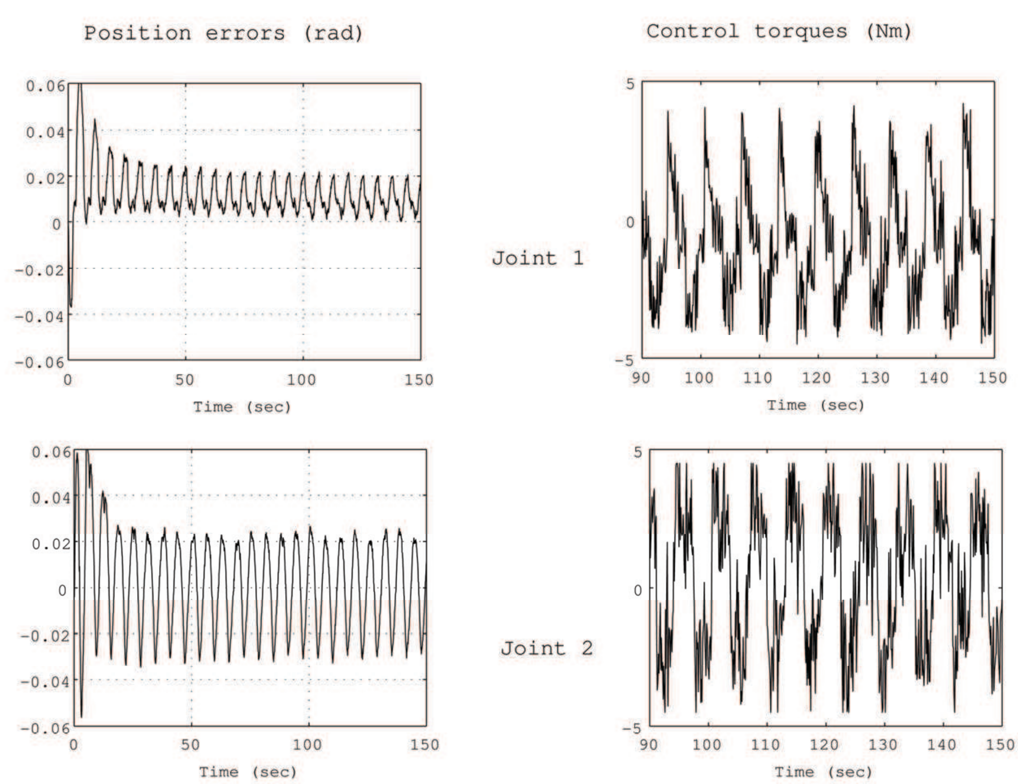

Figure 6. Experimental results for proposed controller (95): Tracking errors of joints (left) and characteristics of control inputs (right)

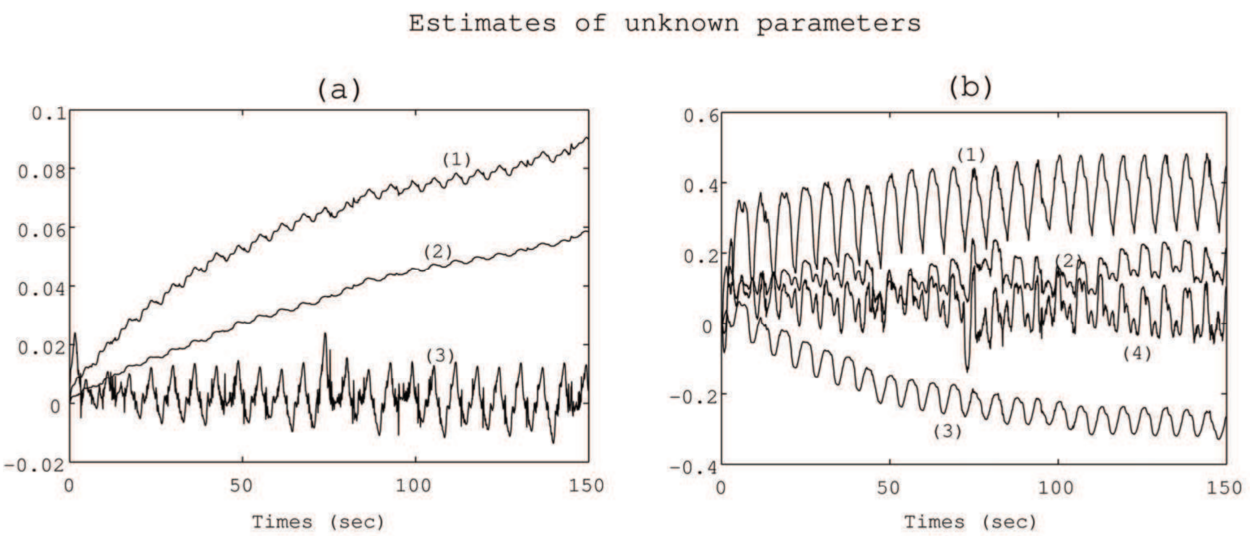

Figure 7. Experimental results: Estimates of unknown parameters with traditional LP adaptive controller (97). (a)-estimate $\hat{\mathbf{a}}$: (1)-$\hat{a}_1$, (2)-$\hat{a}_2$, (3)-$\hat{a}_3$. (b)-estimate $\hat{\boldsymbol{\alpha}}$: (1)-$\hat{\alpha}_1$, (2)-$\hat{\alpha}_2$, (3)-$\hat{\alpha}_3$ (4)-$\hat{\alpha}_4$

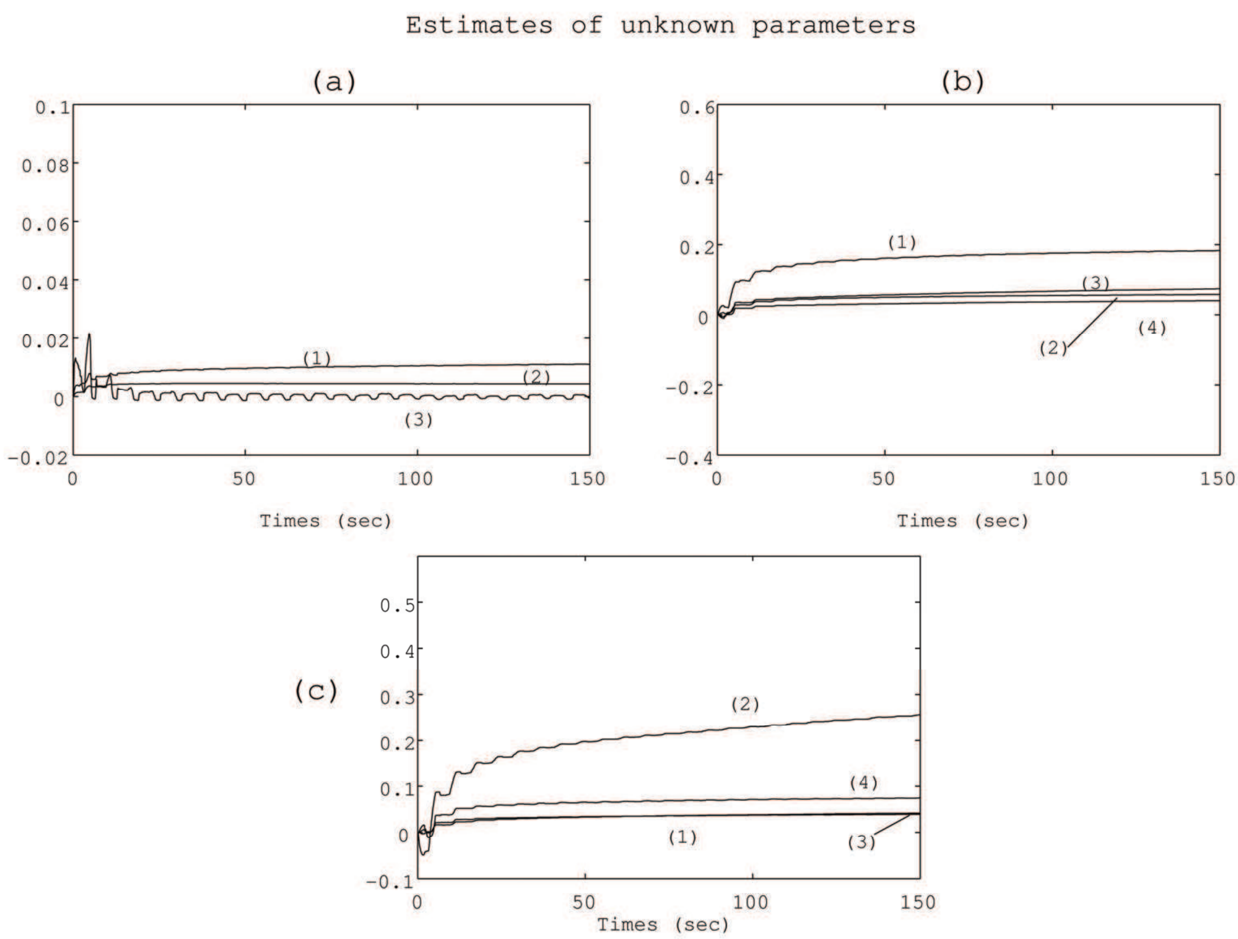

Figure 8. Experimental results: Estimates of unknown parameters with proposed controller (95). (a)-estimate $\hat{\mathbf{a}}$: (1)-$\hat{a}_1$, (2)-$\hat{a}_2$, (3)-$\hat{a}_3$. (b)-estimate $\hat{\boldsymbol{\alpha}}$: (1)-$\hat{\alpha}_1$, (2)-$\hat{\alpha}_2$, (3)-$\hat{\alpha}_3$, (4)-$\hat{\alpha}_4$ estimate $\hat{\boldsymbol{\beta}}$: (1)-$\hat{\beta}_1$, (2)-$\hat{\beta}_2$, (3)-$\hat{\beta}_3$, (4)-$\hat{\beta}_4$

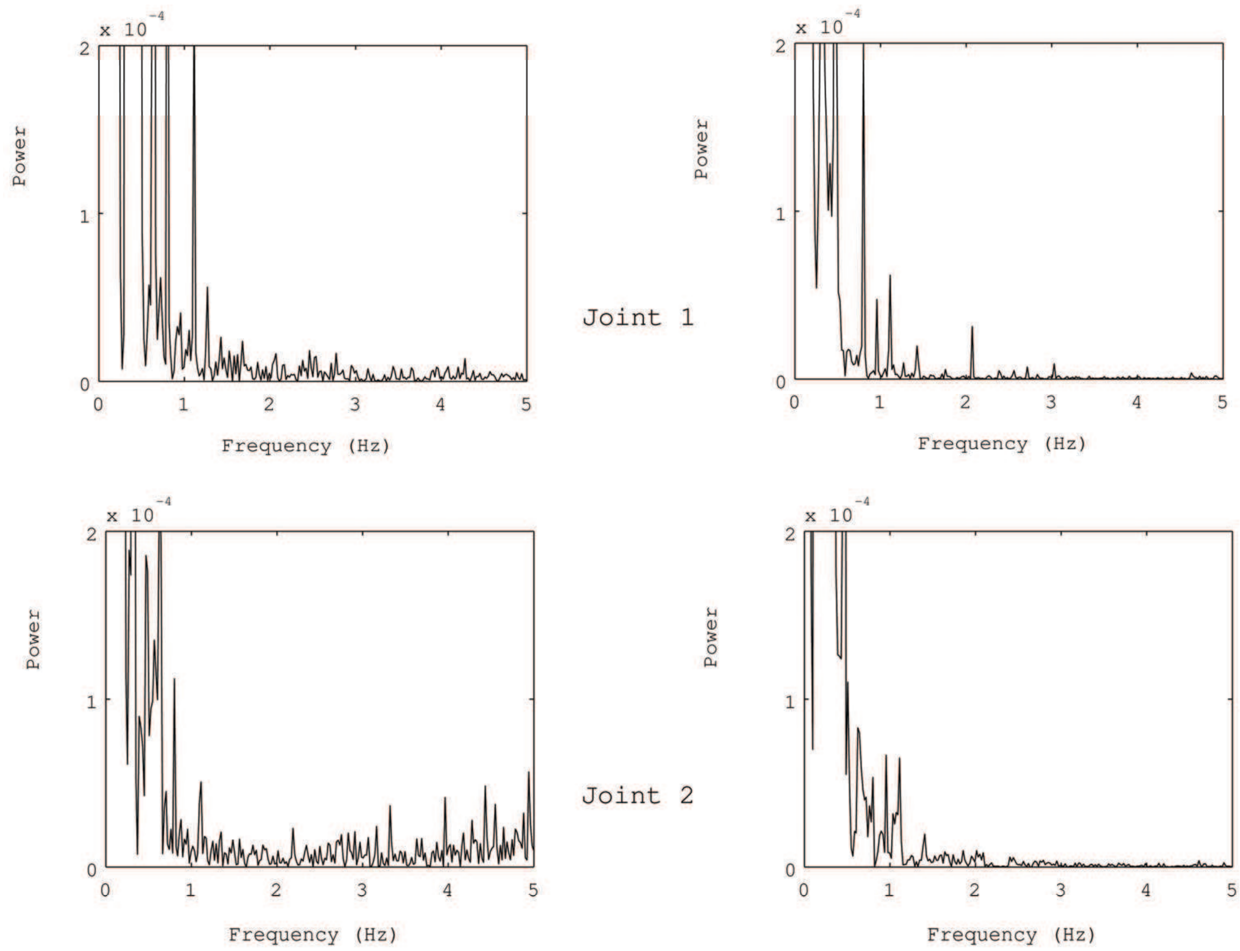

Figure 9. Experimental results: FFT of trajectory tracking errors for traditional LP adaptive controller (97) (left) and proposed controller (95) (right)

5. Conclusions

We have developed a new adaptive control framework which applies to any nonlinearly parameterized system satisfying a general Lipschitzian property. This allows us to extend the scope of adaptive control to handle very general control problems of NP since Lipschitzian parameterizations include as special cases convex/concave and smooth parameterizations. As byproducts, the approach permits also to treat uncertainties in fractional form, multiplicative form and their combinations thereof. Moreover, the proposed control approach allows a flexibility in the design of adaptive control system. This is because the ability of designing 1-dimension estimators provides system designers with more freedom to to balance the dimension of the design estimators and the power required by system control inputs. Otherwise, when it is necessary, simple structure is a key factor enabling the extension of the proposed adaptive controls to more complex control structures. Our next efforts are directed to the following research in order to integrate the proposed adaptive control technique to industrial control systems.

- Mechanisms to control the convergence time of the designed tracking errors. In this context, Lyapunov stability analysis incorporated with dynamic models of signals in the system can be used as an effective synthesis tool.

- Improvement on the robustness of the adaptive schemes toward noise in the system due to un-modeled dynamics or unknown disturbances. In this context, sensing and monitoring the level of noise, and incorporating on-line noise compensation schemes will play an important role.
- Incorporation of the below system's actual working conditions in to the adaptive control system (i) Constrains on the limitation of actuators outputs (ii) Requirement of human-friendly interface (easy-to-tune interface and failure-safe). In this context, control systems need more complex control structure with more intelligent adaptation rules for dealing with wider range of system operation.

6. Appendix

Model and parameters of the manipulator

The equation of motion in joint space for a planar 2DOF manipulator is

$$\mathbf{H}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \boldsymbol{\tau},$$

or,

$$\begin{bmatrix} b_{11}(q_2) & b_{12}(q_2) \\ b_{21}(q_2) & b_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \boldsymbol{\tau}, \tag{98}$$

where,

$$\begin{aligned}
b_{11} &= \underbrace{I_{l_1} + m_{l_1} l_1^2 + kr_1^2 I_{m_1} + m_{l_2} a_1^2 + m_{m_2} a_1^2}_{d_{11}^*} + I_{l_2} + I_{m_2} + m_{l_2} l_2^2 + 2 m_{l_2} a_1 l_2 \cos(q_2), \\
b_{12} &= b_{21} = \underbrace{I_{l_2} + kr_2 I_{m_2} + m_{l_2} l_2^2}_{d_{12}} + m_{l_2} a_1 l_2 \cos(q_2), \\
b_{22} &= \underbrace{I_{l_2} + m_{l_2} l_2^2 + kr_2^2 I_{m_2}}_{d_{22}}, h = -m_{l_2} a_1 l_2 \sin(q_2), \\
c_{11} &= h\,\dot{q}_2, c_{12} = h(\dot{q}_1 + \dot{q}_2), c_{21} = -h\,\dot{q}_1, c_{22} = 0.
\end{aligned}$$

m_{li}, m_{mi} are the masses of link i and motor i, respectively. I_{li}, I_{mi} are the moment of inertia relative to the center of mass of link i and the moment of inertia of motor i. l_i is the distance from the center of the mass of link i to the joint axis. a_i is the length of link i. k_{ri} is the gear reduction ratio of motor i.

A constant vector $\mathbf{a} \in R^3$ of dynamic parameters can be defined as follows:

$$\mathbf{a} = [m_{l_2} a_1 l_2 \quad d_{11}^* \quad d_{22}]^T$$

a_1	a_2	kr_1	kr_2	$m_{l_2} a_1 l_2$	d_{11}^*	d_{22}
0.15	0.15	1	1	0.0043	0.2602	0.0188

Table 3. Parameters of the 2DOF manipulator

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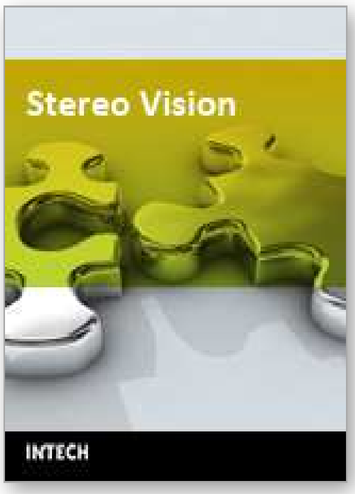

Frontiers in Adaptive Control

Edited by Shuang Cong

ISBN 978-953-7619-43-5
Hard cover, 334 pages
Publisher InTech
Published online 01, January, 2009
Published in print edition January, 2009

The objective of this book is to provide an up-to-date and state-of-the-art coverage of diverse aspects related to adaptive control theory, methodologies and applications. These include various robust techniques, performance enhancement techniques, techniques with less a-priori knowledge, nonlinear adaptive control techniques and intelligent adaptive techniques. There are several themes in this book which instance both the maturity and the novelty of the general adaptive control. Each chapter is introduced by a brief preamble providing the background and objectives of subject matter. The experiment results are presented in considerable detail in order to facilitate the comprehension of the theoretical development, as well as to increase sensitivity of applications in practical problems