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Greedy Algorithms for Spectrum Management in OFDM Cognitive Systems - Applications to Video Streaming and Wireless Sensor Networks

Joumana Farah[1] and François Marx[2]

[1]*Faculty of Sciences & Computer Engineering, Holy-Spirit University of Kaslik, Jounieh,*
[2] *France Telecom, Paris,*
[1]*Lebanon*
[2]*France*

1. Introduction

Since the beginning of regulatory planning in radio communications, new advancements in technology have been driving spectrum management procedures. Sophisticated techniques were introduced to improve the systems spectral efficiency while keeping pace with an increasing demand for new services and higher transmission rates. However, a new paradigm emerged recently in which regulation has driven technology. The exploding success of the first experiments in "open spectrum" using the ISM (Industrial/Scientific/Medical) bands gave rise to a tremendous interest in finding new strategies of spectrum management that will permit a more flexible and opportunistic utilization of the spectrum, without causing harm to existing services. This challenge is of a great concern to the proponents of new generations of communication systems because of the scarcity of spectrum resources.

For this reason, the Federal Communication Commission published, in the last few years, several documents (FCC RO, 2003; FCC NOI and NPRM, 2003) that aimed to improve the radio spectrum allocation, using two different strategies: spectrum leasing and cognitive or smart radio. In the first one, a trading market for spectrum usage is proposed, and users can be dynamically rented the access to a piece of spectrum using a certain arrangement. The second type of dynamic spectrum assignment is the Open Spectrum approach, which allows users to sense available and unallocated spectrum. In this case, the overall spectrum is shared among all users and spectrum etiquette is used to limit harmful interference.

In both cases, but for different motivations - financial stakes in the case of spectrum leasing and voluntary rules for the spectrum etiquette - optimizing the spectrum usage has become of major importance. Therefore, a mechanism must be set in each access point of the communication system in such a way to utilize the unused spectrum in an intelligent way, while not interfering with other incumbent devices in already licensed frequency bands. The spectrum usage will be minimized by an optimization of the channels allocation scheme, so that the spectrum freed by an operator may be used by another operator.

Therefore, new technical challenges must be overcome to enable the success of the cognitive radio paradigm: supporting different air interface standards, operating in multiple environments, adapting to several radio access techniques, counteracting the influence of channel impairments (multipath, fading, noise), coping with user mobility, and guaranteeing a minimum quality of service with an affordable transmission power.

To reach these goals, this chapter investigates the problem of dynamic multiuser subchannel allocation for minimizing spectrum usage. The system overall bandwidth is supposed to be equally divided into a set of frequency bands, therefore assuming Orthogonal Frequency Division Multiplexing (OFDM).

In former studies, most of the work dealing with dynamic spectrum allocation aimed either at maximizing the total system capacity or at minimizing the total transmission power. In (Rhee & Cioffi, 2000), an iterative algorithm was proposed that attributes subchannels to the users in such a way to maximize the smallest user capacity. However, an equal amount of power is allocated to each subcarrier. In (Kim et al., 2004), a maximization of the rate-sum capacity was realized by iterative subcarriers assignment followed by water-filling for power allocation. In (Toufik & Knopp, 2004), a graph theory approach was used to assign a fixed number of subcarriers to each user. Two strategies were considered: maximization of the total transmission rate under the constraint of a fixed amount of transmission power or minimization of the total transmission power while guaranteeing a set of users data rates. The second strategy was also the subject of study in (Wong et al., 1999) and (Kivanc et al., 2003). In (Wong et al., 1999), a set of subcarriers is first assigned to each user based on the Lagrange optimization resolved by parameter relaxation. The transmission power and the number of bits in each subcarrier are then determined by a greedy approach. In (Kivanc et al., 2003), a number of subcarriers is first allocated to each user based on its average Signal-to-Noise Ratio, assuming a flat-fading channel for the user. The best assignment scheme of the subchannels is then determined by an iterative algorithm.

In this chapter, we propose novel techniques based on greedy algorithms for the optimization of the spectrum efficiency of an OFDM transmission system. The aim is to minimize the total allocated bandwidth while guaranteeing a certain transmission data rate to each user, under the constraint of a total transmission power.

We begin, in section 2, by a description of the overall downlink transmission system using OFDM. Then, in section 3, we present two classical approaches for spectrum management based on Frequency Division Multiple Access (FDMA) and Time Division Multiple Access (TDMA). After discussing the disadvantages of these approaches, we explain how the spectrum allocation can be optimized by a proper formulation of a combinatorial assignment problem. Since exact solutions of this problem are impossible to obtain, we present, in section 4, a solution to this problem based on the Hungarian approach (or Munkres algorithm). Then, we propose an enhanced version of this solution in section 5. A quasi-optimal solution is investigated in section 6, based on a simulated annealing approach. A comparative analysis of the simulation results as well as the computational complexity of the different algorithms can be found in section 7. Finally, section 8 is an overture to different applications of our greedy approaches. For this reason, two application examples are presented: optimization of the terminals autonomy in a Wireless Sensor Network and optimization of a multi-user video streaming system where source and channel encoded video sequences are transmitted using OFDM.

2. Description of the OFDM downlink transmission system

The system consists of K mobile users, each requesting a download data rate R_k ($k = 1, …, K$) and uniformly distributed over the cell coverage area. We assume that all users have access to all time slots and that a given subchannel of the OFDM uplink system is allocated to only one user in each time slot (subchannels sharing is not allowed).

After demultiplexing of each original user's binary sequence, subcarriers' symbols are modulated and transformed in the time domain by inverse fast Fourier transform (Figure 1). Cyclic extension is then added (IEEE, 1999) before pulse shape filtering. After transmission through a frequency selective Rayleigh fading channel, each subcarrier will experience a different channel gain $H_{k,n}$ in the frequency domain (Haykin, 2001). In the case of a multiuser system, these channel gains will constitute a channel matrix as in Figure 2, since each channel is seen "from a different point of view", depending on the user to which it is attributed. We assume that the base station receives channel information from all users and can perfectly estimate the channel state on each subcarrier using pilots inserted in a scattered manner in the transmitted OFDM symbols (IEEE, 1999).
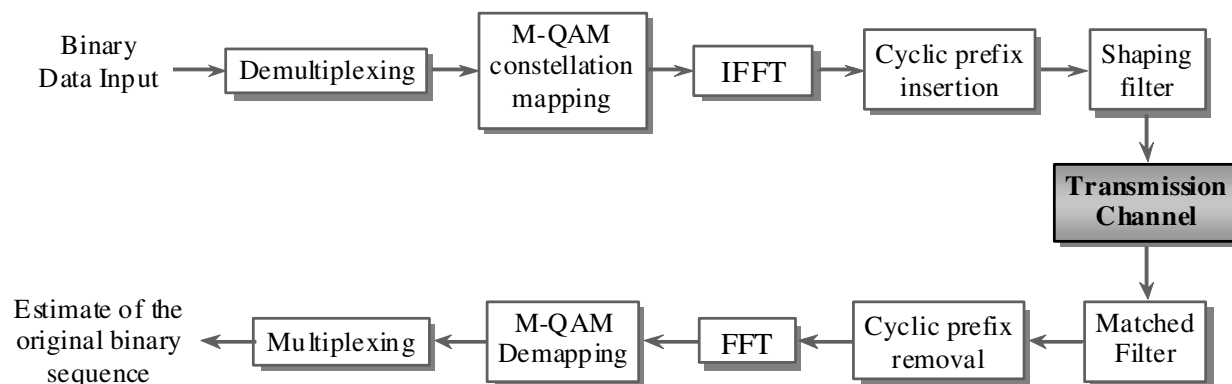

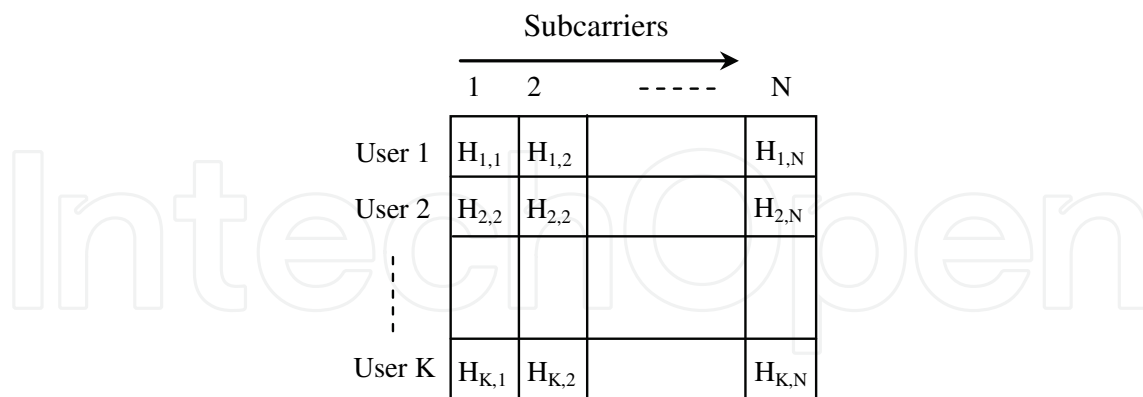Fig. 1. Overall baseband model of an OFDM transceiver.

Fig. 2. Channel matrix of the downlink multiuser OFDM system.

The following notations will be used throughout the paper:
- N is the maximum number of available subchannels which form a set S.
- B is the total system bandwidth.
- S_k is the set of subcarriers allocated to user k.
- P_{max} is the maximum allowed transmission power by the base station.
- $P_{k,n}$ is the power transmitted on the subcarrier n allocated to the user k.

- N_0 is the power spectral density of the additive white Gaussian noise (assumed to be constant over all subcarriers).

3. Classical OFDM spectrum allocation approaches and problem formulation

3.1 OFDM-FDMA approach

In this approach, which uses FDMA in conjunction with OFDM, users are treated sequentially in time (Figure 3): for each user, subchannels are allocated one by one until the user transmission rate becomes at least equal to the target data rate R_k. In this strategy, subchannels are assigned to users without any consideration for the users channel state information. Subcarrier gains are only taken into account in the calculation of the users actual data rates while the transmission powers $P_{k,n}$ are all equal to the same constant, independently from the subcarriers or the users. If we suppose that all users transmit at their capacity limit, the kth user's total transmission rate, at a certain stage of the allocation process, can be written as:

$$R_{k,tot} = \sum_{n \in S_k} \frac{B}{N} log_2 \left(1 + \frac{P_{k,n} H_{k,n}^2}{N_0 \frac{B}{N}} \right)$$

Iterative algorithm for subcarrier allocation in an OFDM-FDMA system

```
        n = 1                              // Initialization
        k = 0
(L1)    k = k + 1
        R_{k,tot} = 0
        while (R_{k,tot} < R_k)
            If (n > N)                     // Test if S is empty
                Break
            End If
            S_k = S_k ∪ {n}                //Attribute subcarrier n to user k
            S = S ∩ {n}^C                  // Remove n from the subcarriers set S
            P_{k,n} = P_max / N            // Equal power allocation
```

$$R_{k,tot} = \sum_j \frac{B}{N} log_2 \left(1 + \frac{P_{k,j} H_{k,j}^2}{N_0 \frac{B}{N}} \right) \qquad \text{// Estimate the user's actual transmission rate}$$

```
            n = n + 1
        End while
        If (S = φ)                         // Test if S is empty
            Return                         // User target rate cannot be reached: End the attribution process
        End If
        If (k = K)
            Return                         // All users target rates were reached: End the attribution process
        Else
            Goto L1
        End If
```

Fig. 3. Description of the iterative subcarrier allocation algorithm in OFDM-FDMA.

3.2 OFDM-TDMA approach

Classically, in an OFDM system using TDMA (Rohling & Grunheid, 1997), the spectrum
bandwidth is entirely allocated to only one user during a certain period of time. The
transmission frame is divided into a number of time slots equal to the number of users K.
However, in order to permit a fair comparison with the optimized greedy approaches, and
for the respect of spectrum etiquette (as explained in section 1), the user must only be
assigned subcarriers that can actually increase its transmission rate. For this purpose, each
step of the allocation algorithm consists (Figure 4) in assigning the user currently under
consideration the best possible subcarrier, i.e. the subcarrier with the highest channel gain
$H_{k,n}$. The subcarrier is then removed from the set of available subcarriers S, and power
allocation is applied to the subcarriers so far allocated to the user (subcarriers in set S_k). This
process is iterated until the user's target rate is achieved, unless the set S is empty.

$$n_c = \arg\max_{n \notin S} H_{k,n}$$

$$S_k = S_k \cup \{n_c\}$$
$$S = S \cap \{n_c\}^C$$

Power allocation for user k on its
so far attributed subcarriers

Estimation of $P_{k,n}, \quad \forall n \in S_k$

$$R_{k,tot} = \sum_{n \in S_k} \frac{B}{N} \log_2 \left(1 + \frac{P_{k,n} H_{k,n}^2}{N_0 \frac{B}{N}} \right)$$

$R_{k,tot} > R_k$ — Yes → User target rate is reached:
End the attribution process

No

$S \neq \varnothing$ — Yes

No

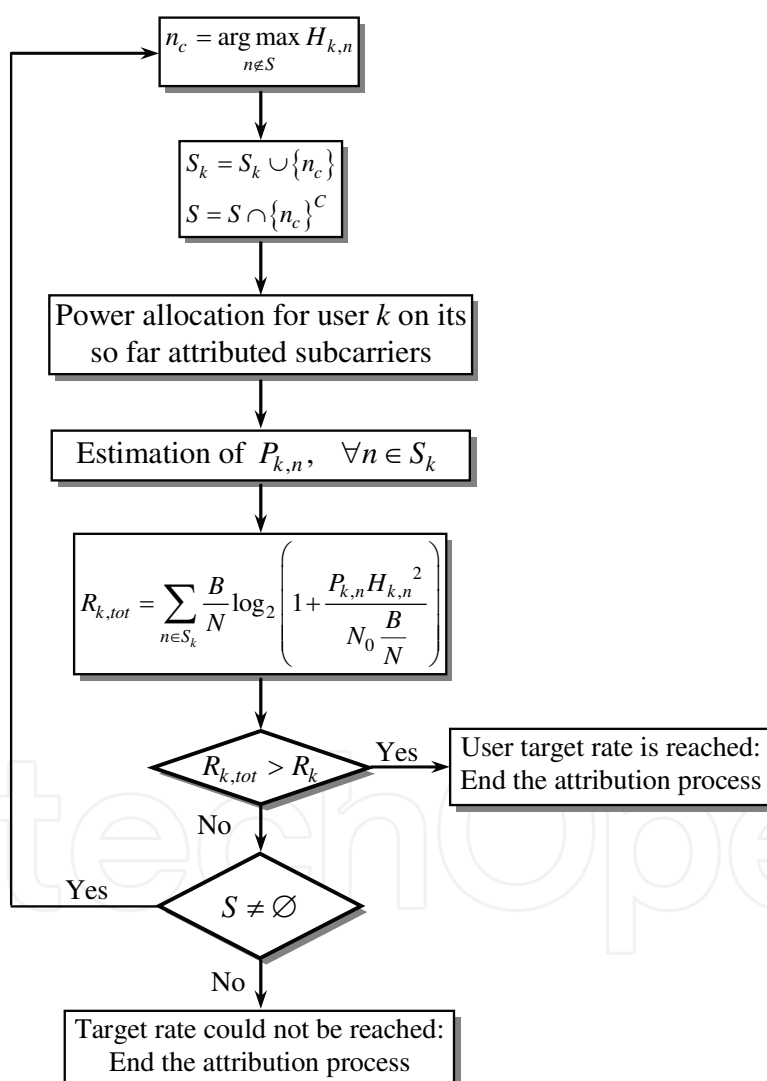Target rate could not be reached:
End the attribution process

Fig. 4. Subcarrier allocation for user k using OFDM-TDMA.

As for the power allocation in OFDM-TDMA, it consists on a distribution of the total
transmission power P_{max} on the user k allocated subcarriers. In other words, $\{P_{k,n}, \ n \in S_k\}$
are to be determined in such a way to maximize the total transmission rate for user k, under
the transmission power constraint:

$$\max_{\{P_{k,n}, n \in S_k\}} \sum_{n \in S_k} \frac{B}{N} log_2 \left(1 + \frac{P_{k,n} H_{k,n}{}^2}{N_0 \frac{B}{N}} \right),$$

under the constaints:

$$P_{k,n} \geq 0, \forall k, \forall n \in S$$

$$\sum_{n \in S_k} P_{n,k} \leq P_{max}$$

It can be proven (Haykin, 2001; Cioffi, 2008) that the solution to this constrained optimization problem is:

$$\begin{cases} P_{k,n} + v_n = \alpha, & \forall n \in S_k \\ \sum_{n \in S_k} P_{n,k} = P_{max} \end{cases}$$

where α is a constant and $v_n = \dfrac{N_0 \cdot B / N}{H_{k,n}{}^2}, \quad \forall n \in S_k$.

An illustration of this solution is given in Figure 5 for the example of five subcarriers attributed to user k.
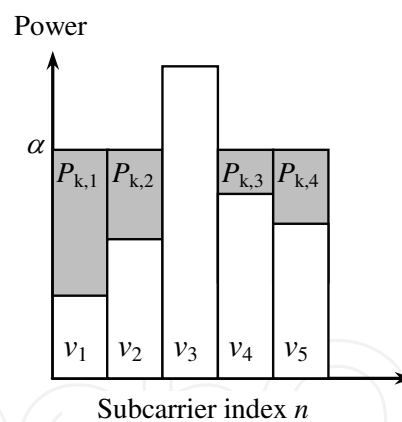

Fig. 5. Representation of the water-filling solution to the power allocation problem in OFDM-TDMA.

This graphical interpretation clearly justifies the name of "water-filling" given to the power allocation technique. It can be seen that highly attenuated subcarriers (small $H_{k,n}$) will be allocated much less power than subcarriers with great amplitudes, therefore the transmission rate on those subcarrier will be much smaller than on others. In case v_n is higher than α (ex: subcarrier 3), no power will be allocated to this subcarrier (i.e. corresponding data rate is null).

In the sequel, we propose a gradual water-filling algorithm for the realization of power allocation. For this purpose, let:

w the current waterline level,

P_{tot} the total transmission power for the current waterline,

P_{tol} the absolute tolerable error on the total transmission power,

l_k the number of subcarriers so far allocated to the user k (l_k is the number of elements in the set S_k), and

$v_{n,min}$ the vector having the smallest amplitude among the vectors v_n, $\forall n \in S_k$.

At the beginning of the power allocation algorithm, the transmission powers $P_{k,n}$ are all set to zero. Water-filling is performed in an iterative way (Figure 6) such that the absolute difference between the powers P_{tot} and P_{max} does not exceed the tolerable error P_{tol}. The initialization of the waterline is chosen in such a way that, in case all vectors v_n are equal, the waterline remains constant and the amount of power attributed to each user is P_{max} / l_k.

$$w = v_{n,min} + P_{max}/l_k$$

Waterline initialization

$$P_{tot} = \sum_{n \in S_k} \max\left(w - v_n, 0\right)$$

$$w = w + \frac{P_{max} - P_{tot}}{l_k}$$

$$P_{tot} = \sum_{n \in S_k} \max\left(w - v_n, 0\right)$$

$$\left|P_{max} - P_{tot}\right| > P_{tol} \quad \text{Yes}$$

No

End the waterfilling process:
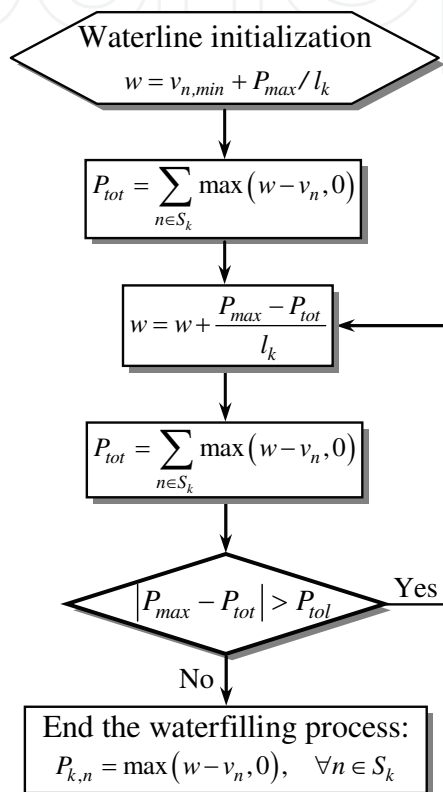$$P_{k,n} = \max\left(w - v_n, 0\right), \quad \forall n \in S_k$$

Fig. 6. Water-filling technique for power allocation in OFDM-TDMA.

3.3 Disadvantages of the classical approaches and formulation of the optimization problem

By analyzing the OFDM-FDMA assignment, we notice that a major drawback of this technique is that a subcarrier n can be assigned to a certain user k_1 while there exists a user k_2 for whom the attribution of this subcarrier would be much more profitable ($H_{k_2,n} > H_{k_1,n}$).

In other words, the additional user rate obtained by the attribution of subcarrier n to user k_1 can be negligible compared to the one that could be achieved by its attribution to user k_2. In our attempt to reach each user's target rate, a large number of subcarriers and a high transmission power will be needed. Furthermore, since users are processed in order, subsequent ones will have smaller chances to reach their target rate. This problem will appear more often as the users target rates increase.

As for the OFDM-TDMA attribution scheme, one of its disadvantages is that each user actually transmits data at a rate of $K \cdot R_k$ [bit/s] during one time slot of the transmission

frame and remains inactive during the other time slots. This effect can be disturbing in the case of real-time applications because of the high latency, especially when the number of users increases. Another important disadvantage is that the necessary number of sub-carriers at a certain moment can reach important values, particularly for high user rates or short time slot durations. Therefore, the risk of failing to satisfy all users target rates is very high.

These disadvantages of the OFDM-FDMA and OFDM-TDMA assignment techniques can be greatly alleviated if the subcarriers were allocated to the users in such a way to take into account the channel states of all users. Therefore, the prohibitive spectrum usage necessitated by those simple techniques, especially for important target rates, can be overcome by applying a dynamic assignment strategy that aims at minimizing the total number of allocated subcarriers under the constraints of the target data rates and the maximum allowed transmission power.

The corresponding optimization problem can be formulated as follows:

$$\min_{P_{n,k},S_k} \sum_{k=1}^{K} card\left(S_k\right)$$

subject to the following constraints:

$$\sum_{n \in S_k} \frac{B}{N} log_2 \left(1 + \frac{P_{k,n}H_{k,n}^{\;2}}{N_0 \frac{B}{N}}\right) = R_k, \forall k$$

$$\sum_{k=1}^{K} \sum_{n \in S_k} P_{n,k} \leq P_{max}$$

$$P_{k,n} \geq 0, \forall k, \forall n \in S$$

$$S_i \bigcap_{i \neq j} S_j = \varnothing$$

$$\bigcup_{k=1}^{K} S_k \subseteq \{1,2,...N\}$$

The first constraint specifies the target transmission rate per user. The second and third conditions specify the power constraints. The last two conditions specify the maximum number of allocated subcarriers and that each subcarrier can be allocated to only one user at a certain time.

We can clearly note that the optimization problem formulated above is a combinatorial assignment problem since set selection is involved. Therefore, it does not form a convex problem. In the literature, several attempts have been made to transform it into a convex optimization problem. In (Kim et al., 2004), specifications were relaxed by introducing a new parameter representing a portion of a subchannel assigned to a user. In (Wong et al., 1999), time sharing of a subcarrier among different users is considered. In either case, the solution obtained constitutes a lower bound to the combinatorial optimization problem. However, a general formulation of this solution is not obvious and since it only provides a lower bound, it is preferable to strive after a quasi-optimal solution to the real assignment problem.

In the following, we describe several possible strategies to determine a quasi-optimal solution of the combinatorial optimization problem, using a greedy approach.

4. Greedy technique for the optimization of spectrum resources using the Munkres algorithm (GOSM)

In order to determine the best system configuration for the optimization problem presented in section 3, we came out with a greedy iterative algorithm that determines the best spectrum allocation scheme by applying a separate optimization algorithm that assigns the subcarriers to the users in such a way to minimize a cost function. This assignment is then followed by power allocation. The optimization algorithm is the well-known Munkres assignment algorithm (Munkres, 1957), also known by the Hungarian method. In our application case, the cost function is the sum of the opposite channel amplitudes $-H_{k,n}$ and it is applied independently from the users actual transmission rates.

At a certain stage of the optimization algorithm, we consider:
U: the set of users whose target data rates have not been reached so far,
k_U: the number of users in U,
S_U: the set of subcarriers attributed to the users in the set U,
l_U: the number of subcarriers in S_U,
P_{rem}: the remaining allowed transmission power after a user has reached its target rate,
P_{tot}: the total transmission power for the current waterline, corresponding to the users in the set U,
P_{tol}: the absolute tolerable error on the total transmission power,
R_{tol}: the required precision on the users target data rates.

At the beginning of the greedy channel allocation, the transmission powers $P_{k,n}$ are all initialized to zero and P_{rem} is initialized to P_{max}.
Our proposed greedy iterative algorithm (Figure 7) can be described as follows:
In each iteration, the Munkres algorithm is used to allocate a subcarrier n_k to each user k that has not reached so far its target data rate R_k (users in the set U). The allocated subcarriers are removed from the set S. Then, water-filling is applied on the allocated subcarriers, using the available remaining power P_{rem}. The water-filling is performed by taking into account only users in the set U. After water-filling, the transmission power is estimated on all allocated subcarriers as well as the actual total transmission rate $R_{k,tot}$ for each user k in the set U. If $R_{k,tot}$ is higher than the target rate R_k, the transmission power on the allocated subcarrier for user k with the least channel amplitude has to be reduced in such a way to adjust the total rate to the exact target rate. Finally, user k is removed from U and the remaining power P_{rem} is updated. The algorithm is iterated with the remaining users, unless the set S of available subcarriers is empty.
By analyzing this allocation technique, it can be seen that, in each iteration, a local optimum is chosen in the hope of reaching the global optimum at the output of the overall algorithm. Therefore, it is indeed a greedy allocation algorithm.
As for the adjustment of the transmission rate for user k before its removal from the set U, it is realized using the following equations:

$$m = \arg\min_{n \in S_k} H_{k,n}$$

$$R_m = R_{k,tot} - \frac{B}{N} log_2 \left(1 + \frac{P_{k,m}}{v_m} \right)$$

$$P_{k,m} = \left(2^{(R_k - R_m).N/B} - 1 \right).v_m$$

Finally, water-filling is performed in the same manner as in Figure 6 except that P_{max} is replaced by P_{rem}, l_k by l_U, and S_k by S_U.
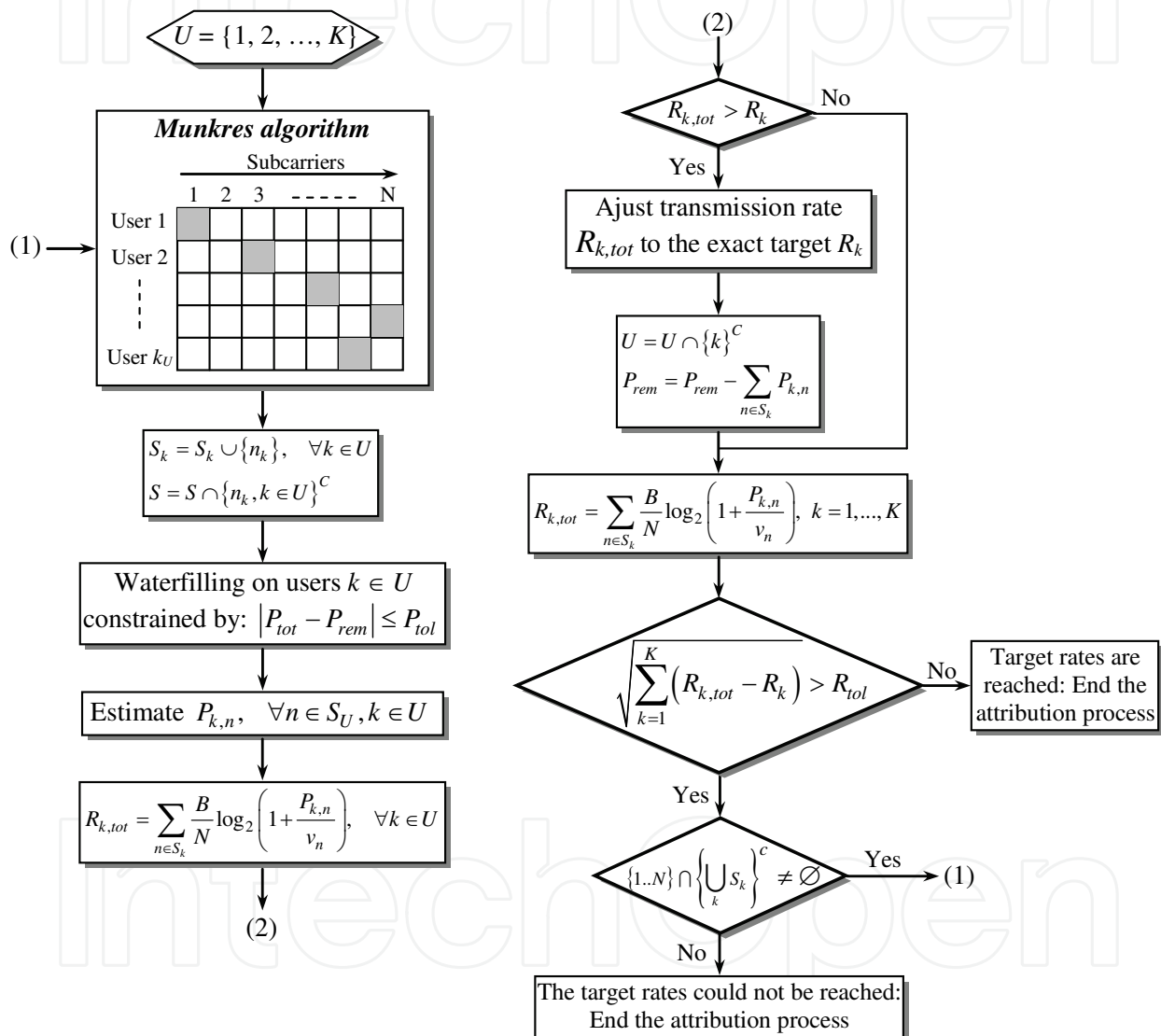

Fig. 7. Greedy iterative technique for dynamic spectrum allocation using the Munkres algorithm (GOSM).

5. Enhanced greedy algorithm for spectrum optimization (EGAS)

As it will be seen in section 7, the GOSM allocation technique has the disadvantage of attributing subchannels to the users without taking into account their actual transmission rates. For this reason, we propose the following enhanced greedy algorithm for dynamic spectrum allocation:

Step 1:   start by identifying the user k_c, whose actual total transmission rate is the farthest
from its target data rate.

$$k_c = \underset{k \in U}{arg\ max}\left(R_k - R_{k,tot}\right).$$

Step 2:   attribute to this user the most favorable subcarrier n_c.

$$n_c = \underset{n \notin \bigcup_k S_k}{arg\ max}\ H_{k_c,n}.$$

$$S_{k_c} = S_{k_c} \cup \left\{n_c\right\}$$

Step 3:   remove n_c from the set S.

$$S = S \cap \left\{n_c\right\}^C$$

Step 4:   perform water-filling on the allocated subcarriers for all users in the set U, using the
available remaining power P_{rem}.

Step 5:   estimate the transmission power on all allocated subcarriers as well as the actual
total data rate for user k_c ($R_{k_c,tot}$).

Step 6:   Check if $R_{k_c,tot}$ exceeds the target rate R_{k_c}. If yes, adjust the transmission power of
user k_c on the subcarrier with the least channel amplitude to reach the exact target
rate R_{k_c} (as described earlier in section 4) and go to *Step 7*. Otherwise, go to *Step 8.*

Step 7:   Remove user k_c from the set U and update the remaining power P_{rem}.

Step 8:   Evaluate all users' actual rates. End the attribution process in case the target rates
have been reached with a precision R_{tol}. Otherwise, loop on *Step 1*, unless no more
subcarriers are available (in this case, the attribution process has failed to satisfy the
users target rates).

6. Optimization of the EGAS algorithm by simulated annealing (SAS)

In the aim of determining an optimal solution to the combinatorial problem presented in
section 3.3, one can think of an exhaustive search that would constitute a lower bound to the
cost function under consideration. Unfortunately, due to the large number of parameters
and constraints involved in the optimization problem, such a strategy appears to be of an
impractical use, especially when the ranges of the system variables (i.e. the number of
subcarriers and users) increase.

On the other side, in the greedy approaches we applied for resolving our optimization
problem, the search can unfortunately get stuck in a local optimum, especially when the
global optimum is hidden among many other poorer local optima. In order to overcome this
difficulty, one possibility is to carry out the iterative process several times starting from
different initial configurations. A similar strategy was applied in statistical mechanics
(Metropolis et al., 1953) for determining the ground state of systems by a simulated
annealing process. In (Kirkpatrick et al., 1983), it was shown how the analogy between

statistical mechanics and multivariate or combinatorial optimization can be exploited to resolve optimization problems of large scales.

This technique, also known by the Metropolis algorithm, simulates an aggregation of atoms in equilibrium at a certain temperature. The cost function is the system energy E. In each step of the simulation algorithm, an atom is randomly selected and displaced. If the resulting change in the system energy ΔE is negative, the modification in the system configuration is accepted, and this new configuration is retained as the initial state of the following iteration. If ΔE is positive, the modification is accepted with probability $\Pr(\Delta E) = \exp(-\Delta E/K_B\lambda)$, where λ is the system temperature and K_B the Boltzmann's constant. Therefore, the system will evolve into a Boltzmann configuration.

As for the temperature, it is used as a control parameter in the same unit as the cost function. When large, the control parameter allows the system to make transitions that would be improbable at low temperatures. Therefore, an annealing process is simulated by first "melting" the system and making coarse changes in the system configuration, and then gradually "freezing" the system by lowering the temperature to develop finer details until the optimal structure is attained.

In this study, we applied the Metropolis algorithm subsequently to the EGAS procedure presented in section 5. The temperature parameter is gradually decreased throughout the iterations.

In each iteration, a first step consists in a random selection of one of the three following possible actions:

Action 1: We randomly select two users k_1 and k_2 and interchange two random subcarriers between the two users. Next, a water-filling is realized separately for each of the two users, on their allocated subcarriers. The water-filling procedure is constrained by the user's target data rate and will be described in the sequel.

Action 2: A randomly chosen subcarrier is removed from a random user k. Water-filling is then performed for k on its allocated subcarriers, with a constraint on its target rate.

Action 3: A free subcarrier is randomly chosen and attributed to a random user k. Water-filling is then performed for k on its allocated subcarriers, with a constraint on its target rate.

The next step is to decide whether the action is accepted. For this purpose, we estimate the new total number of attributed subcarriers $L = \sum_{k=1}^{K} card(S_k)$ as well as the total transmission power $P = \sum_{k=1}^{K} \sum_{n \in S_k} P_{n,k}$. The action is accepted only in the three following possible cases:

Case 1: The total number of subcarriers L was decreased while the constraint on the total transmission power was still respected ($P \leq P_{max}$).

Case 2: The total transmission power P was decreased while maintaining the same number of subcarriers L.

Case 3: Neither the total number of subcarriers L nor the total transmission power P could be decreased. However, $P \leq P_{max}$. In this case, the action is accepted with probability $\Pr(\Delta L) = \exp(-\Delta L/K_B\lambda)$, where ΔL is the increase in the number of subcarriers.

Note that when an action is accepted, the actual system configuration, i.e. the allocation scheme of the subcarriers to the users, is adopted as the initial condition for the subsequent iteration. Besides, due to the stochastic nature of this procedure, the algorithm has to be executed several times in order to increase the chance of finding a global optimum.

As for the water-filling, it is constrained by the user data rate, instead of the available power as in section 3.2. It will be realized using a gradual dichotomy-based approach as described hereafter, where w_{step} is the current waterline step, l_k the number of subcarriers allocated to a user k, etc.
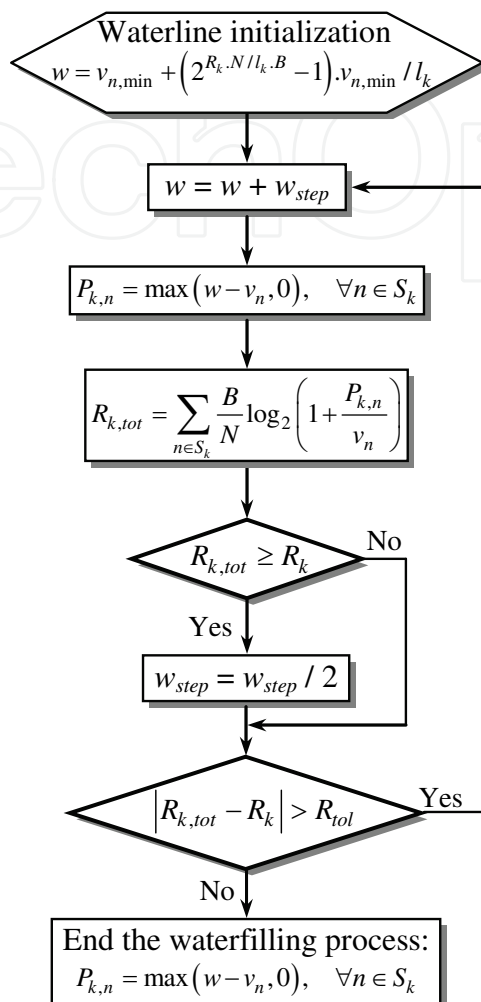

Fig. 8. Gradual dichotomy-based water-filling for the SAS algorithm.

As it can be seen in Figure 8, the waterline is increased using a variable step such that the absolute difference between the achieved data rate and the user's target rate does not exceed R_{tol}. As for the waterline initialization, it is chosen in such a way to satisfy the data rate constraint in the case where all subcarriers have the same SNR. This SNR's value is taken as the highest one among all subcarriers.

7. Performance analysis of the different allocation techniques

7.1 Simulation conditions

The performance of the allocation techniques for spectrum optimization were obtained by simulating a downlink OFDM system with a bandwidth B = 100 MHz and a maximum number of 1024 subcarriers per OFDM symbol. The simulations were conducted in the following conditions:

- The number of users ranges from 10 to 60.

- The total permissible transmission power (by the base station) is P_{max} = 1000 mW.
- The noise power spectral density, constant over all subcarriers, is N_0 = 4·10^{-18} Watt/Hz.
- The absolute tolerable error on the user target rate is R_{tol} = 10^{-3} bit.
- The absolute tolerable error on the total transmission power is P_{tol} = 10^{-5} mW.

The transmission medium is a frequency-selective Rayleigh fading channel with a root mean square delay spread (RMS) of 150 or 500 nanoseconds. The users geographic locations are uniformly distributed in a 10 Km radius cell with a maximum path loss difference of 20 dB between users.

The performance of the different methods is compared in terms of the average total number of allocated subcarriers. In the case of the OFDM-TDMA approach, we measured the median and the maximum of the necessary number of subcarriers (Median OFDM-TDMA and Max OFDM-TDMA).

In Tables 1 and 2, we represent the total number of subcarriers for different user data rates, whereas in Table 3, the results are represented as a function of the number of users. In Tables 1, 2 and 3, we consider that the requested data rates are the same among users. However, in Table 4, the results are obtained for different target rates between users:

$$R_k = R_0 \cdot 0.2 + 0.2 \cdot (k\text{-}1), \ k = 1, ..., K,$$

where R_0 is chosen such that, for K = 20 users, the total transmission rate ranges from 48 to 88 Mbit/s.

7.2 Analysis of the practical results

Rate (Mbit/s)	1	2	3	4	5	6	7	8	9	10
OFDM-FDMA	234	420	529	737	852	-	-	-	-	-
Max OFDM-TDMA	17	41	82	142	253	-	-	-	-	-
Median OFDM-TDMA	13	34	61	94	139	221	501	-	-	-
GOSM	19	36	58	81	111	151	206	249	-	-
EGAS	19	35	56	79	109	146	193	224	312	360
SAS	16	33	55	78	108	144	192	221	307	355

Table 1. Total number of subcarriers necessary to achieve different users data rates for K = 10, channel RMS = 150 ns.

Rate (Mbit/s)	1	2	3	4	5	6
OFDM-FDMA	362	689	-	-	-	-
Max OFDM-TDMA	43	144	-	-	-	-
Median OFDM-TDMA	33	89	193	-	-	-
GOSM	39	81	137	225	350	-
EGAS	40	78	134	214	306	414
SAS	38	76	130	211	303	407

Table 2. Total number of subcarriers necessary to achieve different users data rates for K = 10, channel RMS = 500 ns.

Number of users	10	20	30	40	50	60
OFDM-FDMA	196	455	633	799	-	-
Max OFDM-TDMA	16	44	87	148	334	-
Median OFDM-TDMA	14	34	57	89	139	205
GOSM	19	40	66	95	131	164
EGAS	17	39	71	121	147	177
SAS	16	37	57	88	123	153

Table 3. Total number of subcarriers necessary to achieve a data rate of 1 Mbit/s for different numbers of users.

Total Rate (Mbit/s)	48	58	68	78	88
Median OFDM-TDMA	106	148	268	334	-
GOSM	109	143	189	248	-
EGAS	106	134	169	211	245
SAS	103	131	167	208	243

Table 4. Total number of subcarriers as a function of the total transmission rates for the case of different classes of services between users ($K = 20$).

We can see that our greedy strategies for spectrum optimization clearly outperform the OFDM-FDMA approach in both cases of the channel RMS. For RMS = 150 ns, starting from $R_k = 6$ Mbit/s, the OFDM-FDMA technique fails to satisfy the users target rates under the transmission power constraint. At $R_k = 5$ Mbit/s and $K = 10$, the gain of the three iterative techniques (GOSM, EGAS, and SAS) towards the OFDM-FDMA reaches almost 750 subcarriers (more than 70 % of the available subcarriers). In order to allow a fair comparison, we also tested the performance of the OFDM-FDMA technique in case the subchannels are randomly allocated to users in order to avoid the attribution of several neighboring subcarriers in deep fade to the same user. We noticed that the interleaved OFMA-FDMA technique outperforms the non-interleaved OFDM-FDMA by less than 10 % of the total number of subcarriers. Besides, when the channel RMS increases to 500 ns, both the OFDM-FDMA and OFDM-TDMA techniques fail to satisfy users starting from $R_k = 3$ Mbit/s.

On the other hand, at $R_k = 5$ Mbit/s and $K = 10$, the gain of our greedy techniques is approximately 300 towards the Median OFDM-TDMA. Starting from 9 Mbit/s at RMS = 150 ns and 6 Mbit/s at RMS = 500 ns, even the GOSM technique fails to determine a possible solution for the allocation problem, whereas the EGAS continues to perform well and presents similar results to the quasi-optimal SAS. In fact, the SAS technique permits an amelioration of the system performance, especially when the number of users is important (Tabe 3).

When the users present different data rates, the EGAS approach outperforms the GOSM, especially when the total transmission rate increases. Indeed, as explained in section 5, the EGAS greedy algorithm allows the user whose actual transmission rate is far from its target rate the "right to choose" a favorable subcarrier. Whereas in the GOSM method, all users that have not reached their target rate are allocated a subcarrier at each iteration, regardless of their actual transmission rate, even the ones that are close to reaching their target. This will lead to a higher global amount of allocated subcarriers.

However, as the number of users increases, the GOSM tends to present a slightly better performance than the EGAS (Table 3). This is due to the fact that when the number of

subcarriers per user decreases, there is a higher chance that certain subcarriers are favorable to more than one user. In these conditions, the application of an optimal assignment of the subcarriers by the Munkres algorithm can improve the overall system throughput. As the number of users approaches N, the performance of the GOSM algorithm will tend to be optimal.

As a conclusion to this comparative study, the GOSM technique has the advantage of optimizing the subcarriers allocation between users; however, it does not take into account the users actual transmission rates in the optimization process. Not only does the EGAS take these rates into account, but it also presents a much more affordable complexity towards the GOSM, as will be proven in section 7.3.

7.3 Analysis of the algorithms complexity

Most of the previously presented algorithms for spectrum allocation largely use the water-filling block which constitutes one of the most constraining part in terms of complexity. However, the direct estimation of the complexity of the iterative water-filling procedure presented in section 3.2 is rather impractical, mainly because of its dependence on the tolerance parameter P_{tol}. For this reason, we will replace it, in this part, by the exact water-filling distribution that can be derived as the solution of a linear system of N+1 equations:

$$P_{k,n} + v_n = \alpha, \qquad n = 1,...,N$$

$$\sum_{n=1}^{N} P_{k,n} = P_{max}$$

with N+1 unknowns ($P_{k,n}$ and α).
A simple algorithm was proposed in (Cioffi, 2008) to solve this system. It is summarized in Figure 9.

Fig. 9. Estimation of the water-filling solution for power allocation.

In fact, the sorting step is performed only once. Therefore, the water-filling algorithm complexity is $O(N \cdot \log(N))$ if the sorting step is taken into account and $O(N)$ if not. The latter case will be considered in the sequel.

Now that the water-filling complexity has been studied, we can proceed with the complexity of all spectrum allocation algorithms.

First of all, OFDM-FDMA is basically a loop on each subcarrier; hence, its complexity is $O(N)$.

OFDM-TDMA is a loop on each subcarrier, with a water-filling power allocation each time a new subcarrier is allocated to the user. The number of subcarriers involved in the water-filling procedure is successively 1, 2, …, N. The complexity is therefore $O(N^2)$. However, since the algorithm must be run sequentially for each user, the total complexity of OFDM-TDMA is $O(K \cdot N^2)$.

On the other hand, the complexity of the GOSM technique is dominated by the Munkres assignment algorithm which has a complexity $O((\min(N,K))^2 \cdot \max(N,K))$ (Burgeois, 1971). It assigns K subcarriers at each stage. Since $K<N$, the total complexity of GOSM is $O(N/K \cdot K^2 \cdot N)=O(K \cdot N^2)$.

As for the EGAS technique, a new power allocation distribution (i.e. a water-filling step) is realized for each allocated subcarrier, leading to a total complexity of $O(N^2)$.

Finally, each iteration of the SAS algorithm consists of at least a water-filling step. Therefore, its complexity is approximately $O(n_{iter} \cdot N)$, where n_{iter} is the number of iterations in the SAS algorithm.

Since in general $n_{iter} \gg K \cdot N$, it can be seen from Table 5 that the OFDM-TDMA and GOSM approaches present a similar complexity, which is much smaller than the one for the SAS algorithm, but higher than that of the EGAS approach.

Algorithm	OFDM-FDMA	OFDM-TDMA	GOSM	EGAS	SAS
Complexity	$O(N)$	$O(K \cdot N^2)$	$O(K \cdot N^2)$	$O(N^2)$	$O(n_{iter} \cdot N)$

Table 5. Complexity of the different spectrum allocation approaches.

8. Applications of the greedy spectrum allocation approach in two case studies

8.1 Optimization of wireless sensors' autonomies by greedy spectrum allocation in uplink transmission

Wireless Sensor Networks have lately gained a great deal of attention in the areas of video transmission, surveillance, remote monitoring, etc. In these applications, a certain number of sensors transmit data simultaneously, on a shared medium, to a central base station. Therefore, the terminals batteries are highly solicited by the multimedia functionalities, the radio-frequency part, the real-time execution of increasingly complex algorithms and tasks, etc. Hence, stringent requirements have been put on the wireless terminal battery in order to offer a proper autonomy.

Efficient techniques of dynamic spectrum allocation can help improve the autonomy of wireless terminals, especially in the case of the uplink transmission, where the power amplifier particularly solicits the sensor's battery. For this reason, we propose to apply a greedy approach, similar to the one used in the downlink transmission, to determine the best assignment of subchannels in such a way to maximize the mobiles autonomies by efficiently managing their power consumption. This optimization will enhance the network lifetime defined as the duration of communication between mobiles before a failure occurs due to battery depletion.

Let:

P_{max} the maximum allowed power per user.

Δt the time duration over which the subchannel attribution scheme is valid (the transmission channel response is assumed to be constant over Δt)

E_k the battery level for the kth terminal.

$P_{k,n}$ the power transmitted by user k on the subcarrier n.

The optimization problem is the following:

Minimization of the power consumption of the least privileged user, i.e. the user suffering from the weakest initial battery level or the poorest channel conditions:

$$\max_{P_{n,k},S_k} \min_k \left(E_k - \Delta t \sum_{n \in S_k} P_{n,k} \right)$$

under the following constraints:

$$\sum_{n \in S_k} \frac{B}{N} \log_2 \left(1 + \frac{P_{k,n} H_{k,n}^2}{N_0 \frac{B}{N}} \right) = R_k, \forall k$$

$$\sum_{n \in S_k} P_{n,k} \le P_{max}, \forall k$$

$$P_{k,n} \ge 0, \forall k, \forall n \in S$$

$$S_i \underset{i \neq j}{\bigcap} S_j = \varnothing$$

$$\bigcup_{k=1}^{K} S_k \subseteq \{1, 2, ... N\}$$

A greedy solution for this optimization problem is summarized in Figure 10.
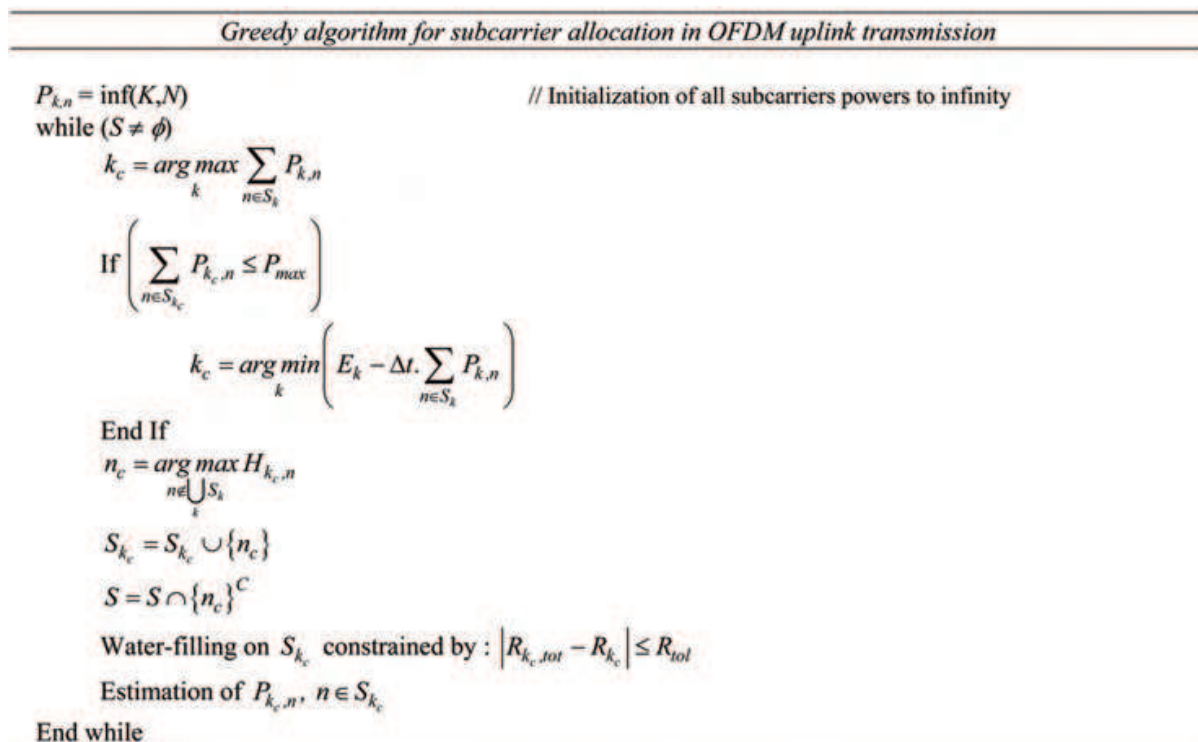
Fig. 10. Description of the greedy subcarrier allocation algorithm in OFDM uplink transmission.

The proposed algorithm can be described as follows:
While the set S of available subcarriers is non empty, we first identify the user k_c with the highest necessary transmission power. If all users required powers respect the power constraint, k_c is then the user with the weakest battery level. In other words, the power constraint has a higher priority over the optimization of the users battery levels. The second step consists in identifying the most interesting subcarrier for user k_c, among all available subcarriers, and assigning it to user k_c. Finally, we determine the power allocation scheme for user k_c in order to reach its target bit rate with an absolute tolerable error R_{tol}. The power allocation for user k_c is realized by performing water-filling on its so far allocated subcarriers. This water-filling is performed using the gradual dichotomy-based approach described in section 6.

In (Farah & Marx, 2007), we propose several enhanced versions of this greedy uplink spectrum allocation approach. We also prove the high superiority of the greedy solution to the classical OFDM-FDMA approach. The gain in the power consumption of the least privileged user is considerable, especially when the number of sensors increases.

8.2 Multiuser video streaming using greedy dynamic channel allocation

Video streaming on demand is becoming a popular application in new generations of wireless and mobile systems. However, due to user mobility and random channel variations, existing networks cannot provide end-to-end quality of service (QoS) to a large number of users using traditional approaches of spectrum allocation. In a former work (Yaacoub et al., 2006; 2007), we showed how optimal rate allocation can improve the overall performance of a multiuser streaming system where a certain number of mobile users, sharing a certain data transmission rate, request different video sequences from a streaming server. This optimization is achieved by unequal error protection (UEP) of the compressed video streams using variable channel coding, in such a way to allocate appropriate transmission rates to users experiencing different channel conditions. This study was conducted with a limited number of users and by assuming Frequency Division Multiplexing of the different users. The results showed a significant improvement in the overall system performance compared to a traditional system where all users are allocated equal channel resources.

In the following, we propose a framework for the optimal distribution of channel resources and transmission power among a large number K of users downloading video sequences from a streaming server, in the context of an OFDM cognitive system. The application of our EGAS allocation approach will permit an optimization of the necessary total bandwidth as well as the users decoding quality.

All video sequences are supposed to be compressed and stored on the streaming server. Each sequence is divided in a certain number of Group Of Pictures (GOP), with an IPPP…P structure (i.e. a GOP consists of an intra-coded I frame followed by a fixed number of predicted P frames). We assume H.264 (ITU-T & ISO/IEC JTC1, 2003) video coding with an error-concealment strategy described as follows: if an error occurs in a decoded frame, this frame and all the following ones in the same GOP are replaced by the last correctly decoded frame.

As for UEP, it is realized by applying a set of different puncturing schemes (Rowitch & Milstein, 2000) to the same channel coder output, in such a way to vary the amount of redundancy bits used to protect different parts of the video stream. This amount will depend on the level of video motion in each part of the stream, as well as on the variable transmission channel conditions.

Let $R_{Ctot,k}$ be the total source coding rate corresponding to the GOP of a user k. $R_{Ctot,k}$ of each user is proportional to the size of the H264-compressed GOP, and therefore to the level of motion in this GOP.

At each stage of the greedy allocation algorithm (Figure 11) , the user k_c whose actual rate is the farthest from its target rate R_{Ctot,k_c} is first identified. This user is attributed the most favorable subcarrier. Then, water-filling is performed on all so-far allocated subcarriers, as in section 5, for users who have not already reached their target rates (i.e. users from the set
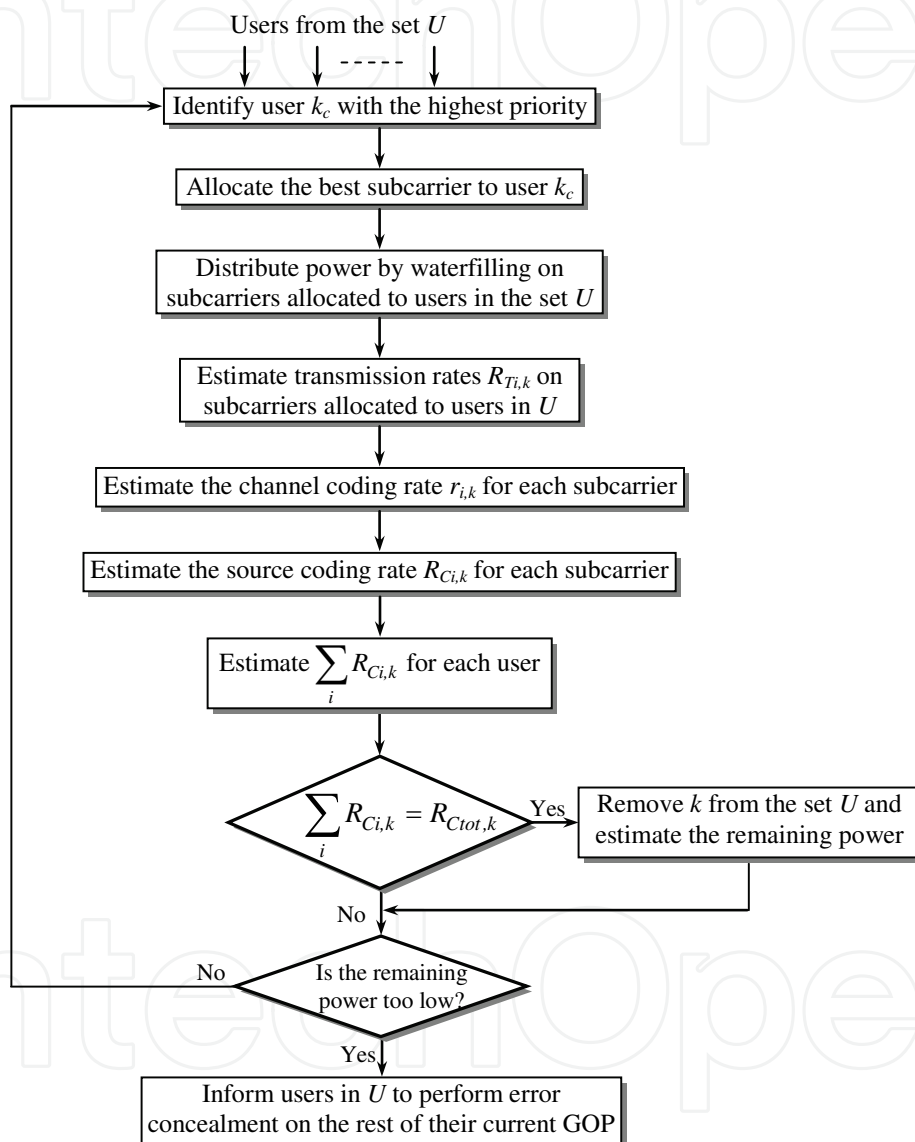
Fig. 11. Greedy channel allocation algorithm for multiuser video streaming in an OFDM cognitive system.

U). After water-filling, the actual transmission rate $R_{Ti,k}$ of each user k is estimated on each of its allocated subcarriers i. This data rate encloses the source coding rate $R_{Ci,k}$ of the GOP part of user k transmitted over the ith subcarrier of user k, as well as the channel coding rate $r_{i,k}$ ($r_{i,kj}$ <1) necessary to achieve an almost correct decoding of the coded stream transmitted on this subcarrier (for ex, with a decoding Bit Error Rate of 10^{-6}):

$$R_{Ti,k} = R_{Ci,k} / r_{i,k}.$$

Note that $r_{i,k}$ depends on the subcarrier i transmission power, on the noise power spectral density N_0, and on the subcarrier attenuation $H_{i,k}$. It can be obtained using pre-determined performance curves of the particular channel coding scheme in use.

Therefore: $R_{Ci,k} = R_{Ti,k} \cdot r_{i,k}$.

A new iteration then begins by identifying the new user k_c such that:

$$k_c = \underset{k \in U}{arg\ max}\left(R_{Ctot,k} - \sum_i R_{Ci,k} \right).$$ Note that in the first iteration: $k_c = \underset{k \in U}{arg\ max}\left(R_{Ctot,k} \right).$

At the end of the iterative process, each user will have his GOP partitioned over a certain number of subcarriers. Besides, the protection level of each part against transmission errors is realized according to the channel state of each allocated subcarrier, thus achieving Unequal Error Protection of the user downloaded stream. Indeed, the first frames in each user GOP will be sent on the best subcarriers (subcarriers with the highest amplitudes), whereas the last ones will be sent on more attenuated subcarriers, but with a higher level of protection. Moreover, users who are treated at the end correspond to those whose GOP contains a low level of motion (i.e. users with a small $R_{Ctot,k}$). It can happen that, before the end of the iterative process, the remaining transmission power (distributed by water-filling) becomes insufficient to insure an acceptable error-protection of the last frames of a few users. In this case, the remaining power will be distributed on some of those frames and users who where not able to complete the download of their GOP are informed by the base station to perform error concealment on the remaining part of their GOP (i.e. replace the last few frames in the GOP by the last correctly received frame).

9. Conclusion

In this chapter, we proposed several greedy approaches for the optimization of spectral resources in a multiuser OFDM system. Through an optimal subcarrier allocation scheme, the total amount of occupied bandwidth can be decreased in the downlink transmission, while ensuring a certain quality of service for a large number of users. Our simulations show that our approaches permit considerable gains towards classical assignment methods based on either FDMA or TDMA techniques. In fact, with our enhanced greedy subcarrier allocation technique, the risk of failing to satisfy the users rate constraints is weak compared to that of the other techniques, especially when the users target rates increase. The achieved performance is almost similar to the one obtained with a quasi-optimum technique based on simulated annealing. However, the complexity of our proposed iterative algorithms is much lower than that of the Metropolis algorithm and certainly lower than the exhaustive exploration. Several application cases can benefit from the application of our greedy iterative approaches for spectrum allocation. For this reason, we proposed two general frameworks for the optimization of power consumption in a wireless sensor network and for the optimization of the decoding quality in the context of multiuser video streaming.

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Greedy Algorithms

Edited by Witold Bednorz

Each chapter comprises a separate study on some optimization problem giving both an introductory look into the theory the problem comes from and some new developments invented by author(s). Usually some elementary knowledge is assumed, yet all the required facts are quoted mostly in examples, remarks or theorems.

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