

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

4,800

Open access books available

122,000

International authors and editors

135M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Nonlinear Control Law for Nonholonomic Balancing Robot

Alicja Mazur and Jan Kędzierski

*Institute of Computer Engineering, Control and Robotics,
Wrocław University of Technology,
ul. Janiszewskiego 11/17, 50-372 Wrocław,
Poland*

1. Introduction

In the paper a new control algorithm for special mobile manipulator, namely for nonholonomic balancing robot, has been presented. A mobile manipulator is defined as a robotic system composed of a mobile platform equipped with non-deformable wheels and a manipulator mounted on the platform. Balancing robot is in fact a mobile robot, which platform can be considered as an inverted pendulum (i.e. rigid manipulator) mounted on the axis with two conventional fixed wheels. Such the axis it is called in the literature a mobile robot with structure of unicycle (Canudas de Wit et al., 1996). The balancing robot considered in this work is presented in Fig. 1.

Taking into account the type of mobility of their components, there are 4 possible configurations of mobile manipulators: type (h,h) - both the platform and the manipulator holonomic, type (h,nh) - a holonomic platform with a nonholonomic manipulator, type (nh,nh) - a nonholonomic platform with a holonomic manipulator, and finally type (nh,h) - both the platform and the manipulator nonholonomic. The balancing robot is a mobile manipulator of (nh,h) type because nonholonomic constraints occur only in the motion of the mobile part (wheels) and the motion of the inverted pendulum (rigid manipulator with only one degree of freedom) is holonomic.

In the literature it can be found control laws for balancing robot but all solutions to this problem use either local linearization of the model (Segbot, 2004) or linear controllers (R. Chi Ooi, 2003). Such linear models and controllers are valid only local, near the desired configuration and therefore their application is limited only to stabilization of the pendulum about $\alpha_d = 0$. However, if the fully nonlinear character of the dynamics is explored, then it is possible to obtain other nonlinear control laws preserving not only point stabilization of the pendulum but the trajectory tracking, too. In this work a new nonlinear control algorithm for balancing robot guaranteeing trajectory tracking for the inverted pendulum is introduced.

This paper is organized as follows. In Section 2 a mathematical model of nonholonomic balancing robot is obtained. Nonholonomic constraints in the model come from an assumption about frictionless motion of robot's wheels. In Section 3 control problem is

formulated. Section 4 is devoted to the design of nonlinear control algorithm. The proof of the convergence of this algorithm is included. Section 5 contains simulation results which illustrate proper working of the proposed nonlinear controller. In Section 6 some conclusions are presented.

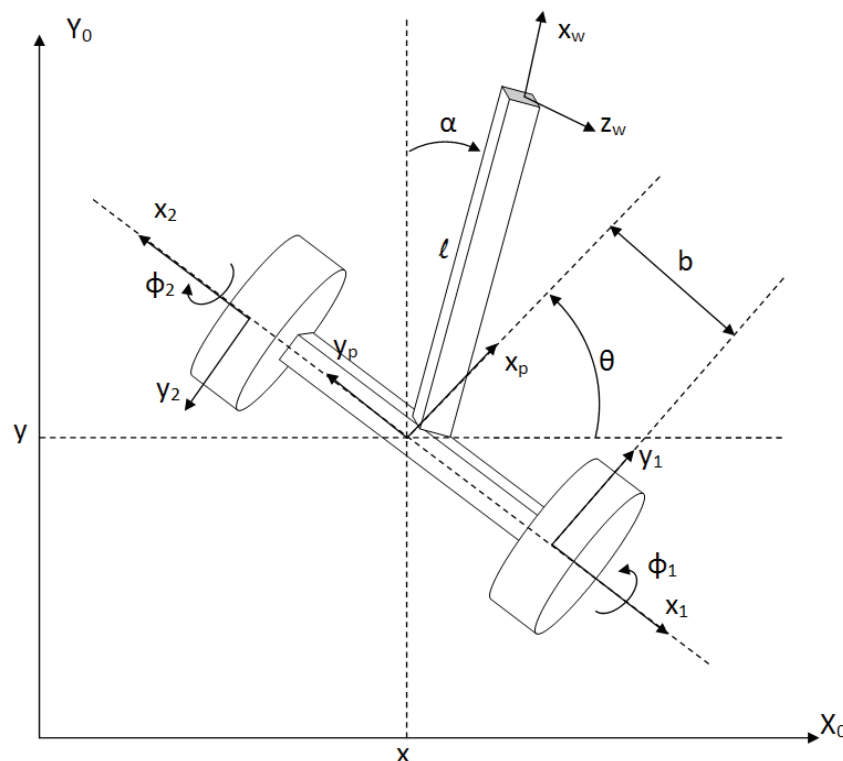


Fig. 1. Balancing robot: inverted pendulum with two fixed wheels on common axis

2. Mathematical model of nonholonomic balancing robot

We consider the mobile manipulator which consists of two subsystems, namely of rigid manipulator (inverted pendulum) and mobile platform (two fixed wheels located on common axis – unicycle).

2.1 Nonholonomic constraints

The motion of wheels can be described using generalized coordinates $q_m \in R^5$ and generalized velocities $\dot{q}_m \in R^5$.

$$q_m^T = (x \quad y \quad \theta \quad \phi_1 \quad \phi_2)$$

where (x, y) denote Cartesian coordinates of the center of the axis relative to the basic frame X_0Y_0 , θ is an angle between X_p and X_0 axis and ϕ_i is a rotation angle of i th wheel. The mobile subsystem should move without slipping of wheels. This assumption implies the existence of 3 independent nonholonomic constraints which can be expressed in the so-called Pfaff's form

$$A(q_m)\dot{q}_m = 0, \quad (1)$$

where $A(q_m)$ is a full rank matrix of (3×5) size. Due to (1), since the platform velocity is always in the null space of A , it is always possible to find a vector of special auxiliary velocities $\eta \in R^m$, $m = 5 - 3 = 2$, such that

$$\dot{q}_m = G(q_m)\eta, \quad (2)$$

where $G(q_m)$ is an 5×2 full rank matrix satisfying $A(q_m)G(q_m) \equiv 0$.

2.2 Dynamic model of the mobile manipulator of (nh, h) type

Let a vector of generalized coordinates of the mobile manipulator be denoted as

$$q = \begin{pmatrix} q_m \\ \alpha \end{pmatrix} \in R^6,$$

where $q_m \in R^5$ is a vector of generalized coordinates for the mobile platform and $\alpha \in R$ describes an angle between the inverted pendulum (axis X_w) and vertical direction. Because of the nonholonomic character of constraints, to obtain the dynamic model of the balancing robot, the d'Alembert Principle should be used

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D(q) = A(q_m)\lambda + B(q)\tau. \quad (3)$$

The model of dynamics (3) can be expressed in more detail as

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{pmatrix} \ddot{q}_m \\ \ddot{\alpha} \end{pmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{pmatrix} \dot{q}_m \\ \dot{\alpha} \end{pmatrix} + \begin{pmatrix} 0 \\ D \end{pmatrix} = \begin{pmatrix} A^T \lambda \\ 0 \end{pmatrix} + \begin{pmatrix} B \tau \\ 0 \end{pmatrix}$$

where

$$M(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix} - \text{inertia matrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} C_{11}(q, \dot{q}) & C_{12}(q, \dot{q}) \\ C_{21}(q, \dot{q}) & 0 \end{bmatrix} - \text{matrix coming from Coriolis and centrifugal forces},$$

$$D(q) = \begin{pmatrix} 0 \\ D \end{pmatrix} - \text{vector of gravity},$$

$\lambda \in R^3$ - vector of Lagrange multipliers,

$$B(q) = \begin{bmatrix} B(q_m) & 0 \\ 0 & 0 \end{bmatrix} - \text{input matrix},$$

$$\tau = \begin{pmatrix} \tau_m \\ 0 \end{pmatrix} - \text{vector of controls}.$$

The model of dynamics (3) of the (nh, h) mobile manipulator is often called a model in generalized coordinates.

Now we want to express the dynamics using auxiliary velocities (2) for the mobile platform. We compute

$$\ddot{q}_m = G(q_m)\dot{\eta} + \dot{G}(q_m)\eta$$

and eliminate from the model (3) the vector of Lagrange multipliers (using the condition $G^T(q_m)A^T(q_m) \equiv 0$) by left-sided multiplying the mobile platform equations by $G^T(q_m)$ matrix. After substituting for \dot{q}_m and \ddot{q}_m we get

$$\begin{bmatrix} G^T M_{11} G & G^T M_{12} \\ M_{21} G & M_{22} \end{bmatrix} \begin{pmatrix} \dot{\eta} \\ \ddot{\alpha} \end{pmatrix} + \begin{bmatrix} G^T (C_{11} G + M_{11} \dot{G}) & G^T C_{12} \\ C_{21} G & C_{22} \end{bmatrix} \begin{pmatrix} \eta \\ \dot{\alpha} \end{pmatrix} + \begin{pmatrix} 0 \\ D \end{pmatrix} = \begin{pmatrix} G^T B \tau_m \\ 0 \end{pmatrix} \quad (4)$$

We introduce a new symbol covering centrifugal and Coriolis forces as well as gravity. Then we obtain the model expressed in more compact form as follows

$$\begin{bmatrix} M_{11}^* & M_{12}^* \\ M_{21}^* & M_{22}^* \end{bmatrix} \begin{pmatrix} \dot{\eta} \\ \ddot{\alpha} \end{pmatrix} + \begin{pmatrix} F_1^* \\ F_2^* \end{pmatrix} = \begin{pmatrix} B^* \tau_m \\ 0 \end{pmatrix} \quad (5)$$

where the arguments of matrices and vectors are omitted for the sake of simplicity. We will refer to the model (5) as the model of dynamics of the (nh, h) mobile manipulator expressed in auxiliary variables.

2.3 Partial global linearization

The dynamic model of nonholonomic balancing robot can be considered as a mobile manipulator with one passive degree of freedom (degree of freedom without actuator). The role of this passive joint plays the inverted pendulum. For such the object it is possible to introduce due to (De Luca et al., 2001) partial global linearization, which transforms the model in auxiliary velocities to a form more convenient from control's point of view. For this reason we extract $\ddot{\alpha}$ from the second matrix equation of (5)

$$\ddot{\alpha} = -(M_{22}^*)^{-1} (M_{21}^* \dot{\eta} + F_2^*) \quad (6)$$

and put it into the first equation, (Ratajczak & Tchoń, 2007). Then we get the following expression

$$\bar{M}(q)\dot{\eta} + \bar{F}(q, \dot{q}) = B^* \tau_m, \quad (7)$$

where

$$\bar{M}(q) = M_{11}^* - M_{12}^*(q)M_{22}^*(q)^{-1}M_{21}^*(q)$$

$$\bar{F}(q, \dot{q}) = F_1^* - M_{12}^*(q)M_{22}^*(q)^{-1}F_2^*(q, \dot{q})$$

Now a linearizing control law with a new control input u should be introduced

$$\tau_m = (B^*)^{-1} \{ \bar{M}(q)u + \bar{F}(q, \dot{q}) \} \quad (8)$$

to get a model (5) defined as a partially linearized control system

$$\begin{cases} \ddot{\alpha} = -(M_{22}^*)^{-1}(M_{21}^*\dot{\eta} + F_2^*) \\ \dot{\eta} = u \end{cases} \quad (9)$$

Such a system is a point of departure to design a new nonlinear control algorithm preserving not only point stabilization but trajectory tracking as well.

3. Control problem statement

In the paper we will find a control law guaranteeing the proper behaviour of the balancing robot. The task of the robot is to follow the desired trajectory $\alpha_d(t)$ (trajectory tracking or point stabilization) of the inverted pendulum without slipping of platform's wheels.

A goal of this paper will be to address the following control problem for balancing robot given by the model (9):

Find control law u such that the balancing robot with the known dynamics follows a desired trajectory $\alpha_d(t)$ without slippage of platform's wheels and tracking errors converge against zero.

To design a controller for the such the mobile manipulator, it is necessary to observe that complete nonholonomic system (9) is a cascade with two subsystems. For this reason the structure of the controller is divided into two parts due to backstepping-like procedure (Krstić et al., 1995):

1. kinematic level - η_r represents an embedded control input, which ensures the convergence the real trajectory α of the inverted pendulum to the desired trajectory $\alpha_d(t)$ for the equation of constraints (6) if the dynamics were not present,
2. dynamic level - as a consequence of cascaded structure of the system model, the pendulum's angle α cannot be commanded directly, as is assumed in the design of control on kinematic level, and instead it must be realized as the output of the partially linearized dynamics driven by u . The dynamic input u intends to regulate η toward the embedded control input η_r , and therefore, attempts to provide control input necessary to track the desired trajectory.

Because there exists a difference between the real velocity of the mobile platform η and the embedded control input η_r at the start position, it is necessary to account for the influence of the error $e_\eta = \eta - \eta_r$ on the behaviour of the full mathematical model of the nonholonomic balancing robot.

4. Nonlinear control law

4.1 Reference auxiliary velocities η_r

Let some reference functions describing desired accelerations of platform's wheels will be defined as follows

$$-(M_{22}^*)^{-1}(M_{21}^*\dot{\eta}_r + F_2^*) = \ddot{\alpha}_d - K_d\dot{e}_\alpha - K_p e_\alpha, \quad K_d, K_p > 0, \quad (10)$$

where

$$e_\alpha = \alpha - \alpha_d$$

is a tracking error of the inverted pendulum. It is obvious that η_r is not unique defined, because this equation is scalar and $\eta_r \in R^2$. However, it is possible to assume some relationship between η_{1r} and η_{2r} (for instance $\eta_{1r} = \eta_{2r}$) and to get unique solution of (10). The motion of wheels with velocities η_r preserves convergence of the inverted pendulum to the desired constant configuration α_d or to the desired trajectory $\alpha_d(t)$. The main problem is that the real velocities of wheels η are not equal to the reference velocities η_r at the beginning of the motion. It means that some errors occur on the dynamic level and disturb the behaviour of the balancing robot. Therefore we want to prove using Lyapunov-like function that the properly chosen control law on dynamic level can guarantee the asymptotic convergence of these errors to zero. As a consequence we obtain stabilization of the pendulum about the desired trajectory (or configuration).

4.2 Nonlinear controller

We consider the model of the balancing robot (9) expressed in auxiliary variables. We assume that we know the solution η_r to the constraints equation (10), which preserves a convergence of real coordinate $\alpha(t)$ of the inverted pendulum to the desired trajectory $\alpha_d(t)$. Then we propose a new control algorithm guaranteeing asymptotic trajectory tracking for all coordinates of the mobile manipulator. This control law is defined by expression

$$u = \dot{\eta}_r - K_m e_\eta, \quad K_m > 0 \quad (11)$$

where K_m is some diagonal regulation matrix and

$$e_\eta = \eta - \eta_r = \begin{pmatrix} \eta_1 - \eta_{1r} \\ \eta_2 - \eta_{2r} \end{pmatrix} = \begin{pmatrix} e_{\eta 1} \\ e_{\eta 2} \end{pmatrix}$$

is an error appearing on dynamic level, if real velocities of wheels are not equal to reference velocities, i.e. $\eta(0) \neq \eta_r(0)$. In such a situation, on the dynamic level we have the dynamic of the closed-loop error given by

$$\dot{e}_\eta + K_m e_\eta = 0 \quad (12)$$

which due to positive definiteness of K_m matrix is exponentially fast convergent to 0. On the other side, on kinematic level (the equation describing constraint, i.e. trajectory of a passive joint) the motion of the inverted pendulum is disturbed in the following way

$$\ddot{\alpha} = -(M_{22}^*)^{-1} M_{21}^* (\dot{\eta}_r - K_m e_\eta) - (M_{22}^*)^{-1} F_2^* = \ddot{\alpha}_d - K_d \dot{e}_\alpha - K_p e_\alpha + (M_{22}^*)^{-1} K_m e_\eta. \quad (13)$$

4.3 Proof of convergence of the control algorithm

Lets consider trajectories of the disturbed closed-loop system (12) and (13). We choose the following Lyapunov-like function

$$V(e_\alpha, \dot{e}_\alpha, e_\eta) = \frac{1}{2}(e_\alpha + \dot{e}_\alpha)^2 + e_\eta^T e_\eta. \quad (14)$$

Now we calculate the time derivative of V

$$\dot{V} = (e_\alpha + \dot{e}_\alpha)(\dot{e}_\alpha + \ddot{e}_\alpha) + e_\eta^T \dot{e}_\eta$$

which along solutions of the closed-loop system (12)-(13) is equal to

$$\begin{aligned} \dot{V} &= (e_\alpha + \dot{e}_\alpha)(\dot{e}_\alpha - K_d \dot{e}_\alpha - K_p e_\alpha + (M_{22}^*)^{-1} M_{21}^* K_m e_\eta) - e_\eta^T K_m e_\eta \\ &= -e_\alpha \dot{e}_\alpha K_2 - K_p e_\alpha^2 - (K_d - 1) \dot{e}_\alpha^2 - e_\eta^T K_m e_\eta + e_\alpha K_1 e_\eta + \dot{e}_\alpha K_1 e_\eta \end{aligned} \quad (15)$$

with parameters defined in the following way

$$K_1 = (M_{22}^*)^{-1} M_{21}^* K_m = \frac{\cos \alpha}{I_p} [K_{m11}, K_{m22}], \quad K_2 = K_p + K_d - 1$$

where I_p is a moment of inertia of the inverted pendulum. Then the time derivative of the V function can be evaluated by the expression

$$\begin{aligned} \dot{V} &\leq -e_\alpha \dot{e}_\alpha K_2 - K_p e_\alpha^2 - (K_d - 1) \dot{e}_\alpha^2 - K_{m11} e_{\eta 1}^2 - K_{m22} e_{\eta 2}^2 + K_3 (e_\alpha + \dot{e}_\alpha) (e_{\eta 1} + e_{\eta 2}) \\ &= \frac{K_2^2}{4} \dot{e}_\alpha^2 + e_\alpha^2 - \left(\frac{K_2}{2} \dot{e}_\alpha + e_\alpha \right)^2 - K_p e_\alpha^2 - (K_d - 1) \dot{e}_\alpha^2 - K_{m11} e_{\eta 1}^2 - K_{m22} e_{\eta 2}^2 + K_3 (e_\alpha + \dot{e}_\alpha) (e_{\eta 1} + e_{\eta 2}) \\ &\leq -(K_p - 1) e_\alpha^2 - \left(K_d - 1 - \frac{K_2^2}{4} \right) \dot{e}_\alpha^2 - K_{m11} e_{\eta 1}^2 - K_{m22} e_{\eta 2}^2 + K_3 (e_\alpha + \dot{e}_\alpha) (e_{\eta 1} + e_{\eta 2}) \end{aligned}$$

with

$$K_3 = \max_{\forall \alpha} \left(\frac{\cos \alpha}{I_p} K_{m11}, \frac{\cos \alpha}{I_p} K_{m22} \right)$$

Using the same method of estimation, we can obtain the following formula

$$\dot{V} \leq - \left(K_p - 1 - \frac{K_3^2}{2} \right) e_\alpha^2 - \left(K_d - 1 - \frac{K_2^2}{2} \right) \dot{e}_\alpha^2 - (K_{m11} - 1) e_{\eta 1}^2 - (K_{m22} - 1) e_{\eta 2}^2 \leq 0. \quad (15)$$

If the regulation parameters K_m , K_p , K_d are properly chosen, i.e.

$$K_{m11} > 1, \quad K_{m22} > 1, \quad K_p > 1 + \frac{K_3^2}{2}, \quad K_d > 1 + \frac{K_2^2}{2},$$

then from LaSalle invariance principle we can deduce that all errors, i.e. $(e_\alpha, \dot{e}_\alpha, e_\eta)$ converge to 0 asymptotically.

5. Simulations

As the object of simulations we have chosen a model of the inverted pendulum on two fixed wheels presented in Fig. 1. The goal of simulations is to examine the behaviour of the presented control algorithm using a full knowledge about the dynamics. The motion of the closed loop system has been examined by simulations which have run with the MATLAB package and the SIMULINK toolbox.

- First, the desired trajectory for inverted pendulum was chosen as a constant configuration $\alpha_d = \pi/3$. The start position of the platform was equal to $(x(0), y(0), \theta(0)) = (0, 0, 0)$ and start position of the manipulator $\alpha(0) = 0$. In Fig. 2b tracking error e_{η_1} for the mobile platform have been shown. The relationship between reference velocities is selected as $\eta_{1r} = \eta_{2r}$ (straightforward motion). Figure 2a presents tracking error e_α for the inverted pendulum. The gains of control parameters used for getting plots presented in Figure 2 are equal to

$$K_m = 50, \quad K_p = 100, \quad K_d = 50.$$

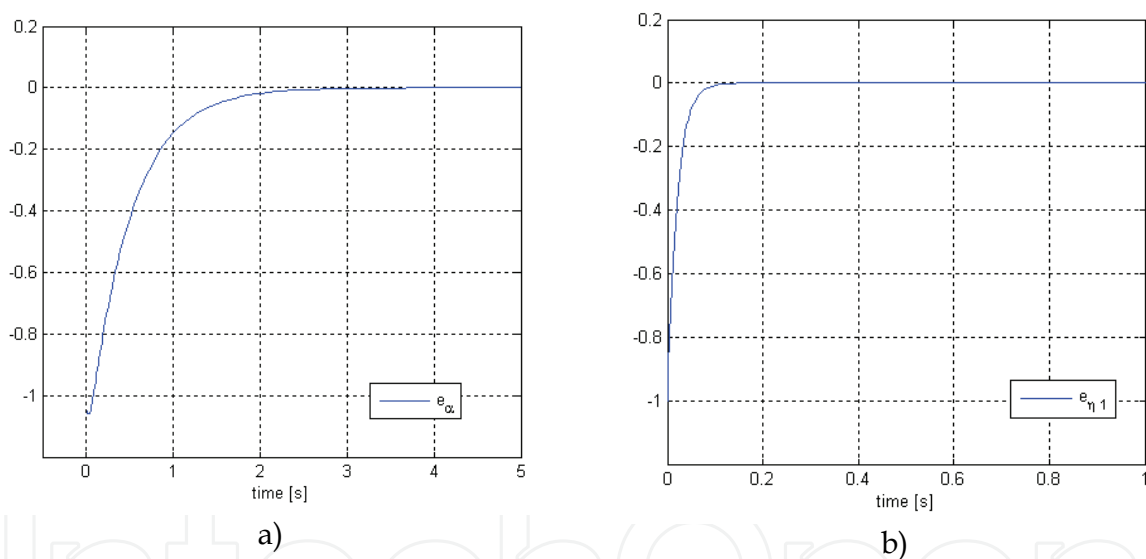


Fig. 2. Tracking errors occurring in the balancing robot during tracking constant configuration: a) e_α b) e_{η_1}

- Next, the desired trajectory for inverted pendulum was chosen as a slowly changing periodic function $\alpha_d(t) = 0.05 \sin(t/10)$. The start position of the platform was equal to $(x(0), y(0), \theta(0)) = (0, 0, 0)$ and start position of the manipulator $\alpha(0) = 0$. In Fig. 3b tracking error e_{η_1} for the mobile platform has been shown. The relationship between reference velocities is selected as $\eta_{1r} = \eta_{2r}$. Figure 3a presents tracking error e_α for the inverted pendulum. The gains of control parameters used for getting plots presented in Fig. 3 are equal to

$$K_m = 50, \quad K_p = 100, \quad K_d = 50.$$

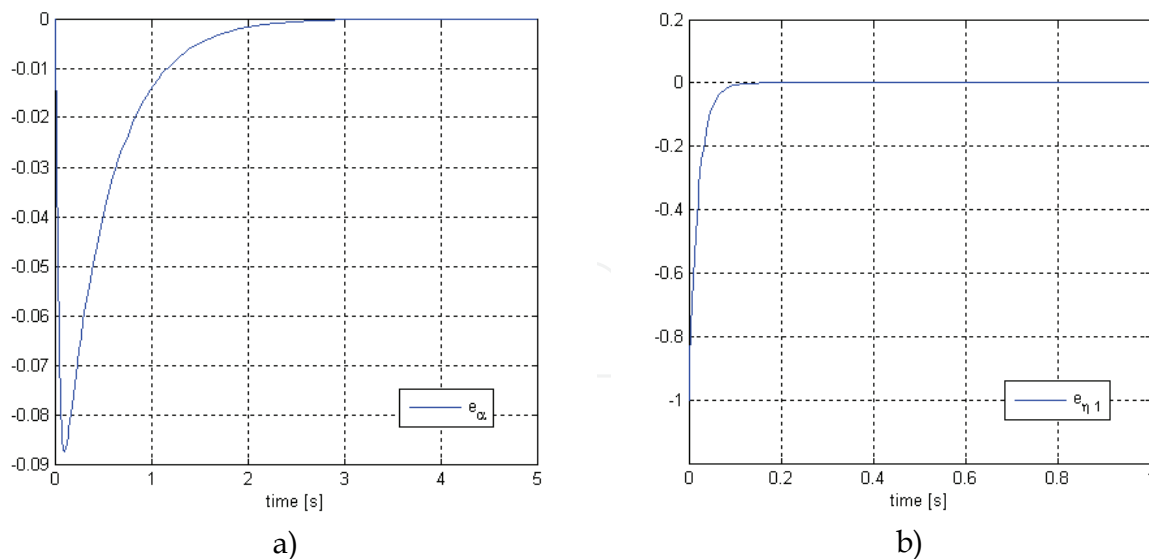


Fig. 3. Tracking errors occurring in the balancing robot during tracking periodic trajectory:
a) e_α b) $e_{\eta 1}$

6. Concluding remarks

In the paper a new control algorithm for nonholonomic balancing robot (inverted pendulum mounted on a two fixed conventional wheels) has been introduced. The algorithm covers not only stabilization of the pendulum about a desired constant configuration α_d , not necessary 0, but the tracking of some time-dependent trajectory as well. Differently from previous works presenting control problem of the balancing robot, the motion of the robot is not restricted to straight-line motion but it is possible to realize more complicated manoeuvres on XY plane without slipping of robot's wheels. It depends on the selection of relationship between reference velocities designed for the wheels, what case of robot's motion will be realized in practice.

In our forthcoming research we will focus on extending the presented approach to other cases of mobile manipulators (nh, h) with different structures of passive joints.

8. References

- C. Canudas de Wit & B. Siciliano & G. Bastin. *Theory of Robot Control*, Springer-Verlag, London, 1996.
- A. De Luca & S. Iannitti & G. Oriolo. Stabilization of the PR planar underactuated robot. *Proc. IEEE International Conference on Robotics and Automation (ICRA 2001)*, pp. 2090–2095, 2001.
- M. Krstić & I. Kanellakopoulos & P. Kokotović, *Nonlinear and Adaptive Control Design*, J. Wiley and Sons, New York, 1995.
- A. Ratajczak & K. Tchoń. Control of underactuated robotic manipulators: an endogenous configuration space approach. *Proc. IEEE Conf. on Methods and Models in Automation and Robotics MMAR 2007*, pp. 985–990, Szczecin, 2007.

Rich Chi Ooi, *Balancing a Two-wheeled Autonomus Robot*, The University of Western Australia; Final Year Thesis, 2003.

Segbot - Final project for the Introduction to Mechatronics class at the University of Illinois
<http://coecsl.ece.uiuc.edu/ge423/spring04/group9/index.htm>, 2004.

IntechOpen

IntechOpen



Automation and Robotics

Edited by Juan Manuel Ramos Arreguin

ISBN 978-3-902613-41-7

Hard cover, 388 pages

Publisher I-Tech Education and Publishing

Published online 01, May, 2008

Published in print edition May, 2008

In this book, a set of relevant, updated and selected papers in the field of automation and robotics are presented. These papers describe projects where topics of artificial intelligence, modeling and simulation process, target tracking algorithms, kinematic constraints of the closed loops, non-linear control, are used in advanced and recent research.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Alicja Mazur and Jan Kedzierski (2008). Nonlinear Control Law for Nonholonomic Balancing Robot, Automation and Robotics, Juan Manuel Ramos Arreguin (Ed.), ISBN: 978-3-902613-41-7, InTech, Available from: http://www.intechopen.com/books/automation_and_robotics/nonlinear_control_law_for_nonholonomic_balancing_robot

INTeCH
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2008 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen