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Chaotic Neural Network with Time Delay Term for Sequential Patterns

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1. Introduction

Recently, chaos is drawing much attention as a method to realize flexible information processing as well as neural networks, fuzzy logic and genetic algorithms. Chaos is a phenomenon which is observed in deterministic nonlinear systems and the behavior is unpredictable. The chaotic behavior exists in many fields such as hydrodynamics, electric circuits and biological systems. In particular, it is considered that chaos plays an important role in the memory and learning of a human brain(Skarda & Freeman, 1987; Yao & Freeman, 1990; Freeman, 1991).

In order to mimic the real neurons, a chaotic neuron model has been proposed by Aihara *et al.*(Aihara et al., 1990). In this model, chaos is introduced by considering the following properties of the real neurons; (1) spatio-temporal summation, (2) refractoriness and (3) continuous output function. It is known that the dynamic (chaotic) association is realized in the associative memories composed of the chaotic neurons(Aihara et al., 1990; Osana & Hagiwara, 1998a; Osana & Hagiwara, 1998b). These models composed of chaotic neurons can generate chaotic response by setting appropriate parameters.

On the other hand, in nervous system, there are two types of synapses; (1) chemical synapse and (2) electrical synapse. The synaptic delay for a chemical synapse is typically about 2 ms, while the synaptic delay for an electrical synapse may be about 0.2 ms. Although the synaptic delay are constant in the most of the artificial neural network models, we can expect that the associative memory which has two types of weights can realize more complex associations.

In this research, we propose a Chaotic Neural Network with Time Delay term for Sequential Patterns (CNNTDSP). It has two types of connection weights; (1) normal weights and (2) weights with time delay, and realize associations of the sequential pattern in short term, and dynamic associations between sequential patterns in long term.

2. Chaotic Neural Network

Here we briefly review a chaotic neuron model and a chaotic neural network(Aihara et al., 1990).

2.1 Chaotic Neuron Model

A chaotic neuron model based on Caianiello-Nagumo-Sato model(Caianiello, 1961; Nagumo & Sato, 1972) has been proposed by Aihara *et al.* (Aihara et al., 1990). In this model, chaos is introduced by considering spatio-temporal summation, refractoriness and continuous output function; they are observed in real neurons. The mathematical expression of this model is given by

$$x(t) = f\left[A(t) - \alpha \sum_{d=0}^{t} k^d x(t-d) - \theta \right] \tag{1}$$

$$f(u) = \frac{1}{1+\exp(-u/\varepsilon)} \tag{2}$$

where $x(t)$ shows the output of the neuron at the time t, $A(t)$ is the strength of the externally applied input at the time t, α is the scaling factor of the refractoriness ($\alpha \geq 0$), k is the damping factor of the refractoriness ($0 \leq k < 1$) and θ is the threshold of the neuron. $f(\cdot)$ is the output function. ε is the steepness parameter. The chaotic neuron model can generate chaotic response by setting appropriate parameters.

2.2 Chaotic Neural Network

A neural network composed of the chaotic neurons described in **2.1** is called a chaotic neural network.

The dynamics of the chaotic neuron i in a neural network composed of N chaotic neurons is represented by the following equation(Aihara et al., 1990):

$$x_i(t+1) = f\left[\sum_{j=1}^{M} v_{ij} \sum_{d=0}^{t} k_s^d A_j(t-d) + \sum_{j=1}^{N} w_{ij} \sum_{d=0}^{t} k_m^d x_j(t-d) - \alpha \sum_{d=0}^{t} k_r^d x_i(t-d) - \theta_i \right] \tag{3}$$

where $x_i(t)$ shows the output of the neuron i at the time t, M is the number of the external input, v_{ij} is the connection weight from the external input j to the neuron i, $A_j(t)$ is the strength of the external input j at the time t, w_{ij} is the connection weight between the neuron i and the neuron j, α is the scaling factor of the refractoriness, k_s, k_m and k_r are the damping factors and θ_i is the threshold of the neuron i.

2.3 Quick Learning

The Quick Learning(Hattori et al., 1994) has been proposed in order to improve the storage capacity of the Bidirectional Associative Memory(Kosko, 1988). The Quick Learning has two phases; (1) Hebbian Learning and (2) PRLAB (Pseudo-Relaxation Learning Algorithm for BAM)(Oh & Kothari, 1994).

Consider the training set $\{(\boldsymbol{X}^{(1)}, \boldsymbol{Y}^{(1)}) \cdots (\boldsymbol{X}^{(p)}, \boldsymbol{Y}^{(p)}) \cdots (\boldsymbol{X}^{(P)}, \boldsymbol{Y}^{(P)})\}$ ($\boldsymbol{X}^{(p)} \in \{1,-1\}^N$, $\boldsymbol{Y}^{(p)} \in \{1,-1\}^M$) is stored in the Bidirectional Associative Memory which has N neurons in the X Layer and M neurons in the Y Layer.

2.4 Hebbian Learning

First, the connection weight between the neuron i in the X Layer and the neuron j in the Y Layer w_{ij} is calculated based on the Hebbian learning.

$$w_{ij} = \sum_{p=1}^{P} X_i^{(p)} Y_j^{(p)} \tag{4}$$

2.5 PRLAB

Next, the connection weights are update based on the PRLAB algorithm. In the PRLAB, the training pair $(\boldsymbol{X}^{(p)}, \boldsymbol{Y}^{(p)})$ is given to the network, and if the conditions shown in Eqs.(5) and (6) are not satisfied, the connection weights and the threshold of the neuron are updated.

$$\left(\sum_{j=1}^{N} W_{ij} X_i^{(p)} - \theta_{Y_j} \right) Y_j^{(p)} > 0 \; \square @ \square @ (j \square = @ 1 \cdots M) \tag{5}$$

$$\left(\sum_{j=1}^{M} W_{ij} Y_j^{(p)} - \theta_{X_i} \right) X_i^{(p)} > 0 \; \square @ \square @ (i \square = @ 1 \cdots N) \tag{6}$$

The connection weights and the threshold are updated as follows:
For the neuron in the X Layer, if $S_{X_i}^{(p)} X_i^{(p)} \leq 0$,

$$\Delta W_{ij} = -\frac{\lambda}{M+1} \left(S_{X_i}^{(p)} - \xi X_i^{(p)} \right) Y_j^{(p)} \tag{7}$$

$$\Delta \theta_{X_i} = \frac{\lambda}{M+1} \left(S_{X_i}^{(p)} - \xi X_i^{(p)} \right) \tag{8}$$

For the neuron in the Y Layer, if $S_{Y_j}^{(p)} Y_j^{(p)} \leq 0$,

$$\Delta W_{ij} = -\frac{\lambda}{N+1} \left(S_{Y_j}^{(p)} - \xi Y_j^{(p)} \right) X_i^{(p)} \tag{9}$$

$$\Delta \theta_{y_j} = \frac{\lambda}{N+1} \left(S_{Y_j}^{(p)} - \xi Y_j^{(p)} \right) \tag{10}$$

where $\xi (\xi > 0)$ is the relaxation factor and $\lambda \in (0,2)$ is the constant. S_{X_i} and S_{Y_j} are given by

$$S_{X_i} = \sum_{j=1}^{M} W_{ij} Y_j^{(p)} - \theta_{X_i} \tag{11}$$

$$S_{Y_j} = \sum_{i=1}^{N} W_{ij} X_i^{(p)} - \theta_{Y_j}. \tag{12}$$

3. Chaotic Neural Network with Time Delay Term for Sequential Patterns

3.1 Outline

The proposed Chaotic Neural Network with Time Delay term for Sequential Patterns (CNNTDSP) has two types of connection weights; (1) normal weights and (2) weights with time delay. The proposed model realizes associations of the sequential pattern in short term and dynamic associations between sequential patterns in long term.

Let consider the case where 2 sequential patterns composed of 3 patterns shown in Fig.1 are memorized in the network. In the network, we can expect that the association of the sequential pattern 1 $(A \rightarrow B \rightarrow C)$ is realized for a certain period, and the association of the sequential pattern 2 $(D \rightarrow E \rightarrow F)$ is realized on the other period as follows.

$$\cdots \rightarrow \underbrace{A \rightarrow B \rightarrow C}_{\text{sequential pattern 1}} \rightarrow \cdots \rightarrow \underbrace{D \rightarrow E \rightarrow F}_{\text{sequential pattern 2}} \rightarrow \cdots$$

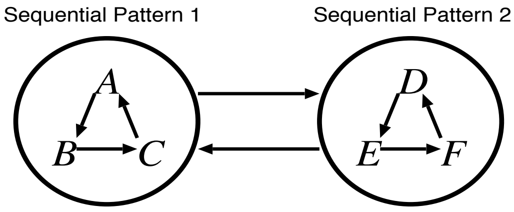


Fig. 1. 2 Sequential Patterns.

3.2 Structure

Figure 2 shows the structure of the proposed model. As shown in Fig.2, the proposed model has two types of weights $\boldsymbol{w}$ and $\boldsymbol{v}$, and all neurons are connected mutually. N neurons in the network are chaotic neuron.

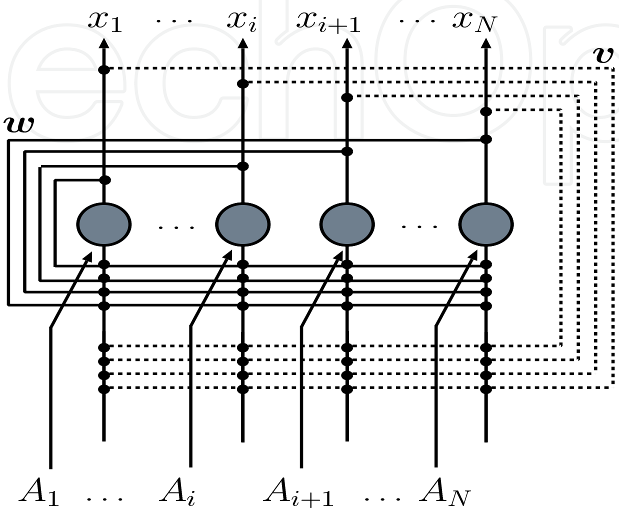

x_1 ... x_i x_{i+1} ... x_N
$\boldsymbol{v}$
$\boldsymbol{w}$
A_1 ... A_i A_{i+1} ... A_N

Fig. 2. Structure of Proposed Model.

3.3 Dynamics

In the proposed model, the dynamics of the neuron i is given by

$$
\begin{aligned}
x_i(t+1) = g\Bigg[& \sum_{d=0}^{t} k_s^d A_i(t-d) + \sum_{j=1}^{N} w_{ij} \sum_{d=0}^{t} k_{m_f}^d x_j(t-d) \\
& + \sum_{j=1}^{N} v_{ij} \sum_{d=0}^{t} k_{m_s}^d x_j(t-T-d) - \alpha \sum_{d=0}^{t} k_r^d x_i(t-d) \Bigg]
\end{aligned}
\tag{13}
$$

$$
g(u) = \tanh(\, u / \varepsilon\,) \tag{14}
$$

where $x_i(t)$ is the output of the neuron i at the time t , $A_i(t)$ is the external input i at the time t , T is the time delay constant, k_s , k_{m_f} , k_{m_s} and k_r are the damping factors, and α is the scaling factor of refractoriness. w_{ij} and v_{ij} are the connection weights from the neuron i to the neuron j . $g(\cdot)$ is the output function, ε is the steepness parameter.

3.4 Learning Process

In the proposed model, the connection weights $\boldsymbol{w}$ and $\boldsymbol{v}$ are trained by the Quick Learning(Hattori et al., 1994). In order to realize the dynamic associations between sequential patterns in long term, the learning patterns are embedded in the connection weights v as the auto associative weights. In order to realize the associations of the sequential pattern in short term, the sequential patterns are memorized in the connection weights w as the hetero associative weights.

Let consider the case where the training set composed of S sequential patterns $(\boldsymbol{X}^{(1,1)} \rightarrow \boldsymbol{X}^{(1,2)} \rightarrow \cdots \rightarrow \boldsymbol{X}^{(1,P(1))}) \rightarrow \cdots \rightarrow (\boldsymbol{X}^{(s,1)} \rightarrow \cdots \rightarrow \boldsymbol{X}^{(s,P(s))}) \rightarrow \cdots \rightarrow (\boldsymbol{X}^{(S,1)} \rightarrow \cdots \rightarrow \boldsymbol{X}^{(S,P(S))})$.

3.4.1 Connection Weights for Auto Association : $\boldsymbol{v}$

The connection weights for auto association $\boldsymbol{v}$ memorizes all patterns in the training set.

Although the Quick Learning was proposed as the learning algorithm for BAM (Kosko, 1988), it can be used to learn the training set for auto association in the single layer network such as the proposed model.

(1) Hebbian Learning

The connection weight v_{ij} is trained as follows:

$$
v_{ij} = \begin{cases} \displaystyle\sum_{s=1}^{S} \sum_{p=1}^{P(s)} X_i^{(s,p)} X_j^{(s,p)}, & \text{if} \quad i \neq j \\ 0, & \text{if} \quad i = j \end{cases} \tag{15}
$$

where S is the number of the sequential patterns, $P(s)$ is the number of the patterns included in the sth sequential pattern and $X_i^{(s,p)}$ is the ith element of training pattern p in the sequence s .

(2) PRLAB

In the PRLAB, all patterns to be stored are given to the network, and if

$$\left(\sum_{j=1}^{N} v_{ij} X_j^{(s,p)}\right) X_i^{(s,p)} \leq 0 \tag{16}$$

is satisfied, the connection weight v_{ij} is updated as follows:

$$\Delta v_{ij} = -\frac{\lambda}{N+1}\left(\sum_{k=1}^{N} v_{ik} X_k^{(s,p)} - \xi X_i^{(s,p)}\right) X_j^{(s,p)} \tag{17}$$

where ξ $(\xi > 0)$ is the pseudo-relaxation constant and λ $(\lambda \in (0,2))$ is the constant. In this learning, the connection weights are updated by Eq.(17) under the constraint that $v_{ij} = v_{ji}$ and $v_{ii} = 0$.

3.4.2 Connection Weights for Hetero Association : *w*

The connection weights for hetero association ***w*** memorizes the sequential patterns as a limit cycle.

Although the Quick Learning was proposed as the learning algorithm for BAM(Kosko, 1988), it can be used to learn hetero associative training set in the single layer network such as the proposed model.

(1) Hebbian Learning

The initial value of connection weight w_{ij} is calculated as follows:

$$w_{ij} = \sum_{s=1}^{S} \sum_{p=1}^{P(s)} X_i^{(s,p+1)} X_j^{(s,p)} \\ \left(X_i^{(s,P(s)+1)} = X_i^{(s,1)}\right). \tag{18}$$

(2) PRLAB

In the PRLAB, all patterns to be stored are given to the network, and if

$$\left(\sum_{j=1}^{N} w_{ij} X_j^{(s,p)}\right) X_i^{(s,p+1)} \leq 0 \tag{19}$$

is satisfied, the connection weight w_{ij} is updated by

$$\Delta w_{ij} = -\frac{\lambda}{N+1}\left(\sum_{k=1}^{N} w_{ik} X_k^{(s,p)} - \xi X_i^{(s,p+1)}\right) X_j^{(s,p)} \tag{20}$$

4. Computer Experiment Results

In this section, we show the computer experiment results to demonstrate the effectiveness of the proposed model.

4.1 Association Result

Here, the training set which has *2* sequential patterns composed of *3* patterns shown in Fig.3 were memorized.

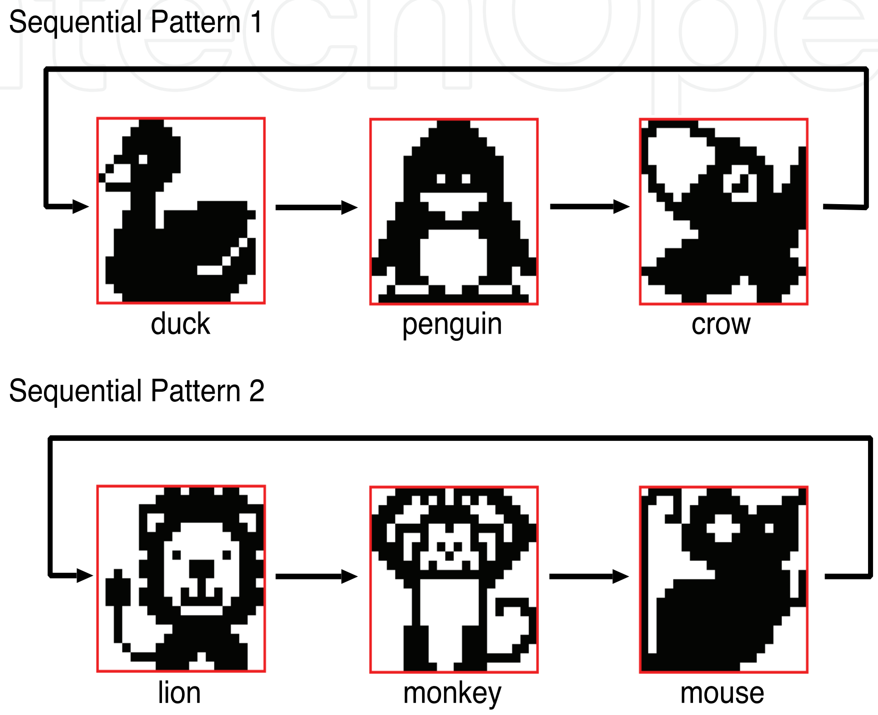


Fig. 3. Stored Patterns.

Figure 4 shows the association result when ``mouse'' in sequential pattern 2 was given to the network as an initial input in the experiment condition of Fig.4.

As shown in Fig.4, from $t=1$, the patterns in the sequential pattern 2 were recalled, from $t=96$, the patterns in the sequential pattern 1 were recalled, and from $t=246$, the patterns in the sequential pattern 2 were recalled again.

Figure 5 shows the association result of the proposed model when ``mouse'' in sequential pattern 2 was given to the network as an initial input by the direction cosine. Here, the direction cosine between the output at the time *t* and the most similar pattern in the sequential pattern *s*, $\cos\theta^s(t)$ is defined as follows:

$$\cos\theta^s(t) = \cos\theta^{s(c)}(t) \tag{21}$$

$$c = \arg\max_i \left|\cos\theta^{s(i)}(t)\right| \tag{22}$$

$$\cos\ \theta^{s(i)}(t) = \frac{\left(\boldsymbol{X}^{s(i)} \cdot \boldsymbol{x}(t)\right)}{\left|\boldsymbol{X}^{s(i)}\right|\left|\boldsymbol{x}(t)\right|} \tag{23}$$

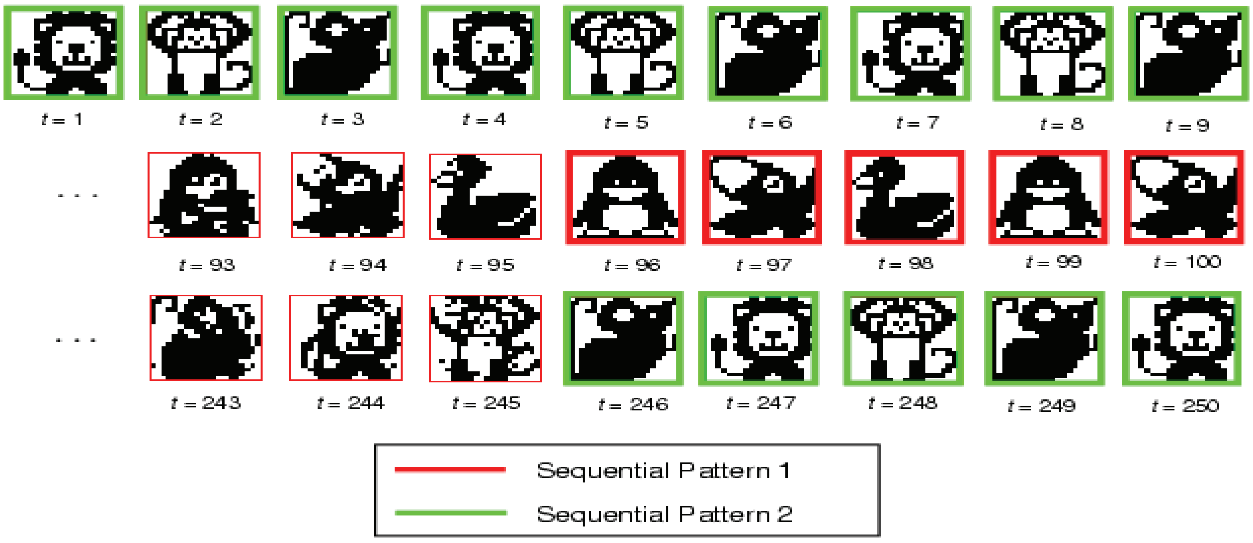


Fig. 4. Association Result when "mouse" in Sequential Pattern 2 was Given.

where $\boldsymbol{X}^{s(i)}$ is the ith stored pattern in the sequential pattern s, $\boldsymbol{X}(t)$ is the output at the time t, $\cos\theta^{s(i)}(t)$ is the direction cosine between the output at the time t, $\boldsymbol{x}(t)$ and the ith pattern in the sequential pattern s, $\boldsymbol{X}^{s(i)}$, and c shows the most similar pattern to the output in the sequential pattern.

As shown in Fig.5, during $t = 1 \sim 66$, $t = 246 \sim 330$, $t = 615 \sim 699$ and $t = 984 \sim 1000$, the sequential pattern 2 were recalled, and during $t = 96 \sim 218$, $t = 353 \sim 429$, $t = 458 \sim 587$, $t = 722 \sim 798$ and $t = 827 \sim 956$, the sequential pattern 1 was recalled. From these results, we confirmed that the proposed model can realize associations of the sequential pattern in short term and dynamic associations between sequential patterns in long term.

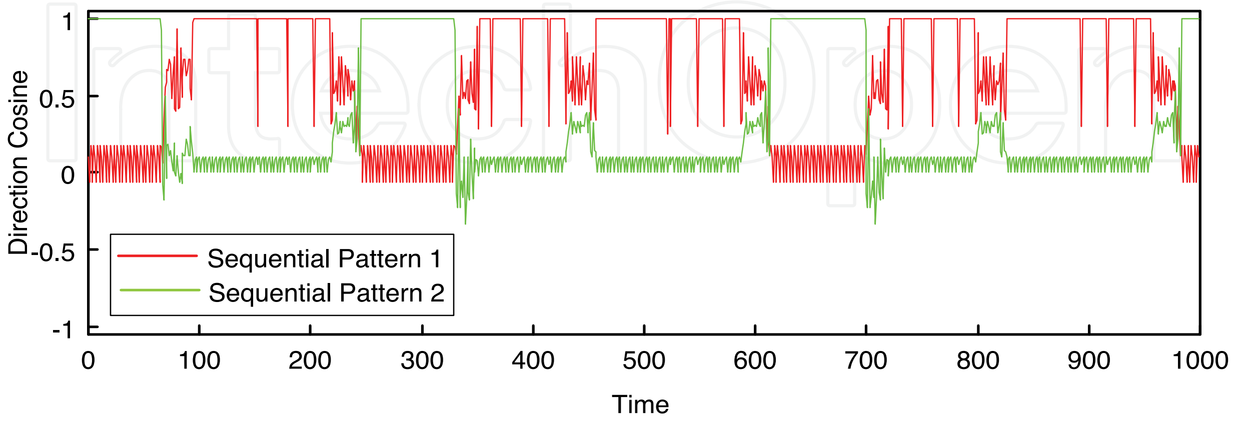


Fig. 5. Association Result when "mouse" in Sequential pattern 2 was Given (Direction Cosine).

4.2 Pattern Dependency of Recall Ability

In the chaotic neural network(Aihara et al., 1990), some patterns are recalled frequently and the other patterns are not recalled or recalled only a few times. However, it is not clear what makes such difference. Here, we examined the pattern dependency of the recall ability.

In this experiment, we used 40 training sets which have 6 patterns (2 sequential patterns composed of 3 patterns). We used 6 patterns shown in Fig.3; (a)duck, (b)penguin, (c)crow, (d)lion, (e)monkey and (f)mouse. When the training set shown in Fig.3 is described as

[abc] [def] ,

40 training sets can be described as follows:

1:	[abc] [def]	2:	[abc] [dfe]	3:	[acb] [def]
4:	[acb] [dfe]	5:	[abd] [cef]	6:	[abd] [cfe]
7:	[adb] [cef]	8:	[adb] [cfe]	9:	[abe] [cdf]
10:	[abe] [cfd]	11:	[aeb] [cdf]	12:	[aeb] [cfd]
			□		
32:	[aed] [bfc]	33:	[adf] [bce]	34:	[adf] [bec]
35:	[afd] [bce]	36:	[afd] [bec]	37:	[aef] [bcd]
38:	[aef] [bdc]	39:	[afe] [bcd]	40:	[afe] [bdc]

In this experiment, each training set was memorized and the proposed model recalled patterns during $t = 1 \sim 1000$ under the condition shown in Table 1.

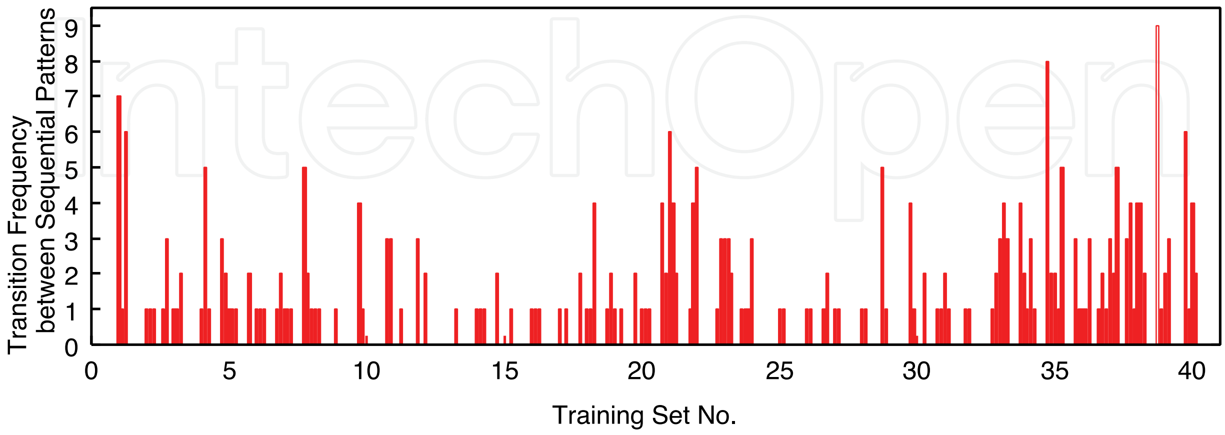
Network Parameters		
The Number of Neurons	N	400
Damping Factor	k_s	0.0
Damping Factor	k_m	0.3
Damping Factor	k_r	0.97
Scaling Factor of Refractoriness	α	82
Time Delay Constant	T	14
External Input	$A_i(t)$	0
Quick Learning Parameters		
Relaxation Constant	ξ	0.1
Pseudo Relaxation Constant	λ	1.8

Table 1. Experiment Conditions.

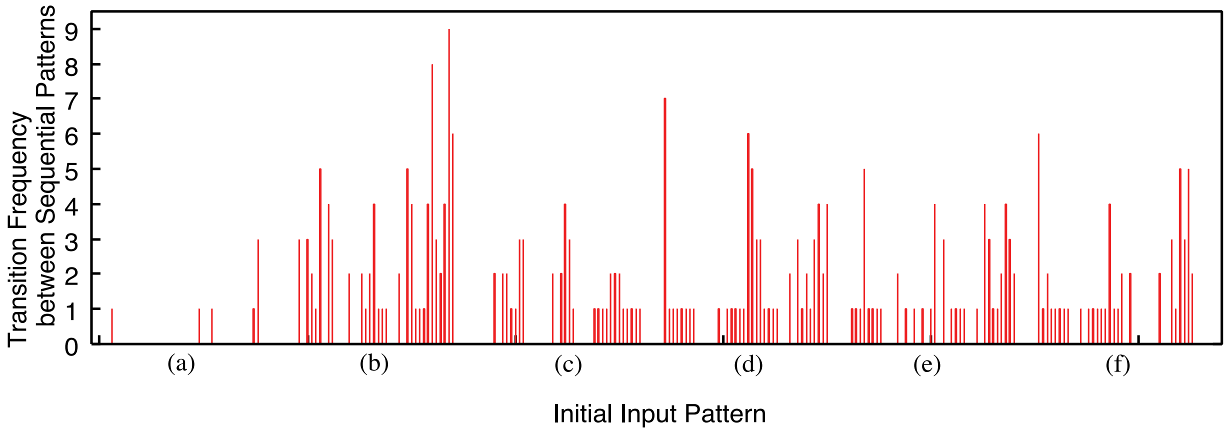
Figures $6 \sim 8$ show the pattern dependency of the recall ability. In Figs.$6 \sim 8$, (a) shows the result sorted by the training set number, and (b) shows the result sorted by the initial input pattern. Figure 6 shows the transition frequency between sequential patterns. As shown in Fig.6, the transition frequency is large when the pattern (b) was given as an initial input.

Figure 7 shows the number of unique stored patterns which were recalled. As shown in Fig.7, when the pattern (a) was given as an initial input, only 3 or 4 patterns were recalled. In contrast, when the patterns (b) ~ (f) were given, all patterns were often recalled.

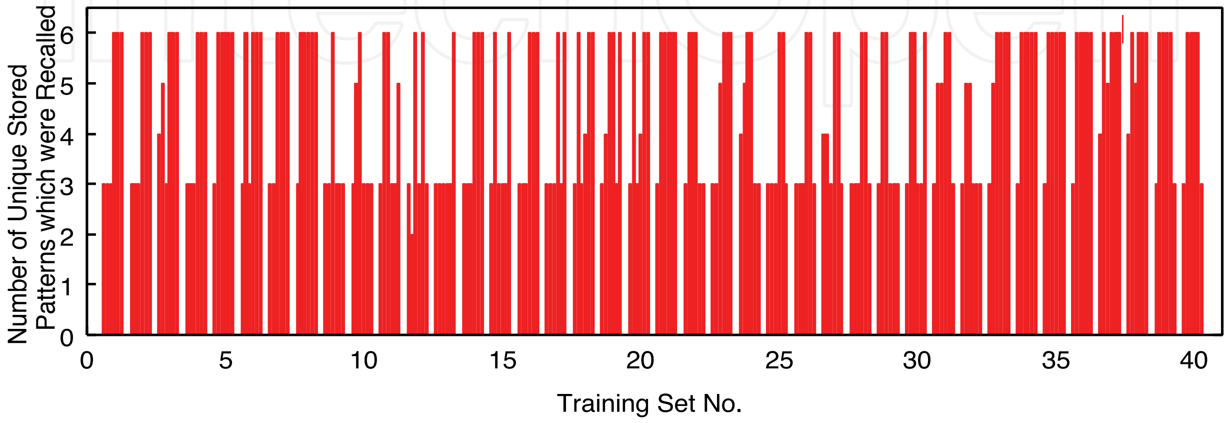
Figure 8 shows the average recall frequency of stored patterns. As shown in Fig.8, the patterns (d) ~ (f) were given as an initial input, the average recall frequency of stored patterns is larger than that for the patterns (a) ~ (c).
From these results, we confirmed that the initial input pattern influences the recall ability in the proposed model.



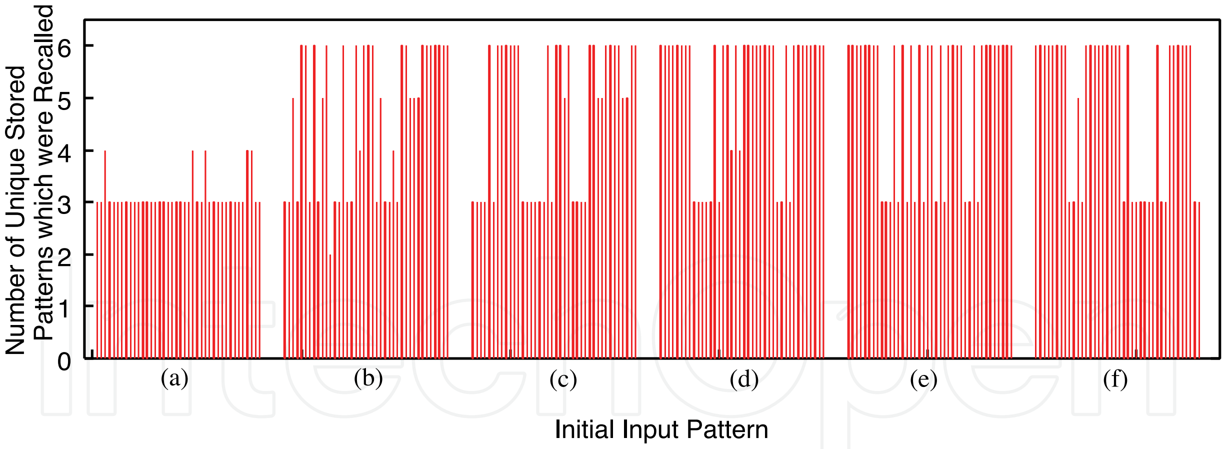
(a) Sorted by Training Set.



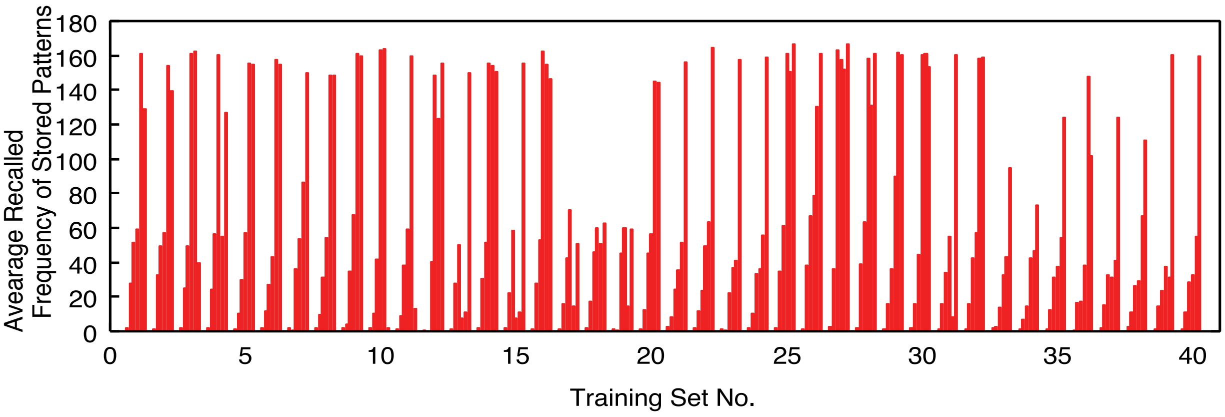
(b) Sorted by Initial Patterns.
Fig. 6. Transition Frequency between Sequential Patterns.



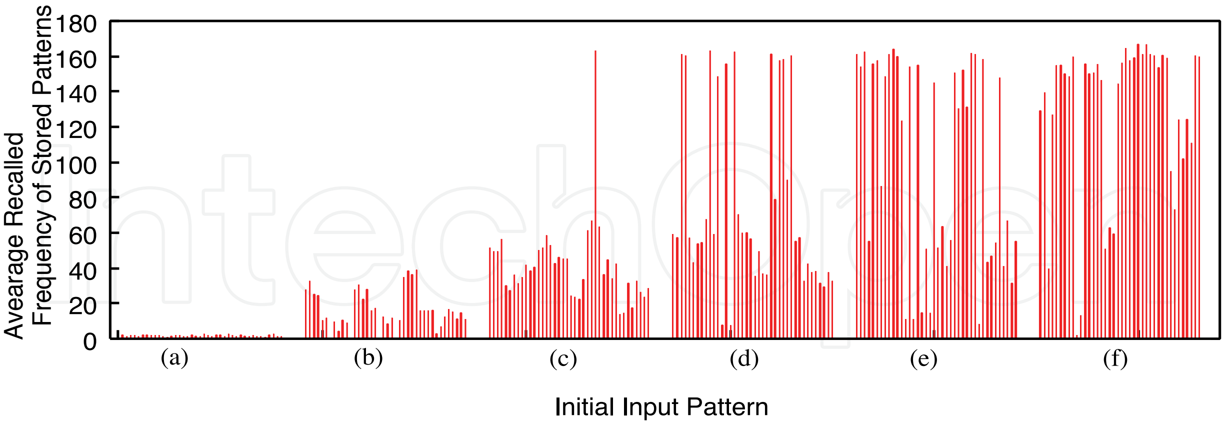
(a) Sorted by Training Set.



(b) Sorted by Initial Patterns.
Fig. 7. Number of Unique Stored Patterns which were Recalled.



(a) Sorted by Training Set.



(b) Sorted by Initial Patterns.
Fig. 8. Average Recalled Frequency of Stored Patterns.

5. Conclusion

In this research, we have proposed the Chaotic Neural Network with Time Delay term for Sequential Patterns (CNNTDSP). The proposed model is based on the conventional chaotic neural network and has two types of connection weights; (1) normal weights for hetero associations and (2) weights with time delay for auto associations. The proposed model deal with the sequential patterns. In the proposed model, associations of the sequential pattern in short term and dynamic associations between sequential patterns in long term are realized. We carried out a series of computer experiments and confirmed the proposed model can realize associations of the sequential pattern in short term and dynamic associations between sequential patterns in long term.

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