22

Motion Behavior of Null Space in Redundant Robotic Manipulators

Tsuyoshi Shibata and Toshiyuki Murakami
Keio University, Department of System Design Engineering
Japan

1. Introduction

As well known, robot manipulator is utilized in many industrial fields. However manipulator motion has a single function at most and is so limited because of low degree-of-freedom motion. To improve this issue, it is necessary for the robot manipulator to have redundant degree-of-freedom motion. The motion spaces of redundant manipulator are divided into work space motion and null space motion. A key technology of the redundant manipulator is dexterous use of null space motion. Then, the important issue is how to design null space controller for the dexterous motion control. However the control strategy of null space motion based on the stability analysis has not been established yet. Focusing on this issue, this chapter shows PID controller based on stability analysis considering passivity in null space motion of redundant manipulator. The PID controller which is passive controller plays a role to compensate disturbance of null space motion without deteriorating asymptotic stability. On the other hand, the work space control is stably achieved by a work space observer based PD control.
In this chapter, work space controller and null space controller are considered separately. In the proposed controller design, the work space observer that compensates the work space disturbance affecting position response of end-effector is one of important key technologies [1]. Furthermore it brings null space motion without calculating kinematic transformation. Then the null space input can be determined arbitrary in joint space. In this chapter, PID control is utilized to synthesize the null space input. The null space motion is also affected by unknown disturbance even if the work space observer compensates the work space disturbance completely. In particular, null space disturbance denotes an interactive effect among null space responses. The large interaction affects the stability of not only the null space but also the work space motion. To increase both the stability margin and robustness of null space motion, this chapter introduces PID controller considering passivity [2]. The validity of the proposed approach is verified by simulations of 4-link redundant manipulator.

2. Kinematics and Dynamics of Redundant Manipulator

2.1 Dynamics Modeling of Redundant Manipulator

This chapter assumes n-link robot manipulator with all revolute-type joints. When the end-effector is free to move, the motion of the manipulator is governed by the Lagrange equation. It is described in terms of joint coordinated $\boldsymbol{q} = (q_1, \cdots, q_n)^T$, that is, the joint angle vector. Then the motion equation of the robot manipulator

$$R(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{R}(\boldsymbol{q})\dot{\boldsymbol{q}} + \mathcal{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$

can be obtained by the Lagrange equation. Here $R(\boldsymbol{q})$ is the inertial matrix and symmetric and positive definite. $(\frac{1}{2}\dot{R}(\boldsymbol{q})\dot{\boldsymbol{q}} + \mathcal{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}})$ denotes the Coriolis and centrifugal torques. $\boldsymbol{\tau} \in R^n$ is the vector of input torques generated at joint actuators. $\mathcal{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is skew-symmetric, since it is expressed in

$$\mathcal{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = \frac{1}{2}\left[\dot{R}(\boldsymbol{q})\dot{\boldsymbol{q}} - \frac{\partial}{\partial \boldsymbol{q}}\left\{\dot{\boldsymbol{q}}^T R(\boldsymbol{q})\dot{\boldsymbol{q}}\right\}^T\right]. \tag{2}$$

$\mathcal{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ satisfies

$$\dot{\boldsymbol{q}}^T \mathcal{S}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} = 0. \tag{3}$$

$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial U(\boldsymbol{q})}{\partial \boldsymbol{q}}$ is the gravity force vector, where $U(\boldsymbol{q})$ denotes a potential energy. In (1), robot manipulator is passive between input $\boldsymbol{\tau}$ and output $\dot{\boldsymbol{q}}$. This passivity relation plays an important role in building a bridge between the energy conservation law in physics and the operational input-output relation in systems theory [2].

2.2 Kinematic Modeling of Redundant Manipulator

In *n*-link manipulator, the kinematic relation can be written by (4)-(5). $\boldsymbol{x} \in R^m$ $(n > m)$ is a position vector in work space motion.

$$\dot{\boldsymbol{x}} = J_{aco}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{4}$$

$$\ddot{\boldsymbol{x}} = J_{aco}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \dot{J}_{aco}(\boldsymbol{q})\dot{\boldsymbol{q}} \tag{5}$$

Here $J_{aco} \in R^{m \times n}$ is the Jacobian matrix, and it is shown in (6).

$$J_{aco}(\boldsymbol{q}) = \frac{\partial \boldsymbol{f}(\boldsymbol{q})}{\partial \boldsymbol{q}} \tag{6}$$

The inverse kinematics of (5) is given as

$$\ddot{\boldsymbol{q}} = J_{aco}^{+}(\ddot{\boldsymbol{x}} - \dot{J}_{aco}\dot{\boldsymbol{q}}) + (I - J_{aco}^{+}J_{aco})\ddot{\boldsymbol{\phi}}. \tag{7}$$

J_{aco}^{+} is the pseudo inverse matrix of Jacobian, which is defined by

$$J_{aco}^{+} = J_{aco}^{T}(J_{aco}J_{aco}^{T})^{-1}. \tag{8}$$

In the right-hand side of (7), $\ddot{\boldsymbol{\phi}}$ is the input vector of null space. Using (7), the acceleration references of work space are resolved into joint space of redundant manipulator.

2.3 Acceleration Reference for Null Space Motion

From the kinematic relation, an equivalent motion equation is derived in the work space [1]. In the proposed approach, the joint acceleration reference is given by

$$\ddot{q}^{ref} = J_{aco}^{+}\ddot{x}^{ref}. \tag{9}$$

The superscript ^{ref} denotes motion command in the work space. To consider the null space motion based on the joint acceleration reference, arbitrary reference of the joint acceleration $\ddot{q}_{null}^{ref}$ space motion is added to (9). Then, the joint acceleration reference $\ddot{q}^{ref}$ is redefined as follows.

$$\ddot{q}^{ref} = J_{aco}^{+}\ddot{x}^{ref} + \ddot{q}_{null}^{ref} \tag{10}$$

Using (5) and (10), the equivalent acceleration error $\ddot{e}_{work}$ in the work space is given as follows. Here, in (5), joint acceleration controller $(\ddot{q} = \ddot{q}^{ref})$ is assumed by joint disturbance observer as shown in Fig. 2.

$$\begin{aligned} \ddot{e}_{work} &= \ddot{x}^{ref} - \ddot{x} \\ &= -J_{aco}\ddot{q}_{null}^{ref} - \dot{J}_{aco}\dot{q} \end{aligned} \tag{11}$$

When the acceleration error $\ddot{e}_{work}$ is calculated or estimated as $\hat{\ddot{x}}^{dis} = \ddot{e}_{work}$, it may be fed back to compensate the acceleration error in the work space. Then, (10) is rewritten as follows. Practically, $\ddot{e}_{work}$ is estimated by a work space observer shown in Fig. 2.

$$\ddot{q}^{ref} = J_{aco}^{+}(\ddot{x}^{ref} + \hat{\ddot{x}}^{dis}) + \ddot{q}_{null}^{ref} \tag{12}$$

Assuming that $\hat{\ddot{x}}^{dis} = \ddot{e}_{work}$ and substituting (11) into (12),

$$\ddot{q}^{ref} = J_{aco}^{+}(\ddot{x}^{ref} - \dot{J}_{aco}\dot{q}) + (I - J_{aco}^{+}J_{aco})\ddot{q}_{null}^{ref} \tag{13}$$

is obtained. (13) shows that the feedback of the estimated acceleration error of the work space brings the null space motion without calculating the matrix $(I - J_{aco}^{+}J_{aco})$.
When $J_{aco}^{+}(\ddot{x}^{ref} + \hat{\ddot{x}}^{dis})$ is defined as $\ddot{q}_{work}^{ref}$, (12) is expressed in

$$\ddot{q}^{ref} = \ddot{q}_{work}^{ref} + \ddot{q}_{null}^{ref}. \tag{14}$$

(14) shows that work space controller and null space controller are separetely designed to determine each acceleration reference. To achieve this, the work space observer is employed in the work space to estimate the acceleration error given by (11) [1].

3. Design of Work Space Controller and Null Space Controller

This section describes design procedure of work space and null space controllers. In the work space, the work space observer based PD control is utilized. The work space observer suppresses work space disturbance $\hat{\ddot{x}}^{dis}$. Hence the work space constitutes robust acceleration control system. As described before, the work space and the null space motion are determined separately according to the final acceleration reference of joint space in (13). Also the disturbance of null space is considered independently without affecting the work space disturbance. In the proposed approach, PID control is used as null space controller to suppress the null space disturbance. Here this paper proves that disturbance of null space is compensated by PID controller. Fig.l shows a whole block diagram of the proposed method.

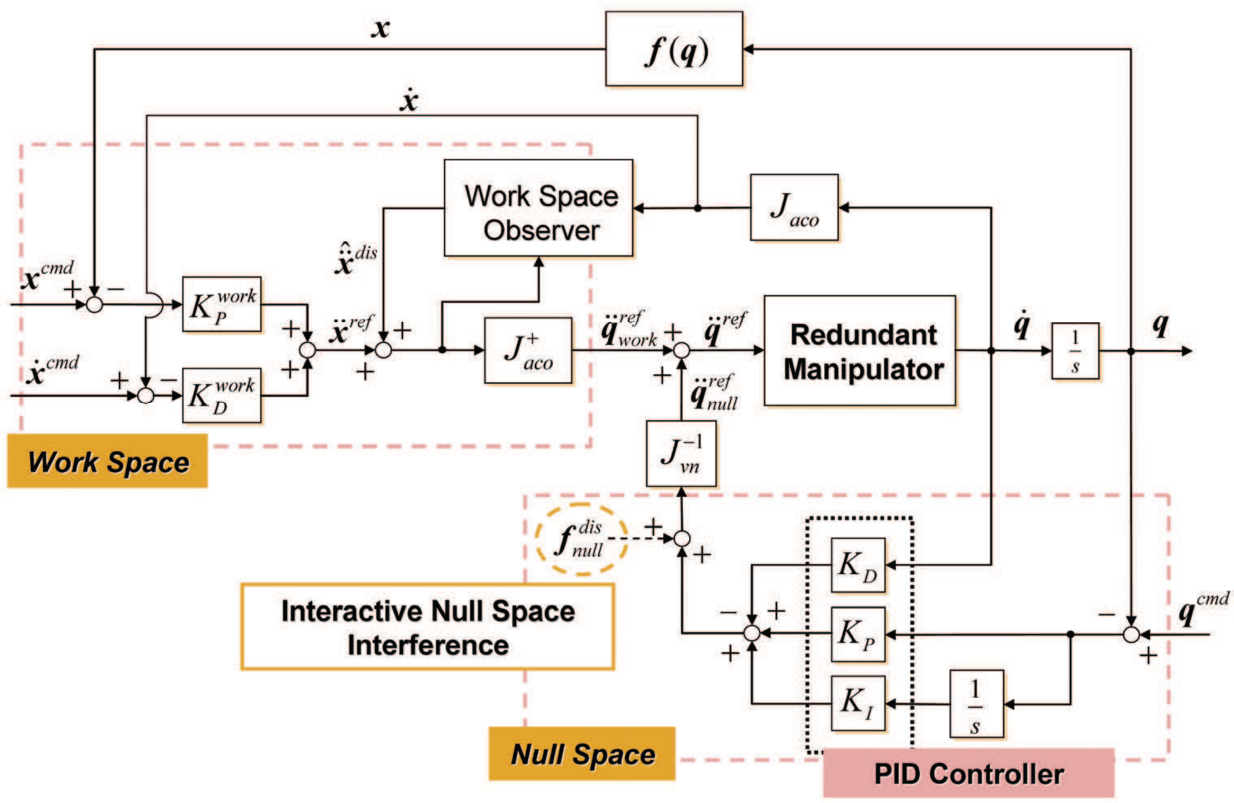

Figure 1. A whole block diagram of proposed method

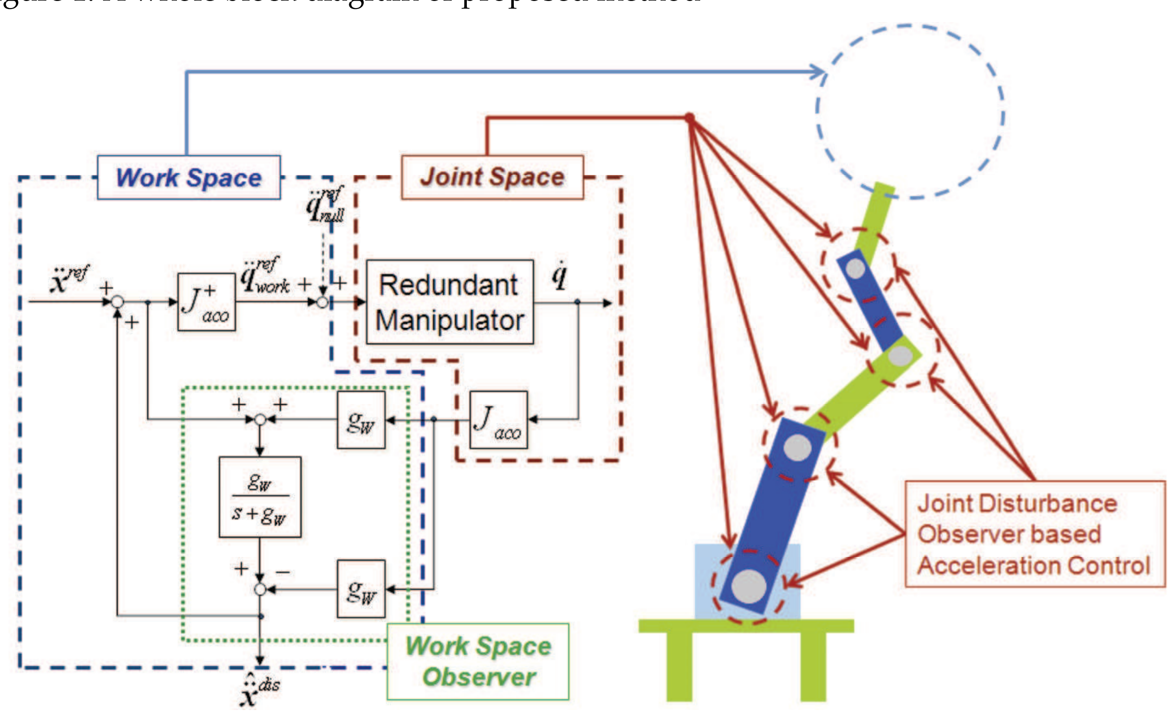

Figure 2. Work space observer

3.1 Work Space Controller

In the work space controller, the acceleration reference is given by

$$\ddot{\boldsymbol{x}}^{ref} = K_P^{work}(\boldsymbol{x}^{cmd} - \boldsymbol{x}) + K_D^{work}(\dot{\boldsymbol{x}}^{cmd} - \dot{\boldsymbol{x}}) \tag{15}$$

where $\boldsymbol{x}^{cmd} \in R^m$ denotes a position command vector. K_P^{work}, $K_D^{work} \in R^{m\times m}$ are positive definite diagonal matrices. Furthermore the work space observer is employed, and the estimated disturbance acceleration $\hat{\ddot{\boldsymbol{x}}}^{dis}$ is fed back as shown in Fig.2 [1].

3.2 Null Space Controller

Passivity of PID Controller

In this section, passivity of PID controller is shown. This is given by H. K. Khalil [4] and B. Brogliato *et al.* [5]. The general formulation of PID controller is given in

$$\boldsymbol{\tau} = K_P\boldsymbol{e} + K_I\int_0^t \boldsymbol{e}(\tau)d\tau + K_D\dot{\boldsymbol{e}}, \tag{16}$$

where $\boldsymbol{e} \in R^n$ and $\boldsymbol{\tau} \in R^n$ are error and input respectively. $K_P,\ K_I,\ K_D \in$ are $R^{n\times n}$ parameters and they are positive definite diagonal matrices. In this paper, only a servo or a regulator problem is considered for null space stability, and a tracking problem is not treated. Then, the error $\boldsymbol{e}$. is written in

$$\boldsymbol{e} = \boldsymbol{q}^{cmd} - \boldsymbol{q}, \tag{17}$$

where $\boldsymbol{q}^{cmd} \in R^n$ denotes a command vector of the joint angle. Here, $\boldsymbol{q}^{cmd} = \boldsymbol{0}$ is regulator problem. (16) is rewritten in

$$\boldsymbol{\tau} = K_P(\boldsymbol{q}^{cmd} - \boldsymbol{q}) + K_I\int_0^t (\boldsymbol{q}^{cmd} - \boldsymbol{q}(\tau))d\tau - K_D\dot{\boldsymbol{q}}. \tag{18}$$

Here a state variable vector $\boldsymbol{z}$ is additionally introduced. The integral term of (18) is replaced with

$$\int_0^t (\boldsymbol{q}^{cmd} - \boldsymbol{q}(\tau))d\tau = \boldsymbol{z}. \tag{19}$$

Hence, (18) is expressed in

$$\boldsymbol{\tau} = K_P(\boldsymbol{q}^{cmd} - \boldsymbol{q}) + K_I\boldsymbol{z} - K_D\dot{\boldsymbol{q}}. \tag{20}$$

Then, the storage function of PID controller (20) is expressed in (21).

$$S_c(\boldsymbol{z},\boldsymbol{q}) = \frac{1}{2}\boldsymbol{z}^T K_I \boldsymbol{z} + \frac{1}{2}(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T K_D(\boldsymbol{q}^{cmd} - \boldsymbol{q}) \tag{21}$$

(21) is the positive definite function on $(\boldsymbol{z},\boldsymbol{q})$. Here the storage function (21) is technical function to prove passivity. The time differentiation of (21) brings

$$\dot{S}_c = (\boldsymbol{q}^{cmd} - \boldsymbol{q})^T\boldsymbol{\tau} - (\boldsymbol{q}^{cmd} - \boldsymbol{q})^T K_P(\boldsymbol{q}^{cmd} - \boldsymbol{q}). \tag{22}$$

This equation means that the PID controller given by (20) is passive between input $(\boldsymbol{q}^{cmd} - \boldsymbol{q} = \boldsymbol{e})$ and $\boldsymbol{\tau}$.

Closed-loop Stability Analysis

In the proposed strategy, the PID parameters are decided so that closed-loop system is asymptotically stable. A proof of asymptotical stabilization is considered based on S. Arimoto [2, 3] and M. W. Spong *et al.* [8]. Here the proposed method is applied to the motion equation of joint space. From (1) and (20), the following equation is obtained.

$$\begin{aligned} & R(\boldsymbol{q})\ddot{\boldsymbol{q}} + \frac{1}{2}\dot{R}(\boldsymbol{q})\dot{\boldsymbol{q}} + \mathcal{S}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) \\ & - K_P(\boldsymbol{q}^{cmd} - \boldsymbol{q}) - K_I\boldsymbol{z} + K_D\dot{\boldsymbol{q}} = \boldsymbol{0} \end{aligned} \tag{23}$$

Here an equilibrium point of (23) is $(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) = (K_I^{-1}\boldsymbol{g}(\boldsymbol{q}^{cmd}),\boldsymbol{q}^{cmd},\boldsymbol{0})$ and it is proved that the equilibrium point $(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) = (K_I^{-1}\boldsymbol{g}(\boldsymbol{q}^{cmd}),\boldsymbol{q}^{cmd},\boldsymbol{0})$ of the closed-loop system (23) is asymptotically stable. Here the following assumptions are satisfied.

[Assumption 1]
PID parameters $K_P,\ K_I,\ K_D$ are positive definite diagonal matrices in (20).
Also they are set to high gains.

[Assumption 2]
It is assumed that K_P is high gain so that it satisfies (24).

$$(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T K_P(\boldsymbol{q}^{cmd} - \boldsymbol{q}) \geq 16(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T \boldsymbol{g}(\boldsymbol{q}) \tag{24}$$

[Assumption 3]
A constant value α is set to a small gain. Also K_D is adjusted to satisfy (25).

$$\alpha K_P - K_I \geq \frac{1}{8}\alpha K_P \tag{25}$$

[**Assumption 4]**
α is set to a small gain. Also K_D is chosen to satisfy (26).

$$K_D - \alpha R(\boldsymbol{q}) \geq \frac{1}{4}K_D \tag{26}$$

[Proposition 1]
If the PID parameters $K_P,\ K_I,\ K_D$ are chosen to satisfy Assumption 1-4 with some $\alpha > 0$, then the equilibrium point $(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) = (K_I^{-1}\boldsymbol{g}(\boldsymbol{q}^{cmd}),\boldsymbol{q}^{cmd},\boldsymbol{0})$ of the closed- loop system (23) is asymptotically stable.

(proof) Carrying out the stability analysis, this paper proposes the Lyapunov function candidate $S(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}})$ given by (27).

$$\begin{aligned} S(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) \;=\; & \frac{1}{2}\alpha\boldsymbol{z}^T K_I\boldsymbol{z} + \frac{1}{2}\alpha(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T K_D(\boldsymbol{q}^{cmd} - \boldsymbol{q}) \\ & + \boldsymbol{z}^T K_I(\boldsymbol{q}^{cmd} - \boldsymbol{q}) + \frac{1}{2}\dot{\boldsymbol{q}}^T R(\boldsymbol{q})\dot{\boldsymbol{q}} + U(\boldsymbol{q}) \\ & + \frac{1}{2}(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T K_P(\boldsymbol{q}^{cmd} - \boldsymbol{q}) \\ & - \alpha(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T R(\boldsymbol{q})\dot{\boldsymbol{q}} \end{aligned} \tag{27}$$

The Lyapunov function candidate, that is, (27) consists of the total energy function of the closed-loop system given by (23). $\frac{1}{2}\boldsymbol{z}^T K_I\boldsymbol{z} + \frac{1}{2}(\boldsymbol{q}^{cmd} - \boldsymbol{q})^T K_D(\boldsymbol{q}^{cmd} - \boldsymbol{q})$ denotes the storage function of the PID controller shown in (20). Also $\frac{1}{2}\dot{\boldsymbol{q}}^T R(\boldsymbol{q})\dot{\boldsymbol{q}} + U(\boldsymbol{q})$ is the energy of the robot manipulator given by (1). Passivity of robot manipulator can be proved easily, when the time differentiation of $\frac{1}{2}\dot{\boldsymbol{q}}^T R(\boldsymbol{q})\dot{\boldsymbol{q}} + U(\boldsymbol{q})$ is considered by using (1). As a result, passivity of PID controller and robot manipulator is available. Then, if Assumption 1-4 are satisfied, (27) is semi-positive definite. Here the stability theorem of semi-positive definite Lyapunov function [9] is utilized. The time differentiation of (27) along (23) is calculated as (28), by using (3).

$$\begin{aligned} \dot{S}(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) \; = \; & -(\boldsymbol{q}^{cmd}-\boldsymbol{q})^T \left\{\alpha K_P - K_I\right\}(\boldsymbol{q}^{cmd}-\boldsymbol{q}) \\ & -\dot{\boldsymbol{q}}^T \left\{K_D - \alpha R(\boldsymbol{q})\right\}\dot{\boldsymbol{q}} \\ & +\alpha(\boldsymbol{q}^{cmd}-\boldsymbol{q})^T \left\{-\frac{1}{2}\dot{R}(\boldsymbol{q}) + S(\boldsymbol{q},\dot{\boldsymbol{q}})\right\}\dot{\boldsymbol{q}} \\ & +\alpha(\boldsymbol{q}^{cmd}-\boldsymbol{q})^T \boldsymbol{g}(\boldsymbol{q}) \end{aligned} \tag{28}$$

Considering Assumption 1-4, (28) is rewritten as

$$\begin{aligned} \dot{S}(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) \; \leq \; & -\frac{\alpha}{16}(\boldsymbol{q}^{cmd}-\boldsymbol{q})^T K_P(\boldsymbol{q}^{cmd}-\boldsymbol{q}) - \frac{1}{4}\dot{\boldsymbol{q}}^T K_D\dot{\boldsymbol{q}} \\ & +\alpha(\boldsymbol{q}^{cmd}-\boldsymbol{q})^T \left\{-\frac{1}{2}\dot{R}(\boldsymbol{q}) + S(\boldsymbol{q},\dot{\boldsymbol{q}})\right\}\dot{\boldsymbol{q}}. \end{aligned} \tag{29}$$

Since $R(\boldsymbol{q})$ is constant or a trigonometric function, each entry of $\dot{R}(\boldsymbol{q})$ and $S(\boldsymbol{q},\dot{\boldsymbol{q}})$ decrease as $\dot{\boldsymbol{q}}$ approaches $\mathbf{0}$. Hence, when $\dot{\boldsymbol{q}}$ approaches $\mathbf{0}$, (28) is satisfying (30).

$$\dot{S}(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) \leq -\frac{\alpha}{16}(\boldsymbol{q}^{cmd}-\boldsymbol{q})^T K_P(\boldsymbol{q}^{cmd}-\boldsymbol{q}) - \frac{1}{4}\dot{\boldsymbol{q}}^T K_D\dot{\boldsymbol{q}} \tag{30}$$

Finally, invoking the LaSalle's theorem [6], the equilibrium point $(\boldsymbol{z},\boldsymbol{q},\dot{\boldsymbol{q}}) = (K_I^{-1}\boldsymbol{g}(\boldsymbol{q}^{cmd}),\boldsymbol{q}^{cmd},\mathbf{0})$ of the closed-loop system (23) is asymptotically stable, when dynamic range of manipulator motion is less than natural frequency of mechanical system.

As shown in the above analysis, Assumption 1-4 are designed by PID controller. Because the Lyapunov function candidate (27) consists of the storage function of PID controller. Assumption 2 and 4 show that PID controller can compensate nonlinear terms. Here, in this case, PID controller guarantees local asymptotic stability. However, as redundant degree-of-freedom increases, it becomes easy to decide configuration satisfying Assumption 1. Therefore position control by PID controller is effective use in redundant systems.

Null Space Controller

The null space controller is given as follows.

$$\ddot{\boldsymbol{q}}_{null}^{ref} = J_{vn}^{-1}\left\{K_P(\boldsymbol{q}^{cmd}-\boldsymbol{q}) + K_I\boldsymbol{z} - K_D\dot{\boldsymbol{q}}\right\} \tag{31}$$

where $J_{vn} = I \in R^{n\times n}$ denotes an equivalent inertial matrix. In practical implementation, J_{vn} is gain to adjust acceleration control of each joint depending on a torque limit of joint. (31) shows that PID controller (20) is used as null space controller $\ddot{\boldsymbol{q}}_{null}^{ref}$. PID parameters are decided so that the closed-loop system (23) is asymptotically stable. Here (31) satisfies

Assumptions 1-4 and the null space disturbance can be suppressed by tuning PID parameters.

4. Simulations

In this section, simulations are carried out to confirm the proposed strategy. The kinematic model of 4-link redundant manipulator is shown in Fig.3. Manipulator parameters are summarized in Table 1.

Link	1	2	3	4
Length [m]	0.2650	0.2450	0.1950	0.1785
Mass [kg]	2.7	2.0	1.0	0.4
Reduction ratio	1/100	1/100	1/100	1/100
Resolution of encoder [pulse/rev]	1024	1000	1000	1000
Moment of inertia [kgm²]	0.019	0.0038	0.081	0.043
Torque constant [Nm/A]	20.6	0.213	5.76	4.91

Table 1. Manipulator parameters

The joint angle vector and position vector are defined as $\boldsymbol{q} = (q_1, q_2, q_3, q_4)^T$ and $\boldsymbol{x} = (x, y)^T$ respectively.

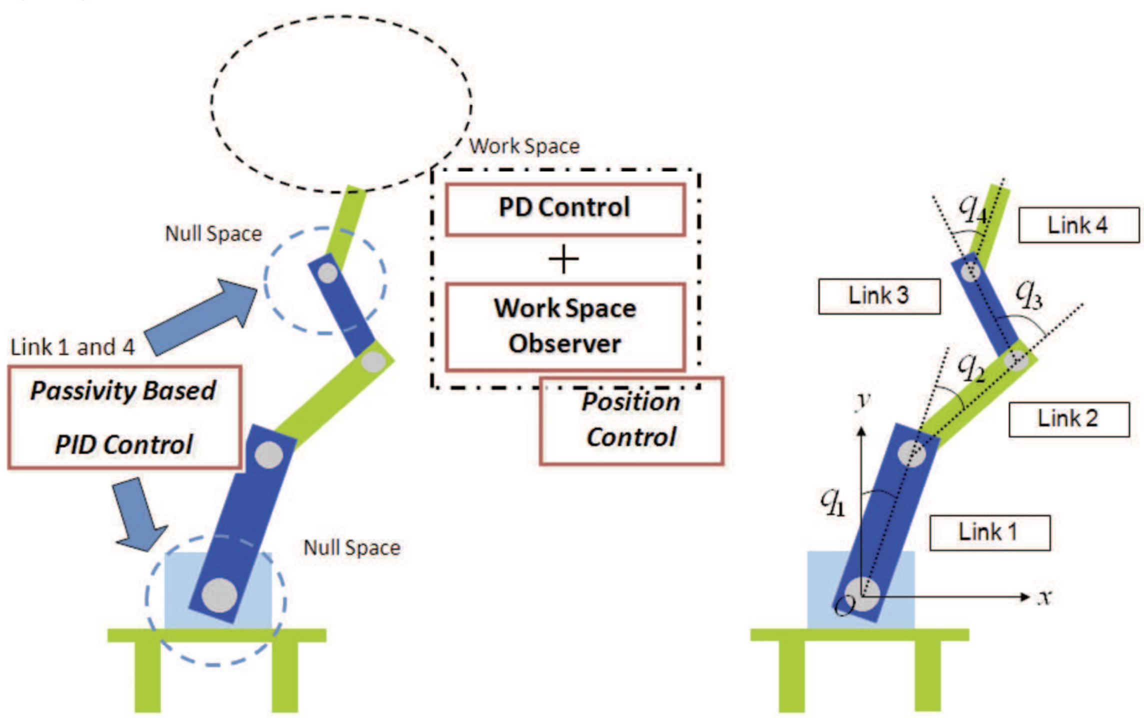

Figure 3. Model of experimental 4-link redundant manipulator

In the simulation, a comparison study of the proposed approach and null space observer is implemented. As described before, the proposed approach is position control in work space and PID control in null space. The compared approach is position control in work space and PD control using null space observer shown in Fig.4. Null space observer is designed based on disturbance observer [7]. Although a structure of null space observer based approach seems PID control, by point of feedback of acceleration disturbance $\hat{\ddot{q}}^{dis}$, it is different from proposed method. The initial configuration of manipulator is as follows.

$$q(0) = \begin{bmatrix} \frac{\pi}{3} & -\frac{\pi}{6} & \frac{\pi}{6} & \frac{\pi}{6} \end{bmatrix}^T$$

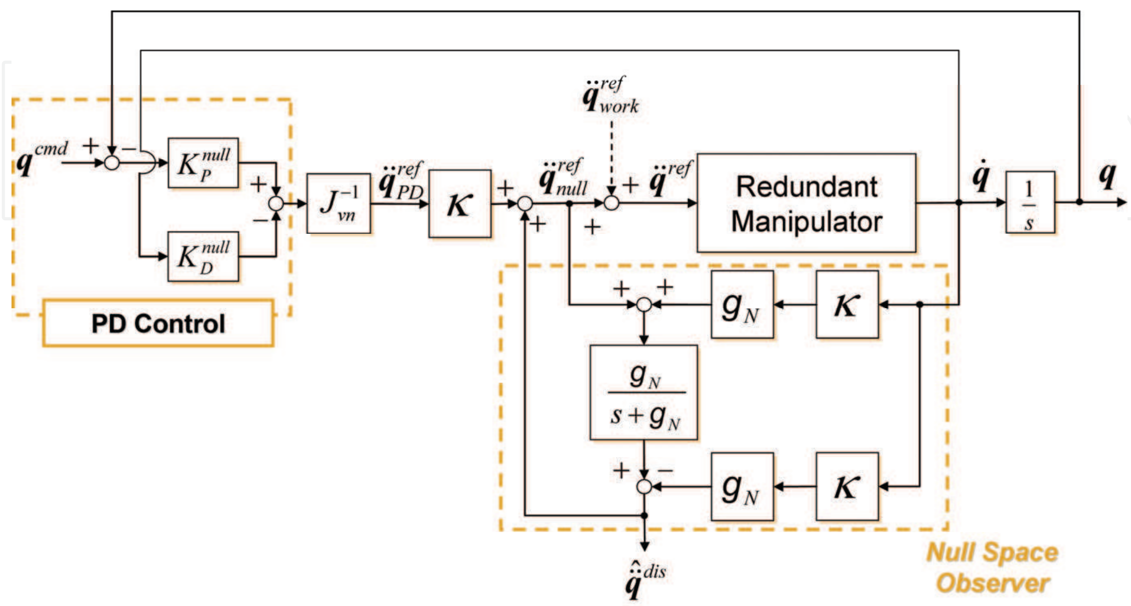

Figure 4. Null space observer based approach

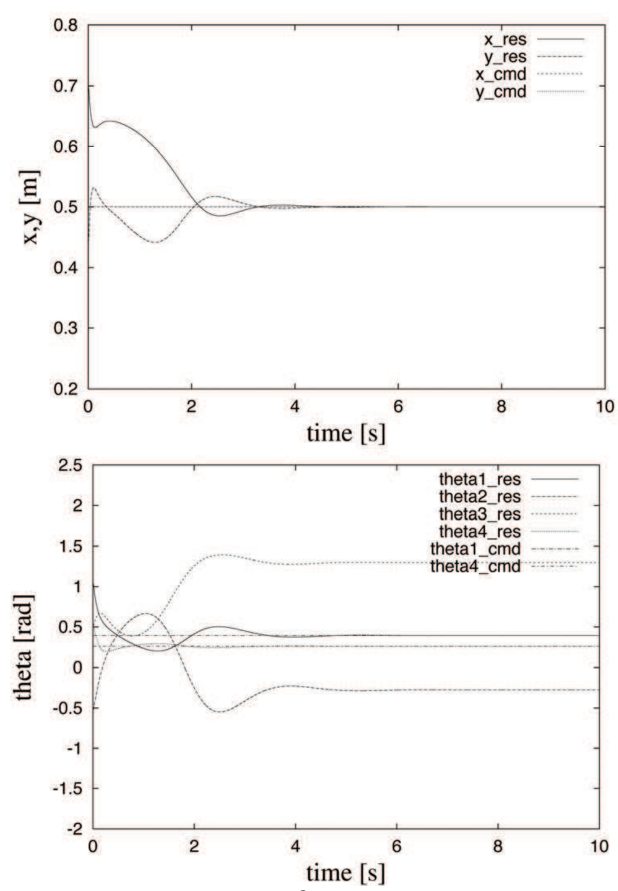

Figure 5. Simulation results of position and θ responses in PID control

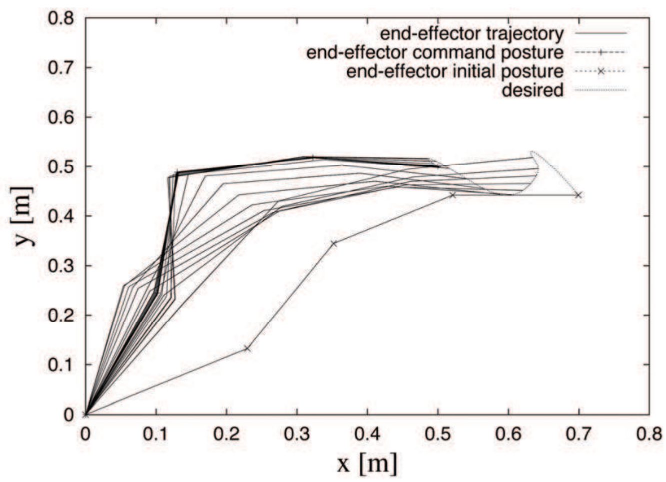

Figure 6. Simulation of manipulator trajectory in PID control

The position commands of end-effector are shown in

$$\boldsymbol{x}^{cmd} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}^T, \; \dot{\boldsymbol{x}}^{cmd} = \begin{bmatrix} 0.0 & 0.0 \end{bmatrix}^T.$$

The controller of work space uses the PD controller (15). K_P^{work} and K_D^{work} are given in

$$K_P^{work} = \begin{bmatrix} 400 & 0 \\ 0 & 400 \end{bmatrix}, \; K_D^{work} = \begin{bmatrix} 60 & 0 \\ 0 & 60 \end{bmatrix}.$$

These parameters are decided by a natural frequency and a damping coefficient.
The gain of work space observer g_W is 50rad/s.
PID controller given by (31) is employed in joint 1 and 4.

$$q_1^{cmd} = \frac{\pi}{8}, \; q_4^{cmd} = \frac{\pi}{12}$$

In joint 2 and 3, velocity feedback control called a null space dumping is implemented for stability improvement of null space motion. Then, the command angle is given as a time function so that the joint angles converge to them in 5.0s.

4.1 PID control

PID parameters are as follows.

$$K_P = \mathrm{diag}(1000, 0, 0, 1000), \; K_I = \mathrm{diag}(2500, 0, 0, 2500),$$
$$K_D = \mathrm{diag}(60, 60, 60, 60)$$

First of all, the higher gain K_I is selected according to system noise. Then other parameters are adjusted by using α. Therefore PID parameters satisfy Assumptions 1-4, sufficiently. Fig.5 and 6 show position and θ response.

4.2 Null Space Observer Based Approach

PD controller of null space is shown in

$$\ddot{\boldsymbol{q}}_{PD}^{ref} = J_{vn}^{-1} \left\{ K_P^{null} (\boldsymbol{q}^{cmd} - \boldsymbol{q}) - K_D^{null} \dot{\boldsymbol{q}} \right\} \tag{32}$$

where $J_{vn} = I \in R^{n \times n}$ denotes the inertia matrix. PD controller given by (31) is set to joint 1 and 4. In joint 2 and 3, the null space dumping is implemented for stability improvement of null space motion. PD parameters are as follows.

$$K_P^{null} = \mathrm{diag}(400, 0, 0, 400), \ K_D^{null} = \mathrm{diag}(60, 60, 60, 60)$$

They are decided so that the first precision improves. The null space observer gain g_N is 50rad/s. Fig.7 and 8 show position and θ response.

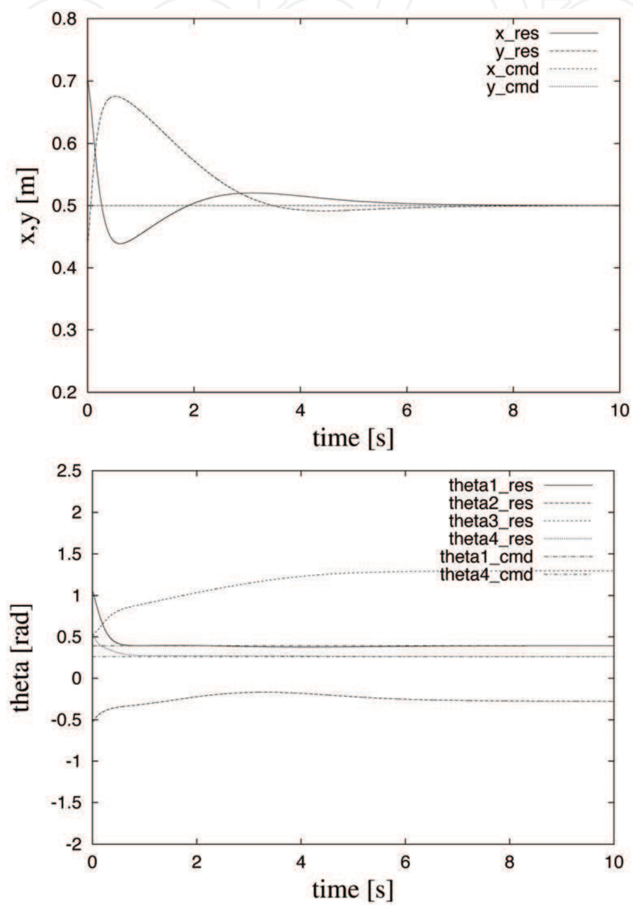

Figure 7. Simulation Results of Position and θ Responses in Null Space Observer Based Approach

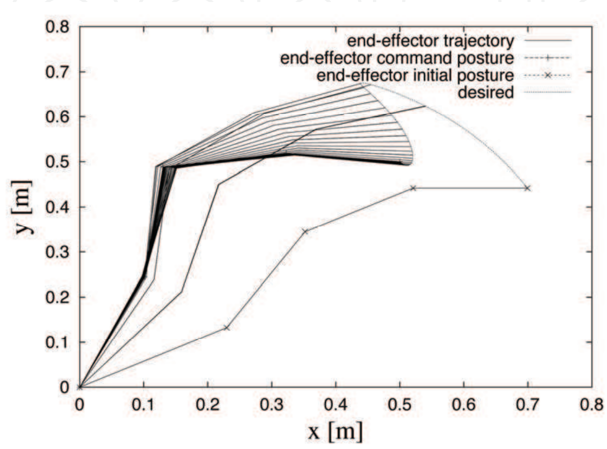

Figure 8. Simulation of Manipulator Trajectory in Null Space Observer Based Approach

4.3 Results and Discussions

In simulation results, the position and θ responses coincided with each command. It is realized that the PID control and null space observer based approach are the same stabilizing ability. Hence, the proposed simple PID controller has an equivalent performance of null space observer.

5. Conclusions

This chapter shows the control design of null space motion by PID controller. When the work space observer is emplyed in work space controller, work space and null space motion are determined independently. Then the PD based work space controller makes work space motion stable, but global stability of null space motion is not always guranteed. To improve the stability and the robustness of null space motion, PID controller considering passivity is useful and the design strategy of PID controller is established for the motion control of null space in this chapter. Several simulations of 4-link manipulator are implemented to confirm the validity of the proposed method. Finally, the feasibility of control design of null space by "classical" PID control and practical/effective use of null space motion are shown.

6. References

T. Murakami, K. Kahlen, and R. W. De Doncker, Robust motion control based on projection plane in redundant manipulator, *IEEE Trans. Ind. Electron.*, vol.49, no.l, pp.248-255, 2002. [1]

S. Arimoto, Control theory of non-linear mechanical systems, *Oxford Science Publications*, 1996. [2]

S. Arimoto, Fundamental problems of robot control: Part I, Innovations in the realm of robot servo-loops, *Robotica*, vol.13, pp.10—27, 1995. [3]

H. K. Khalil, Nonlinear systems, 3rd ed., *NJ:Prentice-Hall*, 2002. [4]

B. Brogliato, R. Lozano, B. Maschke, and O. Egeland, Dissipative systems analysis and control —theory and applications—, *Springer*, 2006. [5]

J. P. LaSalle and S. Lefschetz, Stability by Lyapunov's direct method, *Academic Press*, 1961. [6]

K. Ohnishi, M. Shibata, and T. Murakami, Motion control for advanced mechatronics, *IEEE/ASME Trans. Mech.*, vol.1, no.l, pp.56-67, 1996. [7]

M. W.Spong and M. V. Vidyasagar, *Robot dynamics and control*, John Wiley & Sons, 1989. [8]

R. Sepulchre, M. Jankovic, and P. Kokotovic, *Constructive Nonlinear Control*, Springer, 1997. [9]

Robot Manipulators

Edited by Marco Ceccarelli

ISBN 978-953-7619-06-0

Hard cover, 546 pages

Publisher InTech

Published online 01, September, 2008

Published in print edition September, 2008

In this book we have grouped contributions in 28 chapters from several authors all around the world on the several aspects and challenges of research and applications of robots with the aim to show the recent advances and problems that still need to be considered for future improvements of robot success in worldwide frames. Each chapter addresses a specific area of modeling, design, and application of robots but with an eye to give an integrated view of what make a robot a unique modern system for many different uses and future potential applications. Main attention has been focused on design issues as thought challenging for improving capabilities and further possibilities of robots for new and old applications, as seen from today technologies and research programs. Thus, great attention has been addressed to control aspects that are strongly evolving also as function of the improvements in robot modeling, sensors, servo-power systems, and informatics. But even other aspects are considered as of fundamental challenge both in design and use of robots with improved performance and capabilities, like for example kinematic design, dynamics, vision integration.

InTech Europe

University Campus STeP Ri

Slavka Krautzeka 83/A

51000 Rijeka, Croatia

Phone: +385 (51) 770 447

Fax: +385 (51) 686 166

www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai

No.65, Yan An Road (West), Shanghai, 200040, China

中国上海市延安西路65号上海国际贵都大饭店办公楼405单元

Phone: +86-21-62489820

Fax: +86-21-62489821