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# Robot Control Using On-Line Modification of Reference Trajectories 

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## 1. Introduction

In practice, the robot motion is specified according to physical constraints, such as limited torque input. Thus, the computation of the desired trajectory is constrained to attain the physical limit of actuator saturations. See, e.g., (Pfeiffer \& Johanni, 1987), for a review of trajectory planning algorithms considering the robot model parameters and torque saturation. Besides, trajectories can also be planned irrespectively of the estimated robot model, i.e., simply by using constraints of position, velocity and acceleration at each time instant, see, e.g., (Cao et al., 1997; Macfarlane \& Croft, 2003).
Once that the reference trajectory is specified, the task execution is achieved in real time by using a trajectory tracking controller. However, if parametric errors in the estimated model are presented, and considering that the desired trajectories require to use the maximum torque, no margin to suppress the tracking error will be available. As consequence, the manipulator deviates from the desired trajectory and poor execution of the task is obtained. On-line time-scaling of trajectories has been studied in the literature as an alternative to solve the problem of trajectory tracking control considering constrained torques and model uncertainties.
A technique for time-scaling of off-line planned trajectories is introduced by Hollerbach (1984). The method provides a way to determine whether a planned trajectory is dynamically realizable given actuator torque limits, and a mode to bring it to one realizable. However, this method assumes that the robot dynamics is perfectly known and robustness issues were not considered. It is noteworthy that this approach has been extended to the cases of multiple robots in cooperative tasks (Moon \& Ahmad, 1991) and robot manipulators with elastic joints (De Luca \& Farina, 2002). In order to tackle the drawback of the assumption that the robot model in exactly known, Dahl and Nielsen (1990) proposed a control algorithm that result in the tracking of a time-scaled trajectory obtained from a specified geometric path and a modified on-line velocity profile. The method considers an internal loop that limits the slope of the path velocity when the torque input is saturated. Other solutions have been proposed in, e.g., (Arai et al., 1994; Eom et al., 2001; Niu \& Tomizuka, 2001).
In this chapter, an algorithm for tracking control of manipulators under the practical situation of limited torques and model uncertainties is introduced. The proposed approach consists of using a trajectory tracking controller and an algorithm to obtain on-line time-
scaled reference trajectories. This is achieved through the control of a time-scaling factor, which is motivated by the ideas introduced by Hollerbach (1984). The new method does not require the specification of a velocity profile, as in (Dahl \& Nielsen, 1990; Dahl, 1992; Arai et al., 1994). Instead, we formulate the problem departing from the specification of a desired path and a desired timing law. In addition, experiments in a two degrees-of-freedom directdrive robot show the practical feasibility of the proposed methodology.

## 2. Robot model, desired motion description and control problem formulation

### 2.1 Robot model

The dynamics in joint space of a serial-chain $n$-link robot manipulator, considering the presence of friction at the robot joints, can be written as (Sciavicco \& Siciliano, 2000)

$$
\begin{equation*}
M(q) \ddot{q}+C(q, \dot{q}) \dot{q}+g(q)+f(\dot{q})=\tau \tag{1}
\end{equation*}
$$

where $q$ is the $n \times 1$ vector of joint displacements, $\dot{q}$ is the $n \times 1$ vector of joint velocities, $\tau$ is the $n \times 1$ vector of applied torque inputs, $M(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(q, \dot{q}) \dot{q}$ is the $n \times 1$ vector of centripetal and Coriolis torques, $g(q)$ is the $n \times 1$ vector of gravitational torques, and $f(\dot{q})$ is the vector of forces and torques due to the viscous and Coulomb friction, which depends on joint velocities. Besides, the $i$ th element of the friction vector $f(\dot{q})$ satisfies,

$$
\left|f_{i}(\dot{q})\right| \leq f_{C i}+f_{v i}\left|\dot{q}_{i}\right|
$$

where $f_{\mathrm{C} i}$ and $f_{v i}$ are positive constants, and $i=1, \ldots, n$.
Let $\Psi$ denotes the torque space, defined as

$$
\begin{equation*}
\Psi=\left\{\tau \in R^{n}:-\tau_{i}^{\max }<\tau_{i}<\tau_{i}^{\max }\right\}, i=1, \ldots, n \tag{2}
\end{equation*}
$$

with $\tau_{i}^{\max }>0$ the maximum torque input for the $i$ th joint. It is assumed that

$$
\begin{equation*}
f_{\mathrm{C} i}+\left|g_{i}(q)\right|<\tau_{i}^{\max } i=1, \ldots, n \tag{3}
\end{equation*}
$$

where $g_{i}(q)$ is the $i$ th element of the vector $g(q)$. Assumption (3) ensures that any joint position $q$ will be reachable.

### 2.2 Desired motion description and control problem formulation

The desired motions of a robot arm can be formulated in terms of a desired geometric path in the joint space, and an associated timing law. A desired path is given as a function

$$
\begin{equation*}
q_{d}(s):\left[s_{0}, s_{f}\right] \rightarrow R^{n}, \tag{4}
\end{equation*}
$$

where $s_{f}>s_{0}>0$, and $s$ is called path parameter. It is assumed that the desired path attains

$$
\frac{d}{d s} q_{d}(s) \neq 0
$$

This assumption is required in the problem design of time optimal trajectories (see e.g. (Pfeiffer \& Johanni, 1987)) and also will be necessary in the results exposed in this chaper. The motion specification is completed by specifying a timing law (Hollerbach, 1984):

$$
\begin{equation*}
s(t):\left[t_{0}, t_{f}\right] \rightarrow\left[s_{0}, s_{f}\right] \tag{5}
\end{equation*}
$$

which is a strictly increasing function. The timing law (5) is the path parameter as a time function. As practical matter, it is possible to assume that the path parameter $s(t)$ is a piecewise twice differentiable function. The first derivative $\dot{s}(t)$ is called path velocity and the second derivative $\ddot{s}(t)$ is called path acceleration (Dahl, 1990).
A nominal trajectory is obtained from the path $q_{d}(s)$ and the timing law, i.e.,

$$
\begin{equation*}
q_{r}(t)=q_{d}(s(t)) . \tag{6}
\end{equation*}
$$

### 2.3 Control problem formulation

Let us suppose that only estimations of the robot model (1) are available, namely $\hat{M}(q)$ for the inertia matrix, $\hat{C}(q, \dot{q})$ for the centripetal and Coriolis matrix, $\hat{g}(q)$ for the vector of gravitational forces, and $\hat{f}(\dot{q})$ for the vector of friction forces.
The control problem is to design an algorithm so that forward motion along the path $q_{d}(\sigma(t))$, with $\sigma(t)$ a strictly increasing function, is obtained as precise as possible, i.e.,

$$
\begin{equation*}
\left\|q_{d}(\sigma(t))-q(t)\right\| \leq \varepsilon, \forall t \geq t_{0} \tag{7}
\end{equation*}
$$

where $\varepsilon$ is a positive constant, and considering that the input torque $\tau$ must be into the admissible torque space, i.e., $\tau \in \Psi$.
It is important to remark that in our approach is not necessary that the specified nominal trajectories (6), which are encoded indirectly by the desired path $q_{d}(s)$ and the timing law $s(t)$, evaluated for the estimated robot model produce torque into the admissible torque space $\Psi$ in (2). In other words, it is not necessary the assumption


Figure 1. Block diagram of the classical approach of tracking control of nominal trajectories

This is because the proposed algorithm generate on-line new trajectories that produce admissible control inputs.

## 3. Tracking control of nominal trajectories

Trajectory tracking control is the classical approach to solve the motions tasks specified by the nominal trajectory (6). Since we have assumed the partial knowledge of the robot model, a controller that guarantees robust tracking of the nominal trajectory $q_{r}(t)=q_{d}(s(t))$ must be used. Many algorithms have been proposed with this aim, between them we have the following computed torque-based controller (Lewis et al., 1993)

$$
\begin{equation*}
\tau=\hat{M}(q)\left[\frac{d^{2}}{d t^{2}} q_{d}(s)+K_{v} \dot{e}_{p}+K_{p} e_{p}\right]+\hat{C}(q, \dot{q}) \dot{q}+\hat{g}(q)+\hat{f}(\dot{q}) \tag{8}
\end{equation*}
$$

where

$$
\begin{gathered}
e_{p}(t)=q_{d}(s(t))-q(t), \\
\dot{e}_{p}(t)=\dot{q}_{d}(s(t))-\dot{q}(t), \\
\frac{d^{2}}{d t^{2}} q_{d}(s)=\frac{d^{2} q_{d}(x)}{d x^{2}}(s) \dot{s}^{2}+\frac{d q_{d}(x)}{d x}(s) \ddot{s},
\end{gathered}
$$

and $K_{p}, K_{v}$ are $n \times n$ symmetric positive definite matrices.
Considering unconstrained torque input, the control law (8) guarantees that the signals $e_{p}(t)$ and $\dot{e}_{p}(t)$ are uniformly ultimately bounded (see e.g. (Lewis et al., 1993). Figure 1 shows a block diagram of implementation of the trajectory tracking controller (8), having as inputs the path parameter $s(t)$, the path velocity $\dot{s}(t)$, and the path acceleration $\ddot{s}(t)$. With respect to the performance requirement (7), in the case of the classical trajectory tracking control, we have that $\sigma(t)=s(t)$.
Notwithstanding, the torque capability of the robot actuators is limited, and frequently desired trajectories are planned using this fact. Thus, it does not exist room for extra torque of the feedback control action for compensating the model disturbances; hence a poor tracking performance (7) is presented.

## 4. Tracking control of on-line time-scaled reference trajectories

Let us define a new path parameter $\sigma$ in the following way

$$
\begin{equation*}
\sigma(t)=c(t) s(t) \tag{9}
\end{equation*}
$$

where $c(t)$ is a scalar function called time-scaling factor, and $s(t)$ is the path parameter given as a time function in an explicit way. The desired trajectory is obtained using the new path parameter (1) into the path (4) as a function of time:

$$
\begin{equation*}
\bar{q}_{r}(t)=q_{d}(\sigma(t))=q_{d}(c(t) s(t)) . \tag{10}
\end{equation*}
$$

The signal $c(t)$ in (9) is a time-scaling factor of the nominal trajectory $\bar{q}_{r}(t)$ in (10). The introduction of a time-scaling factor $c(t)$ is also used in the non robust algorithm proposed in (Hollerbach, 1984). Let us notice that the nominal (desired) value of the time scaling factor $c(t)$ is one. It is easy to see that when time-scaling is not present, the time evolution of $\sigma(t)$ is identical to $s(t)$, and $\bar{q}_{r}(t)=q_{r}(t)$, where $q_{r}(t)$ is given by (6). An important point in the definition of the time-scaling factor $c(t)$ is that if $c(t)>1$ the movement is sped up and if $c(t)<1$ the movement is slowed down. Thus, movement speed can be dynamically changed to compensate errors in the resulting tracking performance of trajectory (10).
The time derivative of $\sigma(t)$ in (9) is given as follows

$$
\dot{\sigma}=\dot{c} s+c \dot{s}
$$

We call to the signal $\dot{c}(t)$ time-scaling velocity of the nominal trajectory. Further, the path acceleration $\ddot{\sigma}$ is given by

$$
\begin{equation*}
\ddot{\sigma}=\ddot{c} s+\gamma, \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
\gamma=2 \ddot{c} \dot{s}+c \ddot{s} \tag{12}
\end{equation*}
$$

The signal $\ddot{c}(t)$ in equation (11) is called time-scaling acceleration.
Let us consider the trajectory tracking controller given by

$$
\begin{equation*}
\tau=\hat{M}(q)\left[\frac{d^{2}}{d t^{2}} q_{d}(c s)+K_{v} \dot{\bar{e}}_{p}+K_{p} \overline{\bar{e}}_{p}\right]+\hat{C}(q, \dot{q}) \dot{q}+\hat{g}(q)+\hat{f}(\dot{q}) \tag{13}
\end{equation*}
$$

where

$$
\begin{gathered}
\bar{e}_{p}(t)=\bar{q}_{d}(c(t) s(t))-q(t) \\
\dot{\bar{e}}_{p}(t)=\dot{\bar{q}}_{r}(c(t) s(t))-\dot{q}(t), \\
\frac{d^{2}}{d t^{2}} q_{d}(c s)=\frac{d^{2} q_{d}(x)}{d x^{2}}(c s) \underbrace{[\dot{c} \dot{s}+c \dot{s}]^{2}}_{\dot{\sigma}^{2}}+\frac{d q_{d}(x)}{d x}(c s) \underbrace{[\ddot{c} s+\gamma]}_{\dot{\sigma}}
\end{gathered}
$$

and $K_{p}, K_{v}$ are $n \times n$ symmetric positive definite matrices. Thus, under the control scheme (13) we have that performance requirement (7) is given with $\sigma(t)=c(t) s(t)$.

It is worth noting that controller (13) can be written in a parametric form by factoring the time scaling acceleration $\ddot{c}$, i.e.

$$
\begin{equation*}
\tau=\beta_{1}(c, s, q) \ddot{c}+\beta_{2}(c, \dot{c}, s, \dot{s}, q, \dot{q}) \tag{14}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta_{1}(c, s, q)=\hat{M}(q) \frac{d q_{d}(x)}{d x}(c s) s \tag{15}
\end{equation*}
$$

$$
\begin{align*}
& \beta_{2}(\sigma, \dot{\sigma}, q, \dot{q})=\hat{M}(q) \frac{d q_{d}(x)}{d x}(c s) \gamma \\
& \quad+\hat{M}(q)\left[\frac{d^{2} q_{d}(x)}{d x^{2}}(c s)[\dot{c} s+c \dot{s}]^{2}+\right.  \tag{16}\\
& \left.\quad+K_{v} \dot{e}_{p}+K_{p} e_{p}\right] \\
& \quad+\hat{C}(q, \dot{q}) \dot{q}+\hat{g}(q)+\hat{f}(\dot{q}),
\end{align*}
$$

and $\gamma$ is defined in (12).
The path acceleration $\ddot{\sigma}$ in equation (11) is expressed in terms of the time-scaling factor $c(t)$, the parameter $s(t)$, and successive time derivatives. In this way, instead of constraining the path acceleration $\ddot{\sigma}(t)$, as done in (Dahl \& Nielsen, 1990; Dahl, 1992), for enforcing the control torque into the admissible limits, it is possible to constrain the scaling acceleration $\ddot{c}(t)$ using the limits of torque input for each joint $i$, that is to say

$$
\ddot{c}_{i}^{\min } \leq \ddot{c} \leq \ddot{c}_{i}^{\max }
$$

The signals $\ddot{c}_{i}^{\text {min }}$ and $\ddot{c}_{i}^{\text {max }}$ are computed in the following way:

$$
\begin{aligned}
& \ddot{c}_{i}^{\max }=\left\{\begin{array}{cc}
{\left[\tau_{i}^{\max }-\beta_{2 i}\right] / \beta_{1 i},} & \forall \beta_{1 i}>0, \\
{\left[-\tau_{i}^{\max }-\beta_{2 i}\right] / \beta_{1 i},} & \forall \beta_{1 i}<0, \\
\infty, & \forall \beta_{1 i}=0,
\end{array}\right. \\
& \ddot{c}_{i}^{\min }=\left\{\begin{array}{cc}
{\left[-\tau_{i}^{\max }-\beta_{2 i}\right] / \beta_{1 i},} & \forall \beta_{1 i}>0, \\
{\left[\tau_{i}^{\text {max }}-\beta_{2 i}\right] / \beta_{1 i},} & \forall \beta_{1 i}<0, \\
-\infty, & \forall \beta_{1 i}=0,
\end{array}\right.
\end{aligned}
$$

where $\beta_{1 i}$ and $\beta_{2 i}$ are given explicitly in equations (15), and (16), respectively.
Let us remark that for computing the values of $\ddot{c}_{i}^{m i n}$ and $\ddot{c}_{i}^{m a x}$, the assumption that $s_{0}>0$ is required to avoid that $\ddot{c}_{i}^{\text {min }}$ and $\ddot{c}_{i}^{\text {max }}$ become undetermined.
The limits of the time scaling acceleration $\ddot{c}$, which depends on $s, \dot{s}, \ddot{s}, c, \dot{c}$, and measured signals $q_{i}, \dot{q}_{i}$, are given by

$$
\begin{align*}
\ddot{c}^{\max } & =\min _{i}\left\{\ddot{c}_{i}^{\max }\right\},  \tag{17}\\
\ddot{c}^{\text {min }} & =\max _{i}\left\{\ddot{c}_{i}^{\text {min }}\right\}, \tag{18}
\end{align*}
$$

The limits (17)-(18) provide a way of modifying the reference trajectory (10), via alteration of $\ddot{c}(t)$, so that the torque limits are satisfied. If the resulting nominal reference trajectory $\bar{q}_{r}(c(t) s(t))$ is inadmissible in the sense of unacceptable torques, then it will be modified by limiting the time scaling acceleration $\ddot{c}(t)$ to satisfy $\tau_{i} \in \Psi$.

Note that when the time-scaling acceleration $\ddot{c}(t)$ is modified, the value of the time-scaling factor $c(t)$ will change. Thus, considering the limits (17) and (18), we must design an internal feedback to drive the time-scaling factor $c(t)$ in a proper way. The proposed internal feedback is given by

$$
\begin{gather*}
\frac{d}{d t} c=\dot{c}  \tag{19}\\
\frac{d}{d t} \dot{c}=\operatorname{sat}\left(u ; \ddot{c}^{\min }, \ddot{c}^{\max }\right) \tag{20}
\end{gather*}
$$

with the initial conditions $c\left(t_{0}\right)=1$, and $\dot{c}\left(t_{0}\right)=0$, the saturation function

$$
\operatorname{sat}\left(u ; ;^{\min }, \ddot{c}^{\max }\right)=\left\{\begin{array}{ccc}
u & \forall & \ddot{c}^{\min } \leq u \leq \ddot{c}^{\max }, \\
\ddot{c}^{\min } & \forall & u<\ddot{c}^{\min }, \\
\ddot{c}^{\max } & \forall & u>\ddot{c}^{\max },
\end{array}\right.
$$

and $u$ properly designed. The input torques are kept within the admissible limits by using the internal feedback (19)-(20), which provides a way to scale in time the reference trajectories $q_{d}(c(t) s(t))$. We adopt the idea of specifying a time varying desired time-scaling factor. Let us consider

$$
\begin{equation*}
u=\ddot{r}+k_{v c} \dot{\tilde{c}}+k_{p c} \tilde{c}, \tag{21}
\end{equation*}
$$

where $k_{v c}$ and $k_{p c}$ strictly positive constants,

$$
\begin{equation*}
\tilde{c}(t)=r(t)-c(t), \tag{22}
\end{equation*}
$$

and $r(t)$ is the desired time-scaling reference. The control law (21) is used in the internal feedback (19)-(20).


Figure 2. Block diagram of proposed approach of tracking control of on-line time-scaled references trajectories
If the generated control torque is admissible, then the equation (20) satisfies $\ddot{c}=u, u$ defined in (21). Therefore we can write

$$
\ddot{\tilde{c}}+k_{v c} \dot{\tilde{c}}+k_{p c} \tilde{c}=0 .
$$

Because $k_{v c}$ and $k_{p c}$ are strictly positive constants, $\tilde{c}(t) \rightarrow 0$ as $t \rightarrow \infty$ in an exponential form.
We define the desired time-scaling factor $r(t)$ in accord to the following equation

$$
\begin{equation*}
\dot{r}(t)=-k_{r} \tilde{c}(t), r\left(t_{0}\right)=1, \tag{23}
\end{equation*}
$$

where $k_{r}$ is a strictly positive constant, and $\tilde{c}$ is defined in (22). The value of $\ddot{r}(t)$ in (21) is computed differentiating (23) with respect to time.


Figure 3. Experimental robot arm
The proposed dynamics for $r(t)$ in (23) obeys the error between $c(t)$ and $r(t)$. Thus, when torque saturation is detected the time-scaling factor $c(t)$ is modified (see equations (19)-(20)) to make the reference trajectory $q_{d}(c(t) s(t))$ feasible, i.e. to make the input torque admissible. The desired time-scaling factor $r(t)$ can be interpreted as an estimation of the actual timescaling factor $c(t)$. Figure 2 depicts a block diagram of the controller and time-scaling algorithm (14), (19) and (20).
It is worthwhile to note that the proposed time-scaling method is independent of the definition of path parameter $s(t)$, Thus, several definitions of the path parameter $s(t)$ considering different characteristics in its time evolution can be used along the lines of the proposed time-scaling algorithm. See, e.g, (Macfarlane \& Croft, 2003; Cao et al., 1997), for planning algorithms of the timing law $s(t)$.

## 5. Experimental results

A planar two degrees-of-freedom direct-drive arm has been built at the CITEDI-IPN Research Center. The system is composed by two DC Pittman motors operated in current mode with two Advanced Motion servo amplifiers. A Sensoray 626 I/O card is used to read encoder signals (with quadrature included), while control commands are transferred through the D/A channels. The control system is running in real-time with a sampling rate of 1 kHz on a PC over Windows XP operating system and Matlab Real-Time Windows Target. Figure 3 shows the experimental robot arm.

### 5.1 Practical considerations

The torque delivered by DC motor actuators is given by

$$
\tau=K v,
$$

where $K=\operatorname{diag}\left\{k_{1}, \ldots, k_{n}\right\}$ is a positive definite matrix that contains the voltage to torque conversion constants of the motors, and $v \in R^{n}$ is a vector that contains the applied voltage. In this situation, the proposed control law (13) can be implemented as

$$
v=\hat{K}^{-1}\left[\hat{M}(q)\left[\frac{d^{2}}{d t^{2}} q_{d}(c s)+K_{v} \dot{\bar{e}}_{p}+K_{p} \bar{e}_{p}\right]+\hat{C}(q, \dot{q}) \dot{q}+\hat{g}(q)+\hat{f}(\dot{q})\right]
$$

where $\hat{K}=\operatorname{diag}\left\{\hat{k}_{1}, \ldots, \hat{k}_{n}\right\}$ is an estimation of $K$. Note that torque limits $\Psi$ in (2) can be indirectly specified by voltage limits

$$
\Upsilon=\left\{v \in R^{n}:-v_{i}^{\max }<\tau_{i}<v_{i}^{\max }\right\}, i=1, \ldots, n
$$

with $v_{i}^{\max }>0$ the maximum voltage input for the $i$ th joint. In order to obtain the real-time time-scaling factor $c(t)$, the parameterization (15) -(16) and limits (17)-(18) should be used with $v_{i}^{\max }$ instead of $\tau_{i}^{\max }$.
For simplicity, we have selected

$$
\hat{K}=\operatorname{diag}\{1,1\}[N m / \operatorname{Volt}] .
$$

A constant matrix for the estimation of the inertia matrix is proposed, i.e.,

$$
\begin{equation*}
\hat{M}=\operatorname{diag}\{0.2,0.01\}\left[\mathrm{Nm} \cdot \mathrm{sec}^{2} / \mathrm{rad}\right], \tag{24}
\end{equation*}
$$

while $\hat{C}(q, \dot{q})=0$ for the centripetal and Coriolis matrix, $\hat{g}(q)=0$ for the vector of gravitational forces, and $\hat{f}(\dot{q})=0$ for the friction forces vector. The voltage limits

$$
\begin{equation*}
v_{1}^{\max }=5[\mathrm{Volt}] \text { and } v_{2}^{\max }=0.5[\mathrm{Volt}] . \tag{24}
\end{equation*}
$$

were specified. We have assumed that with the voltage limits (24), the assumption (3) is attained. In other words, we considered that there is enough voltage room to compensate the disturbances due to the friction $f(\dot{q})$ present in the robot model (1). Note that the vector of gravitational forces $g(q)$ is zero, since the joints of the experimental robot arm move in the horizontal plane.
The requested task is to drive the arm in such a way that its joint position $q(t)$ traces a desired path with prescribed velocity in the joint configuration space. The path should be traced with nominal constant tangent velocity. The desired path is

$$
q_{d}(s)=\left[\begin{array}{l}
r_{0} \cos \left(v_{0} s\right)  \tag{25}\\
r_{0} \sin \left(v_{0} s\right)
\end{array}\right],
$$

where $r_{0}=1[\mathrm{rad}]$ and $v_{0}=2.5[\mathrm{rad} / \mathrm{sec}]$. The timing law, i.e., the path parameter $s(t)$ as function of time, is specified as follows

$$
s(t)=\left\{\begin{array}{l}
\frac{v_{0}}{2} t^{2}+0.001, \text { for } t<1, \\
v_{0}[t-0.5], \text { for } t \geq 1,
\end{array}\right.
$$

which is piecewise twice differentiable. In the simulations, the initial configuration of the arm is $q_{1}(0)=1[\mathrm{rad}], q_{2}(0)=0[\mathrm{rad}]$, and $\dot{q}_{1}(0)=\dot{q}_{1}(0)=0[\mathrm{rad} / \mathrm{sec}]$. It is noteworthy that $s_{0}=s(0)>0$ is satisfied.


Figure 4.Tracking control of nominal trajectories: Applied voltage


Figure 5. Tracking control of nominal trajectories: $q_{2}$ versus $q_{1}$


Figure 6. Tracking control of nominal trajectories: Tracking errors

### 5.2 Tracking control of nominal trajectories

In order to compare the performance of the proposed algorithm, we carried out a simulation with the trajectory tracking controller given by (8). Thus, considering the error signal

$$
e_{p}(t)=q_{r}(t)-q(t),
$$

where the desired nominal trajectory $q_{r}(t)$ is defined in (6), the resulting trajectory tracking controller is given by

$$
\begin{equation*}
v=\hat{M}\left[\ddot{q}_{r}+K_{v} \dot{e}_{p}+K_{p} e_{p}\right] \tag{26}
\end{equation*}
$$

which results from the above specifications. The used gains were

$$
\begin{align*}
K_{p} & =\operatorname{diag}\{100,400\}\left[1 / \sec ^{2}\right],  \tag{27}\\
K_{v} & =\operatorname{diag}\{10,20\}[1 / \mathrm{sec}] \tag{28}
\end{align*}
$$

The experimental results are presented in Figure 4 that shows the applied voltage, in Figure 5 that depicts the traced path in $q_{1}-q_{2}$ coordinates, and in Figure 6 that describes the time evolution of the tracking errors.
We note in Figure 4 that both control voltages are saturated at the same time intervals. This is undesirable, because it produces deviations from the specified desired path, as it can be shown in Figure 5. On the other hand, tracking errors $e_{p 1}(t)$ and $e_{p 2}(t)$ are large, as
illustrated in Figure 6. Specifically, we have considered $t \geq 2$ [sec] for the steady state regimen, thus $\max _{\forall t \geq 2}\left\{e_{p 1}(t)\right\}=1.38[\mathrm{rad}]$ and $\max _{\forall t \geq 2}\left\{e_{p 2}(t)\right\}=6.5[\mathrm{rad}]$.


Figure 7. Tracking control of on-line time-scaled references trajectories: Applied voltage


Figure 8. Tracking control of on-line time-scaled references trajectories: $q_{2}$ versus $q_{1}$


Figure 9. Tracking control of on-line time-scaled references trajectories: Tracking errors

### 5.3 Tracking control of on-line time-scaled reference trajectories

By taking into account the practical considerations in Section 5.2, we implemented in realtime the proposed scheme of tracking control of on-line time-scaled reference trajectories (14), (19), (20), and (21)-(23). The control gains (27)-(28) were used in the controller (13), the gains $k_{p c}=50$ and $k_{v c}=14$, in the internal feedback (21), and $k_{r}=50$ in the system (23).
Figure 7 shows the applied voltage, Figure 8 illustrates the traced path in $q_{1}-q_{2}$ coordinates, Figure 9 depicts the tracking errors, and finally Figure 10 shows the time evolution of the time-scaling fact


Figure 10. Tracking control of on-line time-scaled references trajectories: Time-scaling factor It is possible to observe that the applied voltage remains within the admissible limits, and simultaneous saturation does not occur. Moreover, tracking errors are drastically smaller than
the tracking errors obtained with tracking control of nominal trajectories (26). See Figure 6 and Figure 9 to compare the performance of the tracking errors $e_{p}(t)$ with respect to the tracking errors $\bar{e}_{p}(t)$. Particularly, $\max _{\forall t \geq 2}\left\{\bar{e}_{p 1}(t)\right\}=0.36$ [rad] and $\max _{\forall t \geq 2}\left\{\bar{e}_{p 2}(t)\right\}=0.1$ [rad], which are drastically smaller than values obtained for the classical trajecory tracking controller. Finally, Figure 10 shows that the time-scaling factor $c(t)$ tends to 0.6 , thus the tracking accuracy is improved because the reference trajectory $q_{d}(c(t) s(t))$ is slowed down.

## 6. Conclusion

An approach for trajectory tracking control of manipulators subject to constrained torques has been proposed. A secondary loop to control the time-scaling of the reference trajectory is used, then the torque limits are respected during the real-time operation of the robot. The proposed algorithm does not require the specification of a velocity profile, as required in (Dahl \& Nielson, 1990; Dahl, 1992; Arai et al. 1994), but instead a timing law should be specified.

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## Robot Manipulators

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ISBN 978－953－7619－06－0
Hard cover， 546 pages
Publisher InTech
Published online 01，September， 2008
Published in print edition September， 2008

In this book we have grouped contributions in 28 chapters from several authors all around the world on the several aspects and challenges of research and applications of robots with the aim to show the recent advances and problems that still need to be considered for future improvements of robot success in worldwide frames．Each chapter addresses a specific area of modeling，design，and application of robots but with an eye to give an integrated view of what make a robot a unique modern system for many different uses and future potential applications．Main attention has been focused on design issues as thought challenging for improving capabilities and further possibilities of robots for new and old applications，as seen from today technologies and research programs．Thus，great attention has been addressed to control aspects that are strongly evolving also as function of the improvements in robot modeling，sensors，servo－power systems，and informatics．But even other aspects are considered as of fundamental challenge both in design and use of robots with improved performance and capabilities，like for example kinematic design，dynamics，vision integration．

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Javier Moreno－Valenzuela（2008）．Robot Control Using On－Line Modification of Reference Trajectories，Robot Manipulators，Marco Ceccarelli（Ed．），ISBN：978－953－7619－06－0，InTech，Available from：
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