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Impedance Control of Flexible Robot Manipulators

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1. Introduction

Force control is very important for many practical manipulation tasks of robot manipulators, such as, assembly, grinding, deburring that associate with interaction between the end-effector of the robot and the environment. Force control of the robot manipulator can be break down into two categories: Position/Force Hybrid Control and Impedance Control. The former is focused on regulation of both position and manipulation force of the end-effector on the tangent and perpendicular directions of the contact surface. The later, however, is to realize desired dynamic characteristics of the robot in response to the interaction with the environment. The desired dynamic characteristics are usually prescribed with dynamic models of systems consisting of mass, spring, and dashpot. Literature demonstrates that various control methods in this area have been proposed during recent years, and the intensive research has mainly focused on rigid robot manipulators. On the research area of force control for flexible robots, however, only a few reports can be found.

Regarding force control of the flexible robot, a main body of the research has concentrated on the position/force hybrid control. A force control law was presented based on a linearized motion equation[l]. A quasi-static force/position control method was proposed for a two-link planar flexible robot [2]. For serial connected flexible-macro and rigid-micro robot arm system, a position/force hybrid control method was proposed [3]. An adaptive hybrid force/position controller was resented for automated deburring[4]. Two-time scale position/force controllers were proposed using perturbation techniques[5],[6]. Furthermore, a hybrid position-force control method for two cooperated flexible robots was also presented [7]. The issue on impedance control of the flexible robot attracts many attentions as well. But very few research results have been reported. An impedance control scheme was presented for micro-macro flexible robot manipulators [8]. In this method, the controllers of the micro arm (rigid arm) and macro arm (flexible arm) are designed separately, and the micro arm is controlled such that the desired local impedance characteristics of end-effector are achieved. An adaptive impedance control method was proposed for n-link flexible robot manipulators in the presence of parametric uncertainties in the dynamics, and the effectiveness was confirmed by simulation results using a 2-link flexible robot [9].

In this chapter, we deal with the issue of impedance control of flexible robot manipulators. For a rigid robot manipulator, the desired impedance characteristics can be achieved by changing the dynamics of the manipulator to the desired impedance dynamics through nonlinear feedback control[13]. This method, however, cannot be used to the flexible robot because the number of its control inputs is less than that of the dynamic equations so that the whole dynamics including the flexible links cannot be changed simultaneously as desired by using any possible control methods. To overcome this problem, we establish an impedance control system using a trajectory tracking method rather than the abovementioned direct impedance control method. We aimed at desired behavior of the end-effector of the flexible robot. If the end-effector motion precisely tracks the impedance trajectory generated by the impedance dynamics under an external force, we can say that the desired impedance property of the robot system is achieved. Prom this point of view, the control design process can be divided into two steps. First, the impedance control objective is converted into the tracking of the impedance trajectory in the presence of the external force acting at the end-effector. Then, impedance control system is designed based on any possible end-effector trajectory control methods to guarantee that end-effector motion converses and remains on the trajectory. In this chapter, detail design of impedance control is discussed. Stability of the system is verified using Lyapunov stability theory. As an application example, impedance control simulations of a two-link flexible robot are carried out to demonstrate the usage of the control method.

2. Kinematics and dynamics

We consider a flexible robot manipulator consisted of n flexible links driven by n rigid joints. Let $\theta \in R^n$ be the joint variables, $\mathbf{e} \in R^k$ be the vector of the link flexible displacements. We assume that there is no redundant degree of freedom in the joints, and define vector $\mathbf{P} \in R^n$ to describe the end-effector position and orientation in the n-dimension workspace. The kinematical relationship among $\mathbf{P}$, θ and $\mathbf{e}$ is non-linear, and can be given as

$$\mathbf{P} = \mathbf{f}(\theta, \mathbf{e}) \tag{1}$$

The relationship among the velocities $\dot{\theta}$, $\dot{\mathbf{e}}$, and $\dot{\mathbf{P}}$ is a linear function given as

$$\dot{\mathbf{P}} = \mathbf{J}_\theta \dot{\theta} + \mathbf{J}_e \dot{\mathbf{e}} \tag{2}$$

where $\mathbf{J}_\theta \in R^{n \times n}$ and $\mathbf{J}_e \in R^{n \times k}$ are the Jacobian matrices of f with respect to θ and $\mathbf{e}$. They are defined as $\mathbf{J}_\theta = \partial \mathbf{f} / \partial \theta$, and $\mathbf{J}_e = \partial \mathbf{f} / \partial \mathbf{e}$. By taking derivation of (2) with respect to time, we have

$$\ddot{\mathbf{P}} = \mathbf{J}_\theta \ddot{\theta} + \mathbf{J}_e \ddot{\mathbf{e}} + \dot{\mathbf{J}}_\theta \dot{\theta} + \dot{\mathbf{J}}_e \dot{\mathbf{e}} \tag{3}$$

The dynamics of the flexible robot can be derived using Lagrange approach and FEM(Finite Element Method). In the case that there is an external force $\mathbf{F}_E \in R^n$ acting at the end-effector, the approximated model with FEM can be given as

$$\mathbf{M}_{11}\ddot{\theta} + \mathbf{M}_{12}\ddot{\mathbf{e}} + \mathbf{D}_{11}\dot{\theta} + \mathbf{D}_{12}\dot{\mathbf{e}} + \mathbf{L}_1 = \tau - \mathbf{J}_\theta^T \mathbf{F}_E \tag{4}$$

$$\mathbf{M}_{21}\ddot{\theta} + \mathbf{M}_{22}\ddot{\mathbf{e}} + \mathbf{D}_{21}\dot{\theta} + \mathbf{D}_{22}\dot{\mathbf{e}} + \mathbf{L}_2 = -\mathbf{J}_e^T \mathbf{F}_E \tag{5}$$

where, (4) and (5) are the motion equations with respect to the joints and links of the robot. $\mathbf{M}_{11} \in R^{n \times n}$, $\mathbf{M}_{12} \in R^{n \times k}$, $\mathbf{M}_{21} \in R^{k \times n}$, and $\mathbf{M}_{22} \in R^{k \times k}$ are inertia matrices; $\mathbf{D}_{11}\dot{\theta} \in R^n$, $\mathbf{D}_{12}\dot{\mathbf{e}} \in R^n$, $\mathbf{D}_{21}\dot{\theta} \in R^k$, and $\mathbf{D}_{22}\dot{\mathbf{e}} \in R^n$ contain the Coriolis, centrifugal, forces; $\mathbf{L}_1 \in R^n$ and $\mathbf{L}_2 \in R^k$ denote elastic forces and gravity; τ presents the input torque produced by the joint actuators.

System representation (4) and (5) can be expressed in a compact form as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{L} = \mathbf{u} - \mathbf{J}^T \mathbf{F}_E \tag{6}$$

where $\dot{\mathbf{x}}^T = [\dot{\theta}^T, \dot{e}^T]^T \in R^m$ $(m = n + k)$, $\ddot{\mathbf{x}} = d\dot{\mathbf{x}}/dt$, $\mathbf{u} = [\tau^T, \mathbf{0}]^T \in R^m$, and $\mathbf{M} \in R^{m \times m}$, $\mathbf{D} \in R^{m \times m}$ and $\mathbf{L} \in R^k$ are consisted of the elements $\mathbf{M}_{ij}$, $\mathbf{D}_{ij}$, and $\mathbf{L}_i$ (i,j = 1,2) respectively. $\mathbf{J}$ is defined as $\mathbf{J} = [\mathbf{J}_\theta, \mathbf{J}_e]$.

Remark 1: In motion equation (6), the inertia matrix $\mathbf{M}$ is a symmetric and positive definite matrix, and $\dot{\mathbf{M}} - 2\mathbf{H}$ is skew symmetric. These properties will be used in stability analysis of the control system in Section 4.

3. Objective of impedance control and problem formulation

3.1 Desirable impedance dynamics

When an external force acts on the end-effector of the flexible robot, the objective of impedance control is to cause the end-effector to respond to the force according to some desirable impedance dynamics. The desirable impedance dynamics can be designed as following second-order equation

$$\mathbf{M}_d(\ddot{\mathbf{P}}_d - \ddot{\mathbf{P}}_c) + \mathbf{D}_d(\dot{\mathbf{P}}_d - \dot{\mathbf{P}}_c) + \mathbf{K}_d(\mathbf{P}_d - \mathbf{P}_c) = -\mathbf{F}_E \tag{7}$$

where, $\mathbf{P}_c$, $\dot{\mathbf{P}}_c$, $\ddot{\mathbf{P}}_c$ are the commanded end-effector position, velocity and acceleration in the workspace; $\mathbf{P}_d$, $\dot{\mathbf{P}}_d$, $\ddot{\mathbf{P}}_d$ are the responses to the external force and commanded position, velocity and acceleration; $\mathbf{M}_d$, $\mathbf{D}_d$, and $\mathbf{K}_d$ are moment of inertia, damping, and stiffness matrices defined by user so that (7) possesses the desired dynamic characteristics. These matrices are usually defined as constant and diagonal ones.

3.2 The Main idea of impedance control using trajectory tracking approach

For a rigid robot manipulator, the desired impedance described above can be achieved by using impedance control to change dynamics of the robot manipulator [13]. This method, however, is difficult to be used to a flexible robot because one cannot change the whole dynamics of the robot including the flexible links at same time using any possible control method. To overcome this problem, the impedance control system is implemented using a trajectory tracking method rather than using the direct method-the abovementioned impedance control method. Let's consider behavior of the end-effector of the flexible robot. If the end-effector motion correctly tracks the desired impedance trajectories, we can say that the desired end-effector impedance is achieved. Prom this point of view, impedance control can be achieved by tracking the desired impedance trajectory generated by the

designed impedance dynamics (7) in the presence of the external force acting at the end-effector. Therefore, the impedance control design can be changed to the trajectory tracking control design. Accordingly, impedance control is implemented using end-effector trajectory control algorithms. Based on this methodology, the design of the control system is divided into three stages. In the first stage, impedance control of the robot is converted into tracking an on-line impedance trajectory generated by the impedance dynamics designed to possess with terms of the desired moment of inertia, damping, and stiffness as (7). On this stage, on-line impedance trajectory generating algorithms is established, and $\mathbf{P}_d$, $\dot{\mathbf{P}}_d$, $\ddot{\mathbf{P}}_d$ are calculated numerically at every sampling time in response to the external force which may vary. The second stage is concerned on the designing of a manifold to prescribe the desirable performance of impedance trajectory tracking and link vibration damping. The third stage is to derive a control scheme such that the end-effector motion of the robot precisely tracks the impedance trajectory. Figure 1 illustrates the concept of impedance control via trajectory tracking.
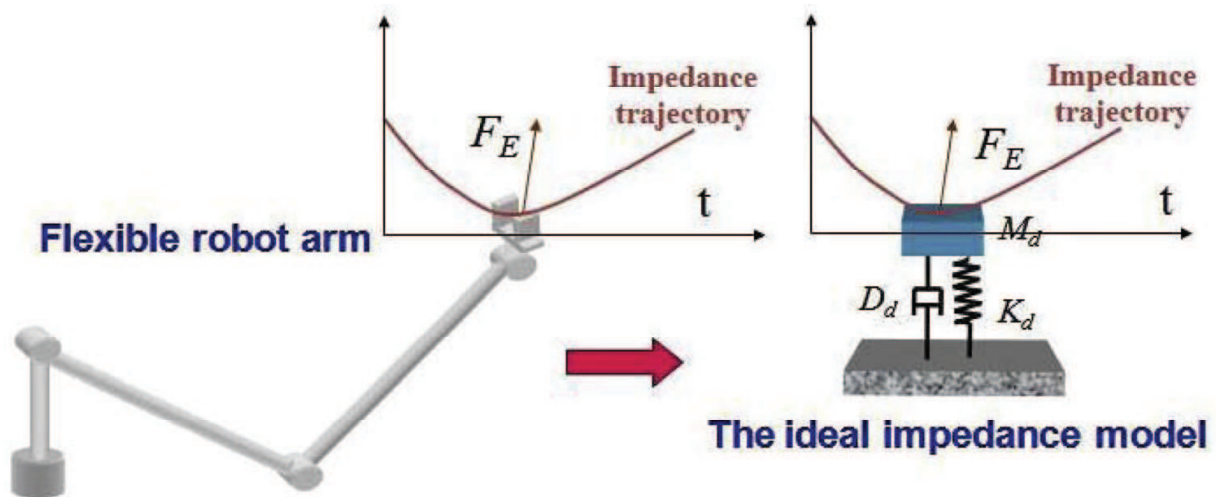

Figure 1. Impedance trajectory generated by the impedance model and tracked by the flexible robot

3.3 The manifold design

In order to achieve good performance in tracking the desired impedance trajectory while keeping the whole system stable, the control system has to include two basic functions. One is link vibration suppressing function and the other one is end-effector trajectory tracking function. We design an ideal manifold
to formulate the desired performance of the system with respect to these two functions. In so doing, let II be the ideal manifold, and divide it into two sub-manifolds HI and n₂ according to the functions abovementioned, and design each of them separately. Now, we define error vectors as follows:

$$\delta \mathbf{P} = \mathbf{P} - \mathbf{P}_d \qquad (8)$$

$$\delta \mathbf{e} = \mathbf{e} - \mathbf{e}_d \qquad (9)$$

We design the first submanifold to prescribe the desirable performance of end-effector trajectory tracking as follows

$$\mathbf{\Pi}_1 : \{(\delta\mathbf{P}, \delta\dot{\mathbf{P}}) : \delta\dot{\mathbf{P}} + \Lambda_1\delta\mathbf{P} = \mathbf{0}\} \tag{10}$$

where $\Lambda_1 \in R^{n \times n}$ is a constant positive definite matrix.
To describe a desired link flexural behavior, the second submanifold is specified as

$$\mathbf{\Pi}_2 : \{(\delta\mathbf{e}, \delta\dot{\mathbf{e}}) : \delta\dot{\mathbf{e}} + \Lambda_2\delta\mathbf{e}) = \mathbf{0}\} \tag{11}$$

where $\Lambda_2 \in R^{k \times k}$ is a constant positive definite matrix.
By combining $\mathbf{\Pi}_1$ and $\mathbf{\Pi}_2$ together, we have the complete manifold $\mathbf{\Pi}$ as follows

$$\mathbf{\Pi} = \mathbf{\Pi}_1 \cup \mathbf{\Pi}_2 \tag{12}$$

In order to formulate $\mathbf{\Pi}$ in a compact form, we define a new vector $\mathbf{S} \in R^m$ as

$$\mathbf{S} = \left[\begin{array}{c} \delta\dot{\mathbf{P}} + \Lambda_1\delta\mathbf{P} \\ \delta\dot{\mathbf{e}} + \Lambda_2\delta\mathbf{P} \end{array} \right] \tag{13}$$

Taking (2), (8), and (9) into consideration, we have

$$\mathbf{S} = \left[\begin{array}{cc} \mathbf{J}_\theta & \mathbf{J}_e \\ \mathbf{0} & \mathbf{I} \end{array} \right] \left[\begin{array}{c} \dot{\theta} \\ \dot{\mathbf{e}} \end{array} \right] + \left[\begin{array}{c} -\dot{\mathbf{P}}_d + \Lambda_1\delta\mathbf{P} \\ -\dot{\mathbf{e}}_d + \Lambda_2\delta\mathbf{e} \end{array} \right] \tag{14}$$

Obviously, the ideal manifold $\mathbf{\Pi}$ is equivalent to

$$\mathbf{S} = \mathbf{0} \tag{15}$$

We define a transformation matrix as

$$\mathbf{T} = \left[\begin{array}{cc} \mathbf{J}_\theta & \mathbf{J}_e \\ \mathbf{0} & \mathbf{I} \end{array} \right] \tag{16}$$

and rewrite (14) as

$$\mathbf{S} = \mathbf{T}\dot{\mathbf{x}} + \mathbf{b} \tag{17}$$

where

$$\mathbf{b} = \left[\begin{array}{c} -\dot{\mathbf{P}}_d + \Lambda_1\delta\mathbf{P} \\ -\dot{\mathbf{e}}_d + \Lambda_2\delta\mathbf{e} \end{array} \right] \tag{18}$$

Taking time differentiation of $\mathbf{S}$, we have

$$\dot{\mathbf{S}} = \mathbf{T}\ddot{\mathbf{x}} + \Lambda\mathbf{T}\dot{\mathbf{x}} + \mathbf{h} \tag{19}$$

where

$$\Lambda = \left[\begin{array}{cc} \mathbf{\Lambda}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_2 \end{array} \right] \tag{20}$$

$$\mathbf{h} = \left[\begin{array}{c} -\ddot{\mathbf{P}}_d - \Lambda_1 \mathbf{P}_d + \dot{\mathbf{J}}_\theta \dot{\theta} + \dot{\mathbf{J}}_e \dot{\mathbf{e}} \\ -\ddot{\mathbf{e}}_d + \Lambda_2 \dot{\mathbf{e}} \end{array} \right] \tag{21}$$

Remark 2: It should be pointed out that the condition for existence of submanifold (10) is the Condition for Compensability of Flexible Robots.

The concept and theory of compensability of flexible robots are introduced and systematically analyzed by [10], [11], and [12]. The sufficient and necessary condition of compensability of a flexible robot is given as

$$\mathbf{J}_e \subseteq \mathbf{J}_\theta \tag{22}$$

In fact, submanifold (10) is equivalent to

$$\mathbf{J}_\theta \delta \dot{\theta} + \mathbf{J}_e \delta \dot{\mathbf{e}} + \dot{\mathbf{J}}_\theta \delta \theta + \dot{\mathbf{J}}_e \delta \mathbf{e} + \Lambda_1 \mathbf{J}_\theta \delta \theta + \Lambda_1 \mathbf{J}_e \delta \mathbf{e} = \mathbf{0} \tag{23}$$

If (23) exists, it should also exist at the equilibrium state $\dot{\theta} = \mathbf{0}$, $\dot{\mathbf{e}} = \mathbf{0}$, $\delta \dot{\theta} = \mathbf{0}$, $\delta \dot{\mathbf{e}} = \mathbf{0}$, that is

$$\mathbf{J}_\theta \delta \theta + \mathbf{J}_e \delta \mathbf{e} = \mathbf{0} \tag{24}$$

The existence of surface (23) means its equilibrium exists, that is, for any possible flexural displacements $\delta \mathbf{e}$ a joint variable $\delta \theta$ exists such that (24) is satisfied. Therefore, the condition of existence of (24) is the condition of existence of such $\delta \theta$. From [10], [11], and [12] we know the condition of existence of $\delta \theta$ is the compensability condition given by (22).

4. Impedance Control Scheme

In the following, we discuss the derivation of impedance control scheme and the verification of stability of the system. From (17) and (19), we have

$$\dot{\mathbf{x}} = \mathbf{T}^{-1} \mathbf{S} - \mathbf{T}^{-1} \mathbf{b} \tag{25}$$

$$\ddot{\mathbf{x}} = \mathbf{T}^{-1} \dot{\mathbf{S}} - \mathbf{T}^{-1} \Lambda \mathbf{S} - \mathbf{T}^{-1} (\mathbf{h} - \Lambda \mathbf{b}) \tag{26}$$

Using the transformation defined above, the system representation (6) can be rewritten as

$$\bar{\mathbf{M}} \dot{\mathbf{S}} + \bar{\mathbf{H}} \mathbf{S} + \mathbf{Q}^T \mathbf{w} + \mathbf{Q}^T \mathbf{J}^T \mathbf{F}_E = \mathbf{Q}^T \mathbf{u} \tag{27}$$

where $\bar{\mathbf{M}} = \mathbf{Q}^T \mathbf{M} \mathbf{Q}$, $\bar{\mathbf{H}} = \mathbf{Q}^T \mathbf{H} \mathbf{Q}$,

$$\mathbf{w} = \mathbf{M} \mathbf{Q} (\Lambda \mathbf{S} + \mathbf{h} - \Lambda \mathbf{b}) - \bar{\mathbf{H}} \mathbf{b} + \mathbf{Q}^T \mathbf{L} \tag{28}$$

and

$$\mathbf{Q} = \mathbf{T}^{-1} = \left[\begin{array}{cc} \mathbf{J}_\theta^{-1} & -\mathbf{J}_\theta^{-1} \mathbf{J}_e \\ \mathbf{0} & \mathbf{I} \end{array} \right] \tag{29}$$

In (27), $\mathbf{w}$ contains Coriolis, centrifugal and stiffness forces and the nonlinear forces resulting from the coordinate transformation from $\mathbf{x}$ to $\mathbf{S}$. w needed to be compensated in the end-effector trajectory tracking control. However, some elements of $\mathbf{w}$ with respect to the dynamics of the flexible links cannot be compensated by the control torque τ directly. To deal with them, we design an indirect compensating scheme such that stability of the system would be maintained. To do so, we partition $\mathbf{w}$ into the directly compensable term and not directly compensable term as follows

$$\mathbf{w} = [\mathbf{w}_1^T, \mathbf{w}_2^T]^T \tag{30}$$

where $\mathbf{w}_1 \in R^n$, and $\mathbf{w}_2 \in R^k$.

Note that in (27) the ideal manifold is expressed as the origin of the system. Eventually, the purpose of the control design is to derive such a control scheme to insure the motion of the system converging to the origin $\mathbf{S} = \mathbf{0}$. We design the control scheme with two controllers as

$$\tau = \tau_1 + \tau_2 \tag{31}$$

where τ_1 is designed for tracking the desired end-effector impedance trajectories, and τ_2 is for link vibration suppressing. In detail, τ_1 and τ_2 are given as follows

$$\tau_1 = \mathbf{w}_1 - \mathbf{\Gamma}_1 - K_1 \bar{\mathbf{S}}_1 + \mathbf{J}_\theta^T \mathbf{F}_E \tag{32}$$

$$\tau_2 = \frac{v_2}{\bar{\mathbf{S}}_1^T v_2} \bar{\mathbf{S}}_2^T (\mathbf{w}_2 - \mathbf{\Gamma}_2 - K_2 \bar{\mathbf{S}}_2 + \mathbf{J}_e^T \mathbf{F}_E) \tag{33}$$

where $K_1 \in R^{n \times n}$ and $K_2 \in R^{k \times k}$ are the feedback gain matrices to be chosen as constant positive definite ones; $v_2 \in R^n$ is an arbitrary non-zero vector, $\mathbf{\Gamma}_1, \mathbf{\Gamma}_2$ and $\bar{\mathbf{S}}_1, \bar{\mathbf{S}}_2$ are the elements of two new vectors defined as bellow

$$\mathbf{\Gamma} = [\mathbf{\Gamma}_1^T, \mathbf{\Gamma}_2^T]^T = \mathbf{M} \dot{\mathbf{Q}} \mathbf{S} \tag{34}$$

where $\mathbf{\Gamma}_1 \in R^n$, $\mathbf{\Gamma}_2 \in R^k$, and

$$\bar{\mathbf{S}} = [\bar{\mathbf{S}}_1^T, \bar{\mathbf{S}}_2^T]^T = \mathbf{Q} \mathbf{S} \tag{35}$$

where $\bar{\mathbf{S}}_1 \in R^n$, $\bar{\mathbf{S}}_2 \in R^k$.

From the control design expressed above, the following result can be obtained.

Theorem For the flexible robot manipulator that the dynamics is described by (6), the motion of the robot uniformly and asymptotically converges to the ideal manifold (15) under control schemes given by (31) ~ (33).

Proof. From Remark 1, we know $\mathbf{M}$ is positive definite matrix, therefore $\bar{\mathbf{M}} = \mathbf{Q}^T \mathbf{M} \mathbf{Q}$ is also a positive definite matrix. Thus, we choose the following positive scalar function as a Lyapunov function candidate

$$\mathbf{V} = \frac{1}{2} \mathbf{S}^T \bar{\mathbf{M}} \mathbf{S} \tag{36}$$

Taking differentiation of $\mathbf{V}$ with respect to time yields

$$\dot{\mathbf{V}} = \mathbf{S}^T \bar{\mathbf{M}} \dot{\mathbf{S}} + \frac{1}{2} \mathbf{S}^T \dot{\bar{\mathbf{M}}} \mathbf{S} \tag{37}$$

where

$$\dot{\bar{\mathbf{M}}} = \dot{\mathbf{Q}}^T \mathbf{M} \mathbf{Q} + \mathbf{Q}^T \dot{\mathbf{M}} \mathbf{Q} + \mathbf{Q}^T \mathbf{M} \dot{\mathbf{Q}} \tag{38}$$

By substituting (27) into (37) we have

$$\begin{aligned}
\dot{\mathbf{V}} = \mathbf{S}^T(-\mathbf{Q}^T\mathbf{w} - \bar{\mathbf{H}}\mathbf{S} - \mathbf{Q}^T\mathbf{J}^T\mathbf{F}_E + \mathbf{Q}^T\mathbf{u}) \\
+ \frac{1}{2}\mathbf{S}^T(\dot{\mathbf{Q}}^T\mathbf{M}\mathbf{Q} + \mathbf{Q}^T\dot{\mathbf{M}}\mathbf{Q} + \mathbf{Q}^T\mathbf{M}\dot{\mathbf{Q}})\mathbf{S}
\end{aligned} \tag{39}$$

Since $\dot{\mathbf{M}} - 2\mathbf{H}$ is a skew symmetric matrix, it is easy to show $\mathbf{Q}^T\dot{\mathbf{M}}\mathbf{Q} - 2\bar{\mathbf{H}}$ is also a skew symmetric matrix. Thus, (39) can be rewritten as

$$\begin{aligned}
\dot{\mathbf{V}} &= \bar{\mathbf{S}}^T(-\mathbf{w} - \mathbf{J}^T\mathbf{F}_E + \mathbf{u}) + \bar{\mathbf{S}}^T\mathbf{M}\dot{\mathbf{Q}}\mathbf{S} \\
&= \bar{\mathbf{S}}_1^T(-\mathbf{w}_1 - \mathbf{J}_\theta{}^T\mathbf{F}_E + \mathbf{\Gamma}_1) \\
&\quad + \bar{\mathbf{S}}_2^T(-\mathbf{w}_2 - \mathbf{J}_e{}^T\mathbf{F}_E + \mathbf{\Gamma}_2) + \bar{\mathbf{S}}_1\tau
\end{aligned} \tag{40}$$

Combining (31) and (40) yields

$$\dot{\mathbf{V}} = -\bar{\mathbf{S}}^T K \bar{\mathbf{S}} \tag{41}$$

where $\mathbf{K} = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2 \end{bmatrix}$.

Notice both $\mathbf{K}_1$ and $\mathbf{K}_2$ are chosen as positive definite matrices, then $\mathbf{K}$ is a positive definite matrix too. We have

$$\dot{\mathbf{V}} < 0 \tag{42}$$

Thus system (27) is asymptotically stable.

Since M is always bounded, and system (27) is stable, the Lyapunov function (36) is bounded and satisfies

$$\frac{1}{2}\lambda_{min} \|\mathbf{S}\|^2 \leq V \leq \frac{1}{2}\lambda_{max} \|\mathbf{S}\|^2 \tag{43}$$

where, $\lambda_{min}, \lambda_{max} > 0$ are the minimum and maximum singular values of $\mathbf{M}$, $\|*\|$ denotes the Euclidean norm of a vector. Similarly, from (41) we obtain

$$-\zeta_{max} \|\mathbf{S}\|^2 \leq \dot{V} \leq -\zeta_{min} \|\mathbf{S}\|^2 \tag{44}$$

where, $\zeta_{min}, \zeta_{max} > 0$ are the minimum and maximum singular values of matrix $\mathbf{Q}^T\mathbf{K}\mathbf{Q}$.

By combining (43) and (44), we have

$$\dot{V} \leq -\frac{2\zeta_{min}}{\lambda_{max}} V \tag{45}$$

Taking time integration of (45) and noting left side inequality of (43) yields

$$\|\mathbf{S}\| \leq \sqrt{\frac{2V_0}{\lambda_{min}}} e^{-\frac{2\zeta_{min}}{\lambda_{max}} t} \tag{46}$$

where V_0 is the initial value of the Lyapunov function. It is shown that system (27) uniformly asymptotically converges to $\mathbf{S} = \mathbf{0}$, i. e. the motion of system (6) uniformly asymptotically converge to the designed ideal manifold.

It should be emphasized that when all the parameters are unknown, joint space tracking control can still be implemented using suitable adaptive and/or robust approaches. For end-effector trajectory control, however, at less the kinematical parameters, such as link lengths and twist angles which determine connections between links, are necessary when the direct measurement of end-effector motion is not available. Unlike the joint variables which are independent from kinematical parameters, end-effector positions and velocities are synthesized with the joint variables, link flexural variables and such kinematical parameters, so that unless an end-effector position sensor is employed, the kinematical parameters cannot be separated from the end-effector motion in a task space control design.

5. An application example: impedance control of a 2-link flexible robot

As an illustration, we apply the impedance control method to a 2-link flexible robot manipulator shown in Fig. 2. The dynamic model used for the control design and simulations was derived using Hamilton's Principle and Finite Element Method. Two elements for each flexible link were specified to approximate elastic deflections and rotation angle caused by the deflections. The approximated system has 10 degree of freedom.

The coordinate systems are set as shown in Figure 3. $O_0 - X_0 Y_0$ is the base coordinate system with its origin O_0 locating on joint 1, and axes X_0 and Y_0 are along the vertical direction and horizontal direction respectively. $O_1 - X_1 Y_1$, and $O_2 - X_2 Y_2$ are joint 2 coordinate frame and end-effector coordinate frame. Their origins O_1 and O_2 lay on joint 2 i.e. the distal end of link, and end-effector, respectively.

Using 3x3 homogenous transformation matrices, the end-effector position vector in the base coordinate system can be given as

$$\mathbf{P} = \mathbf{T}_1 \mathbf{T}_{1e} \mathbf{T}_2 \mathbf{q}_2 \tag{47}$$

In the above equation, $\mathbf{T}_1$ and $\mathbf{T}_1$ are rotation matrices of joint 1 and 2 give as

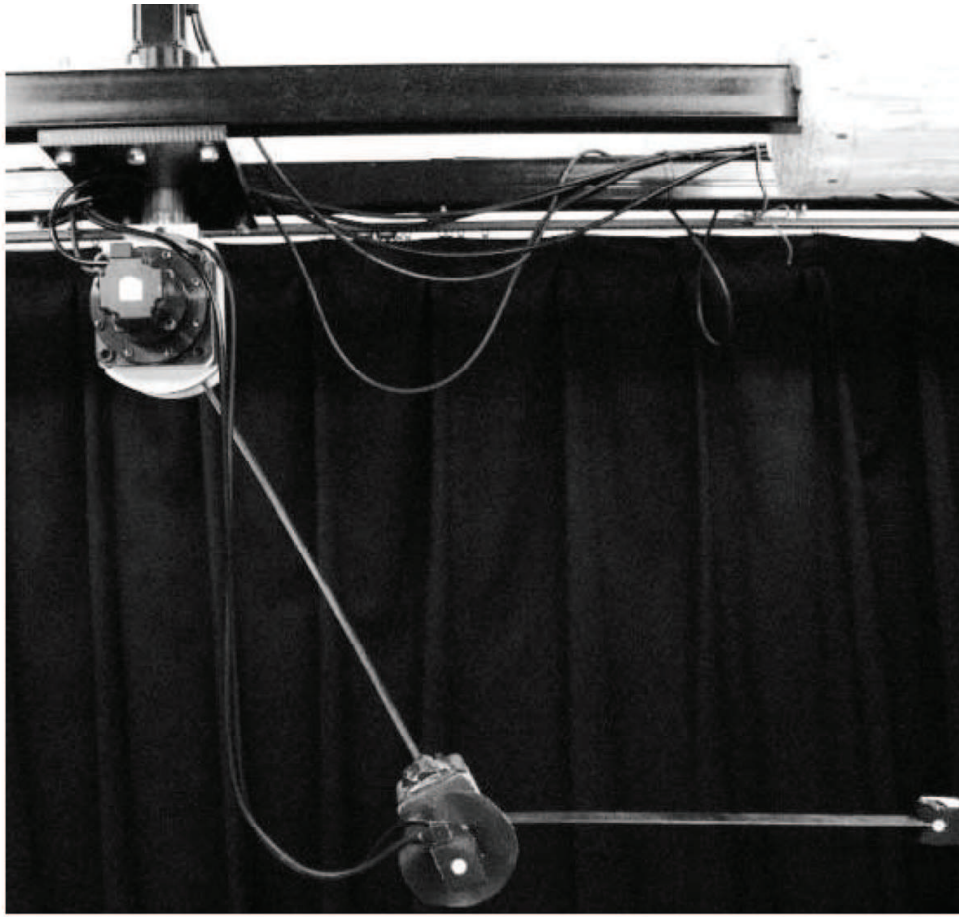$$\mathbf{T}_1 = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{48}$$

Figure 2. The flexible robot manipulator

$$\mathbf{T}_2 = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{49}$$

where $S_1 = \sin\theta_1$, $C_1 = \cos\theta_1$, $S_2 = \sin\theta_2$, and $C_2 = \cos\theta_2$ with θ_1 and θ_2 being the joint angles. $\mathbf{T}_{1e}$ denotes link 1 transformation matrix as

$$\mathbf{T}_{1e} = \begin{bmatrix} C_\phi & -S_\phi & L_1 \\ S\phi & C\phi & \delta_1 \\ 0 & 0 & 1 \end{bmatrix} \tag{50}$$

where $S_\phi = \sin\phi_1$, $C_\phi = \cos\phi_1$, L_1 denotes the length of linkl and δ_1 denotes the deflection at the distal end of link 1. $\mathbf{q}_2$ is a vector defined as

$$\mathbf{q}_2 = [L_2, \delta_2, 1]^T \tag{51}$$

with L_2 and δ_2 denoting the length of Iink2 and deflection at the distal end of Iink2.
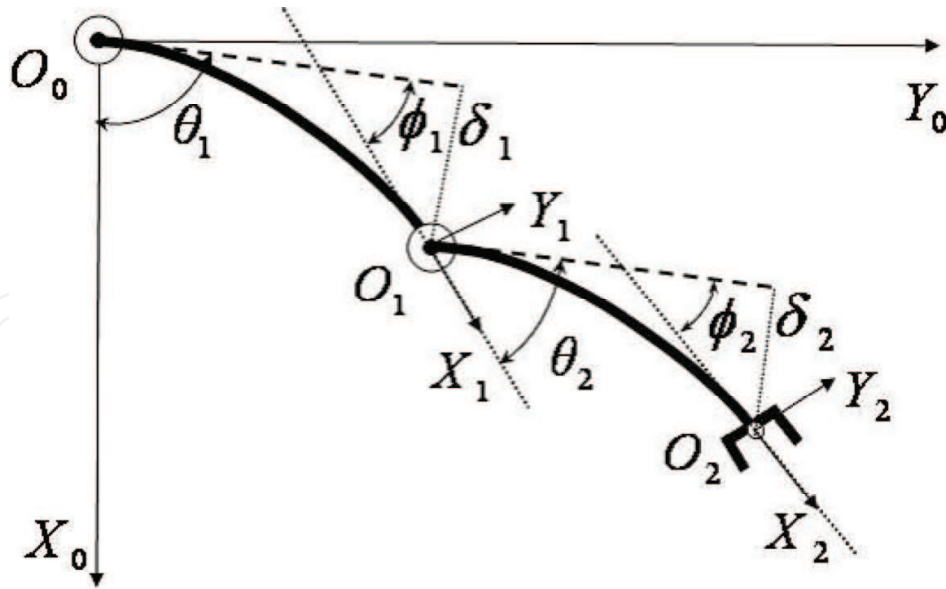
Figure 3. Coordinate systems of two-link robot manipulator

Using the above kinematical analysis results, one can obtain Jacobian matrices with respect to the joint velocity $[\dot{\theta}_1, \dot{\theta}_2]^T$ and link flexural velocity $[\dot{\delta}_1, \dot{\phi}_1, \dot{\delta}_2]^T$ as follows.

$$\mathbf{J}_\theta = \begin{bmatrix} -L_1 S_1 - \delta_1 C_1 - L_2 S_{12\phi} - \delta_2 C_{12\phi} & -L_2 S_{12\phi} - \delta_2 C_{12\phi} \\ L_1 C_1 - \delta_1 S_1 + L_2 C_{12\phi} - \delta_2 S_{12\phi} & L_2 C_{12\phi} - \delta_2 S_{12\phi} \end{bmatrix} \quad (52)$$

and

$$\mathbf{J}_e = \begin{bmatrix} -S_1 & -L_2 S_{12\phi} - \delta_2 C_{12\phi} & -S_{12\phi} \\ C_1 & L_2 C_{12\phi} - \delta_2 S_{12\phi} & C_{12\phi} \end{bmatrix} \quad (53)$$

where $S_{12\phi} = \sin(\theta_1 + \theta_2 + \phi_1)$, $C_{12\phi} = \cos(\theta_1 + \theta_2 + \phi_1)$. The parameters of the flexible robot are shown in Table 1.

	link1	link2	joint1	joint2
L	2.0	2.0		
EJ	200	100		
ρ	2	1		
i			0.01	0.005

L–length(m); EJ–bending rigidity$(N-m^2)$;
ρ–density(kg/m); i–moment of inertia$(kg-m^2)$

Table 1. The parameters of the robot

The simulations are carried out with no natural damping of the flexural behavior of the links. The external force acted at the end-effector is $\mathbf{F}_E=$ $[1.0,1.0]^T$. The inertia, damping, stiffness matrices of the desired impedance dynamics are designed as

$$\mathbf{M}_d = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}, \quad \mathbf{D}_d = \begin{bmatrix} 2.8 & 0 \\ 0 & 2.8 \end{bmatrix}, \quad \mathbf{K}_d = \begin{bmatrix} 4.0 & 0 \\ 0 & 4.0 \end{bmatrix} \quad (54)$$

The parameters of the controller are designed as follows $\Lambda_1 = diag$ (10.0 ,10.0), $\Lambda_2 = diag(10.0,10.0,10.0,10.0,10.0,10.0,10.0,10.0)$, $\mathbf{K}_1 = diag(100.0,100.0)$, $\mathbf{K}_2 = diag(100.0,100.0,100.0,100.0,100.0,100.0,100.0,100.0)$.
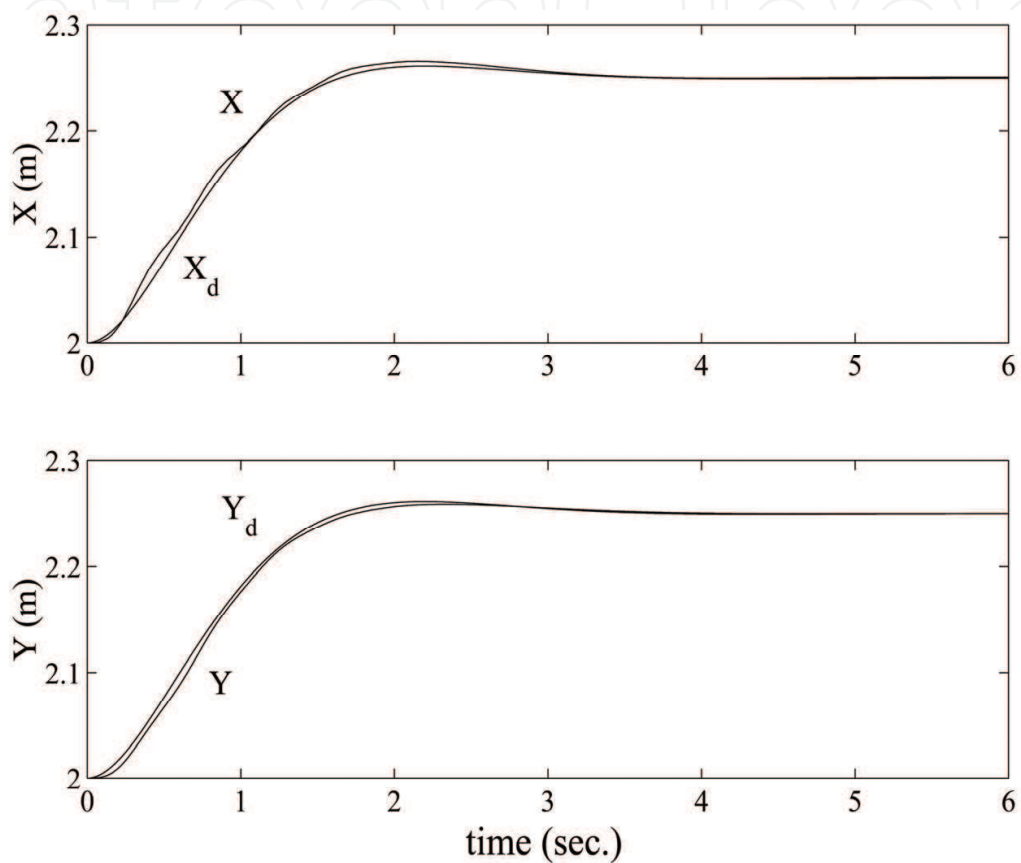

Figure 4. Targeted impedance trajectories and control results
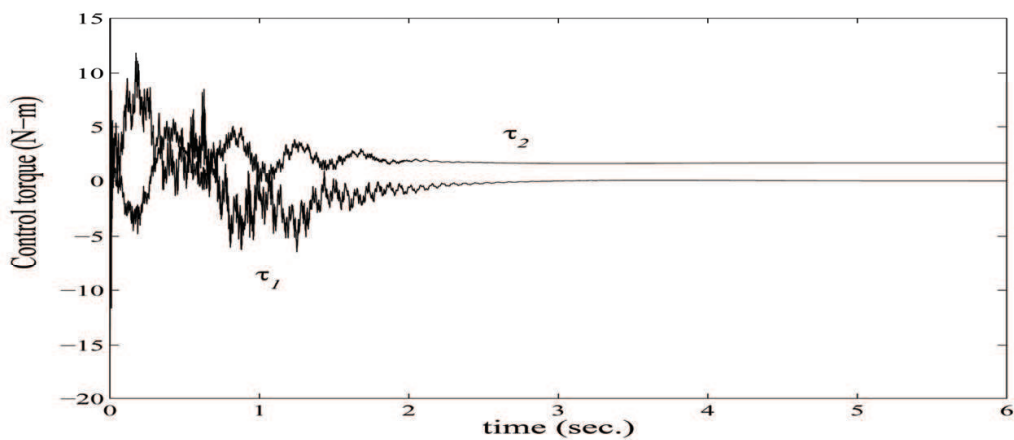
Figure 5. Control torque of joint1 and joint2
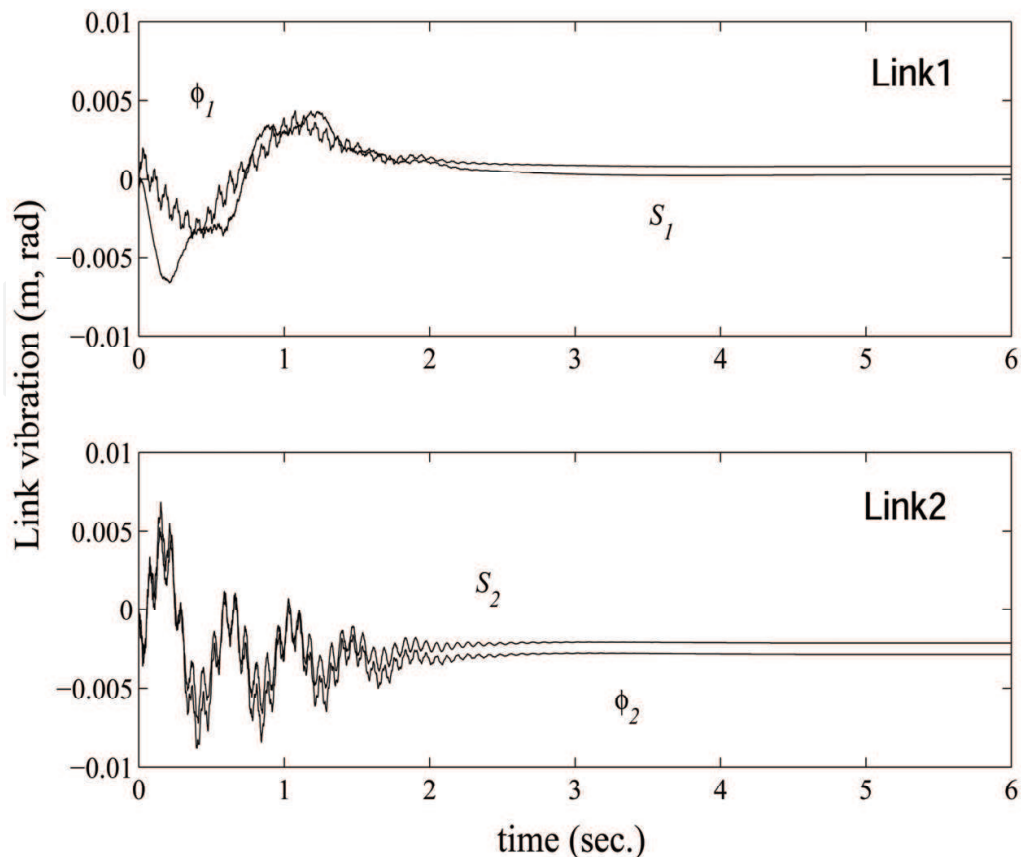
Figure 6. Link deflections and rotations caused by the deflections

Fig 4. shows the desired impedance trajectories and tracking results. The control torque is shown in Fig 5. Fig 6. shows the flexural deflections and the rotation of the cross section caused by the deflections at distal ends of link 1 and link 2.

The simulations validate that the impedance control method using a trajectory tracking approach can achieve good performance of impedance control. By using the proposed control method vibrations of the flexible links are damped effectively stability of the system is guaranteed.

6. Conclusions

An impedance control method for flexible robot manipulators was discussed. The impedance control objective was converted into tracking the end-effector impedance trajectories generated by the designed impedance dynamics. An ideal manifold related to the desired impedance trajectory tracking was designed to prescribe the desirable characteristics of the system. The impeadnce control scheme was derived to govern the motion of the robot system converging and remaining to the ideal manifold in the presence of parametric uncertainties. Using Lyapunov theory it was shown that under the impedance control scheme the motion of the robot is exponentially stable to the designed ideal manifold. Impedance control simulations were carried out using a two-link flexible robot manipulator. The result confirmed the effectiveness and good performance of the proposed adaptive impedance control method.

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In this book we have grouped contributions in 28 chapters from several authors all around the world on the several aspects and challenges of research and applications of robots with the aim to show the recent advances and problems that still need to be considered for future improvements of robot success in worldwide frames. Each chapter addresses a specific area of modeling, design, and application of robots but with an eye to give an integrated view of what make a robot a unique modern system for many different uses and future potential applications. Main attention has been focused on design issues as thought challenging for improving capabilities and further possibilities of robots for new and old applications, as seen from today technologies and research programs. Thus, great attention has been addressed to control aspects that are strongly evolving also as function of the improvements in robot modeling, sensors, servo-power systems, and informatics. But even other aspects are considered as of fundamental challenge both in design and use of robots with improved performance and capabilities, like for example kinematic design, dynamics, vision integration.

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