10

Artificial Coordinating Field based Motion Planning of Mobile Robots

Xing-Jian Jing[1,2*] and Yue-Chao Wang[2]
[1]Automatic Control and Systems Engineering, University of Sheffield,
[2] State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences
[1]U.K., [2]China

1. Introduction

Motion planning of mobile robots in uncertain dynamic environments has been a hot topic in robotic literature. It requires a mobile robot to decide its motion behaviour on line using limited and noised information of the local environment from sensors. There are many methods having been proposed to deal with this problem (Salichs and Moreno 2000, Jing 2005). Noticeably, artificial potential field (APF) based methods have gained increasingly popularity among researchers due to its high safety, simplicity and elegance (Khatib 1986, Rimon and Koditschek 1991, Kant and Zucher 1988, Rimon and Koditschek 1992, Koren and Borenstein 1991, Guldner and Utkin 1995, Ge and Cui 2000, Prassler 1999, Noborio et al 1995, Krogh 1984, Satoh 1993, Louste and Liegeois 2000, Wong and Spetsakis 2000, Singh et al 1997, Tsourveloudis et al 2001, Masoud and Masoud 2000). However, when the involved environment is totally or partially unknown or even dynamically changing, local minima are usually encountered, where the robot is trapped and cannot move on. There may also be unnecessary oscillations on the planned trajectory between multiple obstacles (Koren and Borenstein 1991). These inhibit the practical applications of this methodology to a certain extent. To overcome these problems, there are already some methods having been proposed in literature. For example, Krogh (1984) proposed a generalized potential field, in which the strength of repulsion is directly proportional to the speed of approach and inversely proportional to the minimum avoidance time. Satoh (1993) proposed Laplace potential field, which requires the potential field to be harmonic, and satisfy the Laplace equation. In Louste and Liegeois (2000), the authors used viscous fluid field instead of conventional APF to achieve near optimal path planning. Moreover, electric-like fields (Wong and Spetsakis 2000), magnetic field (Singh et al 1997), electrostatic potential field (Tsourveloudis et al 2001) were all proposed for the navigation and motion planning problems. But all these methods either require some global environment information or only deal with navigation problems in static environments, and only a few take into consideration of the actual dynamic constraints of the mobile robot such as saturations of velocity and acceleration. Moreover, few of the existing potential fields can guarantee the safety and reachability of the mobile robot with consideration of the actual dynamic constraints in uncertain dynamic environments.

[*] The first author has been with the University of Sheffield since Oct 2005.

The reason for the drawbacks of the ACF as mentioned above is that, in our opinion, this simple "repulsive and attractive" information model of the environments in APF methods cannot completely and accurately reflect the actual states and real motion purpose of the mobile robot. Hence, it is difficult or even impossible to decide the optimal or satisfactory motion behaviour in some complicated situations just based on this simple information model of the environments of APF using only attractive and repulsive forces. In order to overcome the drawbacks of the conventional APFs, it does need to change the simple "repulsive and attractive" information model to another more appropriate information model of the environments, and make the new model be adaptable to motion purpose and relative states of the mobile robot with respect to obstacles.

Therefore, an artificial coordinating field (ACF) is proposed in this chapter. In order to overcome the drawbacks of APF, a special force vector called Coordinating Force is defined and added to the conventional APF, and the ACF is designed to be adaptable to the motion purpose and relative states of the mobile robot with respect to obstacles, which includes not only the information of relative positions of the robot with respect to an obstacle, but also the information of the relative velocity, maximum acceleration and velocity of the robot. Decision-making of the robot's behavior when avoiding an obstacle is based on a special variable, called coordinating factor λ, which is simple and in an optimal way. The safety and reachability of the proposed method are theoretically analyzed with some assumptions on the environments. Simulation results are given to illustrate our method.

2. Definition of the ACF

Our study is restricted to the 2-D planar case. Some notations are introduced as follows. The planar U can be denoted as a point set $U=\{p=(x,y)^T \mid x,y \in \mathbb{R}\}$, where point $(x,y)^T$ is a column vector, $(*)^T$ is the transpose of vector $(*)$, $\mathbb{R}$ is the set of all the real numbers. ∂D denotes the boundary of a subset D in U. Without specialty, a bold italic symbol denotes a vector. e($\boldsymbol{A}$) denotes the unitary vector of a vector $\boldsymbol{A}$, *i.e.*, $\mathrm{e}(\boldsymbol{A}) = \mathbf{A}/\|\mathbf{A}\|$, where $\|\boldsymbol{A}\|$ denotes the Euclidian norm of $\boldsymbol{A}$. Difference of two point is a vector, *e.g.*, $\boldsymbol{A}=q_1-q_2$, where $q_1, q_2 \in U$, the direction of $\boldsymbol{A}$ is from q_2 to q_1, *i.e.*, $\mathrm{e}(\boldsymbol{A})=\mathrm{e}(q_1-q_2)$. Moreover, "$a \rightarrow b$" denotes "$a$ is approaching to b nearly or very nearly". On the contrary, "$a>>b$" and "$a<<b$" denote that "a is much larger or smaller than b" respectively.

In addition, the mobile robot can be regarded as a point mass with weight M, its goal is denoted by q_d. An obstacle can be regarded as a point set O or $\mathrm{O_i}$ in U, where the subscript i is to distinguish different obstacles. The obstacle $\mathrm{O_i}$ may also be called obstacle i later on. The distance between two point set $\mathrm{O_i}$ and $\mathrm{O_j}$ is defined as $d(O_1,O_2) = \min_{p\in\partial O_1, q\in\partial O_2} \|p-q\|$.

Define a mapping g_O: $q \rightarrow \partial O$ such that $p= g_O(q)=\arg\min_{p\in\partial O}\|q-p\|$, where $p,q \in U$.

Obviously, p is the nearest point on the boundary of O to q. For an obstacle $\mathrm{O_i}$, this mapping function is also written in short as $g_i(q)$.

The ACF is defined as a force vector field as follows (see Figure 1). The attractive field at the goal q_d of the mobile robot is defined as: $\forall q \in U$

$$\boldsymbol{F_a}(q) = K_a \cdot (q_d - q) \tag{1a}$$

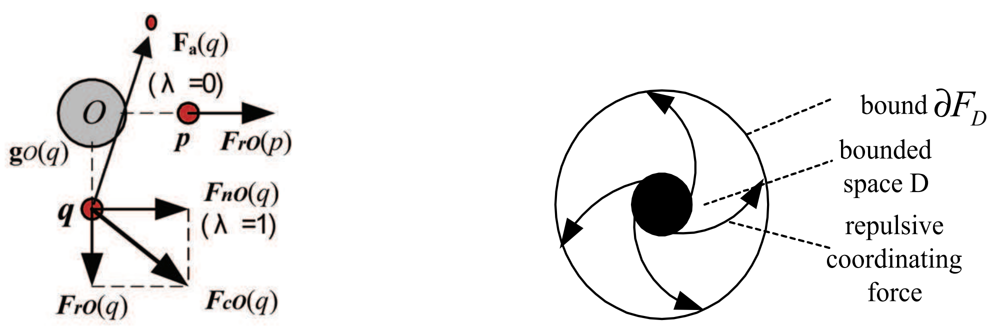
where K_a is to be defined. For an obstacle O, define the ACF as: $\forall q \in U \setminus O$

$$\boldsymbol{F}_{\boldsymbol{cO}}(q) = \boldsymbol{F}_{\boldsymbol{rO}}(q) + \boldsymbol{F}_{\boldsymbol{nO}}(q) \tag{1b}$$

$$\boldsymbol{F}_{\boldsymbol{rO}}(q) = K_{rO} \cdot (q - g_O(q)) \tag{1c}$$

$$\boldsymbol{F}_{\boldsymbol{nO}}(q) = K_{nO} \cdot \lambda \cdot \boldsymbol{T} \cdot (q - g_O(q)) \tag{1d}$$

where $\lambda \in \{1,0,-1\}$ is called coordinating factor, $\boldsymbol{T} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; (1b) is the artificial repulsive-coordinating field of obstacle O, which is also called in short as artificial coordinating field (ACF) in this study; (1c) is the repulsive force vector, K_{rO} is to be defined; (1d) is the coordinating force vector, which is orthogonal to the repulsive force and whose direction is determined by λ, K_{nO} is to be defined. For different obstacle O_i, the aforementioned force vectors are rewritten in short as: $\boldsymbol{F}_{ci}$, $\boldsymbol{F}_{ri}$, $\boldsymbol{F}_{ni}$, respectively, and the corresponding parameters are rewritten as K_{ri}, K_{ni}, λ_i, respectively. If the repulsive force (1c) is substituted by the attractive force (1a), then the new artificial field is called artificial attractive-coordinating field. Moreover, we can also define the ACF in 3-dimensional space using a similar method as above.



(a) Different forces in an ACF (b) A bounded repulsive ACF with λ =1

Figure 1. The repulsive force and attractive force in an ACF

At any time instant t, let the x-coordinate of the dynamic coordinates on the mobile robot with respect to an obstacle O be parallel to the coordinating force vector, and the y-coordinate be parallel to the repulsive force vector. Obviously, the ACF has two-dimensional orthogonal force vectors, thus the mobile robot has two DOF to be controlled in its dynamic coordinates when meeting an obstacle, this may help to realize some desired motion behavior. Compared with ACF, the conventional APF can only exert one-dimensional force to the mobile robot in the dynamic coordinates. Thus the mobile robot can only run away from the APF when meeting an obstacle, but not avoid the obstacle with intention. This may be a major reason that there are local minima in conventional APFs for uncertain dynamic environments. Especially, it is noted that the direction of the coordinating force vector in an ACF at any time is determined by λ. If let λ =0, then $\boldsymbol{F}_{cO}$ (q)=0 (referring to the point p in Figure 1), and there is only a repulsive force at point p in this case, which is right the APF. Hence, APF is only a special case of ACF. Since more environmental information and motion purpose of the mobile robot can be represented in the ACF, the states of a mobile robot can be controlled for some special purposes by using the orthogonal forces in the ACF. Moreover, considering the motion planning problem in uncertain dynamic environments, only the distance between an obstacle and the mobile robot is near enough (*e.g.*, less than a constant R), for the obstacle to be detected by the robot's sensors. Therefore, the radius of the ACF of an obstacle should be less than the distance R around the boundary of the obstacle.

It shall be noted that similar but different work to our ACF defined above can be found in literature. Note in Masoud and Masoud (2000), two orthogonal fields were used, a scalar potential field in normal space and a circular field in tangent space around obstacles, to locally switch the robot from one trajectory to another in order to adapt the unknown changing environments. The idea of orthogonal fields is very similar to ours. The difference is that it needs to solve boundary value problems, and needs also some global environment information. The circular field is only used to shift the robot from one path to another when meeting unknown static obstacles, and the whole field is still a passive one as most existing fields, namely, it cannot be adaptable to the states and motion purpose of the robot in the local environment. Note also in Medio and Oriolo (1999), a vortex field was proposed, which is also a passive field, and has no repulsive force compared with APFs.

3. Properties and Designs of ACF

This section discusses the properties of the ACF and studies how to design the parameters of the ACF to achieve the desired performance in the motion-planning problem of mobile robots in uncertain dynamic environments. More notations are introduced as follows. The position of the mobile robot is denoted by q without specialty, the maximum velocity and acceleration of the mobile robot are V_{max} m/s and a_{max} m/s^2, and the radius within which an obstacle can be effectively detected by the sensors is R. Assume that $R>>2(V_{max})^2/a_{max}$. The region covered by the circle at point $q=(x,y)^T$ with a radius R is called observable region denoted by $P(q)$. All the static and moving obstacles in $P(q)$ that can be detected by the sensors are denoted by sets O_s and O_d, respectively. For instance, if a static obstacle O_i is detected by the sensors, then it can be written as $O_i \in O_s$ or $i \in O_s$. Velocity vectors of the mobile robot and an obstacle O_i are denoted by $\boldsymbol{V_r}$ and $\boldsymbol{V_i}$, respectively, and the relative velocity of the mobile robot with respect to the obstacle O_i is $\boldsymbol{V_{ir}}=\boldsymbol{V_r}-\boldsymbol{V_i}$. In this section, we design the ACF based on the analysis of the dynamics of a point in ACFs.

3.1 Controllability of a mobile robot in the ACFs

For the motion-planning problem, there is an attractive field $\boldsymbol{F_a}(q)$ at the goal point $q_d=(x_d,y_d)^T$, and some ACFs $\boldsymbol{F_{ci}}(q)$ with respect to different obstacles O_i (i=1,2,…,N) in the planar U. Assume that $d(O_i,O_j)>0$, $d(q_d,O_i)>0$ for all i=1,2,…,N. For any time t, the dynamics of the mobile robot in the artificial fields can be written as:

$$M\,\ddot{q} = -K_f\,(\dot{q}-\dot{q}_d) + \mathbf{F_a}(q) + \sum_{i\in O_d\cup O_s}\mathbf{F_{ri}}(q) + \sum_{i\in O_d\cup O_s}\mathbf{F_{ni}}(q) \tag{2}$$

where, $K_f>0$ is a parameter to be defined, $\dot{q}_d$ is the desired velocity of the mobile robot. Equation (2) is called Planning Equation. The first term on the right side of the equality is to balance the dynamics of the mobile robot and control the velocity to a desired level stably. The last three terms stand for all the virtual forces received by the mobile robot, they can be rewritten as

$$\mathbf{F}_{\mathbf{total}} = \mathbf{F}_{\mathbf{a}}(q) + \sum_{i \in O_d \cup O_s} \mathbf{F}_{\mathbf{ri}}(q) + \sum_{i \in O_d \cup O_s} \mathbf{F}_{\mathbf{ni}}(q) \,.$$

Substitute (1) into (2), and transform (2) into state space equation form, we can have:

$$M\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -K_f & 0 & \sum K_{ri} - K_a & -\sum K_{ni}\lambda \\ 0 & -K_f & \sum K_{ni}\lambda & \sum K_{ri} - K_a \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ x \\ y \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \mathbf{u} \tag{3}$$

where

$$\mathbf{u} = K_f \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} + K_a \begin{bmatrix} x_d \\ y_d \end{bmatrix} - \sum_{i \in O_d \cup O_s} K_{ri}\, \mathrm{g_i}(q) + \sum_{i \in O_d \cup O_s} K_{ni} \lambda_i \cdot \mathbf{T} \cdot \mathrm{g_i}(q) \tag{4}$$

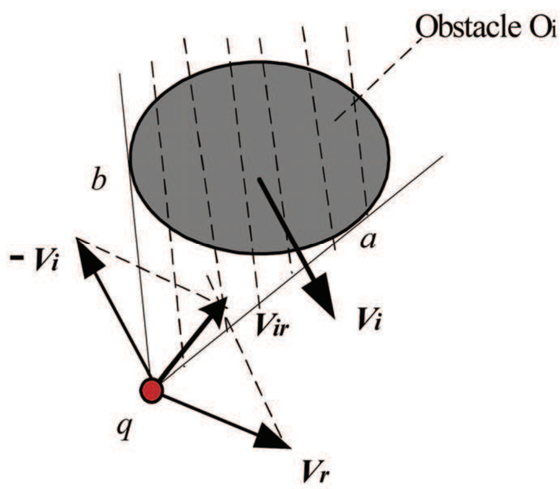


Figure 2. The mobile robot meets a moving obstacle

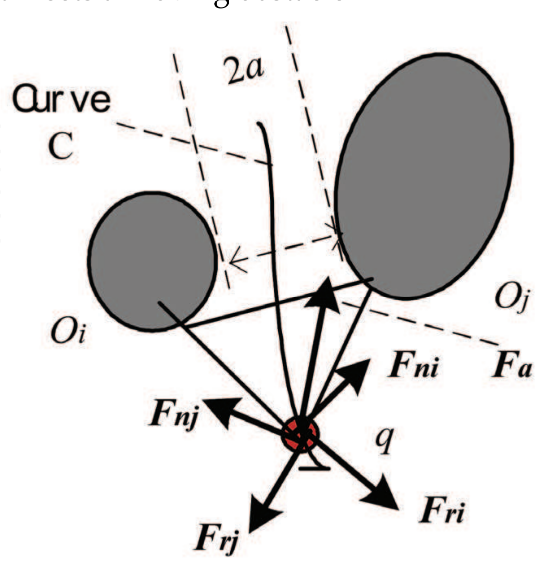


Figure 3. The mobile robot is passing a passage between two obstacles

It can be verified that, (3) is complete state controllability. From (4), the states of (3) is completely controlled by the variables K_a, K_{ri}, K_{ni} and λ_i of the ACFs. Hence, by properly choosing the variables of the ACFs, the desired motion behavior of the mobile robot can be obtained. Without loss of generality, let $\dot{q}_d$ =0 in (2). Specially considering the APF case, *i.e.*, let λ_i =0 in (2-4). $\boldsymbol{F_{total}}$ will be zero in the local minima where the robot may be trapped. However, in ACFs, this is not the case. The local minima can be removed by properly choosing the coordinating factors and other variables of the ACFs such that $\boldsymbol{F_{total}}$ cannot be zero. This is further discussed in the following sections.

3.2 Adaptability of the ACFs

For different collision risk or different relative states of a mobile robot with respect to obstacles, the mobile robot should adopt different motion behavior or strategy according to an optimal evaluation. For this purpose, the ACFs should be adaptable to the collision risk or relative states of the mobile robot and can generate virtual forces of different magnitude and properties corresponding to different situations. This adaptability of ACFs can help the mobile robot to coordinate its motion behavior to avoid different type of obstacles and go to its goal in an optimal or a satisfactory way. To this aim, evaluations of the collision risk of a mobile robot are investigated with only the local information of the environments at first. And then the ACF is designed using these evaluation functions. Note that $\angle$ (*,*) denotes the angle of two vectors later on.

Assume that the mobile robot with velocity $\boldsymbol{V_r}$ and position q meets an obstacle O_i with velocity $\boldsymbol{V_i}$ at time t (See Figure 2). From point q, make two lines tangent to the boundary ∂O_i at point a and b, respectively. If the relative velocity $\boldsymbol{V_{ir}}=\boldsymbol{V_r}-\boldsymbol{V_i}$ is regarded as the velocity of the mobile robot, then the obstacle O_i can be regarded to be static. Let

$$\theta_{v1} = \angle(\mathbf{V}_{\mathbf{ir}}, (\mathrm{e}(a-q)+\mathrm{e}(b-q))/2)\,,\ \theta_{v2} = \angle(\mathrm{e}(a-q), \mathrm{e}(b-q))/2\,.$$

It is easy to verify that, if $\theta_{v1} \le \theta_{v2}$ holds and $\boldsymbol{V_{ir}}$ is also kept unchanging, then the robot must be heading a collision with the obstacle. Let

$$E_{Vi} = \begin{cases} 1 & \theta_{v1} \le \theta_{v2} \\ 0 & else \end{cases},$$

which is called velocity risk with respect to obstacle O_i. Obviously, E_{Vi}=1 implies a possible collision. Define the absolute collision risk with respect to O_i as:

$$E_i = k_{risk_1} \big/ \|q - \mathrm{g}_i(q)\| + k_{risk_2} \cdot E_{Vi} \cdot (1 + (\theta_{v2} - \theta_{v1}) / \sup(\theta_{v2}) + \|\boldsymbol{V_{ir}}\| \big/ \sup(\|\boldsymbol{V_{ir}}\|))$$

where $0<k_{risk1}, k_{risk2}<1$. The total collision risk with respect to all the observable obstacles is

$$E = \sum_{i \in Od \cup Os} E_i\ .$$

Then define the relative collision risk with respect to obstacle O_i as:

$$E_{ri} = E_i / E$$

Using the evaluations of the collision risk with respect to obstacles above, we define the corresponding variables of the ACF as follows:

i. When the collision risk is increasing, the attractive force should be increased accordingly in order to attract the mobile robot to move towards its goal. However, too large magnitude of the attractive force may affect the safety of the robot in complicated situations. Hence, we let

$$K_a = k_a (1 + \min(M_a, E)) \tag{5}$$

where $k_a>0$, $M_a>0$ are constants.

ii. In order to guarantee the safety, the magnitude of the repulsive force should be proportional to the collision risk with respect to an obstacle O_i. Hence, we let

$$K_{ri} = k_{rc} \cdot (E_{ri} + \varepsilon_1) \cdot k_{ri} / \| q - \mathrm{g}_i(q) \| \tag{6}$$

Where $k_{rc}>0$ is a constant, k_{ri} is to be defined, $0 \le \varepsilon_1 < 1$ is the minimal relative collision risk corresponding to different situations to be defined.

iii. As for the magnitude of the coordinating force, it is defined similarly to the repulsive force, and additionally it is defined to be limited:

$$K_{ni} = \min(M_n, k_{nc} \cdot (E_{ri} + \varepsilon_2) \cdot k_{ni} / \| q - \mathrm{g}_i(q) \|) \tag{7}$$

where, $k_{nc}>0$ is a constant, M_n is the upper bound of K_{ni} and satisfies $M_n >> \sup(\|\boldsymbol{F_a}\|)$, $0 \le \varepsilon_2 < 1$ is similar to ε_1, and k_{ni} is to be defined.

It should be noted that from (5-7), the magnitude of all the virtual forces has relation with the collision risk. Especially, the magnitude of an ACF is a function of the relative collision risk. The higher the collision risk with respect to an obstacle is, the larger is the force generated by the ACF of the obstacle. Hence, the robot dynamic is basically dominated by the obstacles with higher collision risks. This helps to guarantee the safety of the mobile robot and can pass the collision risk from one robot to another.

3.3 Safety of a mobile robot in the ACFs

The safety of a mobile robot not only has relation with the complexity of the environments but also is subjected to the dynamic constraints of the mobile robot. Obviously, if $\boldsymbol{V_{ir}} \cdot \mathrm{e}(\mathrm{g}_i(q) - q) \le 0$ whenever $d(q, O_i) \to 0$, then there will be no collision to happen. For this reason, we let k_{ri} in (6) be:

$$k_{ri} = 1 / \left(\mathrm{pos}(d(q, O_i)) - (\mathrm{pos}(\boldsymbol{V_{ir}} \cdot \mathrm{e}(\mathrm{g}_i(q) - q)))^2 / 2a_{\max}) \right)^n \tag{8}$$

where $n \ge 1$, $\mathrm{pos}(x)=\max(0,x)$ (this is directly used later on), $d(q,O_i) = \| q - \mathrm{g}_i(q) \|$.

Based on the designs of the ACF above, the safety of the mobile robot can be guaranteed with some environment constraints. The main results are given as follows.

Proposition 1. Assume $R >> 2(V_{max})^2 / a_{max}$, and at time t the mobile robot is at point q satisfying $d(q,O_i)<R$. Let $\boldsymbol{F_{other}}=\boldsymbol{F_{total}}-\boldsymbol{F_{ri}}$. If $\mathbf{F_{other}} \cdot \mathrm{e}(\mathrm{g}_i(q) - q) << \infty$, and the velocity and acceleration of the obstacle O_i satisfy $\|\boldsymbol{V_i}\| < V_{\max}$ and $\dot{\boldsymbol{V}}_{\boldsymbol{i}} \cdot \mathrm{e}(\mathrm{g}_i(q) - q) \ge 0$ whenever

$d(q, O_i) \to S_0$, then $\forall t, g_i(q) \neq q$, that is there will be no collision to happen between the mobile robot and the obstacle O_i, where S_0= $(pos(\boldsymbol{V_{ir}} \cdot e(g_i(q) - q)))^2 / 2a_{max}$.

Proof. Let v= $\boldsymbol{V_{ir}} \cdot e(g_i(q) - q)$, which is the relative velocity of the mobile robot with respect to the nearest point of obstacle O_i. If $v \leq 0$ then the safety is guaranteed. Consider the case $v>0$ in the following. Since $\|\boldsymbol{V_i}\| < V_{max}$, and $R >> 2(V_{max})^2/a_{max}$, we have $R >> S_0$. Hence, $d(q, O_i) > S_0$ usually holds at the beginning that the robot meets the obstacle. Whenever $d(q, O_i) \to S_0$, if $\dot{\boldsymbol{V}}_{\boldsymbol{r}} \cdot e(g_i(q) - q) = -a_{max}$ holds, and due to the assumptions on the obstacle O_i, we have $\dot{\boldsymbol{V}}_{\boldsymbol{ir}} \cdot e(g_i(q) - q) = (\dot{\boldsymbol{V}}_{\boldsymbol{r}} - \dot{\boldsymbol{V}}_{\boldsymbol{i}}) \cdot e(g_i(q) - q) \leq -a_{max}$. Hence, if the robot can go with an acceleration $-a_{max}$ in the direction of $e(\mathbf{g_i}(q)-q)$, there must be $\boldsymbol{V_{ir}}.e(g_i(q)-q) \leq 0$ whenever $d(q,O_i) \rightarrow 0$, that is, there is no collision to happen between the mobile robot and the obstacle O_i. From (6) and (8), whenever $d(q, O_i) \to S_0$, then $k_{ri} \to \infty$, that is, $\boldsymbol{F_{ri}} \cdot e(g_i(q) - q) = -\infty$. If additionally $\boldsymbol{F_{other}} \cdot e(g_i(q) - q) << \infty$, then $\boldsymbol{F_{total}} \cdot e(q - g_i(q)) = \infty$, that is, $\boldsymbol{F_{total}} \cdot e(q - g_i(q)) - K_f(\dot{q} - \dot{q}_d) >> M.a_{max}$. Then from (2), the mobile robot must go with an acceleration $-a_{max}$ in the direction of $e(g_i(q)-q)$. This completes the proof.

According to Proposition 1, we define **Environment Constraint 1**:

$$\forall i \in O_d,\ \|\boldsymbol{V_i}\| < V_{max} \text{ and } \dot{\boldsymbol{V}}_{\boldsymbol{i}} \cdot e(g_i(q) - q) \geq 0$$

$$\text{whenever } d(q, O_i) \to (pos(\boldsymbol{V_{ir}} \cdot e(g_i(q) - q)))^2 / 2a_{max}$$

In (8), the dynamic constraints of the mobile robot are considered in the design of the magnitude of the repulsive force. It can guarantee the safety of the mobile robot with the environment constraint 1 from Proposition 1. In most of the conventional APF, the repulsive force is only a function of the relative position of the mobile robot with respect to an obstacle. Hence it cannot guarantee the safety of the mobile robot in applications. From the results above, the following result is obvious.

Theorem 1. In static environments, the ACFs, based on the parameter designs in (6) and (8), can guarantee the safety of the mobile robot.

By using contradiction, it is easy to prove Theorem 1. In order to guarantee that the ACF can guarantee the safety of the mobile robot in a dynamic environment, we firstly prove the following proposition.

Proposition 2. Assume two moving obstacles O_i, O_j and the mobile robot are moving in a same line path at time t, the mobile robot is between O_i and O_j. Their velocities are $\boldsymbol{V_i}$, $\boldsymbol{V_j}$ and $\boldsymbol{V_r}$, respectively. And assume $e(\boldsymbol{V_i})=-e(\boldsymbol{V_j})$. Then when the two obstacles are approaching each other, *i.e.*, $d(O_i,O_j) \rightarrow \delta$ as $t \rightarrow \infty$, where δ is a small positive number, the robot will not collide with the obstacles based on the ACFs defined above.

Proof. Let v_i=$\boldsymbol{V_{ir}}.e(g_i(q)-q)$, and v_j=$\boldsymbol{V_{jr}}.e(\mathbf{g_j}(q)-q)$. Whenever $d(O_i,O_j) \rightarrow \delta$, we have $d(q,O_i) \rightarrow pos(v_i)^2/2a_{max}$, $d(q,O_j) \rightarrow pos(v_j)^2/2a_{max}$. According to (6,8), $\boldsymbol{F_{ri}}$ and $\boldsymbol{F_{rj}}$ are both very large for this case. According to (5,7), the attractive force and the coordinating force are both limited, and the coordinating force is orthogonal to the repulsive force at any time, thus

they both can be neglected to consider the safety problem. Then (2) can be rewritten as: $M\ddot{q} \approx -k_f \dot{q} + \boldsymbol{F_{ri}} - \boldsymbol{F_{rj}}$. Obviously, the mobile robot will track the balance point of the two repulsive fields in this case. From the assumptions of this proposition, the balance point of the repulsive forces on the line path is a safe point. This completes the proof.
According to Proposition 2, define the **Environment Constraint 2**:

$$d(O_i, O_j) > 0, \forall O_i, O_j \in O_s \cup O_d$$

In face, the effect of the coordinating forces in the case of Proposition 2 helps to guarantee the safety of the mobile robot, though it is not considered there. By far, we obtain the following result.
Theorem 2. Assume the maximum velocity and acceleration of the mobile robot are V_{max} and a_{max}, respectively. The maximum radius of the sensors within which the obstacles can be effectively detected is $R >> 2(V_{max})^2/a_{\max}$, and assume the environment constraints 1-2 are satisfied. Then the mobile robot is safety in the ACFs using the designs in (5-8).
Proof. If there only one observable obstacle, it is the case in Proposition 1. Otherwise, any other case can be regarded as the typical case in Proposition 2 that the robot is moving between two obstacles. Hence, from Proposition 1 and 2, the mobile robot is safe in the ACFs.

3.4 Reachability of the ACFs

Reachability of the ACFs is the ability of the mobile robot using ACFs to reach its goal provided that there is a safe path from the starting point to the goal in the environment. This requires that there are no local minima in ACFs. In conventional APFs, the attractive force and repulsive force may be balanced at some points where local minima exist. These points are usually between multiple obstacles or on the opposite side of an obstacle with respect to the goal point. However, local minima at these points can be removed by properly using the coordinating forces in ACFs.

- **Using the Coordinating Force to Remove Local Minima**

Let k_{ni} in (7) be

$$k_{ni} = 1 \Big/ \left(\mathrm{pos}(\|q - \mathrm{g_i}(q)\| - (\mathrm{pos}(\mathbf{V_{ir}} \cdot \mathrm{e}(\mathrm{g_i}(q) - q)))^2 / 2a_{\max})\right)^m \tag{9}$$

where $m>0$ is to be defined.
Obviously, it is easy to remove the local minima using the coordinating force if there is only one observable obstacle. As for the multiple obstacles case, the coordinating force should be designed to remove the local minima between any two obstacles in the observable region such that the mobile robot can go through the passage between any two obstacles satisfying the environment constraints 1-2.
Definition 1. Curve C is an equi-repulsive-force curve between two obstacles O_i and O_j, if the following equation holds: $\forall p \in C, \|\mathbf{F_{ri}}(p)\| = \|\mathbf{F_{rj}}(p)\|$ (Referring to Figure 3).
Lemma 1. Neglecting the effects of the attractive and coordinating forces, the mobile robot is moving on the curve C when passing a passage between two obstacles, and the following equations hold: (The proof is omitted)

$$\boldsymbol{V}_{ir} \approx \boldsymbol{V}_{jr}, d(q,O_i) \approx d(q,O_j) \ , \ \mathrm{E_{ri}} \approx \mathrm{E_{rj}}.$$

Proposition 3. Assume that the mobile robot is at point q on the curve C between obstacles O_i and O_j at time t, $d(O_i,O_j)=2a>0$, and the obstacles satisfy environment constraints 1-2. Neglecting the effect of the attractive force, in order for the mobile robot to pass the passage between O_i and O_j, the ACFs should satisfy the following minima-free conditions:
before passing the passage:

$$\lambda_l = \arg\min_{\lambda\in\{1,-1\}} (\angle(\lambda\cdot\mathbf{T}\cdot(\mathrm{g}_l(q)-q),(-\mathbf{F_{rj}}(q)-\mathbf{F_{ri}}(q))/2)),$$

after passing the passage:

$$\lambda_l = \arg\min_{\lambda\in\{1,-1\}} (\angle(\lambda\cdot\mathbf{T}\cdot(\mathrm{g}_l(q)-q),(\mathbf{F_{rj}}(q)+\mathbf{F_{ri}}(q))/2)) \text{ or } 0 \ ,$$

and

$$(k_{rc}/F(q,l)^n)\sqrt{R^2-a^2} \le (k_{nc}/F(q,l)^m)\cdot a$$

where

$$F(q,l)=(\|q-\mathrm{g}_l(q)\|-\mathrm{pos}(\boldsymbol{V}_{lr}\cdot \mathrm{e}(\mathrm{g}_l(q)-q))^2/2a_{\max})\ ,\ l\in\{i,j\}\ .$$

Proof. The repulsive and coordinating forces exerted on the mobile robot by obstacles O_i and O_j are

$$\boldsymbol{F_r} = \boldsymbol{F_{ri}} + \boldsymbol{F_{rj}} = \frac{k_{rc}(E_{ri}+\varepsilon_1)}{F(q,i)^n}\cdot\frac{q-\mathrm{g_i}(q)}{\|q-\mathrm{g_i}(q)\|} + \frac{k_{rc}(E_{rj}+\varepsilon_1)}{F(q,j)^n}\cdot\frac{q-\mathrm{g_j}(q)}{\|q-\mathrm{g_j}(q)\|},$$

$$\boldsymbol{F_n} = \boldsymbol{F_{ni}} + \boldsymbol{F_{nj}} = \frac{k_{nc}(E_{ri}+\varepsilon_2)}{F(q,i)^m}\cdot\frac{q-\mathrm{g_i}(q)}{\|q-\mathrm{g_i}(q)\|} + \frac{k_{nc}(E_{rj}+\varepsilon_2)}{F(q,j)^m}\cdot\frac{q-\mathrm{g_j}(q)}{\|q-\mathrm{g_j}(q)\|},$$

According to Lemma 1, in order for the robot to pass the passage along Curve C, the following equations hold: $F(q,i)\approx F(q,j)$, $E_{ri}\approx E_{rj}$. Let $\theta = \angle(\mathrm{e}(q-\mathrm{g_i}(q)),\mathrm{e}(q-\mathrm{g_j}(q)))$, and according to the cosine lemma in a triangle and the principle to choose λ given in the proposition, we have

$$\|\mathbf{F_r}\| = 2\cdot(\bar{k}_{rc}/F^n)^2 + 2\cdot(\bar{k}_{rc}/F^n)^2\cos\theta,$$

$$\|\mathbf{F_n}\| = 2\cdot(\bar{k}_{nc}/F^m)^2 + 2\cdot(\bar{k}_{nc}/F^m)^2\cos(\pi-\theta) = 2\cdot(\bar{k}_{nc}/F^m)^2 - 2\cdot(\bar{k}_{nc}/F^m)^2\cos\theta.$$

where, $\bar{k}_{rc} = k_{rc}(E_{ri}+\varepsilon_1)$, $\bar{k}_{nc} = k_{nc}(E_{ri}+\varepsilon_2)$, $F=F(q,i)$. Neglecting $\boldsymbol{F_a}$, in order for the robot to pass the passage, only if $\|\boldsymbol{F_r}\| \le \|\boldsymbol{F_n}\|$ holds. Hence, we have

$$(\bar{k}_{rc}/F^n)^2\,(1+\cos\theta) \le (\bar{k}_{nc}/F^m)^2\,(1-\cos\theta),$$

(this inequality is denoted as p1). Due to the maximum detecting radius of the sensors is R, and min($\left\| g_i(q) - g_j(q) \right\|$)=2$a$, applying the cosine lemma to the minimum θ in the triangle (q,$g_i(q)$,$g_j(q)$), we can obtain $R^2+R^2-2R^2\cos\theta=(2a)^2$, which further yields: $\cos(\theta)_{max}=1-(\sqrt{2}a/R)^2$. Substitute it into (p1), and let $\varepsilon_1 = \varepsilon_2$, we obtain

$$(k_{rc}/F^n)\sqrt{R^2 - a^2} \le (k_{nc}/F^m)\cdot a.$$

This completes the proof.
Let m=n, according to Proposition 3 we have

$$k_{nc}/k_{rc} \ge \sqrt{(\tfrac{R}{a})^2 - 1}\,.$$

Note that the smaller a is, the larger is k_{nc}/k_{rc} in this case. It is consistent with the practical fact. If the robot is a circle with radius r, then the corresponding condition should be:

$$k_{nc} \ge k_{rc}\sqrt{\tfrac{R}{r}(\tfrac{R}{r}+2)}\,.$$

This is verified in the simulations. Proposition 3 provides a theoretical view point to the design of the ACFs' parameters, though some assumptions are strict. In fact, the minima-free conditions in Proposition 3 are just sufficient, since the attractive force is neglected in the proof.

- **Online Decision Making Based on Coordinating Factors**

In order to remove the local minima between multiple obstacles in uncertain dynamic environments, the coordinating factors with respect to different obstacles should be properly decided on line such that the coordinating forces can provide actuating forces to the mobile robot to balance the repulsive forces. On the other hand, the wall-following behavior (Lumelshy and Skewis 1999) should be adopted when the mobile robot meets a large obstacle of even nonconvex shape such that the robot can follow the boundary of the obstacle to go until the robot can directly find in free space the direction in which the goal exists. In this case, the coordinating force is used directly as the actuating force for the robot to follow the "wall". For this purpose, the coordinating factors should also be properly decided on line.
The decision making of λ is based on the decision making of the local sub-goal in the observable region of the mobile robot. The local sub-goal (Xu et al 1998) is denoted by $\boldsymbol{e}_{ds}$, which should be an appropriate tradeoff between the collision-avoidance behavior and the going-to-goal behavior. In this study, the mobile robot is expected to avoid an obstacle along the shortest path in local environment. For a static obstacle, the wall-following behavior should be able to be generated; and for a moving obstacle, the robot is expected to run away from the trajectory of the obstacle as fast as possible. To these aims, the local sub-goal is decided as follows with respect to an obstacle O:

$$\text{If } \left\| \boldsymbol{V_O} \right\| \le V_O: \ \boldsymbol{e}_{ds} = \mathrm{e}(\boldsymbol{F}_a) + \kappa\, \mathrm{e}(\boldsymbol{V}_r) \tag{10a}$$

$$\text{else} \quad \boldsymbol{e}_{ds} = -\mathrm{e}(\boldsymbol{V}_O) + \kappa\, \mathrm{e}(\boldsymbol{V}_r) \tag{10b}$$

where $\boldsymbol{V}_r$ and $\boldsymbol{V}_O$ are the velocities of the mobile robot and the obstacle, respectively, $\kappa>0$, V_O is a constant. Then the optimal decision-making of the coordinating factor with respect to the obstacle O is

$$\lambda=\arg(\underset{\lambda\in\{1,-1\}}{\angle}(\lambda\cdot\boldsymbol{T}\cdot(\mathrm{g}_{\mathrm{O}}(q)-q),\boldsymbol{e}_{\boldsymbol{ds}})\leq 90^{0}) \tag{10c}$$

If the velocity of an obstacle is lower than a constant V_O, then it can be regarded as a static obstacle. (10a) is a tradeoff between the going-to-goal behavior and the collision-avoidance behavior, and (10b) provides such a sub-goal that the robot is expected to avoid a moving obstacle as fast as possible. The angle between the optimal direction of the coordinating force and the local sub-goal is less than 90^0 such that the coordinating force can provide an actuating force to the mobile robot. It should be noted that different coordinating factor may correspond to a different motion behavior, the desired motion behavior of the mobile robot is basically determined by the optimal decision-making of the coordinating factors. It can also be verified that the coordinating factor decided by (10c) is consistent with the minima-free conditions in Proposition 3.

- **Realization of the Wall-Following Behavior and no Local Minima**

In order to realize the wall-following behavior with respect to an obstacle, the coordinating factor should be kept constant once the mobile robot meets the obstacle. To show that (10c) can provide a consistent coordinating factor with respect to an obstacle, we have the following results.

Fact 1. Assume the boundary ∂O of an obstacle O is differentiable. q^* is a point outside of O. $\forall p\in\partial O$, $t(p)$ is a tangent line at this point. $\boldsymbol{e}_r(t(p))$ is the unitary vector of the tangent line at point p in anticlockwise, and $\boldsymbol{e}_l(t(p))$ is in clockwise. Then we have $\angle(\boldsymbol{e}_s(t(p)),\boldsymbol{e}_d)\leq 90^0$, where $\boldsymbol{e}_d$=e(q^*-p)+$\boldsymbol{e}_s(t(p))$, s=r or l.

Define **Environment Constraint 3**:

All the obstacles are convex, and their boundaries are one-time differentiable.

Proposition 4. All the obstacles satisfy the environment constraints 1-3. Let f_r=$\boldsymbol{F}_a$e($\boldsymbol{F}_{ri}$), f_n=$\boldsymbol{F}_a$e($\boldsymbol{F}_{ni}$). If choose k_{nc} in (7) such that $\boldsymbol{F}_{ni}$e($\boldsymbol{F}_{ni}$)+f_n>0, then wall-following behavior can be generated based on (10) once the robot meets the obstacle O_i in the case f_r<0.

Proof. Due to the obstacle O_i is convex, its boundary ∂O_i can be classified into two parts according to the sign of f_r. In the side of f_r≥0, the angle between the repulsive and attractive forces is less than 90^0, the mobile robot can run away from the ACF of the obstacle quickly. Hence the wall-following behavior is unnecessary in this case. And for the case f_r<0, the angle between the repulsive and attractive forces is more than 90^0. In the latter case, it can be regarded that f_r=-$\boldsymbol{F}_r$e($\boldsymbol{F}_r$), and then the planning equation in (2) can be rewritten as:

$$M\,\ddot{q}+K_f\,\dot{q}=\boldsymbol{F_a}+\boldsymbol{F_{ri}}+\boldsymbol{F_{ni}}=\left[f_r\cdot e(\boldsymbol{F_{ri}})+\boldsymbol{F_{ri}}\right]+\left[f_n\cdot e(\boldsymbol{F_{ni}})+\boldsymbol{F_{ni}}\right]=f_n\cdot e(\boldsymbol{F_{ni}})+\boldsymbol{F_{ni}},$$

Obviously, the velocity of the mobile robot is basically determined by the coordinating force, and it finally converges to $\dot{q}=\left(f_n\cdot\mathrm{e}(\boldsymbol{F_{ni}})+\boldsymbol{F_{ni}}\right)/K_f$. If $\boldsymbol{F}_{ni}$e($\boldsymbol{F}_{ni}$)+f_n>0 holds, then we have e($\dot{q}$)=e($\boldsymbol{F}_{ni}$). Now utilizing Fact 1, e($\boldsymbol{F}_{ni}$) can be regarded as the direction of the tangle line of the obstacle boundary, *i.e.*, $\boldsymbol{e}_r(t(p))$ or $\boldsymbol{e}_l(t(p))$. If $\|\boldsymbol{V_i}\|\leq V_O$, the sub-goal decided by (10a) is equivalent to $\boldsymbol{e}_d$ in Fact 1 with q^*=q_d, otherwise, q^*=p-$\boldsymbol{V}_i$. In both cases, we always have $\angle(\mathrm{e}(\boldsymbol{F}_{ni}),\boldsymbol{e}_d) \leq 90^0$. That is, if the current coordinating factor $\lambda = 1$ (or -1) with respect to obstacle O_i, then $\lambda = 1$ (or -1) still holds according to (10c) for the next time. The wall-following behavior is generated for this case. This completes the proof.

Further study can show that the wall-following behavior can also be realized for a non-convex obstacle based on (10), this was discussed in Jing and Wang (2004). The repulsive force may prevent the mobile robot to reach its goal if the goal point is very near to an obstacle. To overcome this problem, we let (Wong and Spetsakis 2000) (with respect to a static obstacle)

$$k_{rc} = k_1 \cdot \min\left(\|q - q_d\|^3, \|q - q_d\|^{k_2}\right) \tag{11}$$

where $k_1>0$, $0<k_2\leq 1$ are both constants.

To achieve minima-free ACFs, define **Environment Constraint 4**:

For any two obstacles O_i,O_j, let $\boldsymbol{e}_d=(\mathrm{e}(q_d-p)+\boldsymbol{e}_s(t(p)))/2$, $p\in \partial O_i \cap P(q)$ .Considering the case $g_j(p) \in \partial O_j \cap P(q)$, if $s=l$, clock(e($g_j(p)$ -p),$\boldsymbol{e}_d$)= 1 holds, and if $s=r$, clock(e($g_j(p)$ -p),$\boldsymbol{e}_d$)= -1 holds. Where, clock($\boldsymbol{a}$,$\boldsymbol{b}$) is defined as: it is -1 if $\boldsymbol{b}$ can be obtained by rotating $\boldsymbol{a}$ with the angle $\angle(\boldsymbol{a},\boldsymbol{b})$, otherwise, it is 1.

With the deliberate designs above, it can be seen that the reachibility of the robot can be guaranteed under the environmental contraints 1-4. Due to the attractive force, the mobile robot is always approaching an obstacle that is on the line jointing the current position of the mobile robot and its goal. If the mobile robot meets a static obstacle O_i, then according to Proposition 4, the wall-following behavior is generated. During the wall-following with respect to this static obstacle O_i, if the mobile robot meets another moving obstacle satisfying the environment constraints 1-4, then according to (10) and Proposition 3, the mobile robot either runs away from the trajectory of the moving obstacle as fast as possible, or passes a passage between two obstacles to avoid the moving obstacle. And then, the mobile robot comes back again to the state following the boundary of an obstacle that is on its desired shortest path to the goal. If the mobile robot meets a static obstacle O_j during it is following the boundary of the static obstacle O_i in anti-clockwise (*i.e.*, λ_i =1). Due to the environment constraint 4 and according to Proposition 4, we can have clock(e($\boldsymbol{F}_{rj}(q)$),$\boldsymbol{e}_d$)= 1 and $\boldsymbol{e}_d$=(e(q_d-q)+e($\boldsymbol{V}_r$))/2. Utilizing (10) again, we have $\angle$ ($\boldsymbol{F}_{nj}(q)$,$\boldsymbol{e}_d$) $\leq 90^0$, then clock(e($\boldsymbol{F}_{rj}$),e($\boldsymbol{F}_{nj}$))=1 must hold in this case, *i.e.*, λ_j =-1 with respect to O_j. It is easy to verify that, the minima-free conditions in Proposition 3 are satisfied in this case. That is, the mobile robot can pass the passage between O_i and O_j, and the wall-following behavior can be kept during passing this passage. If the mobile robot follows the boundary of O_i in clockwise, the same conclusion can be made. After the mobile robot avoids obstacle O_i completely, it may meet another obstacle and then the similar process as above is carried out again due to the actuation of the attractive force, until it reaches the goal finally.

4. Motion Planning of the Mobile Robot in Uncertain Dynamic Environments

Assume the goal of the mobile robot is known. The motion planning problem can be written as: *To find the optimal* $\boldsymbol{u}(t)$*, i.e.,*

$$\mathbf{u} = \arg\min_{\mathbf{u}\in U_u}(J(\mathbf{u}))$$

(in the following algorithm, it is transformed to be the optimal decision making equation (10) for λ*, where* U_u *is the decision making space of* $\boldsymbol{u}$ *satisfying the dynamic constraints) such that the mobile robot can go safely from its stating point* q_0 *to its goal point* q_d*, i.e.,*

$$\text{(1) } \forall i,\ q \cap O_i = \Phi,\ \text{(2) } \exists T < \infty,\ \lim_{t\to T} q(t) = q_d\ .$$

Considering the dynamic constraints of mobile robots, the states of (3,4) should be subjected to the saturation constraints of the velocity and acceleration (Jing and Wang 2002). For (3,4), we use the following control law:
If there is no observable obstacle

$$\mathbf{u} = K_f \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} \tag{12a}$$

where

$$\begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} = V_d \cdot \begin{bmatrix} x_d - x \\ y_d - y \end{bmatrix} \Big/ \left\| \begin{matrix} x_d - x \\ y_d - y \end{matrix} \right\|,$$

$$V_d = \min\left(\alpha \cdot V_{\max}, \sqrt{2 \cdot \left\| \begin{matrix} x_d - x \\ y_d - y \end{matrix} \right\| \cdot a_{\max} \cdot \beta} \right),$$

$$0 < \alpha, \beta \le 1,\ K_a=0,\ K_{ri}=0, K_{ni}=0,\ (i=1,2\ldots),\ K_f>0.$$

If there are observable obstacles, let $\dot{q}_d = 0$ and

$$\boldsymbol{u} = K_a \begin{bmatrix} x_d \\ y_d \end{bmatrix} - \sum_{i\in Od\cup Os} K_{ri}\, g_i(q) + \sum_{i\in Od\cup Os} K_{ni} \lambda_i \cdot \boldsymbol{T} \cdot g_i(q) \tag{12b}$$

where, K_a,K_{ri},K_{ni}, λ_i are chosen according to (5-11). The outputs of the planning equation are the desired behavior for the mobile robot to take.

5. Simulations

The planning equation (3) is used in the simulations. The parameters of the mobile robot are set as follows: its radius is r=0.3m, the maximum acceleration a_{max}=0.5 m/s^2, the maximum velocity V_{max}=0.5m/s, the maximum detecting radius of the sensors is R=1.5m>$2(V_{max})^2/a_{\max}$.

The parameters of the ACF are chosen as follows:

Step1. For the parameters of the attractive force (in (5)), let k_a=1, M_a =4. If these parameters are set to be too large, they may affect the safety of the robot.

Step2. For the parameters of the repulsive force (in (6)(11)), let n=2 , ε_1=0.05, $k_{rc} = k_1 \cdot \min\left(\|q - q_d\|^3, \|q - q_d\|^{k_2}\right)$, k_1=0.5□k_2=0.5. The repulsive force should be much larger than the attractive force within the minimum safe radius predefined for the robot.

Step3. For the parameters of the coordinating force (in (7)), let ε_2 =0.05, M_n =200, k_{nc}=6k_r m=2 (in Proposition 3). Based on the parameters chosen for the repulsive force, these parameters for the coordinating force are chosen basically according to Proposition 3.

Step4. For collision risk, let k_{risk1}=0.1, k_{risk2}=0.9. They are chosen according to different inclinations.

Step5. For the decision making of the coordinating factor(in (10)), let V_O=0.2m/s, κ =2. The larger κ is, the larger is the impact of the current velocity on the sub-goal, which further affects the trajectory of the mobile robot.

Step6. For the parameters for the control law(in (12)), let α =1, β =0.5, K_f=10. The larger K_f is, the larger is the damp of the planning equation.

- **The ACFs can reduce oscillations**

Figure 4 is the result using the conventional APFs, and the results of the ACFs are given in Figure 5. The coordinating forces can exert an actuating force to the mobile robot with proper decision making of the coordinating factors, and all the virtual forces in ACFs are proportional to the collision risk. Hence, the ACFs can effectively reduce the oscillation on the trajectory between multiple obstacles. However, the oscillation of "S" shape exists on the trajectory planned by the conventional APFs based methods. It should also be noted that the velocity and acceleration planned for the mobile robot by ACFs are both satisfied with the dynamic constraints.

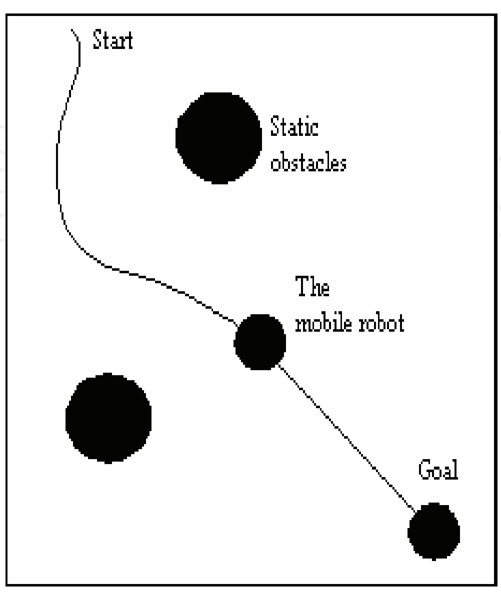


Figure 4. Results of the conventional APFs

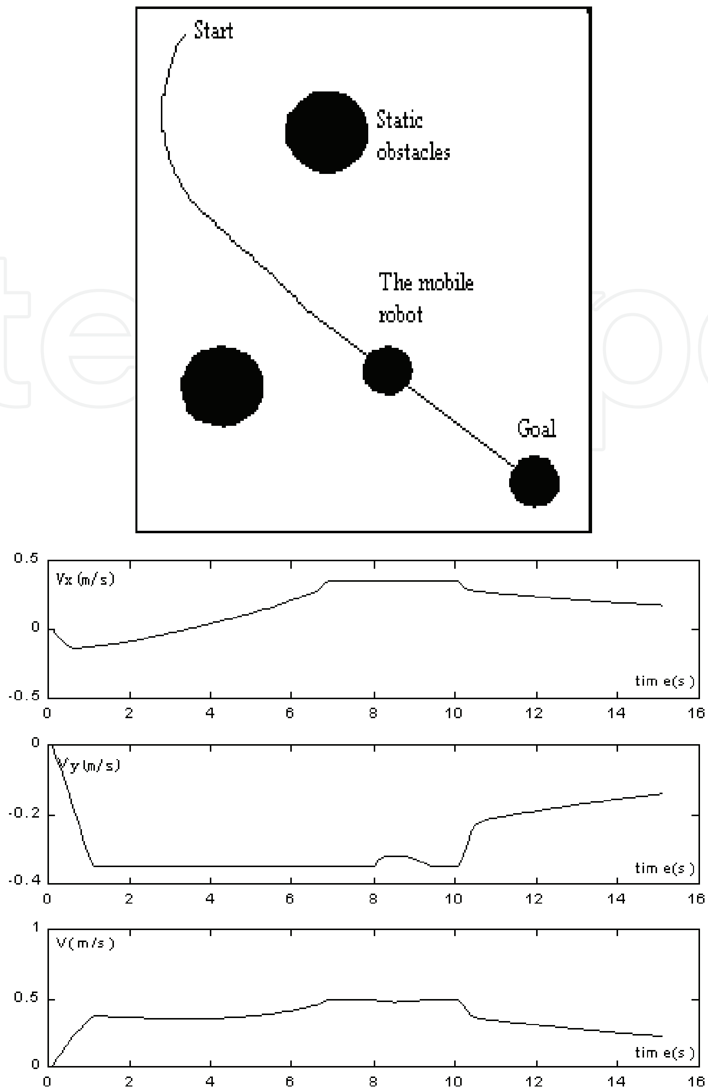
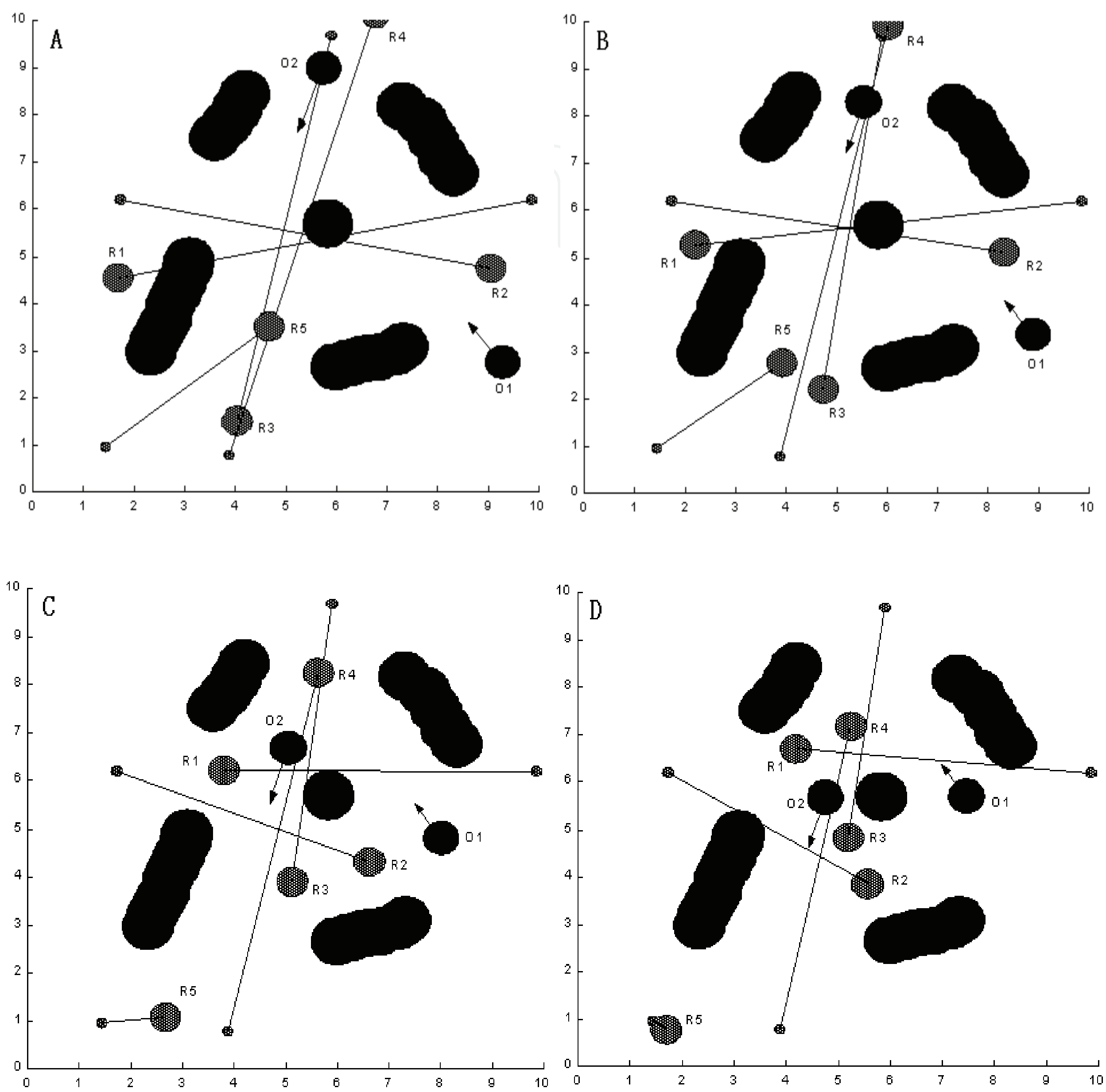


Figure 5. Results of the ACFs

- **The ACFs can improve the autonomy and intelligence of the mobile robot and remove local minima when meeting obstacles and other robots**

The moving obstacle is assumed to be a circle with radius 0.35m and velocity 0.35m/s. A simulation process is in Figure 6 (A-H). In Figure 6, Ri denotes a robot i, Oj denotes a dynamic unknown obstacle j, and others are static obstacles. A line between the current position of a robot and its goal indicates the desired direction. A ray on dynamic obstacle indicates its moving direction. In figure A and B, R3 meets R5, by anti-clockwise rotating they avoid collision with each other, and obviously it is ideal for a shorter collision-free path. In Figure A and B, R1 meets a large static obstacle, wall-following behavior is used. In Figure C, R1 passes a passage between two static obstacles, in conventional APF there may be local minima which will prevent the robot passing. When the robot meets dynamic obstacles, coordinating force can make the collision-avoidance behavior of the robot more intentionally and effectively. See it in Figure C, if no coordinating force, R1 might be pushed back, but in fact from its trajectory in Figure H we can note that a turning-left behavior occurred due to the coordinating force, which makes the motion more effective and rational. In other Figures, we can also see such effective and intelligent collision-avoidance behaviors.

More discussions about this subject can also be referred to Jing and Wang (2003), Jing et al (2004c) and Jing (2004).



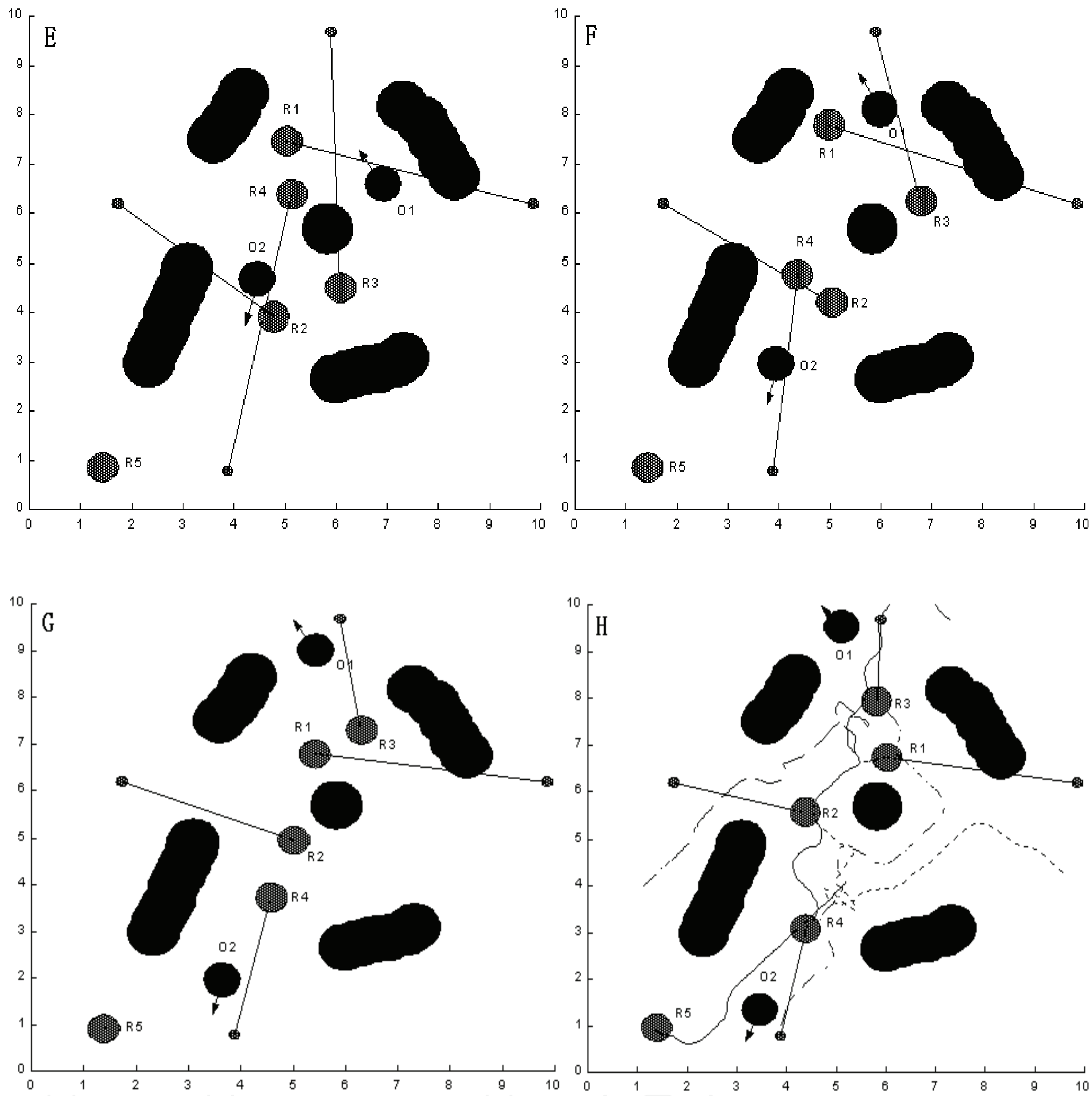


Figure 6. A simulation process (A-H)

6. Conclusions

In order to overcome some noticeable drawbacks of the traditional APF based methods such as local minima and oscillations on the planned trajectory, an artificial coordinating field was proposed recently (Jing et al 2002, 2003, 2004abc). This chapter provides a simple introduction for these newly developed results. A coordinating force is added to the conventional APF which is orthogonal to the repulsive force, and the field parameters are designed with consideration of the states and task of mobile robots under different enviromental situations. These enable the ACF to be more robust and effective for behavior decisions of mobile robots and adaptable to the change of environments when there are different intelligent and unintelligent obstacles. Local minima and unnecessary oscillation in planned trajectory can be avoided. More intelligent coordination between different mobile robots in obstacled environments can also be achieved.

There are three principles in designing the ACFs:

a. All the virtual forces are functions of the motion purpose and relative states of the mobile robot with respect to the local environments,
b. The repulsive force should satisfy the dynamic constraints of the mobile robot,
c. The coordinating force should satisfy the minima-free conditions.

Based on these designs, the ACFs are adaptable to environments and controllable for robots. The ACFs based motion planning can guarantee the safety and reachability under certain environment constraints. Since more information of the robot and environment can be represented, the ACFs are more robust. In the local dynamic coordinates defined in Section 2, the ACF has full-dimensional forces, instead of one-dimensional force in the conventional APF. This is the most important and fundamental difference between ACF and APF, and the conventional APF is just a special case of the ACF.

7. References

Ge S.S. and Cui Y.J (2000). New potential functions for mobile robot path planning. *IEEE trans. Robot. and Automat.* Oct. 16(5): 615-620

Guldner J and Utkin V. I. (1995) Sliding mode control for gradient tracking and robot navigation using artificial potential fields. *IEEE Trans. Robot. and Automat*, April, 11(2):247-253

Jing X.J. (2004), *Stability and planning of internet-based tele-robotic systems* (in Chinese). PhD thesis in the Graduate School of Chinese Academy of Sciences, pp 72-101

Jing X.J. (2005), Behavior dynamics based motion planning of mobile robots in uncertain dynamic environments, *Robotics and Autonomous Systems* 53, 99–123

Jing X. J., Wang Y.C (2002). Artificial Coordinating Field (ACF) Control of Mobile Robot for Collision Avoidance and Navigation. *Proceedings of international conference on Control and Automation*, Xiamen, China: Xiamen University Press, June, pp2478-2482

Jing X. J., Wang Y. (2003), Artificial Coordinating Field based Coordinating Collision Avoidance, in: *IEEE International Conference on Robotics, Intelligent Systems and Signal processing*, Changsha, Hunan, China, pp. 126–130.

Jing X. J. and Wang Y.C. (2004a), Local minima-free design of artificial coordinating fields, *Journal of Control Theory and Applications*, Vol 2, No 4, pp371-380

Jing X. J., Wang Y., Tan D. (2004b), Artificial Coordinating Field and its application to motion planning of robots in uncertain dynamic environments, *Science in China, Ser. E Eng. Mater. Sci.*(both in Chinese and English) 47 (5), 577–594.

Jing X. J., Wang Y., Tan D. (2004c), Artificial coordinating field based real-time collision avoidance planning of multiple mobile robots, *Contr. Theor. Appl.* (in Chinese) 21 (5), 757–764.

Kant K. and Zucher S (1988). Planning collision free trajectories in time-varying environments: a two level hierarchy. *Proc. IEEE Inter. Conf. Robotics and Automation*, pp1644-1649

Khatib O. (1986). Real time obstacle avoidance for manipulators and mobile robots. *International Journal of Robotics Research*, 5(1):90-98

Koren Y., Borenstein J. (1991). Potential field methods and their limitations for mobile robot navigation. *Proc. IEEE Inter. Conf. Robot. and Automat. Sacramento*, California: International House, April 7-12: 1398-1404

Krogh B (1984). A generalized potential field approach to obstacle avoidance control. *In SME Conf. Proc. Robotics Research: The Next Five Years and Beyond*, Bethlehem, Pennsylvania: International house, pp 950-955

Louste C. and Liegeois A (2000). Near optimal robust path planning for mobile robots: the viscous fluid method with friction. *Journal of intelligent and robotic systems*, 27: 99-112

Lumelshy V., Skewis T.(1999) Incorporating range sensing in the robot navigation function. *IEEE Trans. Syst. Man and Cyber*. 30: 2749-1296

Noborio H., Yoshiolka T., Hamaguchi T (1995). On-line deadlock-free path planning algorithms by means of a sensor-feedback tracing. *Proc. IEEE Inter. Conf. Syst. Man and Cyber.* Vancouver: International House, Oct. 22-25:1291-1296

Masoud S. A. and Masoud A. A. (2000) Constrained motion control using vector potential fields. *IEEE trans. on Systems, Man, and Cybernetics—Part A: Systems and Humans*, May, 30(3): 251-272

Medio C. D. and Oriolo G.(1999) Robot obstacle avoidance using vortex fields. In *Advance in robot kinematics*, S.Stifter and J.Lenatc(Eds), Springer-Verlag, Wien, pp227-235

Prassler E. (1999) Navigating a robotic wheelchair in a railway station during rush hour. *The international journal of robotics research*, 8(7): 711-727

Rimon E. and Koditschek D. (1991). The construction of analytic diffeomorphisms for exact robot navigation on star worlds. *Transactions of the American Mathematical Society*, 327(1):71-115.

Rimon E. and Koditschek D.E. (1992) Exact robot navigation using artificial potential functions. *IEEE trans. Robot. and Automat*. Vol.8, no.5, pp501-518.

Salichs M.A. and Moreno L. (2000). Navigation of mobile robots: open questions. *Robotica*, 18:227-234.

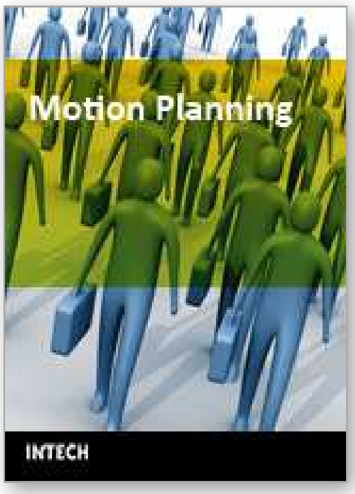
Satoh K.(1993) Deadlock-free motion planning using the laplace potential field. *Adv. Robotics*, 7(5):449-461

Singh L., Wen J., and Stephanou H (1997). Motion planning and dynamic control of a linked manipulator using modified magnetic fields. *Proc. IEEE int. conf. on robotics and automation*, Albuquerque, NM: International House, pp1142-1147

Tsourveloudis N. C., Valavanis K. P. and Hebert T (2001). Autonomous vehicle navigation utilizing electrostatic potential fields and fuzzy logic. IEEE *trans. On Robotics and Automation*, Aug, 17(4): 490-497

Wong B and Spetsakis M (2000) Scene reconstruction and robot navigation using dynamic fields. *Autonomous Robots*, 8: 71-86

Xu W.L., Tso S.K., Fung Y.H (1998). Fuzzy reactive control of a mobile robot incorporating a real/virtual target switching strategy. *Robotics and Autonomous Systems*, 23:171-186



Motion Planning
Edited by Xing-Jian Jing

ISBN 978-953-7619-01-5
Hard cover, 598 pages
Publisher InTech
Published online 01, June, 2008
Published in print edition June, 2008

In this book, new results or developments from different research backgrounds and application fields are put together to provide a wide and useful viewpoint on these headed research problems mentioned above, focused on the motion planning problem of mobile ro-bots. These results cover a large range of the problems that are frequently encountered in the motion planning of mobile robots both in theoretical methods and practical applications including obstacle avoidance methods, navigation and localization techniques, environmental modelling or map building methods, and vision signal processing etc. Different methods such as potential fields, reactive behaviours, neural-fuzzy based methods, motion control methods and so on are studied. Through this book and its references, the reader will definitely be able to get a thorough overview on the current research results for this specific topic in robotics. The book is intended for the readers who are interested and active in the field of robotics and especially for those who want to study and develop their own methods in motion/path planning or control for an intelligent robotic system.