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A Practical Approach for Motion Planning of Wheeled Mobile Robots

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1. Introduction

Wheeled Mobile Robots (WMR) have been widely studied in the past fifteen years. Due to kinematic constraints, many WMR are not integrable (non-holonomic). Therefore, standard techniques developed for robot manipulators are not directly applicable. In particular, the motion planning of WMR is still a relevant issue. Examples of motion planning for WMR are available in the literature (Latombe, 1991; Laumond et al., 1997; Gracia & Tornero, 2003; Borenstein & Koren, 1989). On the other hand, the singularity of WMR kinematics must be avoided since it implies slip or impossible control actions (Gracia & Tornero, 2007a). In the same way, in the vicinity of singularities there is high amplification of active joints' error or high values for active joints. Therefore, the aim of the present research is to develop a practical approach for motion planning of WMR based on avoiding singularities. The chapter is organized as follows. Section 2 presents the kinematics of WMR considering four types of wheels: fixed, centered orientable (hereinafter orientable), castor and Swedish. Afterwards, section 3 discusses the possibilities for motion planning and develops a cost index based on singularity. To illustrate the applications of the proposed motion planning an industrial forklift is considered and several simulation results are shown. Finally, section 4 points out the more outstanding contributions of this research.

2. Kinematics of Wheeled Mobile Robots

Firstly it will be introduced some terminology. Assuming horizontal movement, the position of the WMR body is completely specified by 3 scalar variables (e.g. x, y, θ), referred to in (Campion et al., 1996) as WMR posture, $\mathbf{p}$ in vector form. Its first-order time derivative is called WMR velocity vector $\dot{\mathbf{p}}$ and separately (v_x, v_y, ω) WMR velocities (Muir & Neuman, 1987). Similarly, for each wheel, wheel velocity vector and wheel velocities are defined.

2.1 Kinematic models of wheels

The kinematic modeling of a wheel is used as a previous stage for modeling the whole WMR (Gracia & Tornero, 2007a; Campion et al., 1996; Muir & Neuman, 1987; Alexander & Maddocks, 1989). Here, the four common wheels will be considered: *fixed*, *orientable*, *castor* and *Swedish*. As it is easy to obtain their equations using a vector approach, e.g. see (Gracia

& Tornero, 2007a) among many other possibilities, the detailed development will be omitted. The matrix equation of the off-centered orientable wheel or *castor* wheel is:

$$\mathbf{v}_{\text{slip } i} = \begin{pmatrix} \cos(\beta_i + \delta_i) & \sin(\beta_i + \delta_i) & l_i \sin(\beta_i + \delta_i - \alpha_i) - d_i \cos\delta_i & -d_i \cos\delta_i & 0 \\ -\sin(\beta_i + \delta_i) & \cos(\beta_i + \delta_i) & l_i \cos(\beta_i + \delta_i - \alpha_i) + d_i \sin\delta_i & d_i \sin\delta_i & r_i \end{pmatrix} \begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\beta}_i \\ \dot{\varphi}_i \end{pmatrix}, \qquad (1)$$

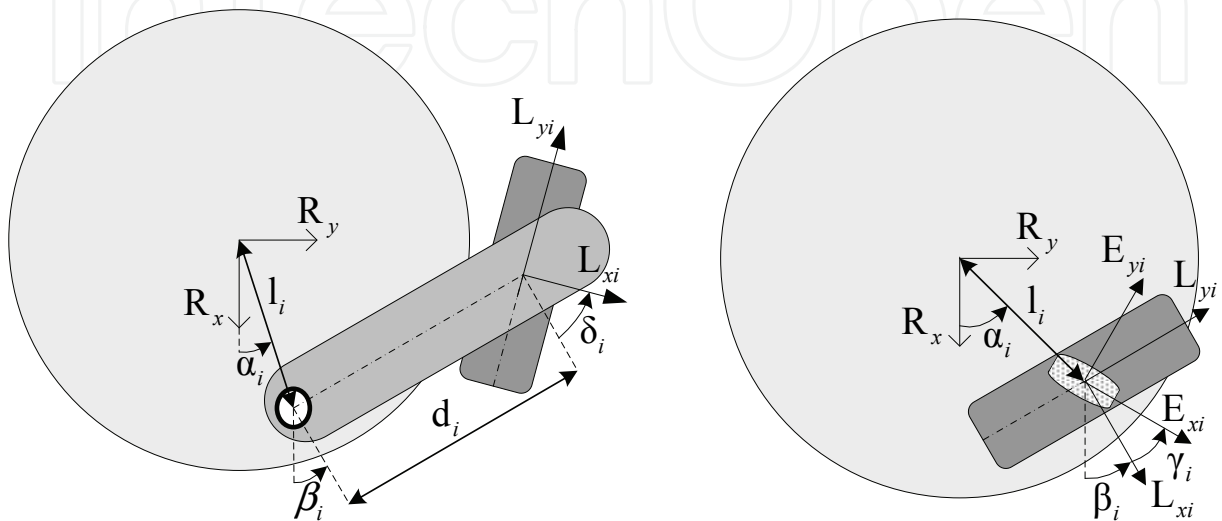where it has been used the parameters of Fig. 1 (a) and the variables of Table 1.

Figure 1. *Castor* wheel parameters: l_i, d_i, α_i, β_i, δ_i *Swedish* wheel parameters: l_i, α_i, β_i, γ_i
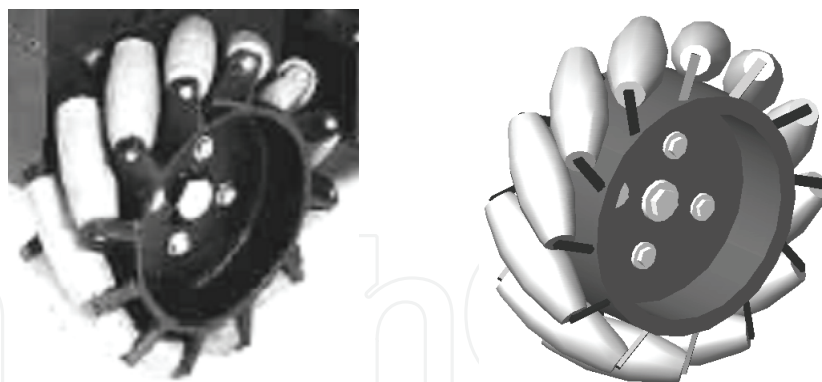

Figure 2. *Swedish* wheel (also called *Mecanum*, *Ilon* or *universal*) with rollers at 45º

The equation of the *orientable* wheel can be obtained from (1) with $d_i = \delta_i = 0$:

$$\mathbf{v}_{\text{slip } i} = \begin{pmatrix} \cos\beta_i & \sin\beta_i & l_i \sin(\beta_i - \alpha_i) & 0 \\ -\sin\beta_i & \cos\beta_i & l_i \cos(\beta_i - \alpha_i) & r_i \end{pmatrix} \begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\varphi}_i \end{pmatrix}. \qquad (2)$$

The previous equation is also valid for *fixed* wheels, where the angle β_i is constant.

The matrix equation of the *Swedish* wheel (see Fig. 2) is (3) where it has been used the parameters of Fig. 1 (b) and the variables and constants of Table 1.

Symbol	Description
R	Frame attached to the robot body with the Z-axis perpendicular to the floor surface
$\bar{\text{R}}$	Frame attached to the floor and instantaneously coincident with the robot frame R. This frame allows to avoid the dependency on a global stationary frame (Muir & Neuman, 1987)
(L_i, E_i)	Frames attached to the wheel i and to the roller of the *Swedish* wheel i, with the *X*-axes coincident with their rotation axle
$\dot{\mathbf{p}}$	WMR velocity vector in coordinate frame $\bar{\text{R}}$, equivalent to $(^{\bar{R}}v_{Rx} \quad ^{\bar{R}}v_{Ry} \quad ^{\bar{R}}\omega_R)^T$ or $(v_x \quad v_y \quad \omega)^T$
$\mathbf{v}_{\text{slip } i}$	Sliding velocity vector of the wheel in coordinate frame L_i (E_i for *Swedish* wheels)
$(\dot{\beta}_i, \dot{\varphi}_i)$	Angular velocity of the steering link and rotation velocity of the wheel in L_{xi}-axis
$\dot{\varphi}_{ri}$	Rotation velocity of the rollers in E_{xi}-axis (it is usually a free wheel velocity)
(r_i, r_{ri})	Wheel equivalent radius and roller radius

Table 1. Frames, variables and constants

$$\mathbf{v}_{\text{slip } i} = \begin{pmatrix} \cos(\beta_i + \gamma_i) & \sin(\beta_i + \gamma_i) & l_i \sin(\beta_i + \gamma_i - \alpha_i) & r_i \sin\gamma_i & 0 \\ -\sin(\beta_i + \gamma_i) & \cos(\beta_i + \gamma_i) & l_i \cos(\beta_i + \gamma_i - \alpha_i) & r_i \cos\gamma_i & r_{ri} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\varphi}_i \\ \dot{\varphi}_{ri} \end{pmatrix} \tag{3}$$

2.2 Kinematic models of wheeled mobile robots

Once the type of WMR wheels and their equations are established, a compound kinematic equation for the WMR may be defined. Using (1), (2), and (3) we can obtain:

$$\mathbf{v}_{\text{slip}} = \begin{pmatrix} \mathbf{v}_{\text{slip } 1} \\ \vdots \\ \mathbf{v}_{\text{slip } N} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{p1} & \mathbf{A}_{w1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}_{pN} & 0 & \cdots & \mathbf{A}_{wN} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{q}}_{w1} \\ \vdots \\ \dot{\mathbf{q}}_{wN} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_p & \mathbf{A}_w \end{pmatrix} \begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{q}}_w \end{pmatrix} = \mathbf{A} \, \dot{\mathbf{q}} \, , \tag{4}$$

where N is the number of wheels; $\mathbf{v}_{\text{slip}}$ is the composite sliding velocity vector; $\dot{\mathbf{q}}_{wi}$ is a vector with all the wheel velocities of wheel i; $\dot{\mathbf{q}}_w$ is the composite vector of all the wheel velocities; $\dot{\mathbf{q}}$ is the vector of all the velocities; $\{\mathbf{A}_{pi}, \mathbf{A}_{wi}\}$ are the multiplying matrices obtained from (1), (2), and (3); $\{\mathbf{A}_p, \mathbf{A}_w\}$ are the composite multiplying matrices; and $\mathbf{A}$ is the WMR kinematic matrix. Under the *no-slip condition*, the kinematic solution for velocity vector $\dot{\mathbf{q}}$ results in:

$$\mathbf{A} \cdot \dot{\mathbf{q}} = \mathbf{0} \tag{5}$$

$$\dot{\mathbf{q}} \in \mathcal{N}(\mathbf{A}) \rightarrow \dot{\mathbf{q}} = \mathbf{B} \cdot \boldsymbol{\eta}, \tag{6}$$

where matrix $\mathbf{B}$ forms a basis of $\mathcal{N}(\mathbf{A})$, $\boldsymbol{\eta}$ is an m-dimensional vector representing WMR mobility, and m is the WMR mobility degree given by the nullity of $\mathbf{A}$:

$$m = \dim(\boldsymbol{\eta}) = \dim(\mathcal{N}(\mathbf{A})) = \dim(\dot{\mathbf{q}}) - \operatorname{rank}(\mathbf{A}) = k - g. \tag{7}$$

In order to use variables with physical meaning, the mobility vector $\boldsymbol{\eta}$ should be replaced with a set of freely *assigned* velocities. Depending on whether wheel velocities or WMR velocities are chosen, a forward or inverse kinematic model is obtained. If a mix of both types of velocities is chosen a mixed solution is achieved. In order to check if an m-set of velocities $\dot{\mathbf{q}}_a$ can be assigned, it must be verified that the determinant of the submatrix they define in (5) is non zero, that is:

$$\begin{pmatrix} \dot{\mathbf{q}}_{na} \\ \dot{\mathbf{q}}_a \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{na} \\ \mathbf{B}_a \end{pmatrix} \cdot \boldsymbol{\eta} \tag{8}$$

$$\text{if } |\mathbf{B}_a| \neq 0 \rightarrow \dot{\mathbf{q}}_{na} = \mathbf{B}_{na} \cdot \mathbf{B}_a^{-1} \cdot \dot{\mathbf{q}}_a, \tag{9}$$

where $\dot{\mathbf{q}}_{na}$ are the remaining non-assigned velocities of $\dot{\mathbf{q}}$.

Alternatively to the previous procedure, based on the null space concept, it is possible to apply another method based on separating the m assigned velocities in (5):

$$\mathbf{A}_{na}\, \dot{\mathbf{q}}_{na} = -\mathbf{A}_a\, \dot{\mathbf{q}}_a. \tag{10}$$

To check if an m-set of velocities could be assigned $\dot{\mathbf{q}}_a$, it must be verified that matrix $\mathbf{A}_{na}$ is, in general, of full rank g:

$$\operatorname{rank}(\mathbf{A}_{na}) = \operatorname{rank}(\mathbf{A}) = g. \tag{11}$$

Therefore, the singularity of a kinematic model is given by $|\mathbf{B}_a| = 0$ in (9) or alternatively when matrix $\mathbf{A}_{na}$ in (10) loses its full rank g. In (Gracia & Tornero, 2007a) it is characterized the singularity of WMR with a generic geometric approach.

On the other hand, (Gracia & Tornero, 2007b) consider a kinematic solution with redundant information ($\dim(\dot{\mathbf{q}}_a) > m$) applying weighted left pseudoinverse to (10):

$$\dot{\mathbf{q}}_{na} = -\left(\mathbf{A}_{na}^{\mathrm{T}}\, \left(\sqrt{\boldsymbol{\mu}_{na}}\right)^{\mathrm{T}} \sqrt{\boldsymbol{\mu}_{na}}\, \mathbf{A}_{na} \right)^{-1} \mathbf{A}_{na}^{\mathrm{T}} \left(\sqrt{\boldsymbol{\mu}_{na}}\right)^{\mathrm{T}} \sqrt{\boldsymbol{\mu}_a}\, \mathbf{A}_a\, \dot{\mathbf{q}}_a. \tag{12}$$

where $\left(\sqrt{\boldsymbol{\mu}_{na}}, \sqrt{\boldsymbol{\mu}_a}\right)$ are the pre-multiplying weight matrices in (10) and, again, singularity arises when matrix $\mathbf{A}_{na}$ loses its full rank or equivalently when $|\mathbf{A}_{na}^{\mathrm{T}}\, \mathbf{A}_{na}| = 0$.

When singularity arises for an m-set of assigned velocities there are two approaches:
- *Loss of degrees of mobility:* in order to avoid incompatibility the assigned velocities are coordinated properly, what implies a loss of degrees of mobility.

- *Kinematics Incompatibility:* no type of coordination for the assigned velocities is considered, so the kinematic incompatibility is not solved. If the assigned velocities are wheel velocities (forward kinematics), slip (due to the incompatibility, not because of accelerations) is inevitable. If they are WMR velocities (inverse kinematics), impossible (infinite) control action values are obtained.

In the same way, the singularity of a redundant forward kinematics (12) would produce an infinite error in the estimation of the WMR velocity vector. Therefore, it is obtained the following criterion: *singularity* (i.e. mobility degree loss, slip, impossible control actions, or infinite error in the estimation) *has to be avoided*. Moreover, *nearness to singularity is neither desirable* since it implies: high amplification of wheel velocities' error (redundant and non-redundant forward kinematics) or high values for wheel velocities (non-redundant inverse kinematics). If the singularity depends on the steering angles of *orientable* or *castor* wheels the previous criterion is a *planning criterion*, i.e. the upper level planner (path generator) has to develop paths not close to singularities, otherwise it becomes *design criterion*.

3. Motion Planning

Given a starting and ending configuration of a given WMR, a motion planning problem consists of automatically computing a collision-free path. This gives rise to the famous *piano mover problem*, i.e. any solution appears as a path in the admissible (i.e. collision-free) *configuration space*. Many papers have proposed general, exact, approximate, efficient … methods in order to represent and explore this admissible configuration space: e.g. cellular decomposition, polygon representation, etc. (see (Latombe, 1991) for a synthesis of these approaches). One classical approach is based on *tree graphs* whose leafs are the WMR posture and whose branches are the paths from one posture to another. Then, the planner checks, during the construction of the tree graph, if the goal has been achieved. In order to avoid the high computational cost of the tree-graph method, it was developed the *roadmap* technique that builds a graph whose nodes are collision-free configurations and whose edges denote the presence of collision-free paths between two configurations. The roadmaps tend to capture both the coverage and connectivity of the configuration space and replace the concept of deterministic completeness by the concept of probabilistic completeness.

However, numerous classical methods work only when the WMR is holonomic and not when there is some *non-holonomic constraint* between its configuration parameters. In order to overcome this, in (Laumond et al., 1997) it is developed a planner that firstly generates a collision-free path ignoring the non-holonomic constraints and afterwards the path is transformed into one that is feasible with respect to these constraints.

On the other hand, other planners are *specific for one task*, e.g. in (Gracia & Tornero, 2003) it is presented a planner for parallel parking based on a geometric characterization for collision avoidance. Moreover, other types of approaches do not *explicitly* generate collision-free paths; instead, they integrate the WMR motion planning with the WMR control using tools like *fuzzy*, neural networks, reactive architecture, etc. For example, in (Borenstein & Koren, 1989) it is used artificial potential fields: the WMR is attracted by the objective configuration and repelled by the obstacles. If a time value is associated to each point of the path it becomes a trajectory; otherwise, a forward constant velocity is usually used across the path.

3.1 Proposed cost index

This research introduces a *cost index* based on kinematics singularity that is useful for many types of planners (based on tree graphs, roadmaps, etc.), since it allows to choose the path with minimum cost index among several possible collision-free paths. In the cost index it will be weighted the nearness to singularity of forward and inverse kinematics. This will allow avoiding singularity and nearness to singularity, i.e. high amplification of the WMR velocities' error or high values for wheel velocities. Similarly to robotic manipulators, the singularity of *inverse kinematic models* can be deal with a null velocity on the path at the singularity point, which is equivalent to a loss of degrees of mobility. It implies to stop the WMR in order to reorientate it and/or its wheels, as it is pointed out in (Gracia & Tornero, 2008) for the five types of WMR classified according to (Campion et al., 1996). This may be appropriate when there is not much space available (e.g. for parking maneuvers) but not in a general case, since it involves an important waste of time. Therefore, this option will not be considered here. The nearness to singularity of *forward kinematic models* produces high amplification of the WMR velocities' error, what implies a tracking error if the assigned wheel velocities are actuated wheel velocities or an estimation error if they are sensed wheel velocities. Both types of forward models will be considered in the cost index. Therefore, it is proposed the following *cost index*:

$$J = \sum_{i=1}^{N} \left(\frac{1}{f_1\left(\left|\mathbf{B}_a\right|_{\text{inv}\,i}\right)} + \frac{f_2(N-i+1)}{f_3\left(\left|\mathbf{B}_a\right|_{\text{fwd act}\,i}\right)} + \frac{f_4(N-i+1)}{f_5\left(\left|\mathbf{A}_{na}^{T}\,\mathbf{A}_{na}\right|_{\text{fwd sensed}\,i}\right)} + f_6\left(\dot{\boldsymbol{\beta}}_{o\,i}\right) \right) + f_7(D), \quad (13)$$

where N is the number of branches/edges of the path in the tree-graph/roadmap; f_i is a generic non-linear function; $\left|\mathbf{B}_a\right|_{\text{inv}\,i}$ is the singularity of the inverse kinematic model; $\left|\mathbf{B}_a\right|_{\text{fwd act}\,i}$ is de singularity of the forward models with actuated wheel velocities as assigned; $\left|\mathbf{A}_{na}^{T}\,\mathbf{A}_{na}\right|_{\text{fwd sensed}\,i}$ is de singularity of the forward model with redundant sensed wheel velocities; $\dot{\boldsymbol{\beta}}_{o\,i}$ is the steering velocity vector of all the *orientable* wheels; and D is the length or distance of the collision-free path. Note that, the singularity of forward models has been multiplied by f_j (N – i +1) since the tracking/estimation error of the initial branches/edges is more important because it is propagated across the whole path. However, in order to limit the uncertainty of the estimation other global or local position sensors are required. Note also that, it has been introduced the steering velocities of the *orientable* wheels because they are not present in the velocity vector $\dot{\mathbf{q}}$, see (2).

3.2 Example of motion planning

The cost index of previous subsection will be obtained for the case of the industrial forklift of Fig. 3, which is equivalent to the tricycle WMR, where the origin of R (tracking point) has been located at the middle point of the *fixed* wheels. The traction of this industrial forklift is given by both *fixed* wheels, which are *properly coordinated* through a differential mechanism depending on the steering angle of the *orientable* wheel. Moreover, this WMR has three encoders measuring the rotation of both *fixed* wheels and the steering angle of the *orientable* wheel.
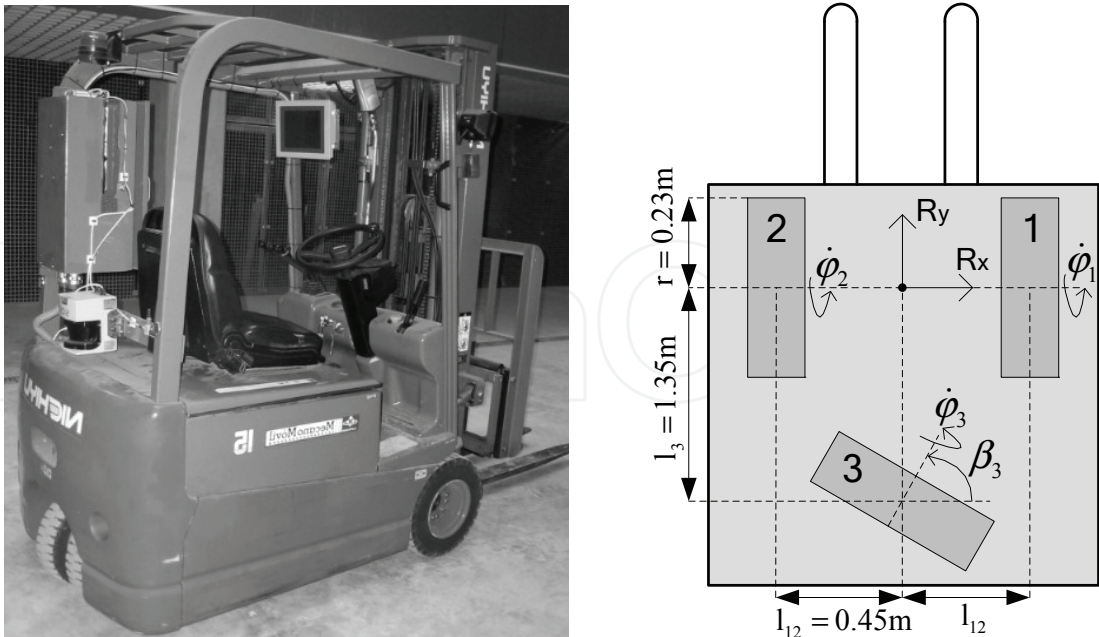
Figure 3. Industrial forklift Nichiyu FBT15 series 65 and equivalent tricycle representation

The composite equation (4) of this WMR results:

$$
\mathbf{v}_{slip} = \begin{pmatrix} \mathbf{v}_{slip\,1} \\ \mathbf{v}_{slip\,2} \\ \mathbf{v}_{slip\,3} \end{pmatrix} = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & l_{12} & r & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -l_{12} & 0 & r & 0 \\ \cos\beta_3 & \sin\beta_3 & l_3 \cos\beta_3 & 0 & 0 & 0 \\ -\sin\beta_3 & \cos\beta_3 & -l_3 \sin\beta_3 & 0 & 0 & r \end{array} \right) \begin{pmatrix} {}^{\bar{R}}\dot{\mathbf{p}} \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{pmatrix}. \tag{14}
$$

Under the no-slip condition, a kinematic solution (8) is:

$$
\begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -l_3 \cos\beta_3 \\ \sin\beta_3 \\ (L\sin\beta_3 - l_3 \cos\beta_3)/r \\ (L\sin\beta_3 + l_3 \cos\beta_3)/r \\ l_3/r \end{pmatrix} \eta. \tag{15}
$$

For the redundant forward kinematics, (10) is particularized to:

$$
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & l_{12} \\ 0 & 1 & -l_{12} \\ \cos\beta_3 & \sin\beta_3 & l_3 \cos\beta_3 \end{pmatrix} \dot{\mathbf{p}} = - \begin{pmatrix} 0 & 0 \\ r & 0 \\ 0 & r \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix} \rightarrow \mathbf{A}_{na\,p}\, \dot{\mathbf{p}} = - \begin{pmatrix} 0 & 0 \\ r & 0 \\ 0 & r \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix}. \tag{16}
$$

where it has been considered together the first and third equation of (14), and the last equation (used only to compute $\dot{\varphi}_3$) has been obviated. Therefore, the kinematics singularity is given by:

$$\eta = v_y \rightarrow \left| \mathbf{B}_a \right|_{\text{inv}} = -1_3\cos\beta_3 = 0 \rightarrow \beta_3 = \pm 90^\circ$$

$$\eta = \dot{\varphi}_1 \rightarrow \left| \mathbf{B}_a \right|_{\text{fwd act1}} = (1_{12}\sin\beta_3 - 1_3\cos\beta_3)/r = 0 \rightarrow \beta_3 = \begin{cases} \text{atan}(1_3/1_{12}) \\ \text{atan}(1_3/1_{12})+180^\circ \end{cases}$$

$$\eta = \dot{\varphi}_2 \rightarrow \left| \mathbf{B}_a \right|_{\text{fwd act2}} = (1_{12}\sin\beta_3 + 1_3\cos\beta_3)/r = 0 \rightarrow \beta_3 = \begin{cases} -\text{atan}(1_3/1_{12}) \\ -\text{atan}(1_3/1_{12})+180^\circ \end{cases}$$

$$\dot{\mathbf{q}}_a = (\dot{\varphi}_1, \dot{\varphi}_2) \rightarrow \left| \mathbf{A}_{\text{na p}}^{\text{T}} \, \mathbf{A}_{\text{na p}} \right|_{\text{fwd sensed}} = 0 \rightarrow \text{No solution, never singular.}$$

(17)

The cost index (13) will be particularized to:

$$J = \sum_{i=1}^{N} \left(\frac{\text{K}_1}{\left| \mathbf{B}_a \right|_{\text{inv}\,i}^2} + \frac{\text{K}_2\,(N-i+1)}{\max\left(\left| \mathbf{B}_a \right|_{\text{fwd act1}\,i}^2, \text{M} \right)} + \frac{\text{K}_3\,(N-i+1)}{\max\left(\left| \mathbf{B}_a \right|_{\text{fwd act2}\,i}^2, \text{M} \right)} + \frac{\text{K}_4\,(N-i+1)}{\left| A_{\text{na p}}^{\text{T}} A_{\text{na p}} \right|_{\text{fwd sensed}\,i}^2} \right) + \\ + \sum_{i=1}^{N} \left(\text{K}_5 \left(\dot{\beta}_{3\,i} \right)^2 \right) + D,$$

(18)

where K_j is the weight of each term in the cost index and M is a kind of singularity saturation in order to not reduce in excess the WMR maneuverability. Note that the industrial forklift has one degree of mobility ($m = 1$), i.e. one instantaneous degree of freedom, that allows to specify a forward tracking velocity v_y. It has another non-instantaneous degree of freedom through the angle β_3 of the *orientable* wheel that allows turning. Therefore, this WMR can track 2-dimensional paths. In order to obtain simulation results, it will be considered the *tree graph technique* together with the previous cost index. It will be used a constant forward velocity on the path, e.g. $v_y = 1$ m/s, and the following motion equations between *leaves*/samples:

$$x_{k+1} = x_k + (v_y/\omega)(\sin(\theta_k + \omega\text{T}) - \sin\theta_k)$$
$$y_{k+1} = y_k - (v_y/\omega)(\cos(\theta_k + \omega\text{T}) - \cos\theta_k) \qquad \theta_{k+1} = \theta_k + \omega\text{T},$$

(19)

where T is the sample time, and it has been considered a constant forward motion v_y and a constant turning motion ω between samples. If the WMR angular velocity ω is null, it must be used the following equations:

$$x_{k+1} = x_k + v_y\text{T}\cos\theta_k \qquad y_{k+1} = y_k + v_y\text{T}\sin\theta_k \qquad \theta_{k+1} = \theta_k.$$

(20)

Note that the distance D of each collision-free path results $v_y \cdot \text{T} \cdot N$. For the construction of the tree graph it will be considered three possible steering velocities for the *orientable* wheel: $\{-\dot{\beta}_{3\,\text{max}}, 0, \dot{\beta}_{3\,\text{max}}\}$. During the construction of the tree graph it will be verified if the goal has been achieved within a tolerance. The parameters used for the simulations results of Fig. 4 are: $v_y = 1$ m/s, T = 0.5 s, N = 22, $\dot{\beta}_{3\,\text{max}} = 0.4$ rad/s, M = 0.01, $\text{K}_1 = \text{K}_5 = 18$, $\text{K}_2 = \text{K}_3 = \text{K}_4 = 3$; and it has been considered two rectangular obstacles that represent two warehouse shelves. The goal WMR posture **p** in the three examples of Fig. 4 are: (5.5, 0, 0); (2, 0, 180°); and (5, 0, any) respectively. The continuous thick line is the path with minimum cost index; the dashed thick line is the path with minimum distance; and the continuous thin lines are *some* (a sample) of the collision-free paths.
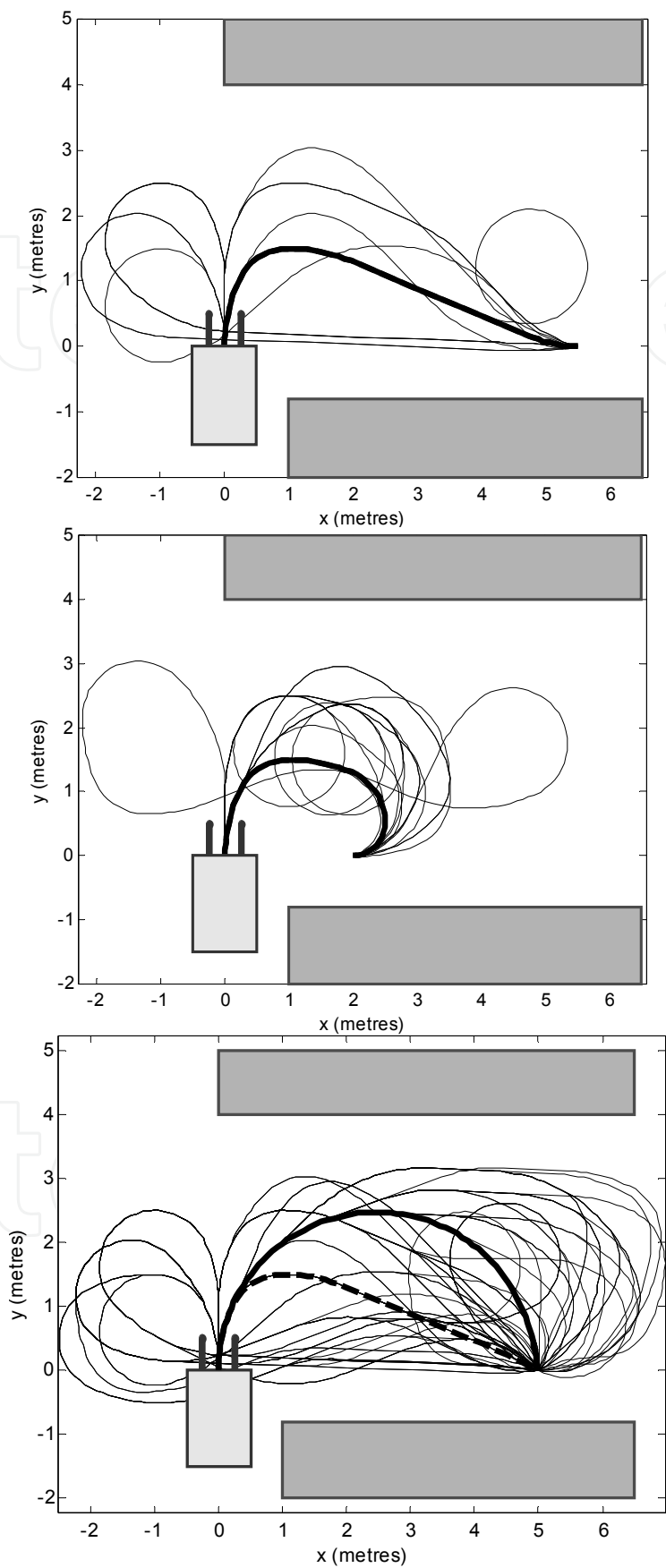
Figure 4. Simulation examples for the industrial forklift in a warehouse environment

4. Conclusion

In the previous work (Gracia & Tornero, 2007a) the authors had characterized the singularity of WMR kinematics. In this chapter it has been shown how to use WMR singularity or nearness to WMR singularity for motion planning. In particular, it has been proposed a cost index that assesses the nearness to singularity of forward and inverse kinematic models. This cost index can be used straightforward for many planning techniques (tree graphs, roadmaps, etc.) in order to choose one path among several possible collision-free paths. Therefore, the chosen path would avoid not only slip and impossible control actions (i.e. the singularity of forward and inverse kinematic models) but also high amplification of wheel velocities' error and high values for wheel velocities (i.e. the nearness to the singularity of forward and inverse kinematic models). To illustrate the applications of the proposed approach it has been considered an industrial forklift that is equivalent to the tricycle WMR. Finally, several results have been shown for this WMR in a simulated environment. It is suggested as further work to integrate the presented motion planning with other classical techniques like artificial potential fields, fuzzy planners, etc.

5. Acknowledgement

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Motion Planning

Edited by Xing-Jian Jing

In this book, new results or developments from different research backgrounds and application fields are put together to provide a wide and useful viewpoint on these headed research problems mentioned above, focused on the motion planning problem of mobile ro-bots. These results cover a large range of the problems that are frequently encountered in the motion planning of mobile robots both in theoretical methods and practical applications including obstacle avoidance methods, navigation and localization techniques, environmental modelling or map building methods, and vision signal processing etc. Different methods such as potential fields, reactive behaviours, neural-fuzzy based methods, motion control methods and so on are studied. Through this book and its references, the reader will definitely be able to get a thorough overview on the current research results for this specific topic in robotics. The book is intended for the readers who are interested and active in the field of robotics and especially for those who want to study and develop their own methods in motion/path planning or control for an intelligent robotic system.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

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