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# Robust and Active Trajectory Tracking for an Autonomous Helicopter under Wind Gust

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## 1. Introduction

High levels of agility, maneuverability and the capability of operating in degraded visual environments and adverse weather conditions are the new trends of helicopter design nowadays. Helicopter flight control system should make these performance requirements achievable by improving tracking performance and disturbance rejection capability. Robustness is one of the critical issues which must be considered in the control system design for such highperformance autonomous helicopter, since any mathematical helicopter model, especially those covering large flight envelope, will unavoidably have uncertainty due to the empirical representation of aerodynamic forces and moments.

The purpose of this chapter is to present the stabilization (tracking) with motion planning of a reduced-order helicopter model having 3DOF (Degrees Of Freedom) (see Fig.1). This last one represents a scale model helicopter mounted on an experimental platform. It deals with the problem of disturbance reconstruction acting on the autonomous helicopter, the disturbance consists in vertical wind gusts. The objective is to compensate these disturbances and to improve the performances of the control. Consequently, a nonlinear simple model with 3DOF of a helicopter with unknown disturbances is used. Three approaches of robust control are then compared via simulations: a robust nonlinear feedback control, an active disturbance rejection control based on a nonlinear extended state observer and a backstepping control.

Design of control of autonomous flying systems has now become a very challenging area of research, as shown by a large literature (Beji & Abichou, 2005) (Frazzoli et al., 2000) (Koo & Sastry, 1998). Many previous works focus on (linear and nonlinear, robust, ...) control, including a particular attention on the analysis of the stability (Mahony & Hamel, 2004), but very few works have been made on the influence of wind gusts acting on the flying system, whereas it is a crucial problem for out-door applications, especially in urban environment: as a matter of fact, if the autonomous flying system (especially when this system is relatively slight) crosses a crossroads, it can be disturbed by wind gusts and leave its trajectory, which could be critical in a highly dense urban context.

In (Martini et al., 2005) and (Martini et al., 2007a), three controllers (nonlinear,  $H_\infty$  and robust nonlinear feedback) are designed for a nonlinear reduced-order model of a 3 DOF helicopter. In (Pflimlin et al., 2004), a control strategy stabilizes the position of the flying vehicle in wind gusts environment, in spite of unknown aerodynamic efforts and is based on robust backstepping approach and estimation of the unknown aerodynamic efforts.

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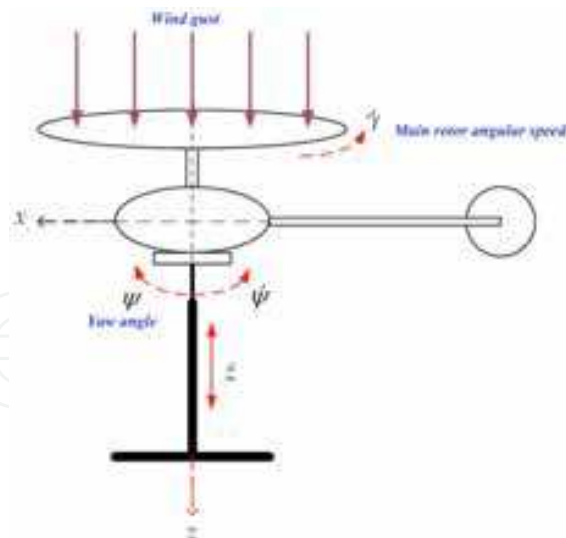


Fig. 1. Helicopter-platform with wind gust

In recent papers, feedback linearization techniques have been applied to helicopter models. The main difficulty in the application of such an approach is the fact that, for any meaningful selection of outputs, the helicopter dynamics are non-minimum phase, and hence are not directly input-output linearizable. However, it is possible to find good approximations to the helicopter dynamics (Koo & Sastry, 1998) such that the approximate system is input-output linearizable, and bounded tracking can be achieved.

Nonlinear control designs previously attempted include neural network based controllers (McLean & Matsuda, 1998), fuzzy control (Sanders et al., 1998), backstepping designs (Mahony & Hamel, 2004), and adaptive control (Dzul et al., 2004). These methods either assume feedback linearizability, which in turn restricts the motion to be around hover, or do not include parametric uncertainties, or realistic aerodynamics. Specific issues such as unknown trim conditions that degrade the performance of the helicopter have not been addressed. While adaptive control schemes have been proposed in the aircraft and spacecraft control context, there is a lack of similar work on helicopter control. The non-minimum phase nature of the helicopter dynamics adds to the challenge of finding a stable adaptive controller. (Wei, 2001) showed the control of nonlinear systems with unknown disturbances, using a disturbance observer based control (DOBC). In (Ifassiouen et al., 2007) a robust sliding mode control structure is designed using the exact feedback linearization procedure of the dynamic of a small-size autonomous helicopter in hover.

This chapter is organized as follows. In section 2, a 3DOF Lagrangian model of the disturbed helicopter mounted on an experimental platform is presented. This model can be seen as made of two subsystems (translation and rotation). In section 3 two approaches of robust control design for the reduced order model are proposed. The application of three approaches of robust control on our disturbed helicopter is analyzed in section 4. Section 6 is devoted to simulation results and the study of model stability is carried out in section 5. Finally some conclusions are presented in Section 7.

## 2. Model of the disturbed helicopter

Helicopters operate in an environment where task performance can easily be affected by atmospheric turbulence. This chapter discusses the airborne flight test of the *VARIO Benzin*

*Trainer* helicopter in turbulent conditions to determine disturbance rejection criteria and to develop a low speed turbulence model for an autonomous helicopter simulation. A simple approach to modeling the aircraft response to turbulence is described by using an identified model of the *VARIO Benzin Trainer* to extract representative control inputs that replicate the aircraft response to disturbances. This parametric turbulence model is designed to be scaled for varying levels of turbulence and utilized in ground or in-flight simulation. Hereafter the nonlinear model of the disturbed helicopter (Martini et al., 2005) starting from a non disturbed model (Vilchis, 2001) is presented. The *Vario* helicopter is mounted on an experimental platform and submitted to a vertical wind gust (see Fig.1). It can be noted that the helicopter is in an Out Ground Effect (OGE) condition. The effects of the compressed air in take-off and landing are then neglected. The Lagrange equation, which describes the system of the helicopter-platform with the disturbance, is given by:

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = Q(q, \dot{q}, u, v_{raf}) \quad (1)$$

where the input vector of the control  $u = [u_1 \ u_2]^T$  and  $q = [z \ \psi \ \gamma]^T$  is the vector of generalized coordinates. The first control  $u_1$  is the collective pitch angle (swashplate displacement) of the main rotor. The second control input  $u_2$  is the collective pitch angle (swashplate displacement) of the tail rotor. The induced gust velocity is noted  $v_{raf}$ . The helicopter altitude is noted  $z$ ,  $\psi$  is the yaw angle and  $\gamma$  is the main rotor azimuth angle.  $M \in R^{3 \times 3}$  is the inertia matrix,  $C \in R^{3 \times 3}$  is the Coriolis and centrifugal forces matrix,  $G \in R^3$  represents the vector of conservative forces,  $Q(q, \dot{q}, u, v_{raf}) = [f_z \ \tau_z \ \tau_\gamma]^T$  is the vector of generalized forces. The variables  $f_z$ ,  $\tau_z$  and  $\tau_\gamma$  represent respectively, the total vertical force, the yaw torque and the main rotor torque in presence of wind gust.

Finally, the representation of the reduced system of the helicopter, which is subjected to a wind gust, can be expressed as (Martini et al., 2005) :

$$\begin{aligned} \dot{x}_1 &= x_2 = \dot{z}, \\ \dot{x}_2 &= \frac{1}{c_0} [c_8 \dot{\gamma}^2 u_1 + c_9 \dot{\gamma} + c_{10} - c_7] + \frac{1}{c_0} c_{16} \dot{\gamma} v_{raf}, \\ \dot{x}_3 &= x_4 = \dot{\psi}, \\ \dot{x}_4 &= \frac{1}{c_1 c_5 - c_4^2} [c_5 c_{11} \dot{\gamma}^2 u_2 - c_4 (c_{14} \dot{\gamma}^2 + c_{15})] \\ &\quad - \frac{c_4}{c_1 c_5 - c_4^2} [(c_{12} \dot{\gamma} + c_{13} + c_8 \dot{\gamma} v_{raf}) u_1 + (\frac{5}{2} c_9 + c_{17} v_{raf}) v_{raf}], \\ \dot{x}_5 &= x_6 = \dot{\gamma}, \\ \dot{x}_6 &= \frac{c_4}{c_1 c_5 - c_4^2} [c_{11} \dot{\gamma}^2 u_2 + c_1 (c_{14} \dot{\gamma}^2 + c_{15})] \\ &\quad + \frac{1}{c_1 c_5 - c_4^2} [c_1 c_4 (c_{12} \dot{\gamma} + c_{13} + c_8 \dot{\gamma} v_{raf}) u_1 + (\frac{5}{2} c_9 + c_{17} v_{raf}) v_{raf}]. \end{aligned} \quad (2)$$

where  $c_i$  ( $i = 0, \dots, 17$ ) are numerical aerodynamical constants of the model given in table 1 (Vilchis, 2001). For example  $c_0$  represents the helicopter weight,  $c_{15} = 2ka_{1s}b_{1s}$  where  $a_{1s}$  and  $b_{1s}$  are the longitudinal and lateral flapping angles of the main rotor blades,  $k$  is the blades stiffness of main rotor.

Table 2 shows the variations of the main rotor thrust and of the main rotor drag torque (variations of the helicopter parameters) operating on the helicopter due to the presence of wind gust. These variations are calculated from a nominal position defined as the equilibrium of helicopter when  $v_{raf} = 0$ :  $\dot{\gamma} = -124.63 \text{ rad/s}$ ,  $u_1 = -4.588 \times 10^{-5}$ ,  $u_2 = 5 \times 10^{-7}$ ,  $T_{M_0} = -77.3 \text{ N}$  and  $C_{M_0} = 4.6 \text{ N.m}$ .

$c_0$	7.5 kg	$c_9$	0.6004 kg m/s
$c_1$	0.4305 kg m <sup>2</sup>	$c_{10}$	3.679 N
$c_2$	$3 \times 10^{-4}$ kg m <sup>2</sup>	$c_{11}$	-0.1525 kg m
$c_3$	-4.143	$c_{12}$	12.01 kg m/s
$c_4$	0.108 kg m <sup>2</sup>	$c_{13}$	$1 \times 10^5$ N
$c_5$	0.4993 kg m <sup>2</sup>	$c_{14}$	$1.206 \times 10^{-4}$ kg m <sup>2</sup>
$c_6$	$-6.214 \times 10^{-4}$ kg m <sup>2</sup>	$c_{15}$	2.642 N
$c_7$	-73.58 N	$c_{16}$	0.1706 kg
$c_8$	3.411 kg	$c_{17}$	0.2132 N s <sup>2</sup> /m

Table 1. 3DOF model parameters

$v_{raf}(m/s)$	$\delta T_M(N)$	$\frac{\delta T_M}{T_{M_o}} \%$	$\delta C_M(N.m)$	$\frac{\delta C_M}{C_{M_o}} \%$
0.68	-14.5	19 %	1.1	25 %
3	-64	83 %	6.5	141 %

Table 2. Variation of forces and torques for different wind gusts

Three robust nonlinear controls adapted to wind gust rejection are now introduced in section 4.1, 4.2 and 4.3 devoted to control design of disturbed helicopter.

### 3. Control design

#### 3.1 Robust feedback control

Fig.2 shows the configuration of this control (Spong & Vidyasagar, 1989) based on the inverse dynamics of the following mechanical system:

$$M(q)\ddot{q} + h(q, \dot{q}) = u. \quad (3)$$

Since the inertia matrix  $M$  is invertible, the control  $u$  is chosen as follows:

$$u = M(q)v + h(q, \dot{q}). \quad (4)$$

The term  $v$  represents a new input to the system. Then the combined system (3-4) reduces to:

$$\ddot{q} = v. \quad (5)$$

Equation (5) is known as the double integrator system. The nonlinear control law (4) is called the inverse dynamics control and achieves a rather remarkable result, namely that the new system (5) is linear, and decoupled.

$$u = \widehat{M}(q)v + \widehat{h}(q, \dot{q}), \quad (6)$$

where  $\widehat{M}, \widehat{h}$  represent nominal values of  $M, h$  respectively. The uncertainty or modeling error, is represented by:  $\Delta M = \widehat{M}(q) - M(q); \Delta h = \widehat{h}(q, \dot{q}) - h(q, \dot{q})$ , with system equation (3) and nonlinear law (6), the system becomes:

$$\widehat{M}v + \widehat{h} = M\ddot{q} + h. \quad (7)$$

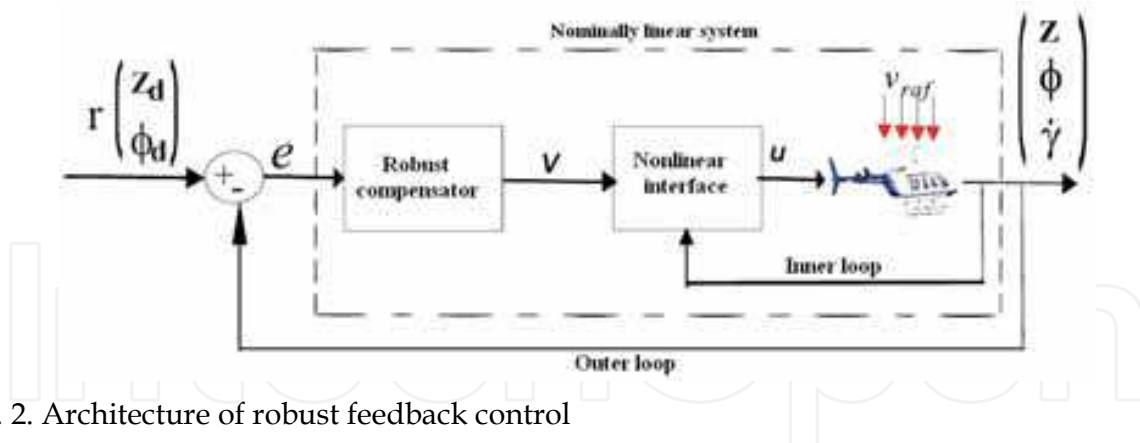


Fig. 2. Architecture of robust feedback control

Thus  $\ddot{q}$  can be expressed as

$$\ddot{q} = M^{-1}\widehat{M}v + M^{-1}\Delta h = v + (M^{-1}\widehat{M} - I)v + M^{-1}\Delta h. \quad (8)$$

Defining  $E = M^{-1}\widehat{M} - I$  and  $\eta = Ev + M^{-1}\Delta h$ , then in state space the system (8) becomes:

$$\dot{\zeta} = A\zeta + B(v + \eta), \quad (9)$$

where:  $A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$ ,  $\zeta = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ .

Using the error vectors  $e = [e_1 \ e_2]^T$ ,  $e_1 = q - q_d$  and  $e_2 = \dot{q} - \dot{q}_d$ , leads to:

$$\dot{e} = Ae + B\{v + \eta - \ddot{q}_d\}. \quad (10)$$

Therefore the problem of tracking the desired trajectory  $q_d(t)$  becomes one of stabilizing the (time-varying, nonlinear) system (10). The control design to follow is based on the premise that although the uncertainty  $\eta$  is unknown, it may be possible to estimate "worst case" bounds and its effects on the tracking performance of the system. In order to estimate a worst case bound on the function  $\eta$ , the following assumptions can be used (Spong & Vidyasagar, 1989):

- Assumption 1:  $\sup_{t \geq 0} \|\ddot{q}^d\| < Q_1 < \infty$ .
- Assumption 2:  $\|E\| = \|M^{-1}\widehat{M} - I\| \leq \alpha < 1$  for some  $\alpha$ , and for all  $q \in R^n$ .
- Assumption 3:  $\|\Delta h\| \leq \psi(e, t)$  for a known function  $\psi$ , bounded in  $t$ .

Assumption 2 is the most restrictive and shows how accurately the inertia of the system must be estimated in order to use this approach. It turns out, however, that there is always a simple choice for  $\widehat{M}$  satisfying Assumption 2. Since the inertia matrix  $M(q)$  is uniformly positive definite for all  $q$  there exist positive constants  $\underline{M}$  and  $\overline{M}$  such that:

$$\underline{M} \leq \|M^{-1}(q)\| \leq \overline{M} \quad \forall q \in R^n. \quad (11)$$

If we therefore choose:  $\widehat{M} = \frac{1}{c}I$  where  $c = \frac{\overline{M} + \underline{M}}{2}$ , it can be shown that:  $\|M^{-1}\widehat{M} - I\| \leq \frac{\overline{M} - \underline{M}}{\overline{M} + \underline{M}} = \alpha < 1$ . Finally, the following algorithm may now be used to generate a stabilizing control  $v$ :

Step 1 : Since the matrix  $A$  in (9) is unstable, we first set:



$$v = \ddot{q}_d - K_1 e_1 - K_2 e_2 + \Delta v, \quad (12)$$

where  $K = [K_1 \ K_2]$ ; and  $K_1 = \text{diag}\{\omega_1^2, \dots, \omega_n^2\}$ ,  $K_2 = \text{diag}\{2\zeta_1\omega_1, \dots, 2\zeta_n\omega_n\}$ . The desired trajectory  $q_d(t)$  and the additional term  $\Delta v$  will be used to attenuate the effects of the uncertainty and the disturbance. Then we have:

$$\dot{e} = \bar{A}e + B\{\Delta v + \bar{\eta}\}, \quad (13)$$

where  $\bar{A} = A - BK$  is Hurwitz and  $\bar{\eta} = E\Delta v + E(\ddot{q}_d - Ke) + M^{-1}\Delta h$ .

Step 2: Given the system (13), suppose we can find a continuous function  $\rho(e, t)$ , which is bounded in  $t$ , satisfying the inequalities:

$$\|\Delta v\| < \rho(e, t), \quad \|\bar{\eta}\| < \rho(e, t). \quad (14)$$

The function  $\rho$  can be defined implicitly as follows. Using Assumptions 1-3 and (14), we have the estimate:

$$\begin{aligned} \|\bar{\eta}\| &\leq \|E\Delta v + E\ddot{q}_d - EKe + M^{-1}\Delta h\| \\ &\leq \alpha\rho(e, t) + \alpha Q_1 + \|K\| \|e\| + \bar{M}\psi(e, t) = \rho(e, t). \end{aligned} \quad (15)$$

This definition of  $\rho$  makes sense since  $0 < \alpha < 1$  and we may solve for  $\rho$  as:

$$\rho(e, t) = \frac{1}{1 - \alpha} \{\alpha Q_1 + \|K\| \|e\| + \bar{M}\psi(e, t)\}. \quad (16)$$

Note that whatever  $\Delta v$  is now chosen must satisfy (14).

Step 3: Since  $A$  is Hurwitz, choose a  $n \times n$  symmetric, positive definite matrix  $Q$  and let  $P$  be the unique positive definite symmetric solution to the Lyapunov equation:

$$\bar{A}^T P + P\bar{A} + Q = 0. \quad (17)$$

Step 4: Choose the outer loop control  $\Delta v$  according to:

$$\Delta v = \begin{cases} -\rho(e, t) \frac{B^T P e}{\|B^T P e\|} & \text{if } \|B^T P e\| \neq 0, \\ 0 & \text{if } \|B^T P e\| = 0, \end{cases} \quad (18)$$

that satisfy (14). Such a control will enable us to remove the principal influence of the wind gust.

### 3.2 Active disturbance rejection control

The primary reason to use the control in closed loop is that it can treat the variations and uncertainties of model dynamics and the outside unknown forces which exert influences on the behavior of the model. In this work, a methodology of generic design is proposed to treat the combination of two quantities, denoted as disturbance. A second order system described by the following equation is considered (Gao et al., 2001) (Hou et al., 2001):

$$\ddot{y} = f(y, \dot{y}, p) + bu, \quad (19)$$

where  $f(\cdot)$  represents the dynamics of the model and the disturbance,  $p$  is the input of unknown disturbance,  $u$  is the input of control, and  $y$  is the measured output. It is assumed that the value of the parameter  $b$  is given. Here  $f(\cdot)$  is a nonlinear function. An alternative method is presented by (Han, 1999) as follows. The system in (19) is initially increased:

$$\dot{x}_1 = x_2; \quad \dot{x}_2 = x_3 + bu; \quad \dot{x}_3 = \dot{f} \quad (20)$$

where  $x_1 = y$ ,  $x_2 = \dot{y}$ ,  $x_3 = f(y, \dot{y}, p)$ .  $f(\cdot)$  is treated as an increased state. Here  $f$  and  $\dot{f}$  are unknown. By considering  $f(y, \dot{y}, p)$  as a state, it can be estimated with a state estimator. Han in Han (1999) proposed a nonlinear observer for (20):

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + Lg(e, \alpha, \delta), \\ \hat{y} &= C\hat{x}, \end{aligned} \quad (21)$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, C = [1 \ 0 \ 0], L = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}. \quad (22)$$

The observer error is  $e = y - \hat{x}_1$  and:

$$g_i(e, \alpha_i, \delta)_{i=1,2,3} = \begin{cases} |e|^{\alpha_i} \text{sign}(e) & |e| > \delta \\ \frac{e}{\delta^{1-\alpha_i}} & |e| \leq \delta \end{cases} \quad \delta > 0. \quad (23)$$

The observer is reduced to the following set of state equations, and is called extended state observer (ESO):

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1 g_1(e, \alpha, \delta), \\ \dot{\hat{x}}_2 = \hat{x}_3 + L_2 g_2(e, \alpha, \delta) + bu, \\ \dot{\hat{x}}_3 = L_3 g_3(e, \alpha, \delta). \end{cases} \quad (24)$$

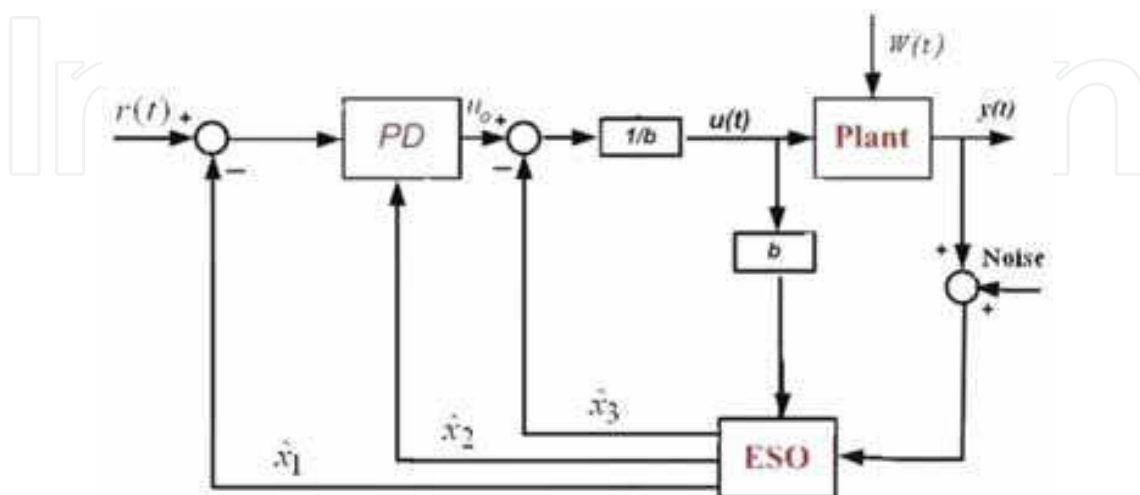


Fig. 3. ADRC structure



The active disturbance rejection control (ADRC) is then defined as a method of control where the value of  $f(y, \dot{y}, p)$  is estimated in real time and is compensated by the control signal  $u$ . Since  $\hat{x}_3 \rightarrow f$ , it is used to cancel actively  $f$  by the application of:  $u = (u_0 - \hat{x}_3)/b$ . This expression reduces the system to:  $\ddot{y} = (f - \hat{x}_3) + u_0 \approx u_0$ . The process is now a double integrator with a unity gain, which can be controlled with a PD controller.  $u_0 = k_p(r - \hat{x}_1) - k_d\dot{\hat{x}}_2$ , where  $r$  is the reference input. The observer gains  $L_i$  and the controller gains  $k_p$  and  $k_d$  can be calculated by a pole placement. The configuration of ADRC is presented in fig.3.

## 4. Control of disturbed helicopter

### 4.1 Robust feedback control

#### 4.1.1 Control of altitude $z$

We apply this robust method to control the altitude dynamics  $z$  of our helicopter. Let us remain the equation which describes the altitude under the effect of a wind gust:

$$\ddot{z} = \frac{1}{c_0} [c_8 \dot{\gamma}^2 u_1 + c_9 \dot{\gamma} + c_{10} - c_7] + \frac{1}{c_0} c_{16} \dot{\gamma} v_{raf}, \quad (25)$$

with  $u_1 = M_1(q)\ddot{z} + h_1(q, \dot{q}, v_{raf})$  where  $M_1(q) = \frac{c_0}{c_8 \dot{\gamma}^2}$  and  $h_1(q, \dot{q}, v_{raf}) = -\frac{1}{c_8 \dot{\gamma}^2} [c_9 \dot{\gamma} + c_{10} - c_7 + c_{16} \dot{\gamma} v_{raf}]$ .

Moreover:  $\underline{\dot{\gamma}} \leq \dot{\gamma} \leq \overline{\dot{\gamma}}$  and  $\underline{v_{raf}} \leq v_{raf} \leq \overline{v_{raf}}$ . It is noted that  $\dot{\gamma}_{trim} = -124,63 \text{rad/s}$  and:

$$\underline{\dot{\gamma}} = -209.4 \text{rad/s} \quad \overline{\dot{\gamma}} = -99.5 \text{rad/s}; \quad \underline{v_{raf}} = -0.68 \text{m/s}; \quad \overline{v_{raf}} = 0.68 \text{m/s}. \quad (26)$$

The value of  $|v_{raf}| = 0.68 \text{m/s}$  corresponds to an average wind gust. In that case, we have the following bounds:  $5 \times 10^{-5} \leq M_1 \leq 22.2 \times 10^{-5}$ ;  $-2, 2 \times 10^{-3} \leq h_1 \leq 1, 2 \times 10^{-3}$ .

Note: We will add an integrator to the control law to reduce the static error of the system and to attenuate the effects of the wind gust which is located in low frequency ( $raf \leq 7 \text{rad/s}$ ). We then obtain (Martini et al., 2007b):

$$v_1 = \ddot{z}_d - K_1 e_1 - K_2 e_2 - K_3 \int_0^t e_1 dt + \Delta v_1, \quad (27)$$

and the value of  $\Delta v$  becomes:  $\Delta v_1 = -\rho_1(e, t) \text{sign}(287e_1 + 220e_2 + 62e_3)$ . Moreover  $\rho_1 = 1.7 \|v_1\| + 184$ .

#### 4.1.2 Control of yaw angle $\psi$ :

The control law for the yaw angle is:

$$u_2 = \frac{1}{c_5 c_{11} \dot{\gamma}^2} [(c_1 c_5 - c_4^2) \ddot{\psi} + c_4 ((c_{12} \dot{\gamma} + c_{13} + c_8 \dot{\gamma} v_{raf}) u_1 + c_{14} \dot{\gamma}^2 + c_{15} + \frac{5}{2} c_9 v_{raf} + c_{17} v_{raf}^2)] \quad (28)$$

We have:

$$\begin{aligned}
u_2 &= M_2(q)\ddot{\psi} + h_2(q, \dot{q}, v_{raf}), \\
M_2(q) &= \frac{c_1 c_5 - c_4^2}{c_5 c_{11} \dot{\gamma}^2}, \\
h_2(q, \dot{q}, v_{raf}, u_1) &= \frac{c_4}{c_5 c_{11} \dot{\gamma}^2} \left[ ((c_{12} \dot{\gamma} + c_{13} + c_8 \dot{\gamma} v_{raf}) u_1 + c_{14} \dot{\gamma}^2 + c_{15} + \frac{5}{2} c_9 v_{raf} + c_{17} v_{raf}^2) \right].
\end{aligned} \tag{29}$$

Using (26) and with  $\underline{u}_1 \leq u_1 \leq \bar{u}_1, \bar{u}_1 = 0, \underline{u}_1 = -0.0112m$ , we find the following values :  $-2.7 \times 10^{-4} \leq M_2 \leq -6.1 \times 10^{-5}; -1.3 \times 10^{-3} \leq h_2 \leq 0.16$ .

We also add an integrator to the control law of the yaw angle (Martini et al., 2007b) :

$$v_2 = \ddot{\psi}_d - K_4 e_1 - K_5 e_2 - K_6 \int_0^t e_1 dt + \Delta v_2, \tag{30}$$

where  $e_1 = \psi - \psi_d$  and  $e_2 = \dot{e}_1, \dot{e}_3 = e_1$ . We obtain :  $\rho_2 = 1.7 \|v_2\| + 1614.6$ , the value of  $\Delta v$  becomes:  $\Delta v_2 = -\rho_2(e, t) \text{sign}(217e_1 + 87e_2 + 4e_3)$ .

On the other hand, the variation of inertia matrices  $M_1(q)$  and  $M_2(q)$  from their equilibrium value (corresponding to  $\dot{\gamma} = -124.63 \text{rad/s}$ ) are shown in table 3. It appears, in this table, that when  $\dot{\gamma}$  varies from  $-99.5$  to  $-209, 4 \text{rad/s}$  an important variation of the coefficients of matrices  $M_1(q)$  and  $M_2(q)$  of about 65% is obtained.

$M_1(s^2/\text{rad}^2)$	$ \frac{\Delta M_1}{M_1} (\%)$	$M_2(m.s^2/\text{rad}^2)$	$ \frac{\Delta M_2}{M_2} (\%)$
$1.42 \times 10^{-4}$	65%	$-1.72 \times 10^{-4}$	65%

Table 3. Variations of the inertia matrices  $M_1$  and  $M_2$

#### 4.2 Active disturbance rejection control

Two approaches are proposed here (Martini et al., 2007a) . The first uses a feedback and supposes the knowledge of a precise model of the helicopter. For the second approach, only two parameters of the helicopter are necessary, the remainder of the model being regarded as a disturbance, as well as the wind gust.

- Approach 1 (ADRC) : Firstly, the nonlinear terms of the non disturbed model ( $v_{raf} = 0$ ) are compensated by introducing two new controls  $v_1$  and  $v_2$  such as:

$$\begin{aligned}
u_1 &= \frac{1}{c_8 \dot{\gamma}^2} [c_0 v_1 - c_9 \dot{\gamma} - c_{10} + c_7], \\
u_2 &= \frac{c_1 c_5 - c_4^2}{c_5 c_{11} \dot{\gamma}^2} \left[ v_2 + \frac{c_4}{c_1 c_5 - c_4^2} ((c_{12} \dot{\gamma} + c_{13}) u_1 + c_{14} \dot{\gamma}^2 + c_{15}) \right].
\end{aligned} \tag{31}$$

Since  $v_{raf} \neq 0$ , a nonlinear system of equations is obtained:

$$\begin{aligned}
\ddot{z} &= v_1 + \frac{1}{c_0} c_{16} \dot{\gamma} v_{raf}, \\
\ddot{\psi} &= v_2 - \frac{c_4 c_0 v_{raf}}{(c_1 c_5 - c_4^2) \dot{\gamma}} v_1 - \frac{c_4 v_{raf}}{c_1 c_5 - c_4^2} \left[ \frac{c_7 - c_{10}}{\dot{\gamma}} + \frac{5}{2} c_9 + c_{17} v_{raf} \right].
\end{aligned} \tag{32}$$

- Approach 2 (ADRCM): By introducing the two new controls  $\acute{u}_1$  and  $\acute{u}_2$  such as:

$$\begin{aligned}
\acute{u}_1 &= \frac{c_8 \dot{\gamma}^2}{c_0} u_1, \\
\acute{u}_2 &= \frac{c_5 c_{11}}{c_1 c_5 - c_4^2} \dot{\gamma}^2 u_2,
\end{aligned}$$

a different nonlinear system of equations is got:

$$\begin{aligned}\ddot{z} &= \dot{u}_1 + \frac{1}{c_0} [c_9\dot{\gamma} + c_{10} - c_7 + c_{16}\dot{\gamma}v_{raf}], \\ \ddot{\psi} &= \dot{u}_2 - \frac{c_4}{(c_1c_5 - c_4^2)} [(c_{12}\dot{\gamma} + c_{13} + c_8\dot{\gamma}v_{raf}) \frac{c_0\dot{u}_1}{c_8\dot{\gamma}^2} + c_{15} \\ &\quad + c_{14}\dot{\gamma}^2 + \frac{5}{2}c_9v_{raf} + c_{17}v_{raf}^2].\end{aligned}\quad (33)$$

The systems (32) and (33) can be written as the following form:

$$\ddot{y} = f(y, \dot{y}, p) + bu, \quad (34)$$

with  $b = 1$ ,  $u = v_1$  or  $v_2$  for the approach 1, whether:

$$\begin{aligned}\text{For } y = z &\Rightarrow f_z(y, \dot{y}, p) = \frac{1}{c_0} c_{16}\dot{\gamma}v_{raf}, \\ \text{For } y = \psi &\Rightarrow f_\psi(y, \dot{y}, p) = -\frac{c_4c_0v_{raf}}{(c_1c_5 - c_4^2)\dot{\gamma}} v_1 \\ &\quad - \frac{c_4v_{raf}}{c_1c_5 - c_4^2} \left[ \frac{c_7 - c_{10}}{\dot{\gamma}} + \frac{5}{2}c_9 + c_{17}v_{raf} \right],\end{aligned}\quad (35)$$

and  $b = 1$ ,  $u = \dot{u}_1$  or (ADRC)  $\dot{u}_2$  for the approach 2, whether:

$$\begin{aligned}\text{For } y = z &\Rightarrow \dot{f}_z(y, \dot{y}, p) = \frac{1}{c_0} [c_9\dot{\gamma} + c_{10} - c_7 + c_{16}\dot{\gamma}v_{raf}], \\ \text{For } y = \psi &\Rightarrow \dot{f}_\psi(y, \dot{y}, p) = -\frac{c_4}{(c_1c_5 - c_4^2)} \left[ (c_{12}\dot{\gamma} + c_{13} + c_8\dot{\gamma}v_{raf}) \frac{c_0\dot{u}_1}{c_8\dot{\gamma}^2} \right. \\ &\quad \left. + c_{15} + c_{14}\dot{\gamma}^2 + \frac{5}{2}c_9v_{raf} + c_{17}v_{raf}^2 \right].\end{aligned}\quad (36)$$

Concerning the first approach, an observer is built:

- for altitude  $z$ :

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1g_1(e_z, \alpha_1, \delta_1), \\ \dot{\hat{x}}_2 = \hat{x}_3 + L_2g_2(e_z, \alpha_2, \delta_2) + bv_1, \\ \dot{\hat{x}}_3 = L_3g_3(e_z, \alpha_3, \delta_3), \end{cases}\quad (37)$$

where  $e_z = z - \hat{z}_1$  is the observer error,  $g_i(e_i, \alpha_i, \delta_i)$  is defined as exponential function of modified gain:

$$g_i(e_z, \alpha_{iz}, \delta_i)_{i=1,2,3} = \begin{cases} |e_z|^{\alpha_{iz}} \text{sign}(e_z), & |e_z| > \delta_i, \\ \frac{e_z}{\delta_i^{1-\alpha_{iz}}}, & |e_z| \leq \delta_i, \end{cases}\quad (38)$$

with  $0 < \alpha_i < 1$  and  $0 < \delta_i \leq 1$ , a *PID* controller is used in stead of *PD* in order to attenuate the effects of disturbance:

$$v_1 = \ddot{z}_d - \lambda_1(\hat{x}_1 - z_d) - \lambda_2(\hat{x}_2 - \dot{z}_d) - \lambda_3 \int_0^t (x_1 - z_d)dt - \hat{x}_3. \quad (39)$$

The control signal  $v_1$  takes into account of the terms which depend on the observer ( $\hat{x}_1, \hat{x}_2$ ). The fourth part, which also comes from the observer, is added to eliminate the effect of disturbance in this system.

- for the yaw angle  $\psi$ :

$$\begin{cases} \dot{\hat{x}}_4 = \hat{x}_5 + L_4 g_4(e_\psi, \alpha_4, \delta_4), \\ \dot{\hat{x}}_5 = \hat{x}_6 + L_5 g_5(e_\psi, \alpha_5, \delta_5) + b v_2, \\ \dot{\hat{x}}_6 = L_6 g_6(e_\psi, \alpha_6, \delta_6), \end{cases} \quad (40)$$

where  $e_\psi = \psi - \hat{x}_4$  is the observer error, with  $g_i(e_\psi, \alpha_{i\psi}, \delta_i)$  is defined as exponential function of modified gain:

$$g_i(e_\psi, \alpha_{i\psi}, \delta_i)_{i=4,5,6} = \begin{cases} |e_\psi|^{\alpha_{i\psi}} \text{sign}(e_\psi), & |e_\psi| > \delta_i, \\ \frac{e_\psi}{\delta_i^{1-\alpha_{i\psi}}}, & |e_\psi| \leq \delta_i, \end{cases} \quad (41)$$

and

$$v_2 = \ddot{\psi}_d - \lambda_4(\hat{x}_4 - \psi_d) - \lambda_5(\hat{x}_5 - \dot{\psi}_d) - \lambda_6 \int_0^t (x_4 - \psi_d) dt - \hat{x}_6, \quad (42)$$

$z_d$  and  $\psi_d$  are the desired trajectories. *PID* parameters are designed to obtain two dominant poles in closed-loop: for  $z$   $\begin{cases} \omega_{c1} = 2 \text{rad/s} \\ \xi_1 = 1 \end{cases}$  and for  $\psi$   $\begin{cases} \omega_{c2} = 5 \text{rad/s} \\ \xi_2 = 1 \end{cases}$ . The approach 2 uses the same observer with the same gain, simply  $(-\hat{x}_3)$  and  $(-\hat{x}_6)$  compensate respectively  $\dot{f}_z(y, \dot{y}, p)$  and  $\dot{f}_\psi(y, \dot{y}, p)$ .

### 4.3 Backstepping control

To control the altitude dynamics  $z$  and the yaw angle  $\psi$ , the steps are as follows:

1. Compensation of the nonlinear terms of the nondisturbed model ( $v_{raf} = 0$ ) by introducing two new controls  $V_z$  and  $V_\psi$  such as:

$$\begin{aligned} u_1 &= \frac{1}{c_8 \dot{\gamma}^2} [c_0 V_z - c_9 \dot{\gamma} - c_{10} + c_7], \\ u_2 &= \frac{1}{c_5 c_{11} \dot{\gamma}^2} [(c_1 c_5 - c_4^2) V_\psi + c_4 ((c_{12} \dot{\gamma} + c_{13}) u_1 + c_{14} \dot{\gamma}^2 + c_{15})], \end{aligned} \quad (43)$$

with these two new controls, the following system of equations is obtained:

$$\ddot{z} = V_z + d_1(\dot{\gamma}, v_{raf}), \quad (44)$$

$$\ddot{\psi} = V_\psi + d_2(V_z, \dot{\gamma}, v_{raf}). \quad (45)$$

2. Stabilization is done by backstepping control, we start by controlling the altitude  $z$  then the yaw angle  $\psi$ .

#### 4.3.1 Control of altitude $z$

We already saw that  $\ddot{z} = V_z + d_1(\dot{\gamma}, v_{raf})$ . The controller, generated by backstepping, is generally a *PD* (Proportional Derived). Such *PD* controller is not able to cancel external disturbances with non zero average unless they are at the output of an integrating process. In order to attenuate the errors due to static disturbances, a solution consists in equipping the regulators obtained with an integral action (Benaskeur et al., 2000). The main idea is to

introduce, in a virtual way, an integrator in the transfer function of the process and to carry out the development of the control law in a conventional way using the method of backstepping. The state equations of  $z$  dynamics which are increased by an integrator, are given by:

$$\begin{aligned}x_1 &= z, \\ \dot{x}_1 &= x_2 = \dot{z}, \\ \dot{x}_2 &= x_3 + d_1(\dot{\gamma}, v_{raf}), \\ \dot{x}_3 &= w_1,\end{aligned}\tag{46}$$

where  $V_z = \int w_1 = x_3$ . The introduction of an integrator into the process only increases the state of the process. Hereafter the control by backstepping is developed:

**Step 1:** Firstly, we ask the output to track a desired trajectory  $x_{1d}$ , one introduces the trajectory error:  $\xi_1 = x_{1d} - x_1$ , and its derivative:

$$\dot{\xi}_1 = \dot{x}_{1d} - \dot{x}_1 = \dot{x}_{1d} - x_2,\tag{47}$$

which are both associated to the following Lyapunov candidate function:

$$V_1 = \frac{1}{2}\xi_1^2.\tag{48}$$

The derivative of Lyapunov function is evaluated:  $\dot{V}_1 = \xi_1\dot{\xi}_1 = \xi_1(\dot{x}_{1d} - x_2) = \xi_1(\dot{x}_{1d} - x_2)$ . The state  $x_2$  is then used as intermediate control in order to guarantee the stability of (47). We define for that a virtual control:  $\alpha_1 = a_1\xi_1 + \dot{x}_{1d}$ , where:  $\dot{\alpha}_1 = a_1\dot{\xi}_1 + \ddot{x}_{1d}$ .

**Step 2:** It appears a new error:  $\xi_2 = \alpha_1 - x_2 = a_1\xi_1 + \dot{\xi}_1$ . Its derivative is written as follows:

$$\dot{\xi}_2 = \dot{\alpha}_1 - \dot{x}_2 = a_1\dot{\xi}_1 + \ddot{x}_{1d} - x_3 - d_1.\tag{49}$$

In order to attenuate this error, the precedent candidate function (48) is increased by another term, which will deal with the new error introduced previously:

$$V_2 = \frac{1}{2}\xi_1^2 + \frac{1}{2}\xi_2^2,\tag{50}$$

its derivative:  $\dot{V}_2 = \xi_1\dot{\xi}_1 + \xi_2\dot{\xi}_2 = -a_1\xi_1^2 + \xi_2(\dot{\alpha}_1 - x_3 + \xi_1) - \xi_2d_1$ . The state  $x_3$  can be used as an intermediate control in (49). This state is given in such a way that it must return the expression between bracket equal to  $-a_2\xi_2$  for  $d_1(\dot{\gamma}, v_{raf}) = 0$ . The virtual control obtained is:  $\alpha_2 = a_2\xi_2 + \dot{\alpha}_1 + \xi_1 = (1 + a_1a_2)\xi_1 + (a_2 + a_1)\dot{\xi}_1 + \ddot{x}_{1d}$ , its derivative:  $\dot{\alpha}_2 = (1 + a_1a_2)\dot{\xi}_1 + (a_2 + a_1)\ddot{\xi}_1 + x_{1d}^{(3)}$ .

**Step 3:** Still here, another term of error is introduced:

$$\begin{aligned}\xi_3 &= \alpha_2 - x_3 \\ &= (1 + a_1a_2)\xi_1 + (a_2 + a_1)\dot{\xi}_1 + \ddot{\xi}_1 + d_1,\end{aligned}\tag{51}$$

and the Lyapunov function (50) is augmented another time, to take the following form:

$$V_3 = \frac{1}{2}\xi_1^2 + \frac{1}{2}\xi_2^2 + \frac{1}{2}\xi_3^2,\tag{52}$$

its derivative:

$$\begin{aligned}
 \dot{V}_3 &= \xi_1 \dot{\xi}_1 + \xi_2 \dot{\xi}_2 + \xi_3 \dot{\xi}_3 \\
 &= -a_1 \xi_1^2 + \xi_2 (\dot{\alpha}_1 - x_3 + \xi_1) - \xi_2 d_1 (\dot{\gamma}, v_{raf}) + \xi_3 (\dot{\alpha}_2 - \dot{x}_3) \\
 &= -a_1 \xi_1^2 + \xi_2 (\dot{\alpha}_1 + \xi_3 - \alpha_2 + \xi_1) + \xi_3 (\dot{\alpha}_2 - \dot{x}_3) - \xi_2 d_1 (\dot{\gamma}, v_{raf}) \\
 &= -a_1 \xi_1^2 - a_2 \xi_2^2 + \xi_3 (\xi_2 + \dot{\alpha}_2 - \dot{x}_3) - \xi_2 d_1 (\dot{\gamma}, v_{raf}) \\
 &= -a_1 \xi_1^2 - a_2 \xi_2^2 + \xi_3 (\xi_2 + \dot{\alpha}_2 - w_1) - \xi_2 d_1 (\dot{\gamma}, v_{raf}).
 \end{aligned} \tag{53}$$

The control  $V_z$  should be selected in order to return the expression between the precedent bracket equal to  $-a_3 \xi_3$  for  $d_1 = 0$ :

$$\begin{aligned}
 w_1 &= a_3 \xi_3 + \xi_2 + \dot{\alpha}_2 \\
 &= x_{1d}^{(3)} + (a_3 + a_1 a_2 a_3 + a_1) \xi_1 \\
 &\quad + (2 + a_1 a_2 + a_1 a_3 + a_2 a_3) \dot{\xi}_1 + (a_1 + a_2 + a_3) \ddot{\xi}_1,
 \end{aligned} \tag{54}$$

with the relation (47), we obtain:  $\dot{\xi}_1 = \dot{x}_{1d} - \dot{x}_1 \Rightarrow \ddot{\xi}_1 = \ddot{x}_{1d} - \ddot{x}_1$ . These values, replaced in the control law, gives for  $d_1 (\dot{\gamma}, v_{raf}) = 0$ :

$$\begin{aligned}
 \dot{V}_z &= w_1 \\
 &= x_{1d}^{(3)} + (a_3 + a_1 a_2 a_3 + a_1) \xi_1 \\
 &\quad + (2 + a_1 a_2 + a_1 a_3 + a_2 a_3) \dot{\xi}_1 + (a_1 + a_2 + a_3) \ddot{\xi}_1.
 \end{aligned} \tag{55}$$

If we replace (54) in (53), we obtain finally:

$$\dot{V}_3 = -a_1 \xi_1^2 - a_2 \xi_2^2 - a_3 \xi_3^2 - \xi_2 d_1. \tag{56}$$

**Step 4:** It is here that the design of the control law by the method of backstepping stops. The integrator, which was introduced into the process, is transferred to the control law, which gives the final following control law:

$$V_z = \ddot{x}_{1d} + T_d (\dot{x}_{1d} - \dot{x}_1) + K_c (x_{1d} - x_1) + T_i \int (x_{1d} - x_1) dt \tag{57}$$

where  $s$  is the Laplace variable and:  $T_d = a_1 + a_2 + a_3$ ;  $K_c = 2 + a_1 a_2 + a_1 a_3 + a_2 a_3$ ;  $1/T_i = a_1 + a_3 + a_1 a_2 a_3$ .

#### 4.3.2 Control of yaw angle $\psi$ :

The calculation of the yaw angle control is also based on backstepping control (Zhao & Kanellakopoulos, 1998) dealing with the problem of the attenuation of the disturbance which acts on lateral dynamics. The representation of yaw state dynamics with the angular velocity of the main rotor is:

$$\begin{aligned}
 \dot{x}_4 &= x_5 = \dot{\psi}, \\
 \dot{x}_5 &= \ddot{\psi} = V_\psi + d_2 (V_z, \dot{\gamma}, v_{raf}) = w_2 + \dot{\gamma} + d_2 (V_z, \dot{\gamma}, v_{raf}), \\
 \dot{x}_6 &= \ddot{\gamma} = f_1 (\dot{\gamma}) V_z + f_2 (\dot{\gamma}) w_2 + d_3 (V_z, \dot{\gamma}, v_{raf}).
 \end{aligned} \tag{58}$$

The backstepping design then proceeds as follows:



**Step 1:** We start with the error variable:  $\xi_4 = x_4 - x_{4d}$ , whose derivative can be expressed as:  $\dot{\xi}_4 = x_5 - \dot{x}_{4d}$ , here  $x_5$  is viewed as the virtual control, that introduces the following error variable:

$$\xi_5 = x_5 - \alpha_4 \quad (59)$$

where  $\alpha_4$  is the first stabilizing function to be determined. Then we can represent  $\dot{\xi}_4$  as:

$$\dot{\xi}_4 = \alpha_4 + \xi_5 - \dot{x}_{4d} \quad (60)$$

In order to design  $\alpha_4$ , we choose the partial Lyapunov function  $V_4 = \frac{1}{2}\xi_4^2$  and we evaluate its time derivative along the solutions of (60):  $\dot{V}_4 = \xi_4\dot{\xi}_4 = \xi_4(\alpha_4 + \xi_5 - \dot{x}_{4d})$ . The choice of:  $\alpha_4 = -a_4\xi_4 + \dot{x}_{4d}$ , yields:  $\dot{V}_4 = -a_4\xi_4^2 + \xi_4\xi_5$  and  $\dot{\xi}_4 = -a_4\xi_4 + \xi_5$ .

**Step 2:** According to the computation of step 1, driving  $\xi_5$  to zero will ensure that  $\dot{V}_4$  is negative definite in  $\xi_4$ . We need to modify the Lyapunov function to include the error variable  $\xi_5$ :

$$V_5 = \frac{1}{2}(\xi_4^2 + \xi_5^2). \quad (61)$$

We rewrite  $\dot{\xi}_5$ :

$$\begin{aligned} \dot{\xi}_5 &= \dot{x}_5 - \dot{\alpha}_4 \\ &= \ddot{\psi} - \dot{\alpha}_4 \\ &= w_2 + \dot{\gamma} + d_2 - a_4^2\xi_4 + a_4\xi_5 - \ddot{\psi}_d. \end{aligned} \quad (62)$$

In this equation,  $\dot{\gamma}$  is viewed as the virtual control. This is a departure from the usual backstepping design which only employs state variables as virtual controls. In this case, however, this simple modification is not only dictated by the structure of the system, but it also yields significant improvements in closed-loop system response. The new error variable is  $\xi_6 = \dot{\gamma} - \alpha_5$ , and  $\alpha_5$  is yet to be computed. Then (62) becomes:  $\dot{\xi}_5 = w_2 + d_2 - a_4^2\xi_4 + a_4\xi_5 - \ddot{\psi}_d + \xi_6 + \alpha_5$ ,

$$\begin{aligned} \dot{V}_5 &= \xi_4\dot{\xi}_4 + \xi_5\dot{\xi}_5 \\ &= -a_4\xi_4^2 + \xi_5 \left[ w_2 + \xi_4 - a_4^2\xi_4 + a_4\xi_5 - \ddot{\psi}_d + \alpha_5 + d_2 + \xi_6 \right]. \end{aligned} \quad (63)$$

From (63), the choice of:  $\alpha_5 = (a_4^2 - 1)\xi_4 - (a_4 + a_5)\xi_5 - w_2 + \ddot{\psi}_d$ , provides:

$$\dot{\xi}_5 = -\xi_4 - a_5\xi_5 + \xi_6 + d_2 \Rightarrow \dot{V}_5 = -a_4\xi_4^2 - a_5\xi_5^2 + \xi_5\xi_6 + \xi_5d_2. \quad (64)$$

**Step 3:** Similarly to the previous steps, we will design the stabilizing function  $w_2$  in this step. To achieve that, firstly, we define the error variable  $\xi_6$ :  $\xi_6 = \dot{\gamma} - \alpha_5$ , its time derivative:

$$\begin{aligned} \dot{\xi}_6 &= \ddot{\gamma} - \dot{\alpha}_5 \\ &= f_1(\dot{\gamma})V_z + f_2(\dot{\gamma})w_2 + \frac{c_4}{c_5}\dot{\gamma} - \psi_d^{(3)} - (a_5a_4 + a_5^2 + a_4^2 - 1)\xi_5 \\ &\quad - (2a_4 + a_5 - a_4^3)\xi_4 + (a_4 + a_5)\xi_6 + \dot{w}_2 + (a_4 + a_5)d_2 + d_3. \end{aligned} \quad (65)$$

Therefore, along the solutions of  $\dot{\xi}_4$ ,  $\dot{\xi}_5$  and  $\dot{\xi}_6$ , we can express the time derivative of the partial Lyapunov function  $V_6 = \frac{1}{2}(\xi_4^2 + \xi_5^2 + \xi_6^2)$  as:

$$\begin{aligned} \dot{V}_6 &= \xi_4 \dot{\xi}_4 + \xi_5 \dot{\xi}_5 + \xi_6 \dot{\xi}_6 \\ &= -a_4 \xi_4^2 - a_5 \xi_5^2 + \xi_5 \xi_6 + \xi_5 d_2 + \xi_6 [f_1(\dot{\gamma})V_z + f_2(\dot{\gamma})w_2 \\ &\quad + \frac{c_4}{c_5} \dot{\gamma} - (2a_4 + a_5 - a_4^3)\xi_4 - (a_5 a_4 + a_5^2 + a_4^2 - 1)\xi_5 \\ &\quad + (a_4 + a_5)\xi_6 + \dot{w}_2 + (a_4 + a_5)d_2 + d_3 - \psi_d^{(3)}] \\ &= -a_4 \xi_4^2 - a_5 \xi_5^2 - a_6 \xi_6^2 + (\xi_5 + \xi_6(a_4 + a_5))d_2 + \xi_6 d_3 \\ &\quad + \xi_6 [(a_4 + a_5 + a_6)\xi_6 + f_1(\dot{\gamma})V_z + f_2(\dot{\gamma})w_2 + \frac{c_4}{c_5} \dot{\gamma} + \dot{w}_2 \\ &\quad - (2a_4 + a_5 - a_4^3)\xi_4 - (a_5 a_4 + a_5^2 + a_4^2 - 2)\xi_5 - \psi_d^{(3)}]. \end{aligned} \quad (66)$$

In the above expression (66), our choice of  $\dot{w}_2$  is:

$$\begin{aligned} \dot{w}_2 &= -f_1(\dot{\gamma})V_z - f_2(\dot{\gamma})w_2 - \frac{c_4}{c_5} \dot{\gamma} - (a_4 + a_5 + a_6)\xi_6 \\ &\quad + (2a_4 + a_5 - a_4^3)\xi_4 + (a_5 a_4 + a_5^2 + a_4^2 - 2)\xi_5 + \psi_d^{(3)}. \end{aligned} \quad (67)$$

Then one replaces (67) in (65), to obtain:  $\dot{\xi}_6 = -\xi_5 - a_6 \xi_6 + (a_4 + a_5)d_2 + d_3$ , the derivative of  $V_6$  becomes:

$$\dot{V}_6 = -a_4 \xi_4^2 - a_5 \xi_5^2 - a_6 \xi_6^2 + (\xi_5 + \xi_6(a_4 + a_5))d_2 + \xi_6 d_3 \quad (68)$$

The integral of (67) provides  $w_2$  and  $V_\psi = w_2 + \dot{\gamma}$ . In this way, the yaw angle control is calculated.

## 5. Stability analysis of ADRC control

In this section, the stability of the perturbed helicopter controlled using observer based control law (ADRC) is considered. To simplify this study, the demonstration is done with one input and one output as in (Hauser et al. (1992)) and the result is applicable for other outputs. Let us first define the altitude error using equations (32), (37) and the control (39):

$[e_1 \ e_2]^T = [z \ \dot{z}]^T - [z_d \ \dot{z}_d]^T$ , and  $e_3 = f e_1$ , we can write:

$$\begin{aligned} \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -\lambda_1 & -\lambda_2 & -\lambda_3 \\ 1 & 0 & 0 \end{bmatrix}}_A \times \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ +\lambda_1 & +\lambda_2 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_B \times \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} \quad (69) \\ \dot{\eta} &= \beta(z, \eta, v_{raf}) \end{aligned}$$

Where  $A$  is a stable matrix determined by pole placement, and  $\dot{\eta}$  represents the zero dynamics of our system,  $\eta = \dot{\gamma} - \dot{\gamma}_{eq}$ , where  $\dot{\gamma}_{eq} = -124.63 \text{ rad/s}$  is the equilibrium of the main rotor angular speed:

$$\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}$$

is the observer error. Hereafter, we consider the case of a linear observer, so that:

$$\begin{bmatrix} \dot{\hat{e}}_1 \\ \dot{\hat{e}}_2 \\ \dot{\hat{e}}_3 \end{bmatrix} = \begin{bmatrix} -L_1 & 1 & 0 \\ -L_2 & 0 & 1 \\ -L_3 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{f}(z, \eta, v_{raf}) \end{bmatrix} \quad (70)$$

which can be written as:  $\dot{\hat{e}} = \hat{A}\hat{e} + \dot{f}(z, \eta, v_{raf})$ . Where  $\hat{A}$  is a stable matrix determined by pole placement.

**Theorem:** Suppose that:

- The zero dynamics of the system  $\dot{\eta} = \beta(z, \eta, v_{raf})$  (where is represented by the  $\dot{\gamma}$  dynamics) are locally exponentially stable and
- The amplitude of  $v_{raf}$  is sufficiently small and the function  $\dot{f}(z, \eta, v_{raf})$  is bounded and small enough (i.e.  $\hat{l}_u < 1/5$ , see equation (72) for definition of bound  $\hat{l}_u$ ).

Then for desired trajectories with sufficiently small values and derivatives ( $z_d, \dot{z}_d, \ddot{z}_d$ ), the states of the system (32) and of the observer (37) will be bounded.

**Proof:** Since the zero dynamics of model are assumed to be exponentially stable, a conserve Lyapunov theorem implies the existence of a Lyapunov function  $V_1(\eta)$  for the system:

$\dot{\eta} = \beta(0, \eta, 0)$  satisfying  $k_1\|\eta\|^2 \leq V_1(\eta) \leq k_2\|\eta\|^2$ ,  $\frac{\partial V_1}{\partial \eta} \eta(0, \eta, 0) \leq -k_3\|\eta\|^2$ ,  $\left\| \frac{\partial V_1}{\partial \eta} \right\| \leq k_4\|\eta\|$  for some positive constants  $k_1, k_2, k_3$  and  $k_4$ . We first show that  $e, \hat{e}, \eta$  are bounded. To this end, consider as a Lyapunov function for the error system ((69) and (70)):

$$V(e, \hat{e}, \tilde{\eta}) = e^T P e + \delta \hat{e}^T \hat{P} \hat{e} + \mu V_1(\eta) \quad (71)$$

where  $P, \hat{P} > 0$  are chosen so that:  $A^T P + P A = -I$  and  $\hat{A}^T \hat{P} + \hat{P} \hat{A} = -I$  (possible since  $A$  and  $\hat{A}$  are Hurwitz),  $\mu$  and  $\delta$  are a positives constants to be determined later. Note that, by assumption,  $z_d$  and its first derivatives are bounded:  $\|z\| \leq \|e\| + b_d$ ,  $\|\hat{x}\| \leq \|\hat{e}\| + \|e\| + b_d$ .

The functions,  $\beta(z, \eta, v_{raf})$  and  $\dot{f}(z, \eta, v_{raf})$  are locally Lipschitz (since  $\dot{f}$  is bounded) with  $\dot{f}(0, 0, 0) = 0$ , we have:

$$\begin{aligned} \|\beta(z^1, \eta^1, v_{raf}^1) - \beta(z^2, \eta^2, v_{raf}^2)\| &\leq l_q(\|z^1 - z^2\| + \|\eta^1 - \eta^2\| + \|v_{raf}^1 - v_{raf}^2\|) \\ \|2\hat{P}\dot{f}(z, \eta, v_{raf})\| &\leq \hat{l}_u \end{aligned} \quad (72)$$

with  $l_q$  and  $\hat{l}_u$  2 positive reals. Using these bounds and the properties of  $V_1(\cdot)$ , we have:

$$\begin{aligned} \frac{\partial V_1}{\partial \eta} \beta(z, \eta, v_{raf}) &= \frac{\partial V_1}{\partial \eta} \beta(0, \eta, 0) + \frac{\partial V_1}{\partial \eta} (\beta(z, \eta, v_{raf}) - \beta(0, \eta, 0)) \\ &\leq -k_3\|\eta\|^2 + k_4 l_q \|\eta\| (\|e\| + b_d + \|v_{raf}\|) \end{aligned} \quad (73)$$

Taking the derivative of  $V(\cdot, \cdot, \cdot)$  along the trajectory, we find:

$$\begin{aligned}\dot{V} &= \dot{e}^T P e + e^T P \dot{e} + \delta \hat{e}^T \hat{P} \hat{e} + \delta \hat{e}^T \hat{P} \dot{\hat{e}} + \mu \frac{\partial V_1}{\partial \dot{\gamma}} \beta(z, \eta, v_{raf}) \\ &\leq - \left[ \frac{3}{8} - \frac{\delta \hat{l}_u}{2} \right] \|e\|^2 + \left[ \frac{1}{2} - \delta + \delta \frac{5}{2} \hat{l}_u + 2(\|B\| \|P\|)^2 \right] \|\hat{e}\|^2 - \\ &\quad - \left[ \frac{3}{4} \mu k_3 - 4(\mu k_4 l_q)^2 + \frac{1}{2} \mu k_4 l_q \right] \|\eta\|^2 + \mu \frac{(k_4 l_q b_d)^2}{k_3} + \frac{1}{2} (\delta b_d \hat{l}_u)^2 + \\ &\quad + \left( \delta \hat{l}_u + \frac{1}{2} \mu k_4 l_q \right) \|v_{raf}\|^2\end{aligned}$$

Define:  $\mu_0 = \frac{k_3}{16(k_4 l_q)^2}$ ,  $\delta_1 = \frac{2}{8 \hat{l}_u}$ ,  $\delta_2 = \frac{1+4(\|B\| \|P\|)^2}{1-5 \hat{l}_u}$  and  $\hat{l}_u < 1/5$ . Then, for all  $\mu \leq \mu_0$  and for  $\delta_2 \leq \delta \leq \delta_1$ , we have:

$$\begin{aligned}\dot{V} &\leq - \frac{\|e\|^2}{4} - \frac{\delta}{2} \|\hat{e}\|^2 - \frac{\mu k_3 \|\eta\|^2}{2} + \left[ \mu \frac{(k_4 l_q)^2}{k_3} + \right. \\ &\quad \left. + \frac{1}{2} (\delta \hat{l}_u)^2 \right] b_d^2 + \left( \delta \hat{l}_u + \frac{1}{2} \mu k_4 l_q \right) \|v_{raf}\|^2\end{aligned}$$

Thus,  $\dot{V} < 0$  whenever  $\|e\|$ ,  $\|\hat{e}\|$  and  $\|\eta\|$  is large which implies that  $\|\hat{e}\|$ ,  $\|e\|$  and  $\|\eta\|$  and, hence,  $\|z\|$ ,  $\|\hat{x}\|$  and  $\|\eta\|$  are bounded. The above analysis is valid in a neighborhood of the origin. By choosing  $b_d$  and  $v_{raf}$  sufficiently small and with appropriate initial conditions, we can guarantee the state will remain in a small neighborhood, and which implies that the effect of the disturbance on the closed-loop can be attenuated. Moreover, if  $v_{raf} \rightarrow 0$  then  $\hat{l}_u \rightarrow 0$  and  $\delta_1 \rightarrow \infty$ ;  $\delta_2 \rightarrow 1 + 4(\|B\| \|P\|)^2$ , so that the constraint  $\hat{l}_u < 1/5$  is naturally satisfied for small  $v_{raf}$ .

## 6. Results in simulation

Robust nonlinear feedback control (RNFC), active disturbance rejection control based on a nonlinear extended state observer (ADRC) and backstepping control (BACK) are now compared via simulations.

1. RNFC: The various numerical values for the (RNFC) are the following:
  - For state variable  $z$ :  $\{K_1 = 84, K_2 = 24, K_3 = 80\}$  for  $\omega_1 = 2\text{rad/s}$  which is the bandwidth of the closed loop in  $z$  (the numerical values are calculating by pole placement).
  - For state variable  $\psi$ : We have  $\{K_4 = 525, K_5 = 60, K_6 = 1250\}$  for  $\omega_2 = 5\text{rad/s}$  which is the bandwidth of the closed loop in  $\psi$ .
2. ADRC: The various numerical values for the (ADRC) are the following:
  - a. For state variable  $z$ :  $k_1 = 24, k_2 = 84$  and  $k_3 = 80$  (the numerical values are calculating by pole placement). Choosing a triple pole located in  $\omega_{0z}$  such as  $\omega_{0z} = (3 \sim 5) \omega_{c1}$ , one can choose  $\omega_{0z} = 10 \text{ rad/s}$ ,  $\alpha_1 = 0.5$ ,  $\delta_1 = 0.1$ , and using pole placement method the gains of the observer for the case  $|e| \leq \delta$  (i.e linear observer) can be evaluated:

$$\begin{aligned}\frac{L_1}{\delta_1^{1-\alpha_1}} &= 3\omega_{0z}, \\ \frac{L_2}{\delta_1^{1-\alpha_1}} &= 3\omega_{0z}^2, \\ \frac{L_3}{\delta_1^{1-\alpha_1}} &= \omega_{0z}^3,\end{aligned}\tag{74}$$

which leads to:  $L_i = \{9.5, 94.87, 316.23\}$ ,  $i \in [1, 2, 3]$ .

- b. For state variable  $\psi$ :  $k_4 = 60$ ,  $k_5 = 525$ ,  $k_6 = 1250$ ,  $\omega_{0\psi} = 25 \text{ rad/s}$ ,  $\alpha_2 = 0.5$  and  $\alpha_2 = 0.025$ . And by the same method in (74) one can find the observer gains:  $L_i = \{11.86, 296.46, 2.47 \times 10^3\}$ ,  $i \in [4, 5, 6]$ .
3. *BACK*: The regulation parameters ( $a_1$ ;  $a_2$ ;  $a_3$ ;  $a_4$ ;  $a_5$ ;  $a_6$ ) for the (*BACK*) controller was calculated to obtain two dominating poles in closed-loop such as  $\omega_1 = 2 \text{ rad/s}$ , which defines the bandwidth of the closed-loop in  $z$ , and  $\omega_2 = 5 \text{ rad/s}$  for  $\psi$ .
- a. The closed-loop dynamics of the  $z$ -dynamics with  $d_1(\dot{\gamma}, v_{raf}) = 0$  is given by (Benaskeur et al., 2000):

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 \\ -1 & -a_2 & 1 \\ 0 & -1 & -a_3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = A_0 \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix}. \quad (75)$$

Eigenvalues of  $A_0$  can be calculated solving:

$$|Is - A_0| = s^3 + (a_1 + a_2 + a_3)s^2 + (a_1a_2 + a_1a_3 + a_2a_3 + 2)s + a_1 + a_3 + a_1a_2a_3 = 0. \quad (76)$$

If one gives as a desired dynamics specification, one dominant pole in  $-\kappa$  and the two other poles in  $-10\kappa$ , one must solve:

$$\beta_d(s) = (s + \kappa)(s + 10\kappa)^2 = s^3 + 21\kappa s^2 + 120\kappa^2 s + 100\kappa^3, \quad (77)$$

which leads to:

$$\begin{aligned} 21\kappa &= a_1 + a_2 + a_3, \\ 120\kappa^2 &= 2 + a_1a_2 + a_1a_3 + a_2a_3, \\ 100\kappa^3 &= a_1 + a_3 + a_1a_2a_3. \end{aligned}$$

For  $\omega_1 = 2 \text{ rad/s}$ , and resolving the above equations, we find 4 positive solutions for every parameter (see Table 1). The solution:  $a_1 = 21$ ,  $a_2 = 19$ ,  $a_3 = 1.95$  has been used for simulation.

$a_2$	$a_3$	$a_1$	$a_5$	$a_6$	$a_4$
21	19	1.95	51	49	5
	1.95	19		4.97	49
19	1.95	21	49	51	4.97
	21	1.95		4.97	51
1.88	20+0.4i	20-0.4i	4.95	50+0.54i	6-0.54i
	20-0.4i	20+0.4i		50-0.54i	6+0.54i

Table 1. Regulation parameters of  $z$  and  $\psi$ -dynamics

- b. The closed-loop dynamics of the  $\psi$ -dynamics with  $d_3(V_z, \dot{\gamma}, v_{vraf}) = 0$  is given by:

$$\begin{bmatrix} \dot{\xi}_4 \\ \dot{\xi}_5 \\ \dot{\xi}_6 \end{bmatrix} = \begin{bmatrix} -a_4 & 1 & 0 \\ -1 & -a_5 & 1 \\ 0 & -1 & -a_6 \end{bmatrix} \begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} = B_0 \begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix}. \tag{78}$$

Eigenvalues of  $B_0$  can be calculated by solving:

$$|Is - B_0| = s^3 + (a_4 + a_5 + a_6)s^2 + (a_4a_5 + a_4a_6 + a_5a_6 + 2)s + a_4 + a_6 + a_4a_5a_6 = 0. \tag{79}$$

By using the same development as for z-dynamics, one can write:

$$\begin{aligned} a_4 + a_5 + a_6 &= 21\kappa, \\ 2 + a_4a_5 + a_4a_6 + a_5a_6 &= 120\kappa^2, \\ a_4 + a_6 + a_4a_5a_6 &= 100\kappa^3. \end{aligned}$$

For  $\kappa = \omega_2 = 5 \text{ rad/s}$  and resolving the above equations, we find again 4 positive solutions for every parameter (see Table 1). As justified in annex ?? the solution:  $a_4 = 4.97, a_5 = 49, a_6 = 51$  has been used for simulation.

The induced gust velocity operating on the principal rotor is chosen as (G.D.Padfield, 1996):

$$\begin{aligned} v_{raf} &= v_{gm} \sin(0.042t_{d1}) & 70 \leq t \leq 24\pi + 130 \\ v_{raf} &= -v_{gm} \sin(0.042t_{d2}) & 220 \leq t \leq 297 \end{aligned} \tag{80}$$

where  $t_{d1} = t - 70$  and  $t_{d2} = t - 220$ , the value of 0.042 represent  $\frac{2\pi V}{L_u}$  where  $V$  in  $m/s$  is the height rise speed of the helicopter and  $v_{gm} = 0.68m/s$  is the gust density. This density corresponds to an average wind gust, and  $L_u = 1.5m$  is its length (see Fig.5). The take-off time at  $t = t_{off} = 50 \text{ s}$  is imposed and the following desired trajectory is used (Vilchis et al., 2003):

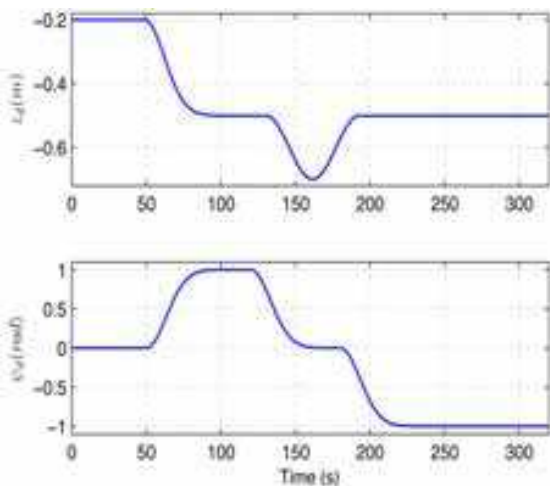


Fig. 4. Trajectories in  $z$  and  $\psi$

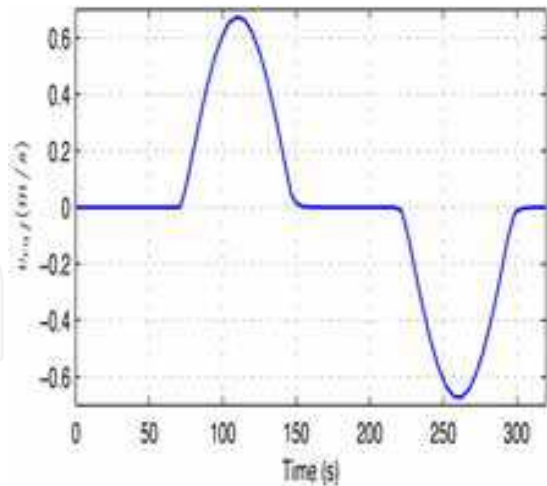


Fig. 5. Induced gust velocity  $v_{raf}$

$$\begin{aligned} z_d &= -0.2, & 0 \leq t \leq t_{off}, \\ z_d &= -0.2 + 0.3 \left[ \exp^{-\frac{(t-t_{off})^2}{350}} - 1 \right], & t_{off} < t \leq t_a, \\ z_d &= -0.6 + 0.1 \cos \left[ \frac{(t-130)}{10} \right], & t_a < t < t_b, \\ z_d &= -0.5, & t \geq t_b, \end{aligned} \tag{81}$$



where  $t_a = 130s$  and  $t_b = 20\pi + 130s$ ,

$$\begin{aligned} \psi_d &= 0, & t < t_{off}, \\ \psi_d &= 1 - \exp^{-(t-t_{off})^2/350}, & t_{off} \leq t < t_c, \\ \psi_d &= \exp^{-(t-t_{off})^2/350}, & t_c \leq t < t_d, \\ \psi_d &= -1 + \exp^{-(t-180)^2/350}, & t \geq t_d, \end{aligned} \quad (82)$$

and  $t_c = 120s$  and  $t_d = 180s$ . The following initial conditions are applied:  $z(0) = -0.2m$ ,  $\dot{z}(0) = 0$ ,  $\psi(0) = 0$ ,  $\dot{\psi}(0) = 0$  and  $\dot{\gamma}(0) = -99.5 \text{ rad/s}$ . A band limited white noise of variance  $3mm$  for  $z$  and  $1^\circ$  for  $\psi$ , has been added respectively to the measurements of  $z$  and  $\psi$  for the three controls. The compensation of this noise is done using a Butterworth second-order low-pass filter. Its crossover frequency for  $z$  is  $\omega_{cz} = 12 \text{ rad/s}$  and for  $\psi$  is  $\omega_{c\psi} = 20 \text{ rad/s}$ . Fig.4 shows the desired trajectories in  $z$  and  $\psi$ .

One can observe that  $\dot{\gamma} \rightarrow -124.6 \text{ rad/s}$  remains bounded away from zero during the flight. For the chosen trajectories and gains  $\dot{\gamma}$  converges rapidly to a constant value (see Fig.7). This is an interesting point to note since it shows that the dynamics and feedback control yield flight conditions close to the ones of real helicopters which fly with a constant  $\dot{\gamma}$  thanks to a local regulation feedback of the main rotor speed (which does not exist on the VARIO scale model helicopter). One can also notice that the main rotor angular speed is similar for the three controls as illustrated in Fig.7. The difference between the three controls appears in Fig.6 where the tracking errors in  $z$  are less significant by using the (BACK) and (ADRC) control than (RNFC) control. For  $\psi$  it is the different. This is explained by the use of a PID controller for the (RNFC) and (ADRC) but a PD controller for the (BACK) controller of  $\psi$  (Fig.6). Here, the (ADRC) and (BACK) controls show a robust behavior in presence of noise.

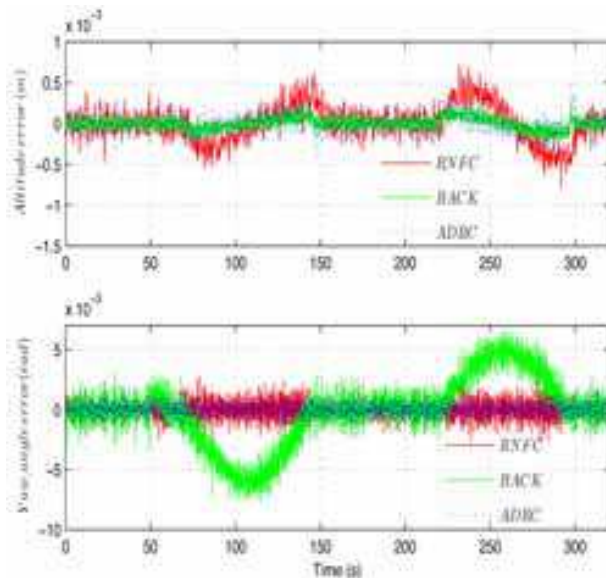


Fig. 6. Tracking error in  $z$  and in  $\psi$ .

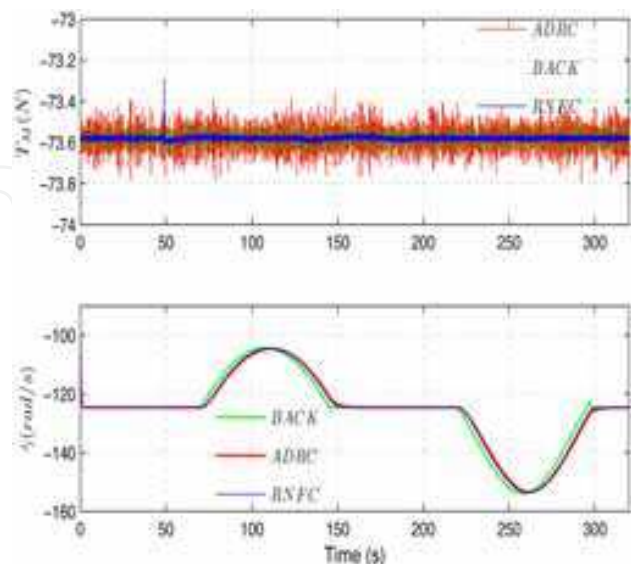


Fig. 7. Variations of the main rotor thrust  $T_M$  and the main rotor angular speed  $\dot{\gamma}$ .

One can see in Fig.7 that the main rotor thrust converges to values that compensate the helicopter weight, the drag force and the effect of the disturbance on the helicopter. The (RNFC) control allows the main rotor thrust  $T_M$  to be less away from its balance position than the other controls, where the RNFC control is less sensitive to noise. Fig.9 represent the effectiveness of the observer:  $\hat{x}_3$  and  $f_z(y, \dot{y}, w)$  are very close and also  $\hat{x}_6$  and  $f_\psi(y, \dot{y}, w)$ . Observer errors are presented in the Fig.8.

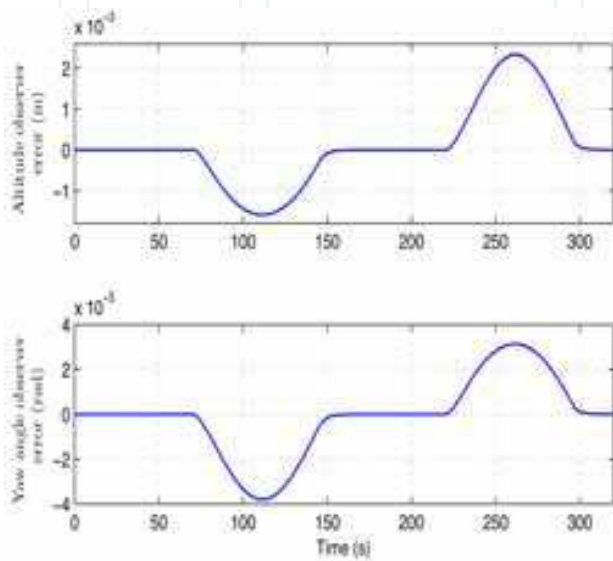


Fig. 8. Observer error in  $z$  and in  $\psi$

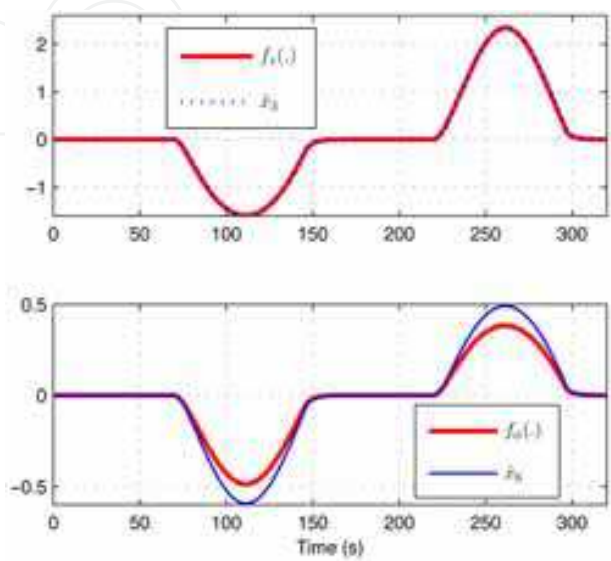


Fig. 9. Estimation of  $f_z$  and of  $f_\psi$

If one keeps the same parameters of adjustment for the three controls and using a larger wind gust ( $v_{raf} = 3m/s$ ), we find that the control (BACK) give better results than the two controls (ADRC) and (RNFC) (see Fig.10).

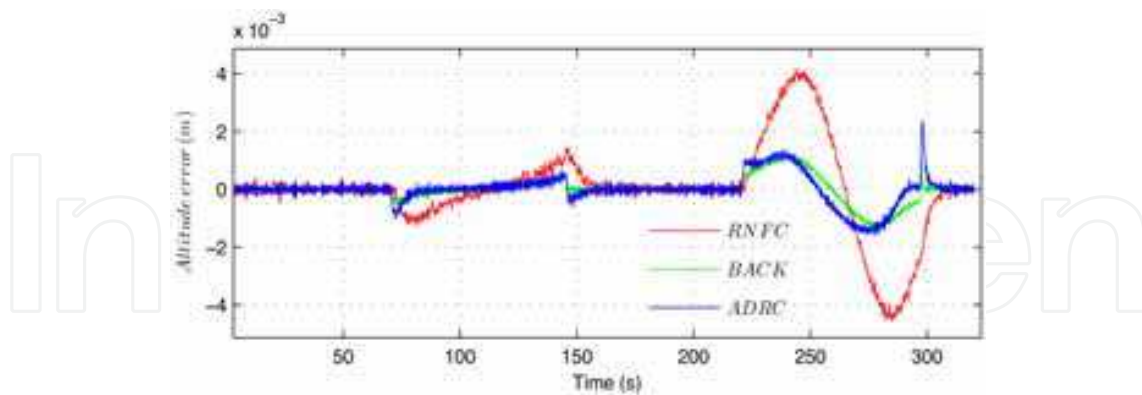


Fig. 10. Large disturbance  $v_{raf} = 3m/s$

Fig.11 shows the tracking error in  $z$  and  $\psi$  for two different ADRC controls. These errors are quite similar for approach 1 (ADRC) and approach 2 (ADRCM). Nevertheless ADRCM induces larger error at the take off, which can be explained by the fact that the control depends directly on the angular velocity of the main rotor: this last one need a few time to reach its equilibrium position as seen in Fig.6. The same argument can be invoked to explain the saturation of ADRCM control  $u_1$  and  $u_2$  as illustrated in Fig.12.

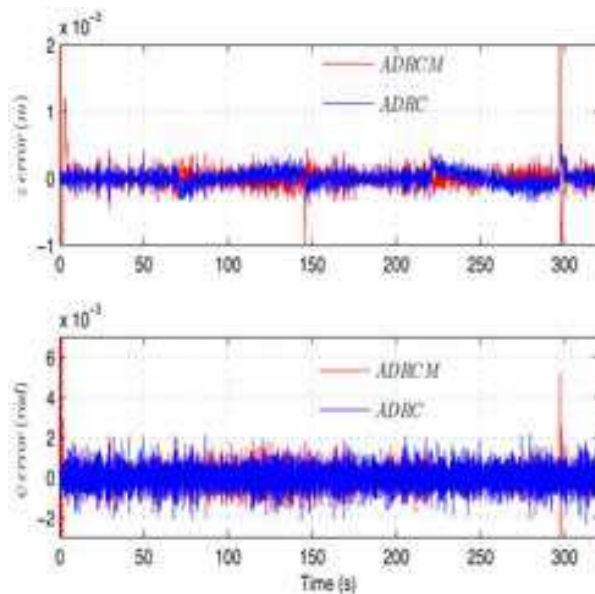


Fig. 11. Tracking error in  $z$  and  $\psi$  for both approaches 1 and 2 of ADRC control.

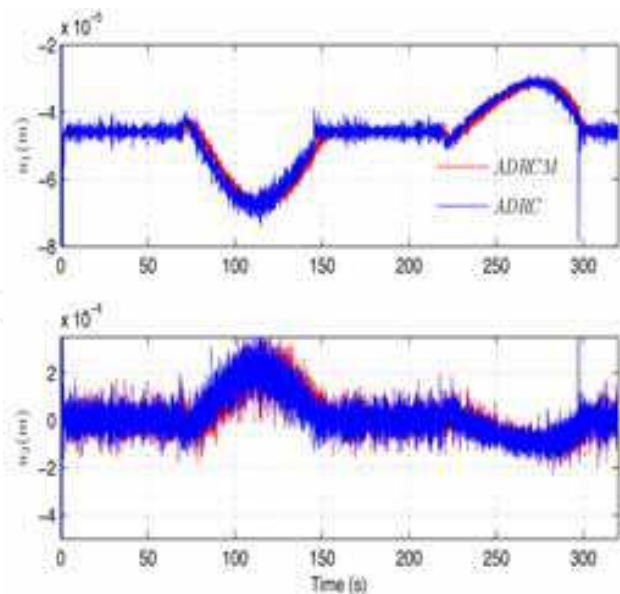


Fig. 12. Inputs  $u_1$  and  $u_2$  for both approaches 1 and 2 of ADRC control.

## 7. Conclusion

In this chapter, a robust nonlinear feedback control (*RNFC*), an active disturbance rejection control based on a nonlinear extended state observer (*ADRC*) and backstepping control (*BACK*) have been applied for the drone helicopter control disturbed by a wind gust. The technique of a robust nonlinear feedback control use the second method of Lyapunov and an additional feedback provides an extra term  $\Delta v$  to overcome the effects of the uncertainty and disturbances. The basis of *ADRC* is the extended state observer. The state estimation and compensation of the change of helicopter parameters and disturbance variations are implemented by *ESO* and *NESO*. By using *ESO*, the complete decoupling of the helicopter is obtained. The major advantage of the proposed method is that the closed loop characteristics of the helicopter system do not depend on the exact mathematical model of the system.

The backstepping technique should not viewed as a rigid design procedure, but rather as a design philosophy which can be bent and twisted to accommodate the specific needs of the system at hand. In the particular example of an autonomous helicopter, we were able to exploit the flexibility of backstepping with respect to the selection of virtual controls, initial stabilizing functions and Lyapunov functions. Comparisons were made in detail between the three methods of control.

It is concluded that the three proposed controls algorithms produces satisfactory dynamic performances. Even for large disturbance, the proposed backstepping (*BACK*) and (*ADRC*) control systems are robust against the modeling uncertainties and external disturbance in various operating conditions. It is also indicated that (*BACK*) and (*ADRC*) achieve a better tracking and stabilization with prescribed performance requirements.

For practical reasons, the second *ADRC* approach is the best one because it only requires to know some aerodynamic parameters of the helicopter (dimensions of the blades of the main and tail rotor and the helicopter weight), whereas the other approaches (first *ADRC*

approach, *RNCF* and *BACK*) depend on all the aerodynamic parameters which generate the forces and the couples that act on the helicopter. For first *ADRC* control, a stability analysis has been carried out where boundness of states of helicopter and observer are proved in spite of the presence of wind gust.

As illustrated in tables 2 and 3, wind gust induces large variation of helicopter parameters, and the controls quoted in this work can efficiently treat these parameter deviations.

As perspective, this work is carried on a model of a 7DOF VARIO helicopter, where *ADRC* and linearizing control will be tested in simulation. The first results using *ADRC* control on this 7DOF helicopter have been recently obtained (see (Martini et al., 2008) ). Moreover, our control methodologies will be also implemented on a new platform to be built using a Tiny CP3 helicopter.

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## **Robotics Automation and Control**

Edited by Pavla Pecherkova, Miroslav Flidr and Jindrich Dunik

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This book was conceived as a gathering place of new ideas from academia, industry, research and practice in the fields of robotics, automation and control. The aim of the book was to point out interactions among various fields of interests in spite of diversity and narrow specializations which prevail in the current research. The common denominator of all included chapters appears to be a synergy of various specializations. This synergy yields deeper understanding of the treated problems. Each new approach applied to a particular problem can enrich and inspire improvements of already established approaches to the problem.

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