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Fuzzy Stabilization of Fuzzy Control Systems

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1. Introduction

Recently there has been significant growth in the use of fuzzy logic in industrial and consumer products (J. Yen 1995). However, although fuzzy control has been successfully applied to many industrial plants that are mostly nonlinear systems, many critics of fuzzy logic claim that there is no such thing as a stability proof for fuzzy logic systems in closed-loop control (Reznik 1997; Farinwata, Filev et al. 2000). Since fuzzy logic controllers are classified as "non-linear multivariable controllers" (Reznik 1997; Farinwata, Filev et al. 2000), it can be argued that all stability analysis methods applicable to these controller types are applicable to fuzzy logic controllers. Unfortunately, due to the complex non-linearities of most fuzzy logic systems, an analytical solution is not possible. Furthermore, it is important to realize that real, practical problems have uncertain plants that inevitably cannot be modelled dynamically resulting in substantial uncertainties. In addition the sensors noise and input signal level constraints affect system stability. Therefore a theory that is able to deal with these issues would be useful for practical designs. The most well-known time domain stability analysis methods include Lyapunov's direct method (Wu & Ch. 2000; Gruyitch, Richard et al. 2004; Rubio & Yu 2007) which is based on linearization and Lyapunov's indirect method (Tanaka & Sugeno 1992; Giron-Sierra & Ortega 2002; Lin, Wang et al. 2007; Mannani & Talebi 2007) that uses a Lyapunov function which serves as a generalized energy function. In addition many other methods have been used for testing fuzzy systems stability such as Popov's stability criterion (Katoh, Yamashita et al. 1995; Wang & Lin 1998), the describing function method (Ying 1999; Aracil & Gordillo 2004), methods of stability indices and systems robustness (Fuh & Tung 1997; Espada & Barreiro 1999; Zuo & Wang 2007), methods based on theory of input/output stability (Kandel, LUO et al. 1999), conicity criterion (Cuesta & Ollero 2004). Also there are methods based on hyper-stability theory (Piegat 1997) and linguistic stability analysis approach (Gang & Laijiu 1996).

Fuzzy logic uses approximate reasoning and in this chapter a practical algorithm to improve system stability by using a fuzzy stabilizer block in the feedback path is introduced. The fuzzy stabilizer is tuned such that its nonlinearity lies in a bounded sector resulting from the circle criterion theory (Safonov 1980). The circle criterion presents the sufficient condition for absolute stability (Vidyasagar 1993). An appealing aspect of the circle criterion is its geometric nature, which is reminiscent of the Nyquist criterion. It is a frequency domain method for stability analysis and has been used by Ray et al (1984) to ensure fuzzy system stability (Ray, Ghosh et al. 1984; Ray & Majumderr 1984).

Throughout this chapter we use a practical approach to stabilize fuzzy systems with the aid of the circle criterion theory using a Takagi-Sugeno fuzzy block in the feedback loop of the closed system. The new technique is used to ensure stability for the proposed robot fuzzy controller. Furthermore, the study indicates that the fuzzy stabilizer can be integrated, with minor modifications, into any fuzzy controller to enhance its stability. As a result, the proposed design is suitable for hardware implementation even permitting relatively simple modification of existing designs to improve system stability. In addition an extension to the approach to stabilize MIMO (Multi-input Multi-output) systems is also presented.

2. Problem formulation and analysis

This chapter concentrates on the stability of a closed loop nonlinear system using a Takagi-Sugeno (T-S) fuzzy controller. Fuzzy control based on Takagi-Sugeno (T-S) fuzzy model (Babuska, Roubos et al. 1998; Buckley & Eslami 2002) has been used widely in nonlinear systems because it efficiently represents a nonlinear system by a set of linear subsystems. The main feature of the T-S fuzzy model is that the consequents of the fuzzy rules are expressed as analytic functions. The choice of the function depends on its practical applications. Specifically, the T–S fuzzy model is an interpolation method, which can be utilized to describe a complex or nonlinear system that cannot be exactly modelled mathematically. The physical complex system is assumed to exhibit explicit linear or nonlinear dynamics around some operating points. These local models are smoothly aggregated via fuzzy inferences, which lead to the construction of complete system dynamics.

Takagi-Sugeno (T-S) fuzzy controller is used in the feedback path as shown in Fig.1, so that it can change the amount of feedback in order to enhance the system performance and its stability.
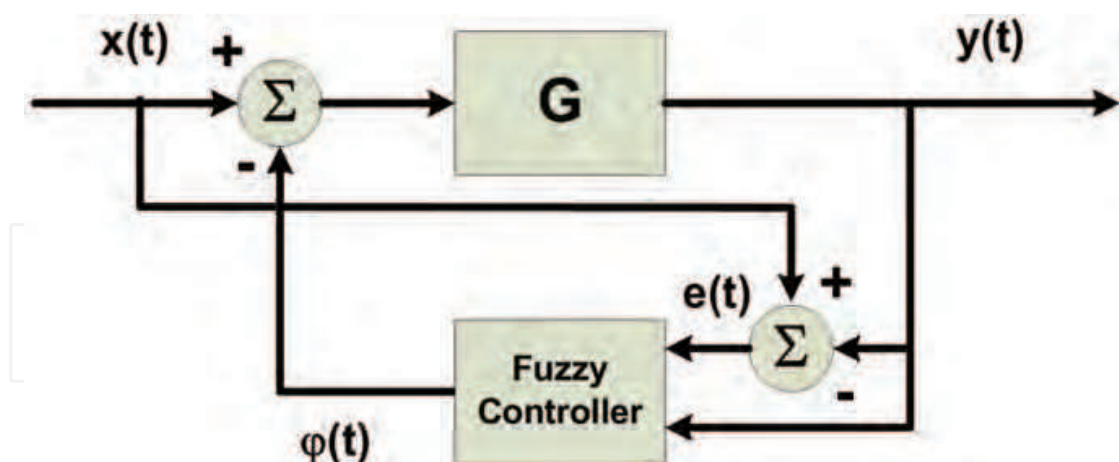

Fig. 1 The proposed System block diagram

The proposed fuzzy controller is a two-input one-output system: the error e(t) and the output y(t) are the controller inputs while the output is the feedback signal $\varphi(t)$. The fuzzy controller uses symmetric, normal and uniformly distributed membership functions for the rule premises as shown in Fig.2(a) and 2(b). Labels have been assigned to every membership function such as NBig (Negative Big) and PBig (Positive Big) etc. Notice that the widths of the membership functions of the input are parameterized by L and h which are used to tune the controller and limited by the physical limitations of the controlled system.
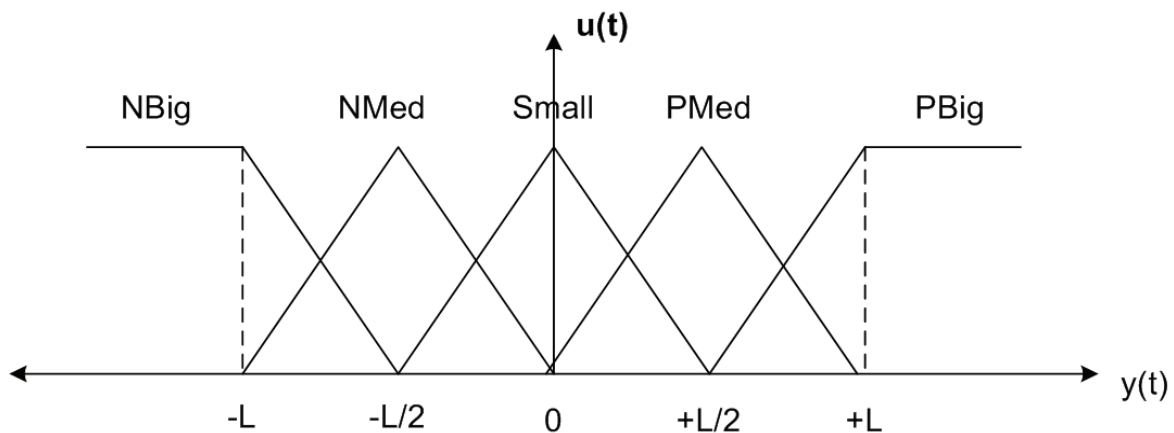
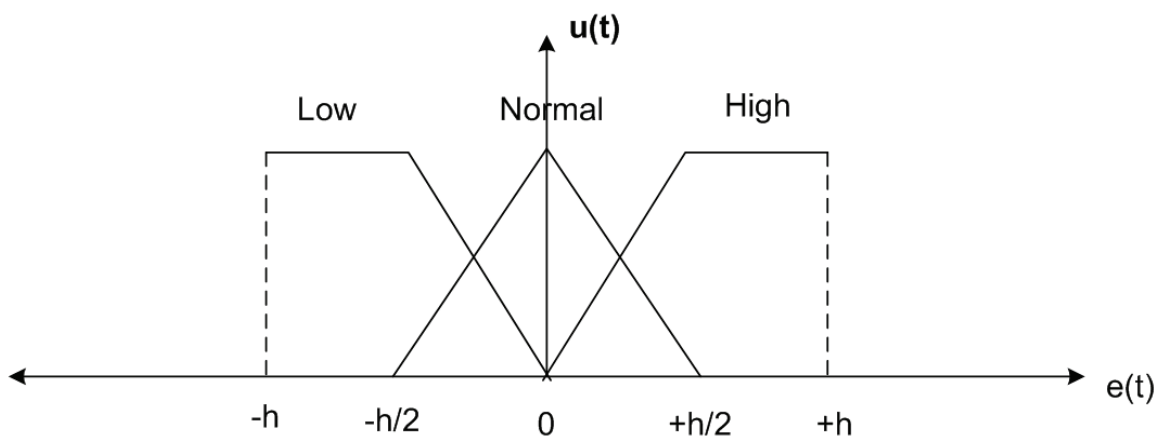Fig. 2 (a) The membership distribution of the 2nd input, open loop output y(t)

Fig. 2 (b) The membership distribution of the 1st input, the error e(t)

While using the T-S fuzzy model (Buckley & Eslami 2002), the consequents of the fuzzy rules are expressed as analytic functions which are linearly dependent on the inputs. In present case, three singleton fuzzy terms are assigned to the output such that the consequent part of the ith rule φ^c_i is a linear function of one input y(t) which can be expressed as:

$$\varphi^c_i(t) = r_i \, M \, y(t) \tag{1}$$

where r_i takes the values -1, 0, 1
                 (depends on the output's fuzzy terms)
         y(t)   is the 2nd input to the controller
         M   is a parameter used to tune the controller.

The fuzzy rules are formulated such that the output is a feedback signal inversely proportional to the error signal as follow:

IF   the error   is   High   THEN $\varphi^c_1 = M \, y(t)$

IF   the error   is   Normal   THEN $\varphi^c_2 = 0$

IF   the error   is   Low   THEN $\varphi^c_3 = - M \, y(t)$

The fuzzy controller is adjusted by changing the values of L, h and M which affect the controller nonlinearity map. Therefore, the fuzzy controller implements these values

equivalent to the saturation parameters of standard saturation nonlinearity (Jenkins & Passino 1999).

Before studying the system stability, a general model of a Sugeno fuzzy controller is defined (Thathachar & Viswanath 1997; Babuska, Roubos et al. 1998; Buckley & Eslami 2002) as follows:

For a two-input T-S fuzzy system; let the system state vector at time t be:

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

where z_1, and z_2 are the state variable of the system at time t.

A T-S fuzzy system is defined by the implications such that:

$$R_i : if \quad (z_1 \ is \ S_1^i \ AND \quad z_2 \ is \ S_2^i) \quad then$$
$$\dot{z} = A_n z + B_n$$

and for the proposed system where B_n is taken as a zero matrix and n = 2 for the two-input system, then:

$$R_i : if \quad (z_1 \ is \ S_1^i \ AND \quad z_2 \ is \ S_2^i) \quad then$$
$$\dot{z} = A_1 z_1 + A_2 z_2$$

for   i = 1 ….. N,

where S_1^i, S_2^i are the fuzzy set corresponding to the state variables z_1, z_2 and R_i.

$A_n = [A_1, A_2]$, are the characteristic matrices which represent the fuzzy system.

However the truth value or weight of the implication R_i at time t denoted by $w_i(z)$ is defined as:

$$w_i(z) = \wedge (\mu_{S_1^i}(z_1), \mu_{S_2^i}(z_2))$$

where

$\mu_S(z)$ is the membership function value of fuzzy set S at position z

$\wedge$ is taken to be the min operator

Then the system state is updated according to (Reznik 1997):

$$\dot{z} = \frac{\sum_{i=1}^{N} w_i(z) A_i z}{\sum_{i=1}^{N} w_i(z)} = \sum_{i=1}^{N} \delta_i(z) A_i z \tag{2}$$

where $\delta_i(z) = \dfrac{w_i(z)}{\sum_{p=1}^{N} w_p(z)}$

However, the consequent part of the proposed system rules is a linear function of only one input y(t) as mentioned in the pervious section, and therefore the output of the fuzzy controller is of the form:

$$\dot{y} = \sum_{i=1}^{N} \delta_i(y) M_i y \tag{3}$$

where N is the number of the rules
               M_i is a parameter used for the ith rule to tune the controller

Notice that Eq. 3 directly depends on the input y(t) and indirectly depends on e(t) which affects the weights δ_i. Thus the proposed system can be redrawn as shown in Fig. 3
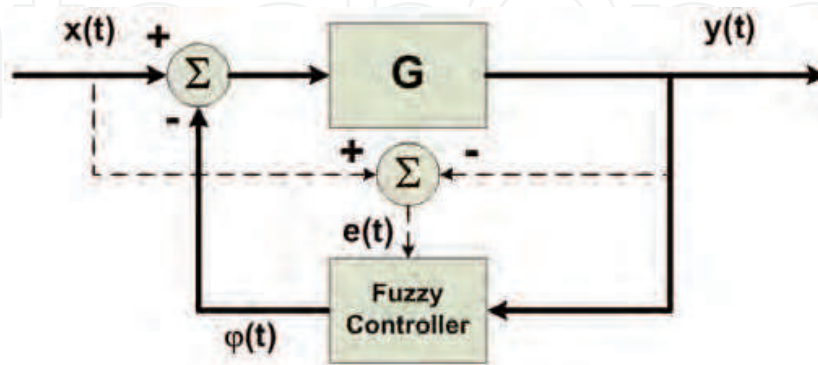

Fig. 3 The equivalent block diagram of the proposed system

The stability analysis of the system considers the system nonlinearities and uses circle criterion theory to ensure stability.

3. Stability analysis using circle criterion

In this section the circle criterion (Ray, Ghosh et al. 1984; Ray & Majumderr 1984; Vidyasagar 1993; Jenkins & Passino 1999) will be used for testing and tuning the controller in order to ensure the system stability and improve its output response. The circle criterion was first used in (Ray, Ghosh et al. 1984; Ray & Majumderr 1984) for stability analysis of fuzzy logic controllers and as a result of its graphical nature; the designer is given a physical feel for the system.

The output of the system given by Eq. 3 can be rewritten as follow:

$$\dot{y} = \sum_{i=1}^{N} \{M_i y - [(1 - \delta_i(y)) M_i y]\} \tag{4}$$

This comprises a separate linear part and nonlinear part denoted as $\varphi(t)$ that can be expressed by (Vidyasagar 1993; Cuesta, Gordillo et al. 1999):

$$\varphi = \sum_{i=1}^{N} [(1 - \delta_i(y)) M_i y] \tag{5}$$

As a result a T-S fuzzy system can be represented according to a LUR'E system (Vidyasagar 1993; Cuesta, Gordillo et al. 1999). Consider a closed loop system, Fig. 4, given a linear time-invariant part G (a linear representation of the process to be controlled) with a nonlinear feedback part φ(t) (represent a fuzzy controller).

The function φ(t) represents memoryless, time varying nonlinearity with:

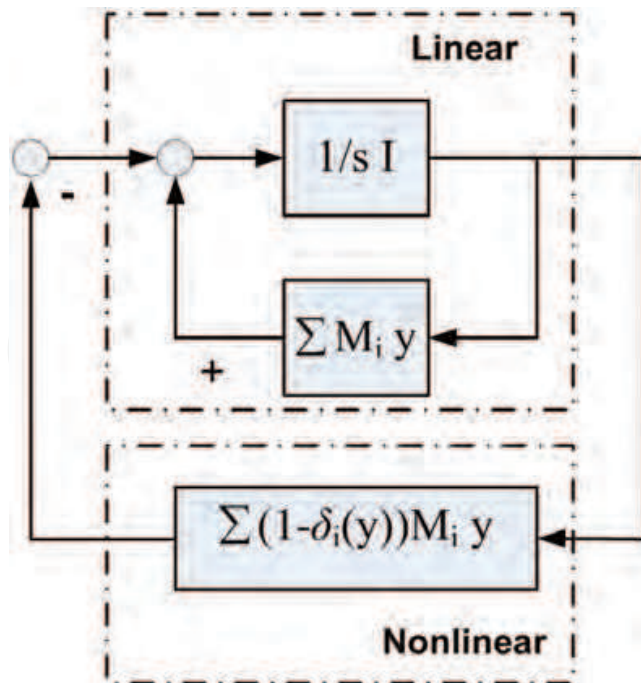$$\varphi : [0, \infty) \times \Re \to \Re$$

Fig. 4 T-S Fuzzy System according to the structure of the problem of LUR'E

If φ is bounded within a certain region as shown in Fig. 5 such that there exist:

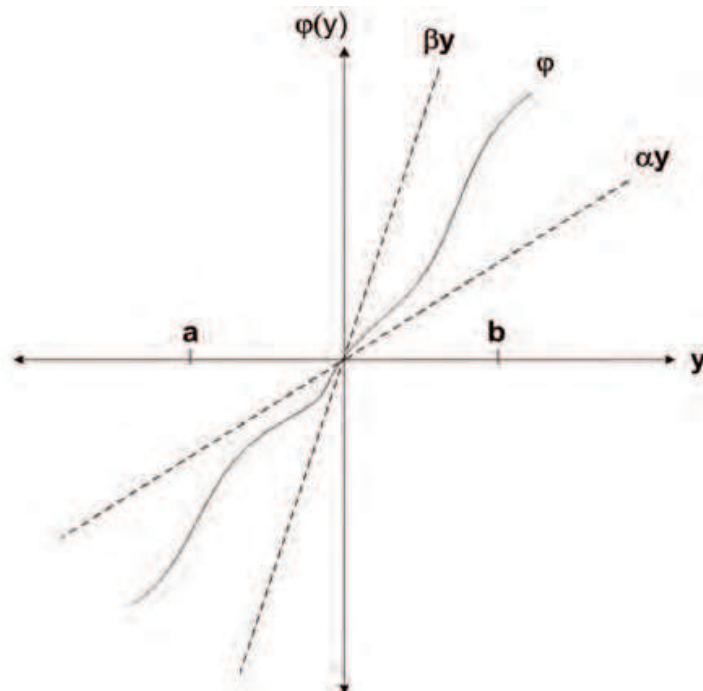α, β, a, b, (β>α, a<0<b) for which:

$$\alpha y \le \varphi(y) \le \beta y \tag{6}$$

Fig. 5 Sector Bounded Nonlinearity

for all t ≥ 0 and all y ∈ [a, b] then: φ(y) is a "Sector Nonlinearity":

If $\alpha y \leq \varphi(y) \leq \beta y$ is true for all y $\in$ (-∞,∞) then the sector condition holds globally and the system is "absolutely stable". The idea is that no detailed information about nonlinearity is assumed, all that known it is that φ satisfies this condition (Vidyasagar 1993).

Let D(α, β) denote the closed disk in the complex plane centred at $-\dfrac{(\alpha+\beta)}{2\alpha\beta}$, with radius

$\dfrac{|\alpha-\beta|}{2|\alpha\beta|}$ and the diameter is the line segment connecting the points $\dfrac{-1}{\alpha}+j0$ and $\dfrac{-1}{\beta}+j0$.

The circle criterion states that when φ satisfies the sector condition Eq.6 the system in Fig.3 is absolutely stable if one of following conditions are met (Vidyasagar 1993):

- If 0 < α < β, the Nyquist Plot of G(jw) is bounded away from the disk D(α, β) and encircles it m times in the counter clockwise direction where m is the number of poles of G(s) in the open right half plane(RHP).
- If 0 = α < β, G(s) is Hurwitz (poles in the open LHP) and the Nyquist Plot of G(jw) lies to the right of the line $s = \dfrac{-1}{\beta}$.

- If α < 0 < β, G(s) is Hurwitz and Nyquist Plot of G(jw) lies in the interior of the disk D(α, β) and is bounded away from the circumference of D(α, β).

For the fuzzy controller represented by Eq. 2, we are interested in the first two conditions (Ray & Majumderr 1984), and it can be sector bounded in the same manner (Jenkins & Passino 1999) as described next.

Consider the fuzzy controller as a nonlinearity φ and assume that there exist a sector (α, β) in which φ lies, then use the circle criterion to test the stability. Simply, using the Nyquist plot, the sector bounded nonlinearity of the fuzzy logic controller will degenerate, depending on its slope a that is always zero (Jenkins & Passino 1999) and the disk to the

straight line passing through $\dfrac{-1}{\beta}$ and parallel to the imaginary axis as shown in Fig.6 In

such case the stability criteria will be modified as follows (Vidyasagar 1993):

Definition: A single-input single-output (SISO) system will be globally and asymptotically stable provided the complete Nyquist locus of its transfer function does not enter the

forbidden region left to the line passing through $\dfrac{-1}{\beta}$ in an anticlockwise direction as shown

in Fig. 6.

The fuzzy controller is tuned until its parameters lie in the bounded sector, so that the fuzzy system nonlinearity is bounded in this sector. In fact, even if the function φ is approximately linear, the saturation outside this region causes φ to be always nonlinear.

From the above discussion, we conclude that to ensure stability for a closed loop system with known transfer function or nonlinearity sector, one can add a fuzzy block (stabilizer) in the feedback loop tuned in the manner described above and under the condition that the stabilizer block is faster than the controlled system. This concept is used to enhance the performance of existing control systems especially for systems controlled using fuzzy controller in the forward loop. In such cases the feedback fuzzy stabilizer can be integrated in the main fuzzy controller as explained in the next section.
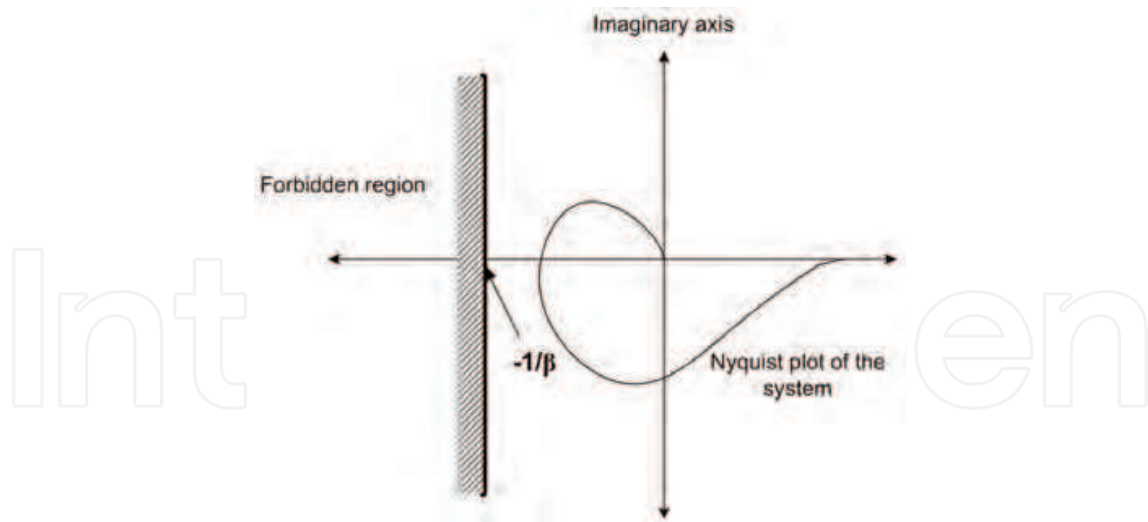
Fig. 6 Nyquist plot with fuzzy feedback system (Ray & Majumderr 1984).

4. Self stabilized fuzzy controller

Figure 7 comprises a plant controlled by a SISO fuzzy controller. In order to guarantee the system stability, a fuzzy stabilizer has been added in the feedback path.
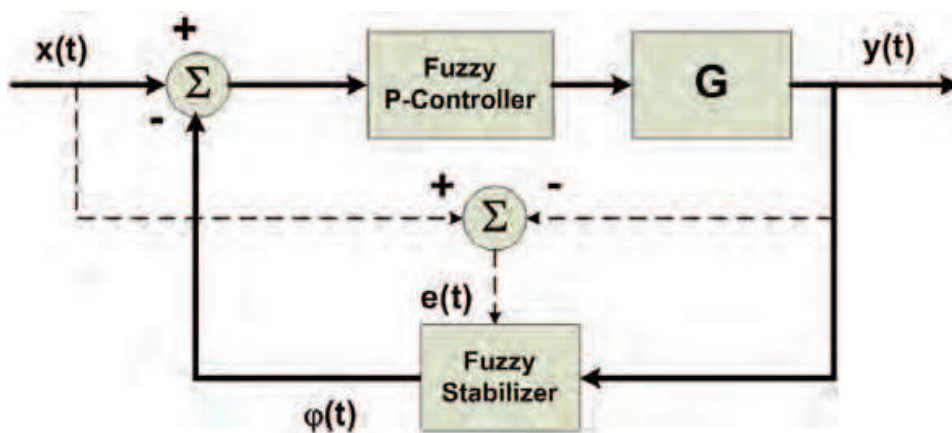

Fig. 7 Block Diagram of the system with Fuzzy-P controller

Only, minor changes are necessary to the above analysis in order to include the SISO fuzzy controller nonlinearities if these have not been included in the previously calculated sector. As a result, the fuzzy stabilizer will be retuned to the new sector which will be the minimum intersection between the fuzzy controller nonlinearity sector and the sector results using the circle criterion. This is understandable as the fuzzy controller represents an odd function (Reznik 1997; Jenkins & Passino 1999) (i.e. $\varphi(-y) = -\varphi(y)$) , so that fuzzy controller can be in the feedback path rather than the feed forward path. Therefore, the dominant nonlinear sector will be the minimum sector. Consequently from analysis, the feedback stabilizer can be built in each fuzzy controller to improve its performance by adding an extra input and modifying the original fuzzy rule base by adding the stabilization rules.

Generally, there are many types of fuzzy reasoning that can be employed in fuzzy control applications, the most commonly used types are Mamdani and Takagi-Sugeno (T-S) type. For Mamdani fuzzy systems (Farinwata, Filev et al. 2000), the same structure can be used

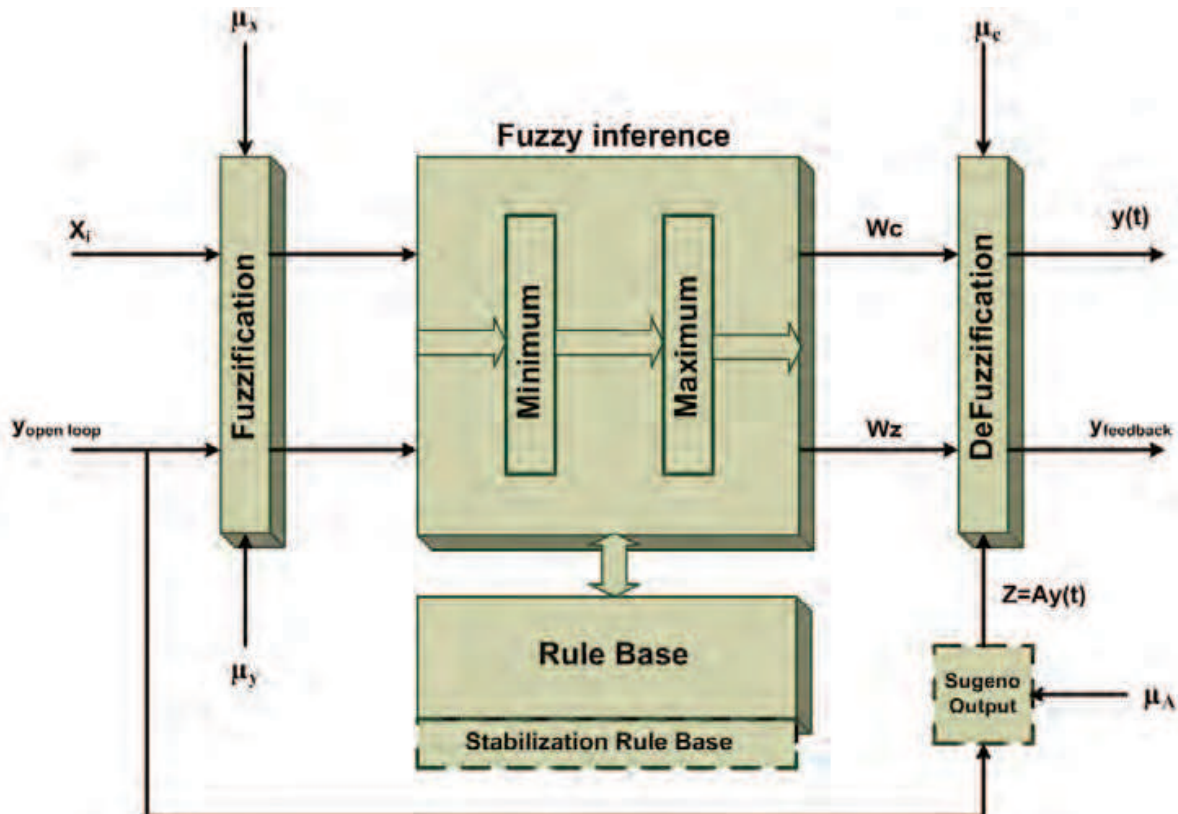except for the addition of another input y(t) and three extra rules to the rule base as shown in Fig. 8.

Fig. 8 The modification to the fuzzy system structure

Where μ_x, μ_c are the input and output fuzzy sets for Mamdani fuzzy system
        μ_y, μ_A are the input and output fuzzy sets for fuzzy stabilizer system
Consequently, less modification is required for T-S type fuzzy systems.
The main reason for integrating the stabilizer into the normal structure of fuzzy controllers is to make them suitable for hardware and software implementation. The same design of the circuits or algorithms will be used without significant modifications.

5. Examples and simulation results

A plant with transfer function:

$$G(s) = \frac{400}{s^3 + 10.4s^2 + 8s + 40},$$

is used to demonstrate the performance of fuzzy stabilizer. The Nyquist plot of G(jw) is shown in Fig. 9.
The system is unstable and has closed loop poles at -12.6 and 1.08± j 5.82, with a gain margin of -19.3dB. If we consider the fuzzy stabilizer as a nonlinearity φ as shown in Fig. 5, then the disk D(α, β) is the line segment connecting the points $\frac{-1}{\alpha} + j0$ and $\frac{-1}{\beta} + j0$. Applying the Circle Criterion and because $a = 0$ the second condition will be used. To find a sector (a, β)

in which φ lies, the system Nyquist plot Fig. 9 is analyzed. The Nyquist plot does not satisfy the second condition as it intersects with the line drawn at $\frac{-1}{\beta} = -9.259$. In order to meet the second condition of the theory the line drawn at $\frac{-1}{\beta}$ will be moved to be at $\frac{-1}{\beta} = -27.5$ such that the Nyquist plot lies to the right of it. As a result, the fuzzy controller will be tuned by choosing M, and L such that its nonlinearities lies in the sector (0,0.036).
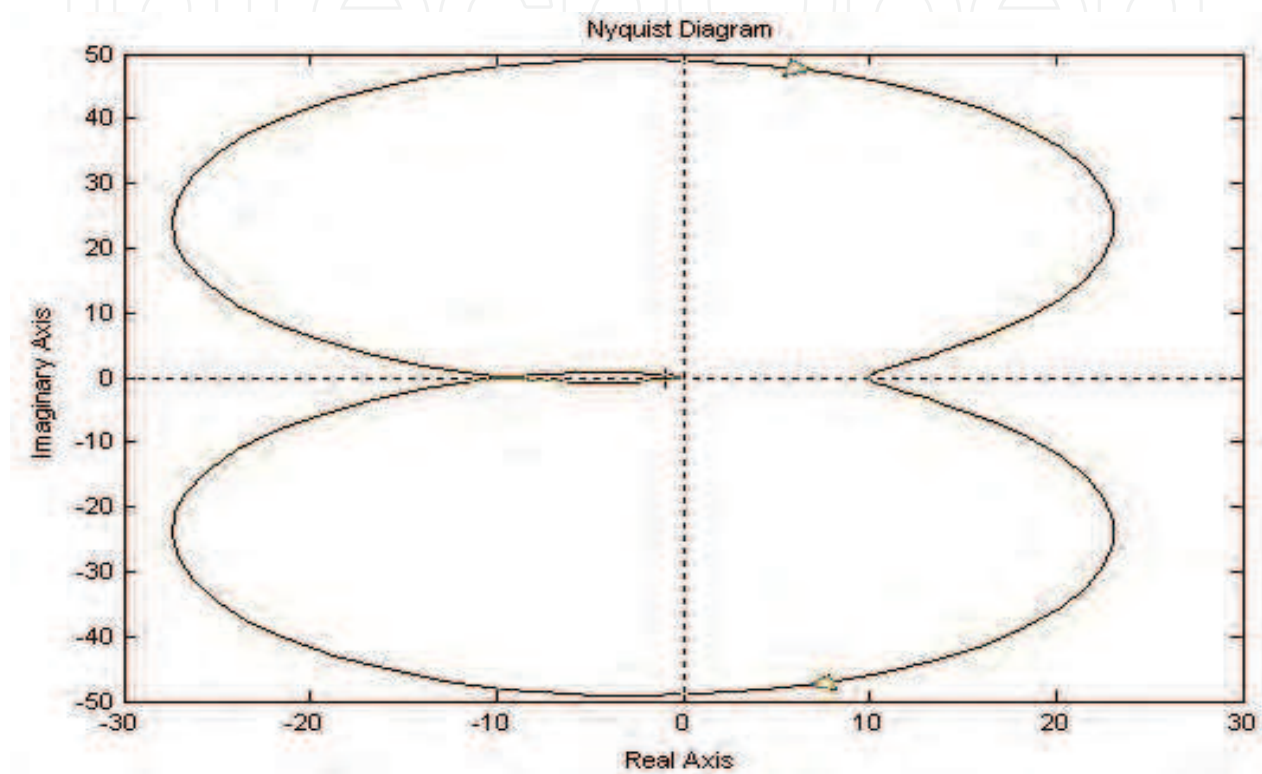

Fig. 9 The plant Nyquist plot

In order to satisfy the circle criterion condition, the ratio M/L will be kept less than β (i.e M/L < 0.036) by choosing M = 0.68 and L = 20 .

A traditional fuzzy like proportional controller (Reznik 1997) is used to control the system with a normal feedback loop as we saw in Fig. 7 in order that a comparison can be made between the results with and without a fuzzy stabilizer in the feedback loop. In order to retune the fuzzy stabilizer, the fuzzy P-controller has a ratio M_c/L_c or $\beta_c = 1$.

However $\beta = 0.036$ for the plant, and therefore the minimum sector for the stabilizer to be tuned is: $(a, \beta) = (0, 0.036)$.

The system step response (solid line) results with and without the use of the stabilizer (dashed line) are shown Fig. 10. The results shows that the system with the fuzzy P-controller in Fig. 7 yields an unstable output (dashed line) while the use of the stabilizer produces a stable output.

The approach described has provided a quick and easy stabilization process which can allow designers to fine tune their controller's performance without at the same time, being worried about stability issues.

In Fig. 11 (a), and (b), the step responses for different systems, according to the setup in Fig. 3, are shown. The simulations show the tested system for a normal feedback without the stabilizer and with adding the stabilizer in the feedback loop as in illustrated in Fig. 3.
Using the same algorithm given a transfer function, a nonlinearity sector and the tuned values of M and L of the fuzzy stabilizer, the stabilizer has been tuned.
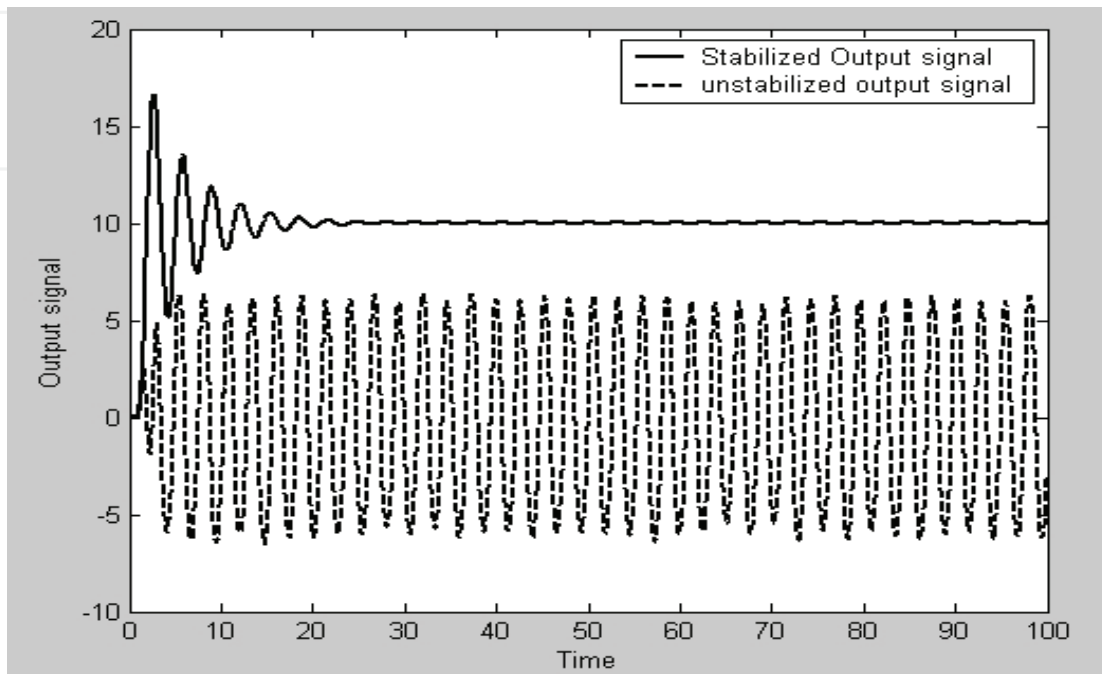

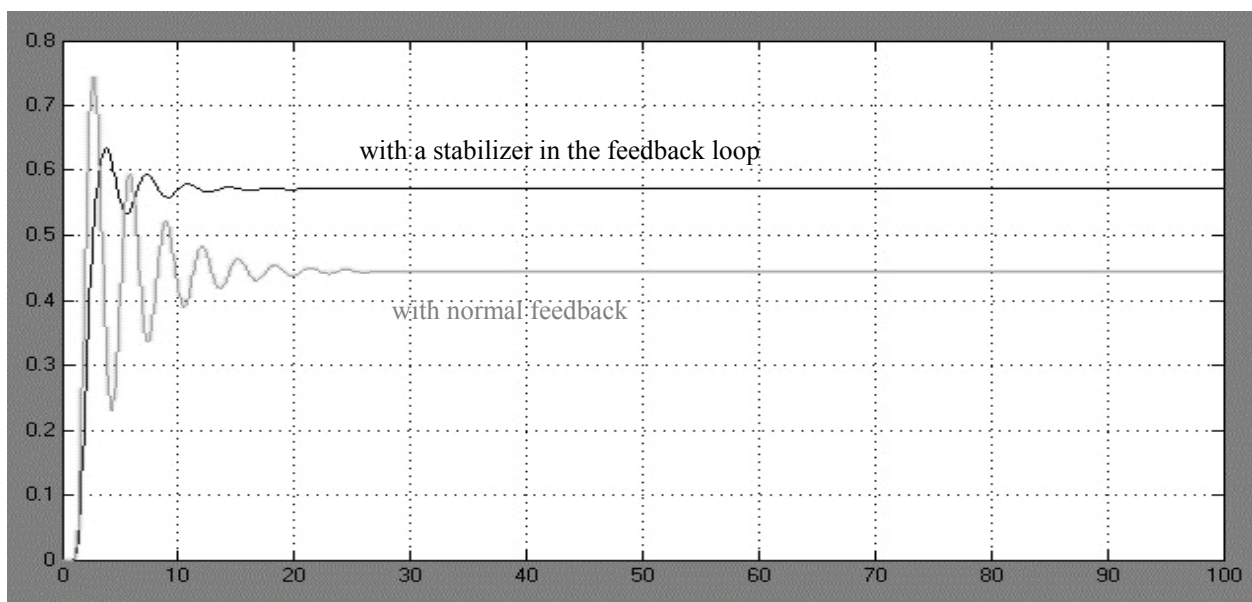Fig. 10 The simulated step response of the two compared systems

Fig. 11(a) The step Response of the controller with following parameters (Black curve):

$$G(s) = \frac{12}{s^3 + 7s^2 + 7s + 15} \text{ , } (\alpha, \beta) = (0, 0.3), \text{ L=1, M= 0.3}$$
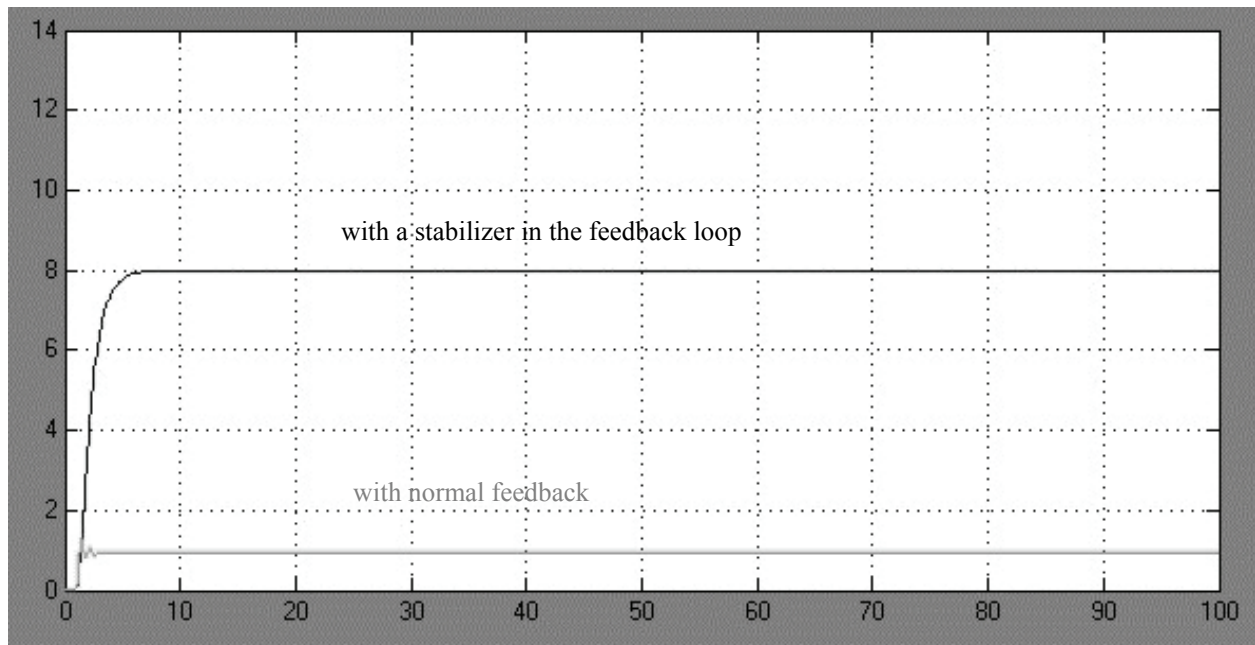
Fig. 11(b) The step Response of the controller with following parameters (Black curve):

$$G(s) = \frac{40}{s^2 + 4s + 3} \ , (\alpha, \beta) = (0, 1.3), \ L=1, M = 1.3$$

6. Extension to MIMO fuzzy systems

The stability analysis of multi-inputs multi-outputs (MIMO) is a nontrivial task due to the complexity of the system (Safonov 1980), however, many algorithms have been proposed to tackle the problem; K. Ray and D. Majumder (Ray & Majumderr 1984) extended their approach of using circle criteria to MIMO systems but restricted the result to square systems only. The conicity theory has been used by others (Kang, Kwon et al. 1998; Cuesta, Gordillo et al. 1999; Cuesta & Ollero 2004) to study the stability of MIMO fuzzy systems but it suffers from the nontrivial problem of determining the candidate centre. Linear matrix inequalities (LMI) technique is also used (Wang, Tanaka et al. 1996; Lam & Seneviratne 2007) but has the disadvantage of high number of LMI used which make the analysis more complicated (Cuesta, Gordillo et al. 1999). The description function is also used to study the stability of MIMO systems (Abdelnour, Cheung et al. 1993; Aracil & Gordillo 2004).

6.1 Stability analysis of open loop MIMO systems

In order to extend the proposed approach fuzzy stabilizer to MIMO (Multi-input Multi-output) systems, an additively decomposition technique (Ying 1996) is used. According to the structure of the classical problem of LUR'E (Vidyasagar 1993; Cuesta, Gordillo et al. 1999) shown in Fig. 4 , and referring to the analysis in section 3, a T-S fuzzy system can be represented as linear and nonlinear part as follows:

Consider a T-S fuzzy system with N rules (Cuesta, Gordillo et al. 1999):

$$R_r \ : \ if \ \ (z_1 \ \ is \ \ S_1^i \ \ AND \ \ \ z_2 \ is \ \ S_2^j) \ \ \ \ then$$
$$\dot{z} \ = \ M_{ij} \ z$$

with:

$$M_{ij} = \begin{bmatrix} a_i & b_j \\ c_i & d_j \end{bmatrix} \text{ and } z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

for   r = 1 ….. m×n, where

S^i_1 , S^i_2 are the fuzzy membership function corresponding to the state variables

z_1, z_2 ,   which represented by linguistic terms with membership functions such that:

$$\mu_{S^1_p}(z_1 = 0) = 1, \quad \mu_{S^1_i}(z_1 = 0) = 0$$

$$i \neq p, i = 1, …, m$$

and

$$\mu_{S^2_q}(z_2 = 0) = 1, \quad \mu_{S^2_j}(z_2 = 0) = 0$$

$$j \neq q, j = 1, …, n$$

and $M_{ij} \in R^{2 \times 2}$, is the characteristic matrices which represents the fuzzy system.

Similar to the analysis in section 2, the system state is updated according to:

$$\dot{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \delta_{ij}(z) M_{ij} z \tag{7}$$

where $$\delta_{ij}(z) = \frac{w_{ij}(z)}{\sum_{k=1}^{m} \sum_{p=1}^{n} w_{kp}(z)}$$

and $w_{ij}(z)$ is the truth value or weight of the implication R_{ij} at time t

Then Eq. 7 can be rewritten as:

$$\dot{z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ M_{ij} z - \left[(1 - \delta_{ij}(z)) M_{ij} z \right] \right\} \tag{8}$$

Eq. 8 shows the system has been split into linear and nonlinear part, Fig. 4. Notice that the first column of M_{ij} depends on i while the second column depends on j Hence, the resulting nonlinear part φ(z) such that:

$$\varphi(z) = \sum_{i=1}^{m} \sum_{j=1}^{n} (1 - \delta_{ij}(z)) M_{ij} z \tag{9}$$

is additively decomposable (Cuesta, Gordillo et al. 1999), that is:

$$\varphi(z) = \varphi(z_1, z_2) = \varphi(z_1, 0) + \varphi(0, z_2) \tag{10}$$

(see (Cuesta, Gordillo et al. 1999) for the proof)

Eq. 10 implies that the nonlinear part φ is additively decomposable, and therefore techniques used for stability analysis of SISO system can be used to stabilize the multi-input multi-output systems. This can be done by adding a number of small fuzzy systems equal to the number of the output variables in the feedback loop of the MIMO system for each input variable as shown in Fig. 12. In this way all the nonlinearities of the fuzzy system can be included within a bounded sector.
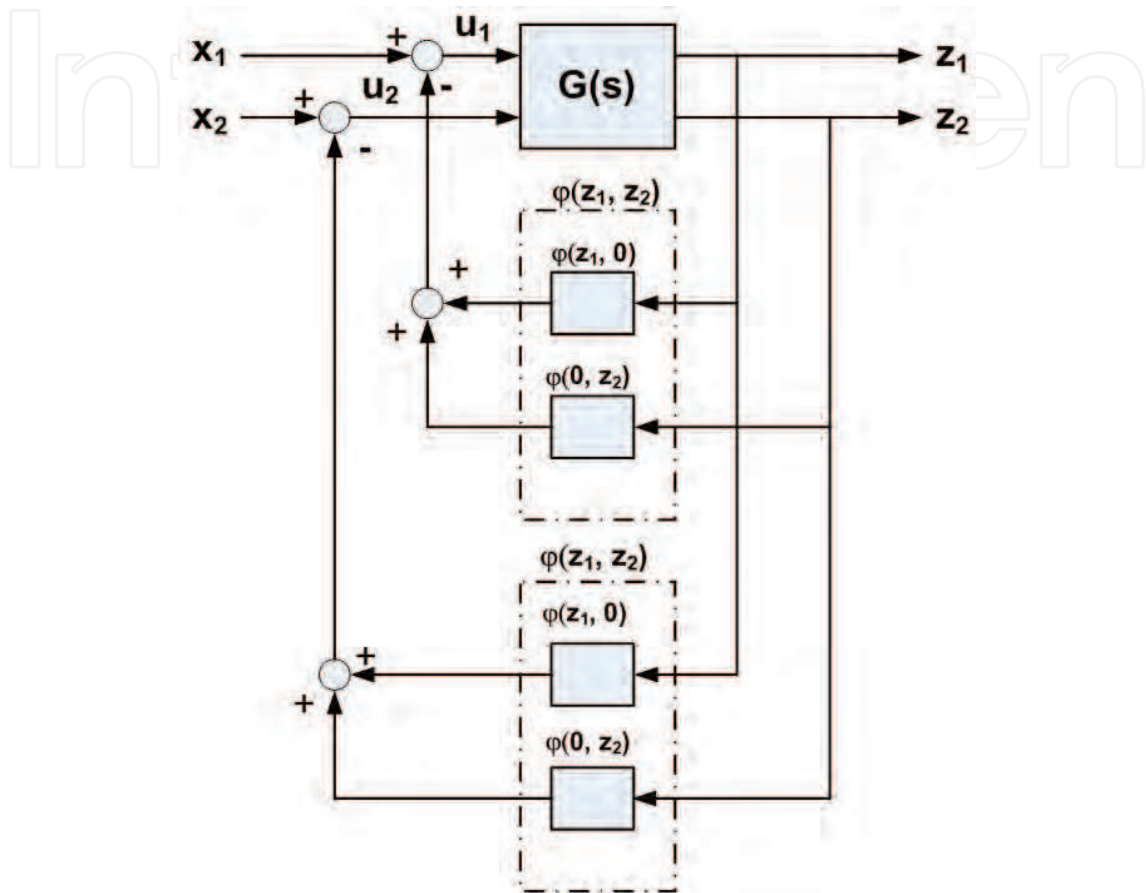

Fig. 12 The proposed MIMO fuzzy feedback system

6.2 Stability analysis of closed loop MIMO system

A simple stability analysis for closed loop system is shown in Fig. 13 (a). In this system the proposed fuzzy stabiliser is placed on each feedback loop for each input as shown in Fig. 13(b). That includes all the nonlinearities of the system.
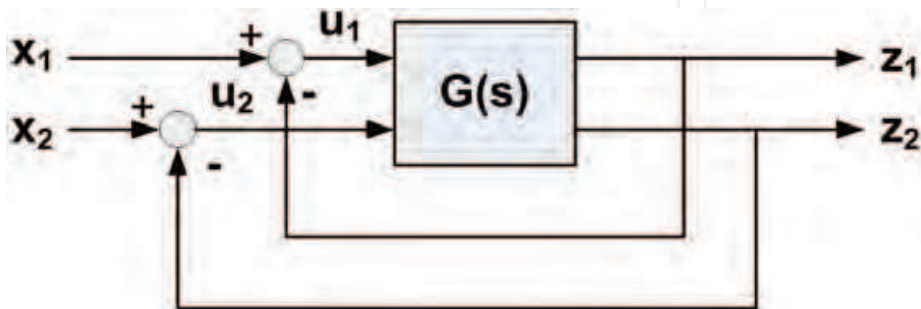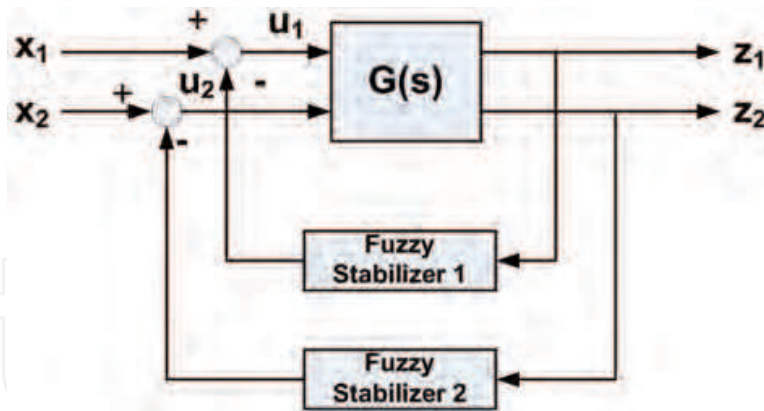

Fig. 13.(a) MIMO closed loop system

Fig. 13.(b) MIMO closed loop system with fuzzy stabilizers

6.3 Simulation example

Consider a MIMO system with a state space representation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -7 & -7 & -50 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

In our problem we will find a transfer function of the model of the form:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where

$$G_{11} = \frac{s^2}{s^3 + 7s^2 + 7s + 50}$$

$$G_{21} = \frac{-s}{s^3 + 7s^2 + 7s + 50}$$

$$G_{12} = \frac{7s + 50}{s^3 + 7s^2 + 7s + 50}$$

$$G_{22} = \frac{s^2 + 7s}{s^3 + 7s^2 + 7s + 50}$$

Using the analysis described in section 5 and by aid of Nyquist plot of the system as shown in Fig. 14 we can determine $-\dfrac{1}{\beta} = -6.45$, as a result M/L ≤ 0.155.

Note that, for all the components of the system (G11, G12, G21, and G22), the denominator in each case remains the same, since it holds the key to the system stability.
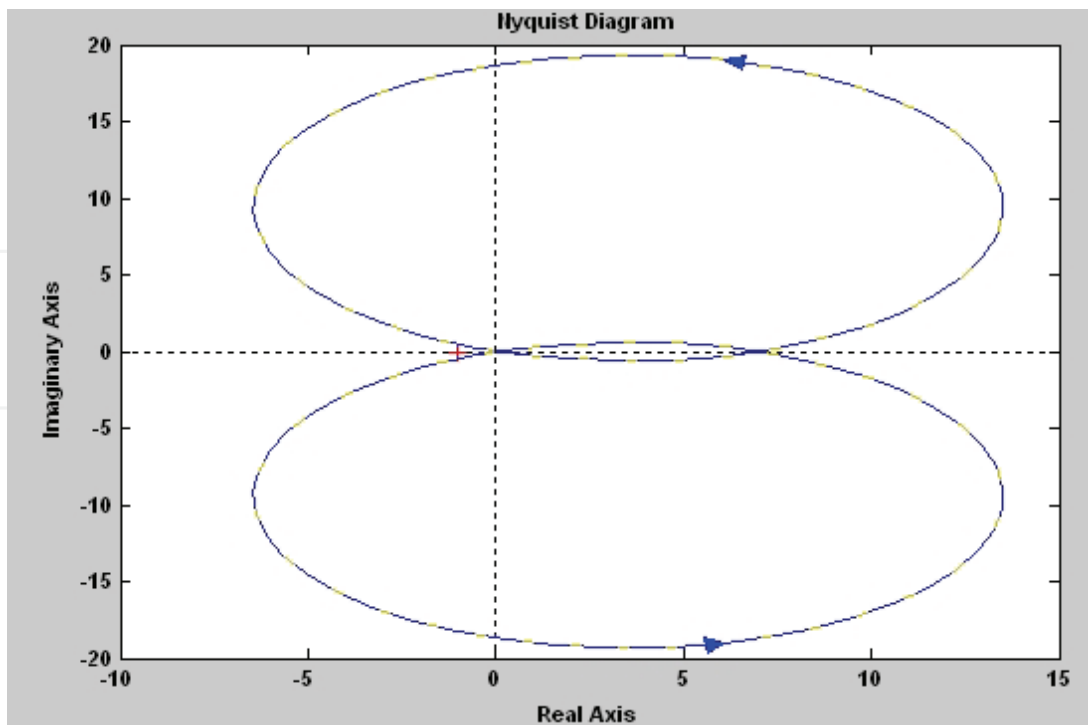
Fig. 14 The Nyquist plot of the simulated system

The outputs of the open loop system show the system instability as shown in Fig. 15.
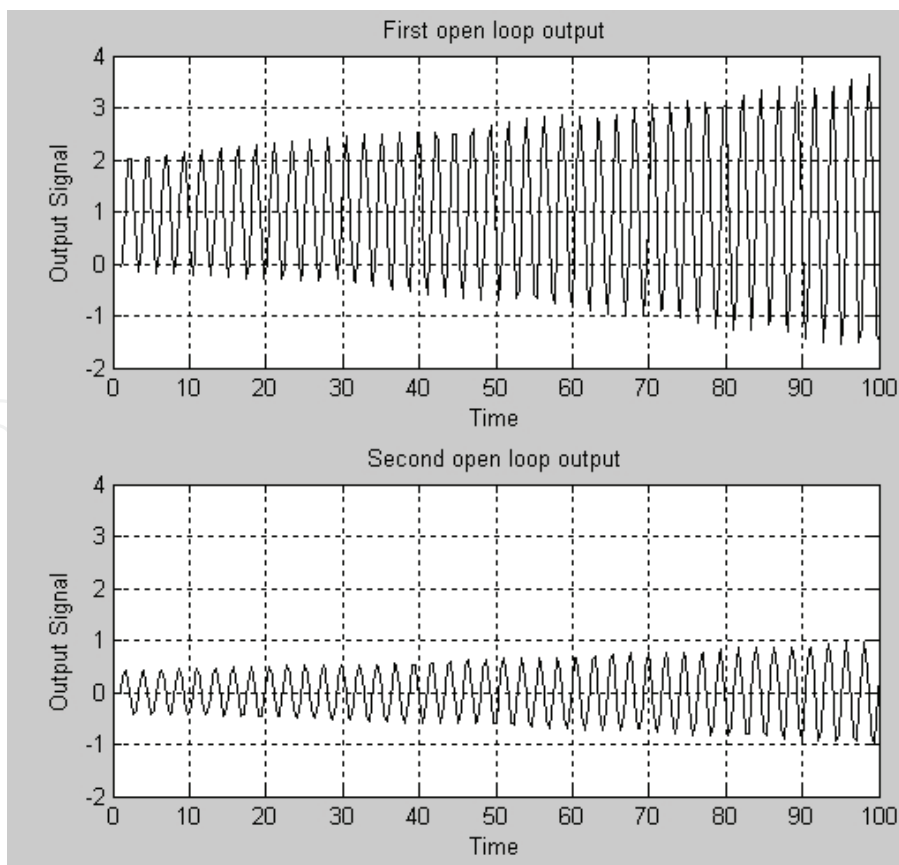

Fig. 15 The open loop response of the simulated system

When the fuzzy stabilizers are added to the system according to Fig. 12 and the fuzzy parameters are set such that the ratio M/L ≤ 0.007 is kept the same as follow:

Stabilizer (1)   M11= 3.1   , L11= 25
Stabilizer (2)   M12= 1.8   , L12= 12
Stabilizer (3)   M21= 0.031   , L21= 0.25
Stabilizer (4)   M22= 0.018   , L22= 0.12

The simulation results in Fig. 16 show the output of the stabilized system.
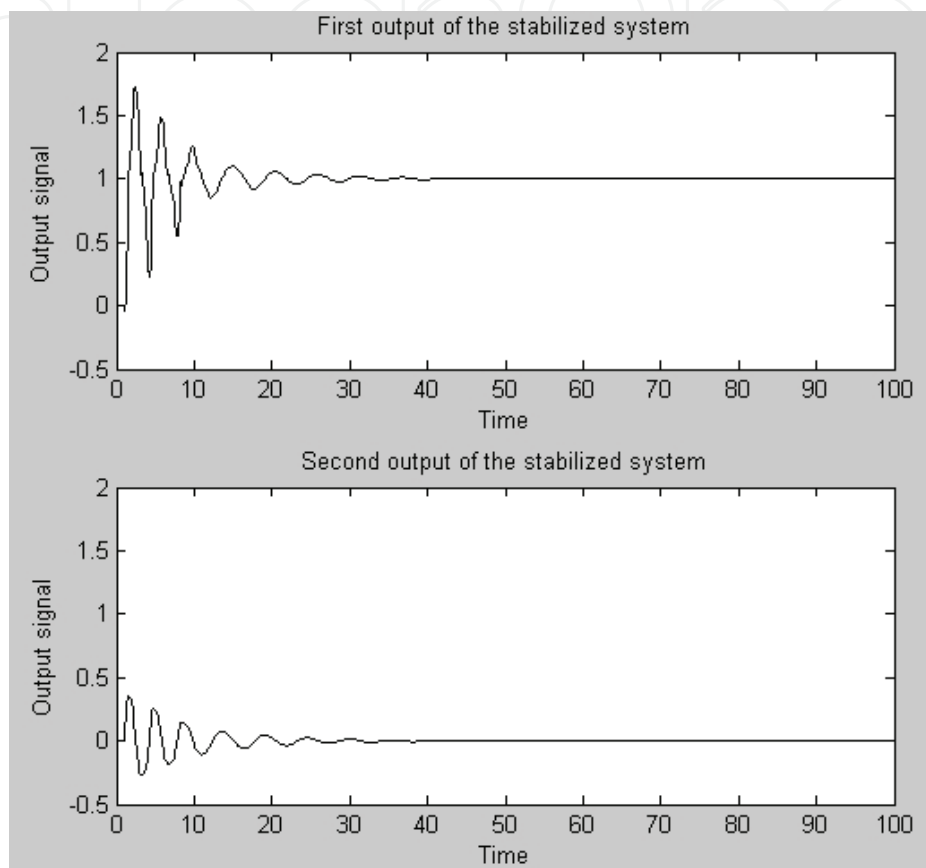

Fig. 16 The outputs of the stabilized simulated system

The proposed technique has the advantage of keeping the system stable even if the system nonlinearities have been changed provided that they still remain within the bounded sector proposed.

7. Conclusion

This chapter presented a practical approach to stabilize fuzzy systems based on adaptive nonlinear feedback using a fuzzy stabilizer in the feedback loop. For this we needed to identify the nonlinearity range of the system. The fuzzy stabilizer is tuned so that the system nonlinearities lie in a bounded sector as delivered by using the circle criterion theory. Because of circle criterion's graphical nature; the designer is given a physical feel for the system. The concept has been used to ensure stability of a car-like robot controller. In addition, the idea has been extended to stabilize MIMO systems based on the additively decomposition technique.

The advantage of the proposed approach is the simplicity of the design procedure especially for the MIMO systems analysis and implementation. The use of the fuzzy system to control the feedback loop using its approximate reasoning algorithm gives a good opportunity to handle the practical system uncertainty. The approach described have provided a quick and easy stabilization process which can allow designers to fine tune their controllers performance without at the same time, worrying about stability issues It is also shown that the fuzzy stabilizer can be integrated, with small modifications, in any fuzzy controller to enhance its stability. As a result it is suitable for hardware implementation or even to modify existence software and hardware design if required to ensure system stability.

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