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# Principal Screws and Full-Scale Feasible Instantaneous Motions of Some 3-DOF Parallel Manipulators

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## 1. Introduction

With the development of parallel robot, various lower-mobility parallel mechanisms are proposed, especially, the 3-DOF parallel mechanisms have interested many researchers. Hunt (1983) proposed the first 3-DOF 3-RPS parallel mechanism. Lee and Shah (1988) addressed various possible applications of the mechanism. Waldron et al. (1989) studied an ARTISAN manipulator. Clavel (1988) proposed the DELTA. Gosselin and Angeles (1988, 1989) proposed an optimum kinematic design for planar and spherical 3-DOF parallel manipulators. Song (1995) studied a force-compensating device based on 3-RPS mechanism. Huang and Fang (1996) proposed some novel 3-DOF parallel mechanisms. Di Gregorio (1999) discussed the influence of flexibility of a 3-DOF parallel mechanism on the platform motion.

We know that a primary and basic step towards understanding a mechanism is to find all the feasible instantaneous motions or twists it can produce. In other words, it needs to determine both the range of the twist pitches and the distribution of the twist axes. It is important to correctly use a robot manipulator and plan its trajectory.

In robotics practice, for a six-DOF manipulator its end-effector has infinite moving possibility and can undergo any given twist in 3D space. For a lower-mobility manipulator it also has infinite moving possibility, however, it is clear that there are many motions impossible to realize. To find all the feasible instantaneous motions of a lower-mobility serial robot is easier than that of the 6-DOF one. The possible twists of a lower-mobility serial robot are obtained only by the linear combination of its joint screws. Nevertheless, for lower-mobility parallel mechanism, it is extraordinary difficult. To solve the problem we use the screw theory.

One hundred years ago Ball (1900) published his classical work on screw theory. Hunt (1978) further developed screw theory. He discussed all the screw systems. The screw systems were distinguished as general and special cases basically according to the pitches of principal screws. Gibson & Hunt (1990 a & b) classified first-order, second-order and third-order screw system further by means of projective geometry and gave the planar representation of the general three-system. Any screw motion of a 3-DOF rigid body can be expressed by a linear combination of its three principal screws in the three-system. The spatial distribution of axes of all the screws of the screw system in three-dimension space is

regular. For example, all the screws of the second-order screw system lie on a cylindroid. For a third-order screw system or three-system, all the screws with the same pitch may lie on a hyperboloid of one sheet. The cylindroid or hyperboloid depicts the distributions of the positions and orientations of all screw axes of that screw system.

The key to determine both the range of the twist pitches and the distribution of the twist axes in three-dimension space is to get principal screws of the screw system. The principal screw is a very important concept in screw theory. After the principal screws are obtained, it is easy to know all possible motions of the mechanisms at a the given instant. In a second-order screw system there are two principal screws. For a third-order screw system, there exist three principal screws. The third-order screw system is the most important and complicated one.

The twist screw system of a 3-DOF parallel mechanism is a third-order screw system. To study the mechanism, we need to determine its three principal screws. Parkin (1990) specified the principal screws of the three-system from three given screws by adopting the mutual moment operation. Tsai & Lee (1993) studied the principal screws from three known screws by means of eigenvector. Zhang & Xu (1998) constructed the principal screws from three known screws by using algebraic method.

From another point of view, we put forward a directly and simply analytical method for identifying the principal screws for a 3-DOF parallel mechanism and obtained a full-scale feasible instantaneous motion of that mechanism. Fang & Huang (1998) firstly established the important relationship between the principal screws and Jacobian matrix of the 3-DOF mechanism, and identified the principal screws of the third-order screw system using the quadratic equation degenerating theory. After that, based on the relationship between the pitch/axis and the Jacobian matrix of the mechanism two equations are obtained. Then, another simpler and more effective principle (Huang & Wang 2001), the quadric degenerating theory, was further proposed for identifying the principal screws. For applying the principle to lower-mobility parallel mechanisms, corresponding two  $3 \times 3$  Jacobian matrices are needed to establish firstly. This can be realized by using the imaginary-mechanism method proposed by Yan & Huang (1985) and developed by Huang & Wang (1992).

In this Chapter two typical examples are also discussed. One is a 3-DOF 3-RPS parallel manipulator (Huang & Wang, 2002). Another example is a special 3-UPU parallel mechanism (Huang, Li & Zuo, 2004). The analysis discovers this mechanism has some interesting and exceptional characteristics. All above analysis are important for enriching the mechanism theory and beneficial the mechanical design of the similar mechanical system.

## 2. Principal screws

The principal screw principle may be used to study the feasible instant motion. Its important merit is that it can illustrate a full-scale feasible instantaneous motion of a mechanism at any given configuration.

### 2.1 The screw representation

In screw theory (Ball, 1900; Hunt, 1978 & Huang, et al, 2006), a straight line in 3D space can be expressed by two vectors,  $\mathbf{S}$  and  $S_0$ . Their dual combination is called a line vector,  $(\mathbf{S}, S_0)$ . The line vector can be expressed as

$$\mathcal{S} = (\mathbf{S}; \mathbf{S}_0) = (\mathbf{S}; \mathbf{r} \times \mathbf{S}) = (l \ m \ n; p \ q \ r)$$

where  $\mathbf{S}$  is a unit vector along the straight line;  $l$ ,  $m$ , and  $n$  are three direction cosines of  $\mathbf{S}$ ;  $p$ ,  $q$ , and  $r$  are the three elements of the cross product of  $\mathbf{r}$  and  $\mathbf{S}$ ;  $\mathbf{r}$  is a position vector of any point on the line or the line vector.  $(\mathbf{S}; \mathbf{S}_0)$  is also called Plücker coordinates of the line vector and it consists of six components in total. For a line vector we have  $\mathbf{S} \cdot \mathbf{S}_0 = 0$ . When  $\mathbf{S} \cdot \mathbf{S} = 1$ , it is a unit line vector. When  $\mathbf{S}_0 = \mathbf{0}$ , the line vector,  $(\mathbf{S}; \mathbf{0})$ , passes through the origin point.

When  $\mathbf{S} \cdot \mathbf{S}_0 \neq 0$ , it is defined as a screw

$$\mathcal{S} = (\mathbf{S}; \mathbf{S}_0) = (\mathbf{S}; \mathbf{r} \times \mathbf{S} + h\mathbf{S}) \quad (1)$$

When  $\mathbf{S} \cdot \mathbf{S} = 1$ , it is a unit screw. The pitch of the screw is

$$h = \mathbf{S} \cdot \mathbf{S}_0 / \mathbf{S} \cdot \mathbf{S}$$

If the pitch of a screw is equal to zero, the screw degenerates into a line vector. In other words, a unit screw with zero-pitch ( $h = 0$ ) is a line vector. The line vector can be used to express a revolute motion or a revolute pair in kinematics, or a unit force along the line in statics. If the pitch of a screw goes to infinity,  $h = \infty$ , the screw is expressed as

$$\mathcal{S} = (\mathbf{0}; \mathbf{S}) = (0 \ 0 \ 0; l \ m \ n)$$

and called a couple in screw theory. That means a unit screw with infinity-pitch,  $h = \infty$ , is a couple. The couple can be used to express a translation motion or a prismatic pair in kinematics, or a couple in statics.  $\mathbf{S}$  is its direction cosine.

Both the revolute pair and prismatic pair are the single-DOF kinematic pair. The multi-DOF kinematic pair, such as cylindrical pair, universal joint or spherical pair, can be considered as the combination of some single-DOF kinematic pairs, and represented by a group of screws.

The twist motion of a robot end-effector can be described by a screw. The linear velocity  $\mathbf{v}_P$  of a selected reference point  $P$  on the end-effector and the angular velocity  $\boldsymbol{\omega}$  of the end-effector are given according to the task requirements. Therefore, the screw of the end-effector can be expressed by the given kinematic parameters  $\mathbf{v}_P$  and  $\boldsymbol{\omega}$

$$\mathcal{S}_i = \boldsymbol{\omega} + \epsilon \mathbf{v}^o = \boldsymbol{\omega} + \epsilon (\mathbf{v}_P + \mathbf{r}_P \times \boldsymbol{\omega})$$

where  $\epsilon$  is a dual sign;  $\mathbf{v}^o$  is the velocity of the point coincident with the original point in the body;  $\mathbf{r}_P$  is a positional vector indicating the reference point on the end-effector of the manipulator. When the original point of the coordinate system is coincident with point  $P$ , the pitch and axis can be determined by the following two equations

$$h = \frac{\boldsymbol{\omega} \cdot \mathbf{v}_P}{\boldsymbol{\omega} \cdot \boldsymbol{\omega}} \quad (2)$$

$$\mathbf{r} \times \boldsymbol{\omega} = \mathbf{v}_p - h\boldsymbol{\omega} \quad (3)$$

If a mechanism has three DOF, the order of the screw system is three. The motion of the three-order mechanism can be determined by three independent generalized coordinates. These independent generalized coordinates are often selected as three input-pair rates. The  $\mathbf{v}_p$  and  $\boldsymbol{\omega}$  of a robot can then be determined by these three input joint rates

$$\begin{aligned} \mathbf{v}_p &= [G]\dot{\mathbf{q}} \\ \boldsymbol{\omega} &= [G']\dot{\mathbf{q}} \end{aligned} \quad \dot{\mathbf{q}} = \{\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3\}^T \quad (4)$$

where  $[G]$  and  $[G']$  are  $3 \times 3$  first-order influence coefficient matrices (Thomas & Tesar, 1983). Substituting Eq. (4) into Eqs. (2, 3), the screw can also be described as the function of the joint rates

$$h = \frac{\dot{\mathbf{q}}^T [G']^T [G] \dot{\mathbf{q}}}{\dot{\mathbf{q}}^T [G']^T [G'] \dot{\mathbf{q}}} \quad (5)$$

$$[\mathbf{r}][G']\dot{\mathbf{q}} = ([G] - h[G'])\dot{\mathbf{q}} \quad (6)$$

where  $[\mathbf{r}]$  is a skew-symmetrical matrix of vector  $\mathbf{r} = (x \quad y \quad z)^T$ . Suppose we give the following expressions

$$\mathbf{u} = \dot{q}_1 / \dot{q}_3 \quad ; \quad \mathbf{w} = \dot{q}_2 / \dot{q}_3 \quad (7)$$

and then

$$\dot{\mathbf{q}} = (u \quad w \quad 1)\dot{q}_3$$

In this case, the pitch and the axis equations are given by

$$h = \frac{\{u \quad w \quad 1\}[G']^T [G] \{u \quad w \quad 1\}^T}{\{u \quad w \quad 1\}[G']^T [G'] \{u \quad w \quad 1\}^T} \quad (8)$$

$$[\mathbf{r}][G']^T \{u \quad w \quad 1\}^T = ([G] + [\mathbf{r}_p][G'] - h[G']) \{u \quad w \quad 1\}^T \quad (9)$$

where  $[\mathbf{r}_p]$  is a skew-symmetrical matrix of coordinate of the point  $P$ .

## 2.2 Principal screws of three-order screw system

A third-order screw system has three principal screws. The three principal screws are mutually perpendicular and intersecting at a common point generally. Any screw in the screw system is the linear combination of the three principal screws. In the third-order screw system, two pitches of three principal screws are extremum, and the pitches of all other screws lie between the maximum pitch and the minimum pitch. Therefore to get the

three principal screws is the key step to analyze the full-scale instantaneous motion of any 3-DOF mechanism. For obtaining the three principal screws there are two useful principles, the quadratic curve degenerating theory and quadric degenerating theory.

### 2.2.1 Quadratic curve degenerating theory

Let  $h_\alpha, h_\beta$  and  $h_\gamma$  be pitches of the three principal screws and suppose  $h_\gamma < h < h_\alpha$ . Ball (1900) gave a graph illustrating the full-scale plane representation of a third-order system with quadratic curves, and each quadratic curve has identical pitch. If the pitch of any screw in the system is equal to  $h_\alpha, h_\beta$  or  $h_\gamma$ , the quadratic equation will degenerate. When  $h = h_\alpha$  or  $h = h_\gamma$ , the quadratic equation collapses into two virtual straight lines intersecting at a real point; when  $h = h_\beta$ , the quadratic equation collapses into two real straight lines (Hunt 1978).

Expanding Eq. (8), we have

$$a_{11}u^2 + 2a_{12}uw + a_{22}w^2 + 2a_{13}u + 2a_{23}w + a_{33} = 0 \quad (10)$$

where the coefficient  $a_{ij}, (i, j = 1 \sim 3)$ , is a function of pitch  $h$  and the elements of the matrices  $[G]$  and  $[G']$ . From the quadratic equation degenerating principle, the determinant of the coefficient matrix should be zero, that is

$$\mathbf{D} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0, \quad (a_{ij} = a_{ji}) \quad (11)$$

Expanding the Eq. (11) we have

$$c_1h^3 + c_2h^2 + c_3h + c_4 = 0 \quad (12)$$

where  $c_i, (i = 1 \sim 4)$ , is a function of the elements of  $[G]$  and  $[G']$ . Three roots of the Eq. (12) are pitches,  $h_\alpha, h_\beta$  and  $h_\gamma$ , of the three principal screws. Substituting the pitch of principal screw into Eq. (10), the above quadratic equation degenerates into two straight lines, the root,  $(u_i, w_i)$ , of the two equations is

$$\begin{aligned} u_i &= \frac{a_{22}a_{13} - a_{12}a_{23}}{a_{12}^2 - a_{11}a_{22}} \\ w_i &= -\frac{a_{23}}{a_{22}} - \frac{a_{12}}{a_{22}}u_i \end{aligned} \quad i = 1, 2, 3 \quad (13)$$

Each set of  $(u_i, w_i)$  corresponds three inputs  $(u_i, w_i, 1)$ . Three sets of  $(u_i, w_i)$ ,  $i = \alpha, \beta, \gamma$ , correspond three output twists, i.e., three principal screws.

When the pitches of three principal screws are obtained, substituting the three values into Eq. (9), the axis equations of three principal screws can also be obtained.

### 2.2.2 Quadric degenerating theory

The quadric degenerating theory is an easier method for calculating the principal screws. Eq. (6) can be further simplified as

$$[A] \dot{\mathbf{q}} = 0 \quad (14)$$

where

$$[A] = [\mathbf{r}][G'] - [G] + h[G']$$

is a  $3 \times 3$  matrix.  $[G]$  and  $[G']$  are also  $3 \times 3$  first-order kinematic influence coefficient matrices, which are functions of the structure parameters of the mechanism. Since not all the components of vector  $\dot{\mathbf{q}}$  are zeros in general, the necessary and sufficient condition that ensures the solutions of Eq. (14) being non-zero is that the determinant of the matrix  $[A]$  is equal to zero. Namely (Huang & Wang 2001)

$$\text{Det}[A] = 0 \quad (15)$$

Expanding Eq. (15), we obtain the position equation describing all the screw axes

$$c_{11}x^2 + c_{22}y^2 + c_{33}z^2 + 2c_{12}xy + 2c_{23}yz + 2c_{13}xz + 2c_{14}x + 2c_{24}y + 2c_{34}z + c_{44} = 0 \quad (16)$$

where the coefficients,  $c_{ij}$  ( $i=1, 2, 3, 4, j=1, 2, 3, 4$ ), are the function of pitch  $h$  as well as coefficients  $g_{ij}, b_{ij}$ , the latter are relative with the elements of matrices  $[G]$  and  $[G']$  in Appendix (Huang & Wang 2001). The Eq. (16) is a quadratic equation with three elements,  $x, y$  and  $z$ . It expresses a quadratic surface in space. The spatial distribution of all the screw axes in 3D is quite complex. Generally, all the screw axes lie on a hyperboloid of one sheet if every coefficient in Eq. (16) contains the same pitch  $h$ .

#### 2.2.2.1 Pitches of three principal screws

For a third-order screw system there exist three principal screws  $\alpha, \beta$  and  $\gamma$ . Let  $h_\alpha, h_\beta$  and  $h_\gamma$  be the pitches of the three principal screws, and also suppose  $h_\alpha > h_\beta > h_\gamma$ .

We know that the quadric surface, Eq. (16), collapses into a straight line where the principal screws  $\alpha$  or  $\gamma$  lies, when  $h = h_\alpha$  or  $h = h_\gamma$ . The quadric surface degenerates into two intersecting planes, when  $h = h_\beta$ , and the intersecting line is just the axis of principal screw  $\beta$  (Hunt 1978). According to this nature, we can identify the three principal screws of the three-system.

The quadric has four invariants,  $I, J, D$  and  $\Delta$ , and they are

$$I = c_{11} + c_{22} + c_{33}$$

$$\Delta = \begin{vmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{vmatrix} ; \quad D = \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix} \quad (17)$$

$$J = c_{11}c_{22} + c_{22}c_{33} + c_{11}c_{33} - c_{12}^2 - c_{23}^2 - c_{13}^2 \quad (c_{ij} = c_{ji})$$

Expanding  $D$ , and let it equal to zero,  $D = 0$ , we have the expression

$$a_1 h^3 + a_2 h^2 + a_3 h + a_4 = 0 \quad (18)$$

where the coefficients  $a_i$  ( $i=1, \dots, 4$ ) are also the function of  $g_{ij}$ ,  $b_{ij}$  and  $h$ . Three possible roots can be obtained by solving Eq. (18), and these three roots correspond to pitches of the three principal screws. When the pitch in the system is equal to one of the three principal screw pitches, the invariant  $\Delta$  is zero as well. It satisfies the condition that the quadric degenerates into a line or two intersecting planes. Therefore, the key to identify the principal screws in the third-order system is that the quadric, Eq. (16), degenerates into a line or a pair of intersecting planes.

#### 2.2.2.2 The axes of principal screws and principal coordinate system

The coordinate system that consists of three principal screws is named the principal coordinate system. We know that the most concise equation of a hyperboloid is under its principal coordinate system. Now, we look for the principal coordinate system of the hyperboloid.

Equation (16) represented in the base coordinate system can be transformed into the normal form of the hyperboloid of one sheet in the principal coordinate system. After the pitches of the three principal screws are obtained, the pitch of any screw in the system is certainly within the range of  $h_\gamma < h < h_\alpha$ . The general three-system (Hunt 1978) appears only when three pitches of the three principal screws all are finite and also satisfy  $h_\gamma \neq h_\beta \neq h_\alpha$ . The axes of all the screws with the same pitch in the range from  $h_\gamma$  to  $h_\beta$  or from  $h_\beta$  to  $h_\alpha$  form a hyperboloid of one sheet. In this case the invariant  $D$  is not equal to zero, and the quadrics are the concentric hyperboloids. By solving Eq. (19)

$$\begin{cases} c_{11}x + c_{12}y + c_{13}z + c_{14} = 0 \\ c_{21}x + c_{22}y + c_{23}z + c_{24} = 0 \\ c_{31}x + c_{32}y + c_{33}z + c_{34} = 0 \end{cases} \quad (19)$$

the root of Eq. (19) is just the center point  $o'$  ( $x_0 \ y_0 \ z_0$ ) of the hyperboloid. It is clear that the point  $o'$  is also the origin of the principal coordinate system. The coordinate translation is



$$\begin{cases} x = x' + x_0 \\ y = y' + y_0 \\ z = z' + z_0 \end{cases} \quad (20)$$

The eigenequation of the quadric is

$$k^3 - Ik^2 + Jk - D = 0 \quad (21)$$

Its three real roots  $k_1, k_2, k_3$  are the three eigenvalues, and not all the roots are zeros. In general,  $k_1 \neq k_2 \neq k_3$ . The corresponding three unit eigenvectors  $(\lambda_1 \ \mu_1 \ \nu_1)$ ,  $(\lambda_2 \ \mu_2 \ \nu_2)$  and  $(\lambda_3 \ \mu_3 \ \nu_3)$  are perpendicular each other, and corresponding three principal screws,  $\alpha$ ,  $\beta$  and  $\gamma$ , form the coordinate system ( $o'$ - $x'y'z'$ ). The principal coordinate system ( $o'$ - $\alpha\beta\lambda$ ) can then be constructed by a following coordinate rotation

$$\begin{cases} x = \lambda_1 x' + \lambda_2 y' + \lambda_3 z' \\ y = \mu_1 x' + \mu_2 y' + \mu_3 z' \\ z = \nu_1 x' + \nu_2 y' + \nu_3 z' \end{cases} \quad (22)$$

After the coordinate transformation, the normal form of the hyperboloid is

$$k_1 x^2 + k_2 y^2 + k_3 z^2 + \frac{\Delta}{D} = 0 \quad (23)$$

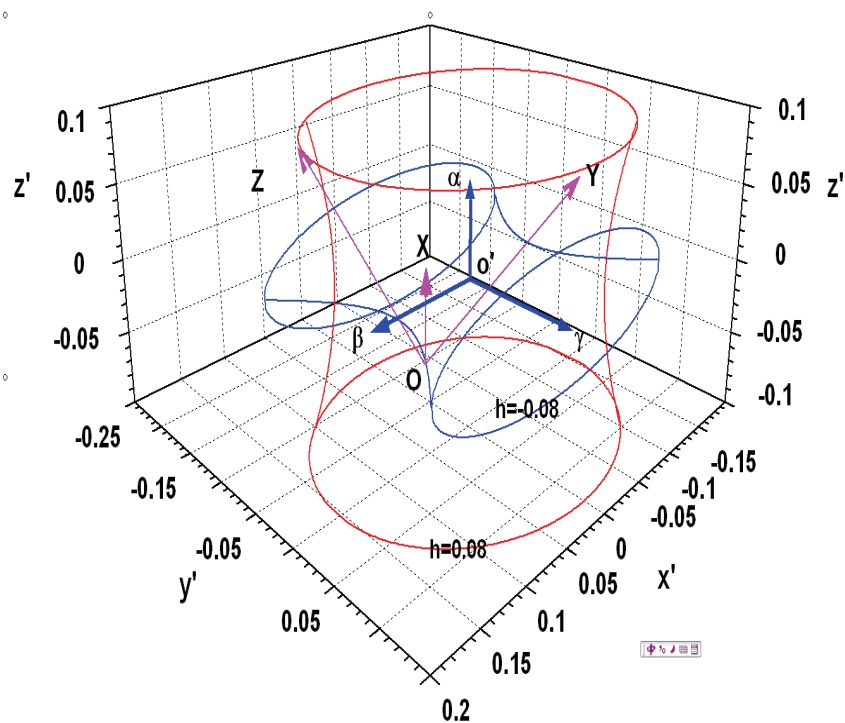


Fig.1. Hyperboloid of one sheet

Hunt (1978) gave that when  $h$  lies within the range  $h_\beta < h < h_\alpha$ , the central symmetrical axis of the hyperboloid is  $\alpha$ , and the semi-major axis of its central elliptical section in the  $\beta\gamma$ -plane always lies along  $\beta$ . For  $h_\gamma < h < h_\beta$ , the central symmetrical axis of the hyperboloid is  $\gamma$ , and the semi-major axis of its central elliptical section in the  $\beta\gamma$ -plane is also along  $\beta$ , Fig.1. Therefore, we may easily determine the three axes of the principal coordinate system.

### 3. Imaginary mechanism and Jacobian matrix

In order to determine the pitches and axes using Eqs. (4-9), the key step is to determine  $3 \times 3$  Jacobian matrices  $[G]$  and  $[G']$ . For a 3-DOF parallel mechanism to determine the  $[G]$  and  $[G']$  is difficult. Here the imaginary-mechanism principle (Yan & Huang, 1985; Huang & Wang, 1992) can solve the issue easily.

Note that, the imaginary-mechanism principle with unified formulas is a general method, and can be applied for kinematic analysis of any lower-mobility mechanism. An example is taken to introduce how to set the matrices  $[G]$  and  $[G']$ .

Fig. 2(a) shows a 3-DOF 3-RPS mechanism consisting of an upper platform, a base platform, and three kinematic branches. Each of its three branches is comprised of a revolute joint R, a prismatic pair P and a spherical pair S, which is a RPS serial chain. The axes of three revolute joints are tangential to the circumcircle of the lower triangle.

The mechanism has three linear inputs,  $\dot{L}_1, \dot{L}_2, \dot{L}_3$ .

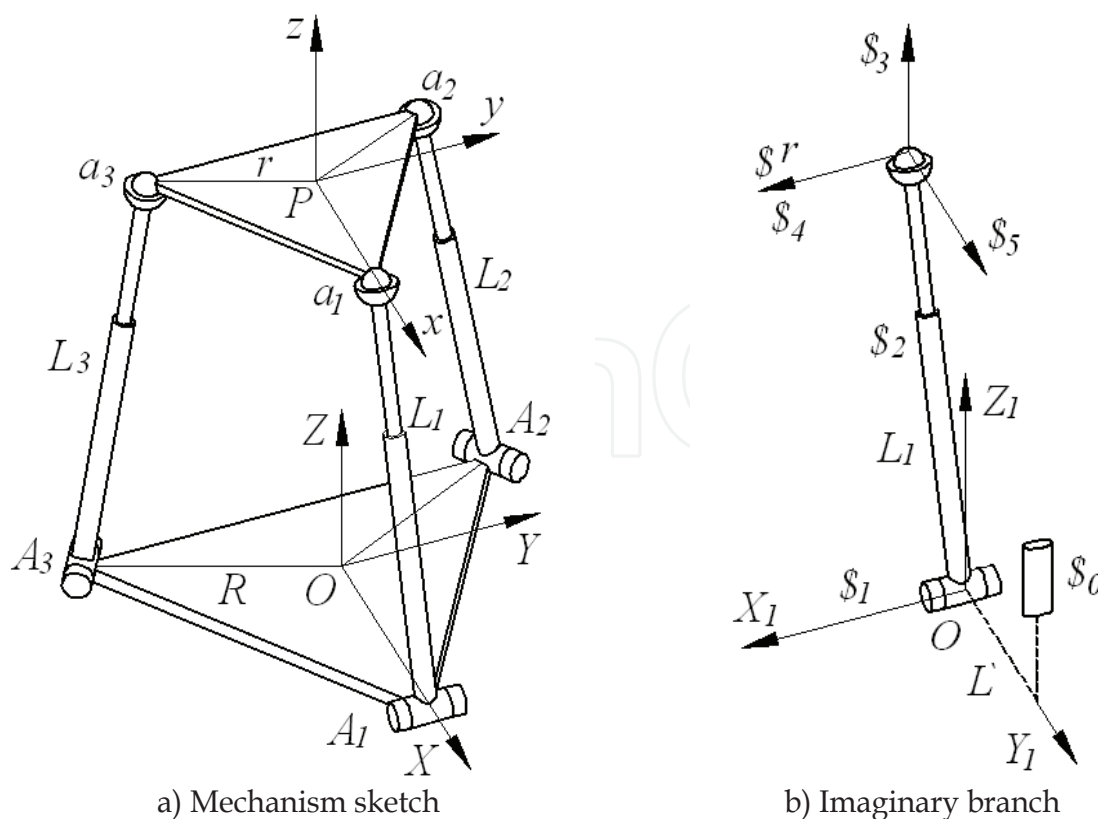


Fig.2. 3-DOF 3-RPS parallel mechanism

### 3.1 Imaginary twist screws of branches

Each kinematic branch of the 3-RPS mechanism may be represented by five single-DOF kinematic pairs as RPRRR. In order to get the Jacobian matrix by means of the method of kinematic influence coefficient of a 6-DOF parallel mechanism (Huang 1985), we may transform this 3-DOF mechanism into an imaginary 6-DOF one in terms of the kinematic equivalent principle. An imaginary link and an imaginary revolute pair,  $\$0$ , with single-DOF, are added to each branch of the mechanism. Then each branch becomes an imaginary 6-DOF serial chain. In order to keep a kinematic equivalent effect, let the amplitude  $\omega_0$  of the imaginary screw  $\$0$  of each branch always be zero; and let each screw system formed by imaginary  $\$0$  and the other five screws of the primary branch RPRRR be linearly independent.

Considering the imaginary pair  $\$0$ , the Plücker coordinates of all six screws shown in Fig. 3b with respect to local  $o-X_1Y_1Z_1$  coordinate system are

$$\begin{aligned} \$1 &= \{1 \ 0 \ 0; 0 \ 0 \ 0\} & \$4 &= \{1 \ 0 \ 0; 0 \ L_0 \zeta \ -L_0 \psi\} \\ \$2 &= \{0 \ 0 \ 0; 0 \ \psi \ \zeta\} & \$5 &= \{0 \ \zeta \ -\psi; -L_0 \ 0 \ 0\} \\ \$3 &= \{0 \ \psi \ \zeta; 0 \ 0 \ 0\} & \$0 &= \{0 \ 0 \ 1; L' \ 0 \ 0\} \end{aligned} \quad (24)$$

where  $\psi$  and  $\zeta$  are directional cosines of the screw axes  $\$2$  and  $\$3$ . The screw matrix of each branch with respect to the local coordinate system is  $[Gg] = \{\$0, \$1, \$2, \$3, \$4, \$5\}$ , and we have  $[G_i^0] = [A_i^0][Gg]$ .

### 3.2 Imaginary Jacobian matrix

For each serial branch, the motion of the end-effector of the 3-RPS mechanism can be represented by the following expression

$$\mathbf{V}_H = [G_i^0] \dot{\boldsymbol{\phi}}^{(i)} \quad i = 1, 2, 3 \quad (25)$$

where  $\mathbf{V}_H = \{\boldsymbol{\omega} \ \mathbf{v}_P\}^T$  is a six dimension vector;  $\boldsymbol{\omega}$  is the angular velocity of the moving platform;  $\mathbf{v}_P$  is the linear velocity of the reference point  $P$  in the moving platform; and  $\dot{\boldsymbol{\phi}}^{(i)} = (\dot{\phi}_0^{(i)} \ \dot{\phi}_1^{(i)} \ \dot{\phi}_2^{(i)} \ \dot{\phi}_3^{(i)} \ \dot{\phi}_4^{(i)} \ \dot{\phi}_5^{(i)})$  is a vector of joint rates. If  $[G_i^0]$  is non-singular

$$\dot{\boldsymbol{\phi}}^{(i)} = [G_i^0]^{-1} \mathbf{V}_H \quad i = 1, 2, 3 \quad (26)$$

where  $[G_i^i] = [G_i^0]^{-1}$

The input rates  $\dot{L}_1, \dot{L}_2, \dot{L}_3$  of the mechanism are known and the rate of each imaginary link is zero, which is equal to known. Then for each branch we have

$$\dot{\boldsymbol{\phi}}^{(i)} = (\dot{\phi}_0 \ \dot{\phi}_1 \ \dot{\phi}_2 \ \dot{\phi}_3 \ \dot{\phi}_4 \ \dot{\phi}_5)^{(i)} = (0 \ \dot{\phi}_1 \ \dot{L}_1 \ \dot{\phi}_3 \ \dot{\phi}_4 \ \dot{\phi}_5)^{(i)} \quad i = 1, 2, 3 \quad (27)$$

Taking the first row and third row from the matrix  $[G_0^i]$  in Eq. (26) of each branch, there are six linear equations. A new matrix equation can be established

$$\dot{\mathbf{q}} = [G_H^q] \mathbf{V}_H \quad \dot{\mathbf{q}} = \{\dot{L}_1 \quad \dot{L}_2 \quad \dot{L}_3 \quad 0 \quad 0 \quad 0\} \quad (28)$$

where

$$[G_H^q] = \left[ \begin{array}{ccc|cc} [G_0^1]_{3:} & [G_0^2]_{3:} & [G_0^3]_{3:} & [G_0^1]_{1:} & [G_0^2]_{1:} & [G_0^3]_{1:} \end{array} \right]^T \in R^{6 \times 6}$$

where  $[G_0^i]_{i:}$  represents the  $i^{\text{th}}$  row of matrix  $[G_0^i]$ . If the matrix  $[G_H^q]$  is non-singular, from Eq. (28)

$$\mathbf{V}_H = [G_q^H] \dot{\mathbf{q}} \quad (29)$$

where

$$[G_q^H] = [G_H^q]^{-1} \quad (30)$$

Since the 3-RPS mechanism has three freedoms, it needs three inputs. The matrix  $[G_L^H]$  formed by taking the first three columns of the matrix  $[G_q^H]$  is a  $6 \times 3$  Jacobian matrix. Therefore

$$\mathbf{V}_H = [G_L^H] \dot{\mathbf{L}} \quad (31)$$

As  $\mathbf{V}_H = \{\boldsymbol{\omega} \quad \mathbf{v}_p\}^T$ , Eq. (31) can be separated into two equations

$$\mathbf{v}_p = [G] \dot{\mathbf{L}} ; \quad \boldsymbol{\omega} = [G'] \dot{\mathbf{L}} \quad (32)$$

where  $[G']$  is the first three rows of  $[G_L^H]$ ;  $[G]$  is the last three rows of  $[G_L^H]$ . Then we obtain the  $3 \times 3$  matrices  $[G]$  and  $[G']$ . From the analysis process we know that the matrices  $[G]$  and  $[G']$  are independent of the chosen of these imaginary pairs.

#### 4. Full-scale feasible instantaneous screws of 3-RPS mechanism

Now, we continue to study the 3-RPS mechanism, Fig. 2, to get the full-scale feasible instantaneous motion. The parameters of the mechanism are :  $R=0.05$  m;  $r=0.05$  m;  $L_0=0.2$  m;  $L'=0.04$  m. Three configurations will be discussed.

##### 4.1 Upper platform is parallel to the base

Substituting given geometrical parameters and expanding Eq. (8), we have Eq. (10)

$$a_{11}u^2 + 2a_{12}uw + a_{22}w^2 + 2a_{13}u + 2a_{23}w + a_{33} = 0 \quad (33)$$

Eq. (33) is a quadratic equation with two variables,  $u$  and  $w$ . It will degenerate, if Equation (11) is satisfied. Expanding Eq. (11) we have the Eq. (12)

$$ah^3 + bh^2 + ch + d = 0 \quad (34)$$

The three roots of Eq. (34) are just three pitches of the three principal screws. Substituting each root  $h$  into Eq. (33) the quadratic equation degenerates into two linear equations expressing two straight lines. The intersecting point  $(u, w)$  of the two lines can be obtained. Then, the axis of the principal screw can also be obtained by using Eq. (9).

When the moving platform is parallel to the fixed one, it follows that:  $a = b = c = d = 0$ ; i.e., all the coefficients of Eq. (34) are zeroes. From algebra, the three roots,  $h$ , can be any constant. For some reasons, which we will present below, however, the three roots of Eq. (34) should be  $(\infty \ 0 \ 0)$ . When  $h \rightarrow \infty$ , we have  $u = 1$ ,  $w = 1$ , then the inputs are  $\dot{\mathbf{L}} = \{u \ w \ 1\} = \{1 \ 1 \ 1\}$ . The output motion is a pure translation, namely  $\mathbf{\$}_{z1} = \{0 \ 0 \ 0 \ ; \ 0 \ 0 \ 1\}$ . When the pitch of the principal screw is zero,  $h = 0$ ,  $u = 0/0$ ;  $w = 0/0$ . Mathematically,  $u$  and  $w$  both can be any value except one. All other roots of Eq. (34) will not be considered, as they are algebraically redundant. Then, the corresponding three principal screws can be written as

$$\begin{aligned} \mathbf{\$}_{z1} &= \{0 \ 0 \ 0; \ 0 \ 0 \ 1\} \\ \mathbf{\$}_{z2} &= \{0 \ 1 \ 0; \ -P_x \ 0 \ 0\} \\ \mathbf{\$}_{z3} &= \{1 \ 0 \ 0; \ 0 \ P_z \ 0\} \end{aligned} \quad (35)$$

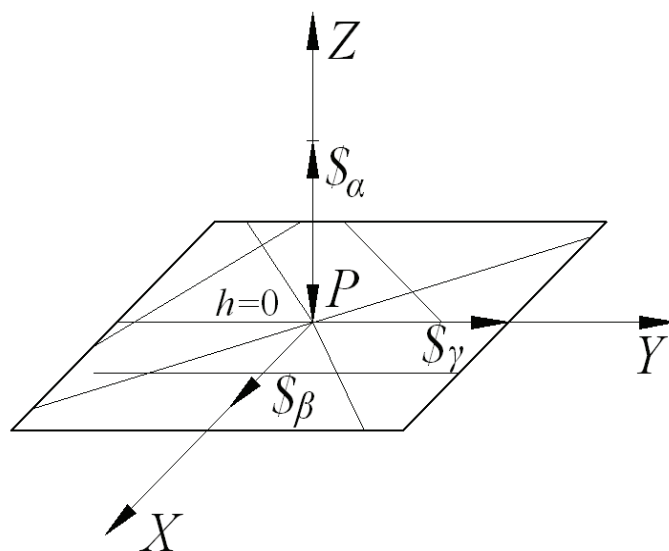


Fig. 3. The spatial distribution of the screws when the upper parallel to the base

Any output motion may be considered as a linear combination of the three principal screws. The full-scale distribution result, Fig.3, of all screws obtained by linear combinations of three principal screws can also be verified by using another method presented in Huang et al.,

(1996), and is identical with the actual mechanism model in our laboratory. The three principal screws belong to the fourth special three-system presented by Hunt (1978).

When the upper platform is parallel to the fixed platform, all possible output twists of the upper platform except the translation along the Z direction are rotations corresponding screws with zero pitch. Their axes all lie in the moving platform and in all the directions. Fig. 3 shows the full-scale possible twist screws with zero-pitch. Therefore from this figure you don't attempt to make the moving platform rotate round any axis not on the plane shown in Fig. 3. That is impossible.

#### 4.2 The upper platform rotates by an angle $\alpha$ about line $a_2a_3$

When the upper platform continually rotates by an angle  $\alpha$  about line  $a_2a_3$ , namely the mechanism is in the configuration that the lengths of the two input links are the same. Note that, for this kind of mechanisms the platform cannot continually rotate about axes lying in the plane shown in Fig.3 except some three axes including  $a_2a_3$ . In other words, it is very often impossible that the platform can continually rotated about an axis lying in the plane, as shown in Fig.3, (Zhao et al, 1999).

The coordinates of point  $a_1$  on the upper platform and point  $A_1$  on the base have the following values

$$a_1 = \{r(3\cos\alpha - 1)/2 \quad 0 \quad L_0 + 3r\sin\alpha/2\} \quad A_1 = \{R \quad 0 \quad 0\} \quad (36)$$

In this configuration, the screw system including the imaginary pair of the first chain corresponding to  $[G_1^0]$  with respect to the fixed coordinate system is

$$\begin{aligned} \mathcal{S}_1 &= \{\mathbf{S}_1; \mathbf{S}_{01}\} = \{\mathbf{S}_1; \mathbf{A}_1 \times \mathbf{S}_1\} \\ \mathcal{S}_2 &= \{\mathbf{S}_2; \mathbf{S}_{02}\} = \{\mathbf{0}; \mathbf{L}_1\} / \|\mathbf{L}_1\| \\ \mathcal{S}_3 &= \{\mathbf{S}_3; \mathbf{S}_{03}\} = \{\mathbf{L}_1; \mathbf{a}_1 \times \mathbf{L}_1\} / \|\mathbf{L}_1\| \\ \mathcal{S}_4 &= \{\mathbf{S}_4; \mathbf{S}_{04}\} = \{\mathbf{S}_1; \mathbf{a}_1 \times \mathbf{S}_1\} \\ \mathcal{S}_5 &= \{\mathbf{S}_5; \mathbf{S}_{05}\} = \{\mathbf{L}_1 \times \mathbf{S}_4; \mathbf{a}_1 \times \mathbf{L}_1 \times \mathbf{S}_4\} / \|\mathbf{L}_1 \times \mathbf{S}_4\| \\ \mathcal{S}_0 &= \{0 \quad 0 \quad 1; 0 \quad -L' \quad 0\} \end{aligned} \quad (37)$$

where  $\mathbf{S}_1 = \mathbf{S}_4 = \{0 \quad -1 \quad 0\}$ ,  $\mathbf{L}_1 = \mathbf{a}_1 - \mathbf{A}_1$ .

The twist screw systems of the other two chains corresponding  $[G_2^0]$  and  $[G_3^0]$  are the same as the case that the upper platform is parallel to the base. Establishing matrices  $[G]$  and  $[G']$ , we can solve principal screws by using the previous method.

Suppose  $\alpha = 30^\circ$ , the pitches of three principal screws can be obtained by solving Eq. (34).

They are  $h_\alpha = 5.13 \times 10^5$ ;  $h_\beta = 0$ ;  $h_\gamma = -5.13 \times 10^5$ . When  $I_2 = 0$ , where  $I_2$  is the two-order determinant of coefficients of the quadratic equation, its two roots are  $h_1 = -0.0057$ ,  $h_2 = 0.0165$ . There are six types of the quadratic curve for the same configuration of the mechanism, as shown in Table 1. The pitch  $h$  varies between  $h_\alpha$  and  $h_\gamma$ .

Each point in Fig. 4 denotes a pitch  $h$  of a twist screw of the moving platform relative the three inputs  $(u, w, 1)$ . You can get the output pitch of the instantaneous twist when three inputs are given. Fig.4 also shows the relation between inputs and the six types of quadratic curves with different pitches in this configuration of the mechanism.

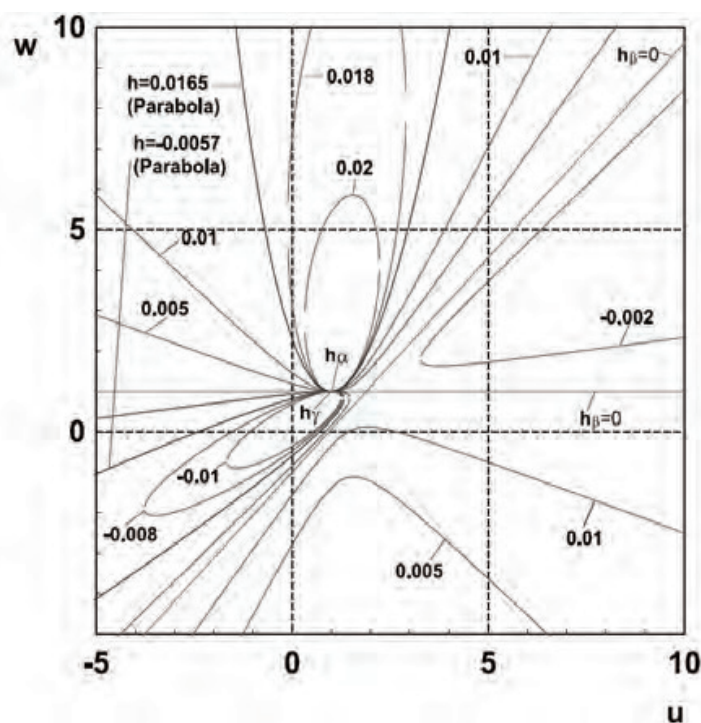


Fig. 4. When the upper platform rotates  $30^\circ$  about  $a_2a_3$

The range of the value of $h$		Type of conics
In $30^\circ$ configuration	In general configuration	
$0.0165256 < h < 5.13 \times 10^5$ or $-5.13 \times 10^5 < h < -0.0057003$	$0.0131215 < h < 4.28 \times 10^5$ or $-4.28 \times 10^5 < h < -0.0160208$	Real ellipse
$h > 5.13 \times 10^5$ or $h < -5.13 \times 10^5$	$h > 4.28 \times 10^5$ or $h < -4.28 \times 10^5$	Imaginary ellipse
$h_\alpha = 5.13 \times 10^5$ or $h_\gamma = -5.13 \times 10^5$	$h_\alpha = 4.28 \times 10^5$ or $h_\gamma = -4.28 \times 10^5$	Dot ellipse
$-0.0057003 < h < 0.0165256$	$-0.0160208 < h < 0.0131215$	Hyperbola
$h_\beta = 0$	$h_\beta = 0.0079$	A pair of intersecting real lines
$h = 0.0165256$ or $h = -0.0057003$	$h = 0.0131215$ or $h = -0.0160208$	Parabola

Table 1. Six types of the quadratic curves

The twist screws with the same pitch,  $h$ , form a quadratic curve. The pure rotations with zero pitch are illustrated as a pair of intersecting real straight lines in the figure.

The two straight lines can also be obtained and proved by using another method proposed by Huang & Fang (1996). The three principal screws are

$$\mathcal{S}_m^1 = \{0 \quad -1 \quad 0 \quad ; \quad 0.2 \quad 0 \quad 0.1\}$$

$$\mathcal{S}_m^2 = \{0.966 \quad 0 \quad 0.259 \quad ; \quad 0 \quad 0.22 \quad 3.96 \times 10^6\} \quad (38)$$

$$\mathcal{S}_m^3 = \{-0.966 \quad 0 \quad -0.259 \quad ; \quad 0 \quad -0.22 \quad 3.96 \times 10^6\}$$

The screw  $\mathcal{S}^m$  with infinite pitch can be obtained by a linear combination of  $\mathcal{S}_m^2$  and  $\mathcal{S}_m^3$

$$\mathcal{S}^m = \{0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1\}$$

It expresses a pure translation along the Z-direction.  $\mathcal{S}_m^1$  with zero pitch is a pure rotation about an axis parallel to the Y-axis.  $\mathcal{S}_m^2$  is a twist screw with  $h \neq 0$  and deviates from the normal direction of  $\mathcal{S}^m$ . The three screws,  $\mathcal{S}^m$ ,  $\mathcal{S}_m^1$  and  $\mathcal{S}_m^2$

$$\mathcal{S}^m = \{0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1\}$$

$$\mathcal{S}_m^1 = \{0 \quad -1 \quad 0 \quad ; \quad 0.2 \quad 0 \quad 0.1\} \quad (39)$$

$$\mathcal{S}_m^2 = \{0.966 \quad 0 \quad 0.259 \quad ; \quad 0 \quad 0.22 \quad 3.96 \times 10^6\}$$

form a set of new principal screws, which is just the seventh special three-system screws presented by Hunt (1978), Tsai and Lee (1993).

#### 4.3 General configuration of the 3-RPS mechanism

In any general configuration, the lengths of three legs of the parallel manipulator are different. The coordinates of the points  $a_1$ ,  $a_2$  and  $a_3$  with respect to the coordinate system  $P$ - $xyz$  are

$$\begin{aligned} \mathbf{a}_1 &= \{r \quad 0 \quad 0\}^T \\ \mathbf{a}_2 &= \{-r/2 \quad \sqrt{3}r/2 \quad 0\}^T \\ \mathbf{a}_3 &= \{-r/2 \quad -\sqrt{3}r/2 \quad 0\}^T \end{aligned} \quad (40)$$

Since the transformation matrix from the system  $P$ - $xyz$  to the fixed system  $O$ - $XYZ$  is  $[T]$ . The coordinates of the points with respect to the fixed coordinate system  $O$ - $XYZ$  are

$$\{\mathbf{P}_i \quad 1\}^T = [T] \{\mathbf{a}_i \quad 1\}^T \quad i = 1, 2, 3 \quad (41)$$

The unit vectors  $u_1$ ,  $u_2$  and  $u_3$  representing revolute axes with respect to the fixed system are



$$\begin{aligned}
\mathbf{u}_1 &= \{0 \ 1 \ 0\}^T \\
\mathbf{u}_2 &= \{-\sqrt{3}/2 \ -1/2 \ 0\}^T \\
\mathbf{u}_3 &= \{\sqrt{3}/2 \ -1/2 \ 0\}^T
\end{aligned} \tag{42}$$

The screw systems of the three serial chains in the fixed system can be expressed as following

$$\begin{aligned}
\mathcal{S}_1^i &= \{\mathbf{S}_1; \mathbf{S}_{01}\} = \{\mathbf{u}_1; \mathbf{A}_i \times \mathbf{u}_1\} \\
\mathcal{S}_2^i &= \{\mathbf{S}_2; \mathbf{S}_{02}\} = \{\mathbf{0}; \mathbf{L}_i\} / \|\mathbf{L}_i\| \\
\mathcal{S}_3^i &= \{\mathbf{S}_3; \mathbf{S}_{03}\} = \{\mathbf{L}_i; \mathbf{P}_i \times \mathbf{L}_i\} / \|\mathbf{L}_i\| \quad i = 1, 2, 3 \\
\mathcal{S}_4^i &= \{\mathbf{S}_4; \mathbf{S}_{04}\} = \{\mathbf{u}_i; \mathbf{P}_i \times \mathbf{u}_i\} \\
\mathcal{S}_5^i &= \{\mathbf{S}_5; \mathbf{S}_{05}\} = \{\mathbf{L}_i \times \mathbf{u}_i; \mathbf{P}_i \times \mathbf{L}_i \times \mathbf{u}_i\} / \|\mathbf{L}_i \times \mathbf{u}_i\|
\end{aligned} \tag{43}$$

Three imaginary revolute pairs added to three branches are supposed all in Z-direction and passing through points  $k_1$ ,  $k_2$  and  $k_3$ , respectively. They are on the lines from original point  $O$  to the points  $A_1$ ,  $A_2$  and  $A_3$ , respectively. All lengths are  $L'$ , then the coordinates of the points  $k_1$ ,  $k_2$  and  $k_3$  are expressed as three vectors

$$\begin{aligned}
\mathbf{k}_1 &= \{L' \ 0 \ 0\} \\
\mathbf{k}_2 &= \{-L'/2 \ \sqrt{3}L'/2 \ 0\} \\
\mathbf{k}_3 &= \{-L'/2 \ -\sqrt{3}L'/2 \ 0\}
\end{aligned} \tag{44}$$

The three corresponding imaginary twist screws are

$$\mathcal{S}_0^i = \{\mathbf{S}_0; \mathbf{k}_i \times \mathbf{S}_0\} \quad i = 1, 2, 3 \tag{45}$$

where  $\mathbf{S}_0 = \{0 \ 0 \ 1\}$ .

The matrices  $[G_i^0]$  corresponding screw systems of the three branches with respect to the fixed coordinate system are

$$[G_i^0] = \{\mathcal{S}_0^i \ \mathcal{S}_1^i \ \mathcal{S}_2^i \ \mathcal{S}_3^i \ \mathcal{S}_4^i \ \mathcal{S}_5^i\} \quad i = 1, 2, 3 \tag{46}$$

When the coordinates of center point of the upper platform with respect to the fixed system are given as

$$X = 0.002 \ m, \ Y = 0.001 \ m, \ Z = 0.22 \ m$$

the pitches of the three principal screws can be obtained as:

$$h_\alpha = 4.28 \times 10^5; h_\beta = 0.0079; h_\gamma = -4.28 \times 10^5.$$

When  $I_2 = 0$ , two possible roots of the pitch are  $h_1 = -0.016$ ,  $h_2 = 0.013$ . There are also six types of conics in this configuration, Table 1. Fig.5 illustrates a planar representation of pitches of all possible twist screws in this case.

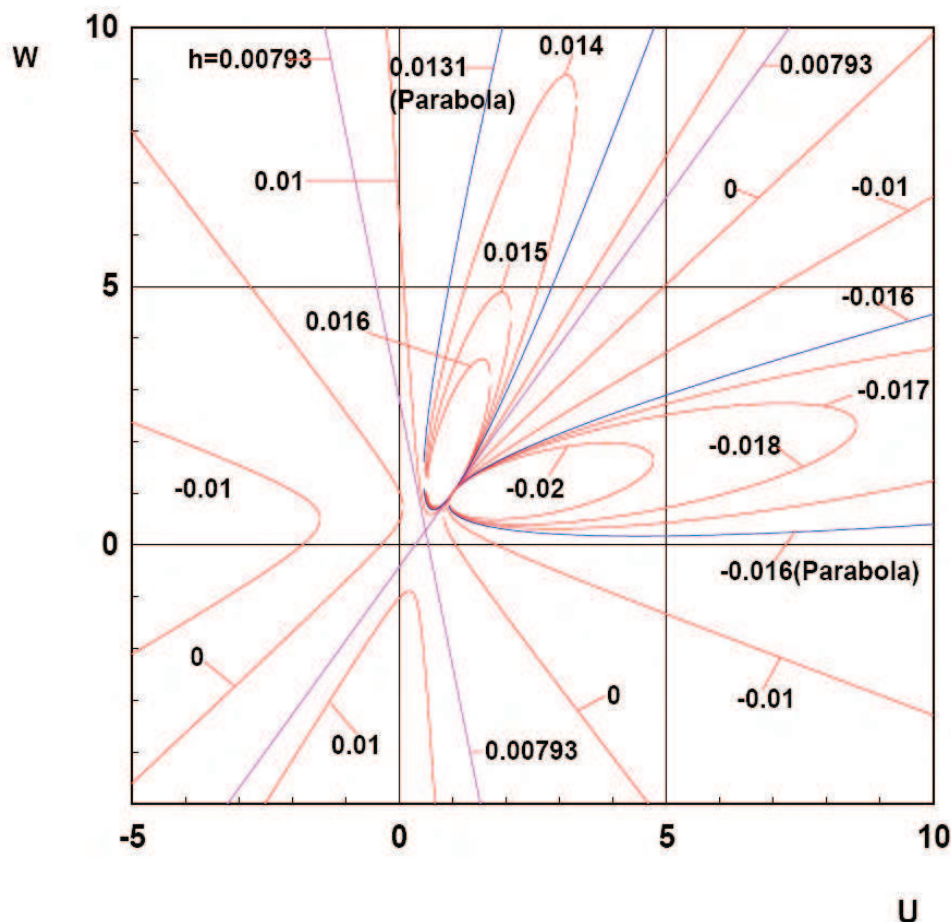


Fig. 5. The planar representations of the twist screws in any general configuration

The coordinates  $(u, w)$  of the principal screw with  $h_\alpha$  are  $(1.0004133965, 1.000387461)$ . The  $(u, w)$  corresponding  $h_\gamma$  are  $(1.0004134267, 1.000387451)$ . They both are too close to be distinguished by naked eye in the figure. The three principal screws can be obtained as

$$\mathcal{S}_m^1 = \{-0.97 \quad 0.23 \quad 0 \quad ; \quad -0.06 \quad -0.22 \quad 0.06\}$$

$$\mathcal{S}_m^2 = \{0.22 \quad 0.95 \quad 0.21 \quad ; \quad -0.204 \quad 0.395 \quad 4.1 \times 10^6\} \quad (47)$$

$$\mathcal{S}_m^3 = \{-0.22 \quad -0.95 \quad -0.21 \quad ; \quad 0.204 \quad -0.395 \quad 4.1 \times 10^6\}$$

The screw  $\mathcal{S}^m = \{0 \quad 0 \quad 0 \quad ; \quad 0 \quad 0 \quad 1\}$  with infinite pitch,  $h^m = \infty$ , can be obtained by the linear combination of  $\mathcal{S}_m^2$  and  $\mathcal{S}_m^3$ .  $\mathcal{S}^m$  expresses a pure translation along the Z direction.

$\mathcal{S}_m^1$  with  $h_m^1 = 0$  is perpendicular to Z-axis.  $\mathcal{S}_m^2$  with  $h_m^2 \neq 0$  deviates from the normal direction of  $\mathcal{S}^m$ . Therefore, the three principal screws,  $\mathcal{S}^m$ ,  $\mathcal{S}_m^1$  and  $\mathcal{S}_m^2$ , also form a seventh special three-system. Therefore, the formation of all linear combinations of  $\mathcal{S}^m$ ,  $\mathcal{S}_m^1$  and  $\mathcal{S}_m^2$  in three-dimensional space, as shown in Fig.6, is a hyperbolic paraboloid.

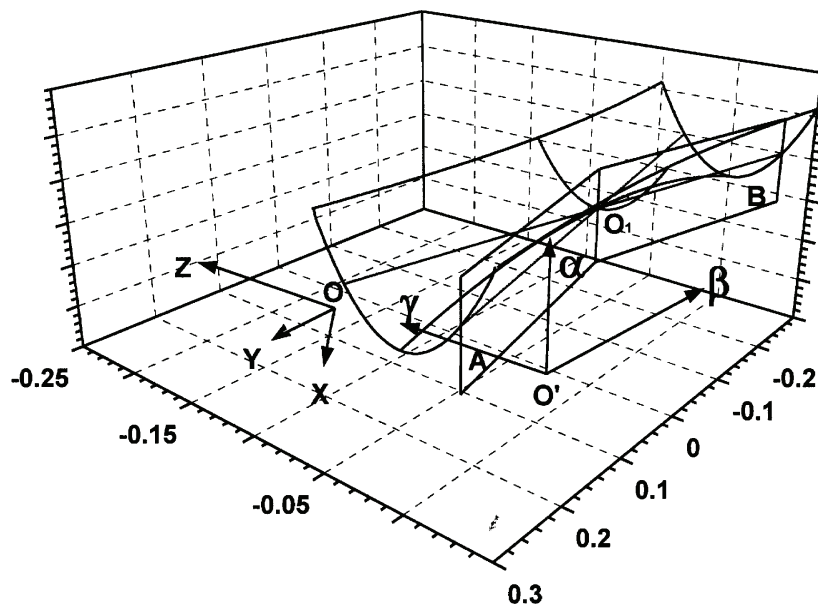


Fig. 6. The spatial distribution of the screws in General configuration

## 5. Full-scale feasible instantaneous screw of a 3-UPU mechanism

In this section we discuss an interesting 3-DOF special 3-UPU mechanism. It has some special inconceivable characteristics.

### 5.1 First-order influence matrices and kinematic analysis

The 3-UPU mechanism, as shown in Fig. 7a, consists of a fixed pyramid  $A_1A_2A_3$ , a moving pyramid  $a_1a_2a_3$  and three UPU kinematic chains. Three centrelines of the three prismatic pairs in the initial position are mutually perpendicular. The middle two revolute pairs,  $\mathcal{S}_2$  and  $\mathcal{S}_4$ , Fig. 7b, adjacent to the prismatic pair in every branch, are mutually perpendicular, moreover they both are perpendicular to the prismatic pair. This is different with general 3-D translational 3-UPU parallel mechanism (Tsai & Stamper, 1996). The base coordinate system is  $O$ - $XYZ$ . The length of each side of the cubic mechanism is  $m$ .

For this special 3-UPU mechanism, each branch of the mechanism has equivalent five single-DOF kinematic pairs. According to the imaginary-mechanism method mentioned in Section 3, an imaginary link and an imaginary revolute pair denoted by a screw with zero pitch,  $\mathcal{S}_{0i}$ , are added to each branch, as shown in Fig. 7c. Then, each branch has six single-DOF kinematic pairs. Note that it is necessary to let the angular velocity amplitude of  $\mathcal{S}_0$  for each branch always be zero.

For each six-DOF serial branch, the motion of the end-effector of the 3-UPU mechanism can be represented as

$$\mathbf{V}_H = [G_i^0] \dot{\phi}^i \quad (i = 1, 2, 3) \quad (48)$$

Based on the Eq. (48) and Section 3, the matrix equation as well as  $[G']$  and  $[G]$  can be obtained

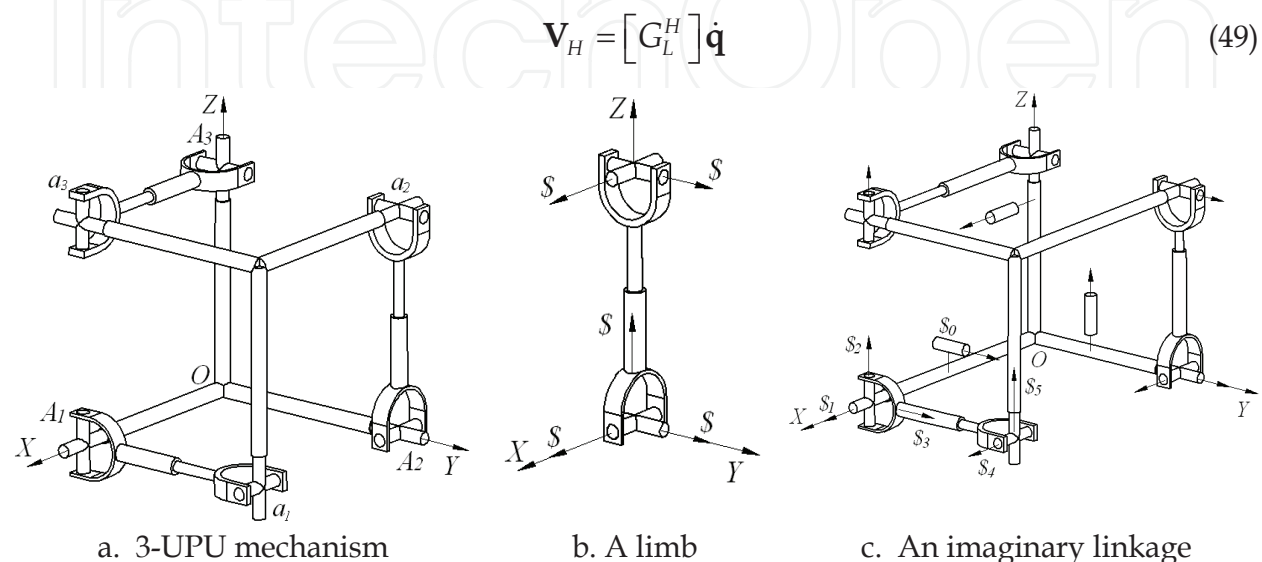


Fig. 7. Initial Position Mechanism Sketch

$$\mathbf{V}_p = [G] \dot{\mathbf{q}} \quad \boldsymbol{\omega} = [G'] \dot{\mathbf{q}} \quad (50)$$

where  $[G']$  is the first three rows of  $[G_L^H]$ ;  $[G]$  is the last three rows of  $[G_L^H]$ . They both are  $3 \times 3$  matrices.

### 5.2. Initial configuration

Fig. 7a shows the initial configuration of the mechanism,  $m = 1.0$  m,  $l = 0.3$  m, and  $d_1 = d_2 = d_3$ . For each branch of the mechanism  $\dot{\phi}_0^i, (\dot{\phi}_0^i = 0)$ , and  $\dot{q}_i, (i = 1, 2, 3)$ , are denoted as inputs.

Assume the three lengths from the origin  $O$  to the centers of three imaginary pairs all to be  $l = m - d_i$ , which lie on the  $X$ -axis,  $Y$ -axis and  $Z$ -axis, respectively.  $d_i$  is the distance between the first two kinematic pairs including the imaginary pair.

The first-order influence coefficient matrices of the three branches are

$$[G_i^0] = (\$0 \quad \$1 \quad \$3 \quad \$4 \quad \$5), \quad (i = 1, 2, 3)$$

According to Eq. (32), we obtain the two matrices

$$[G'] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad [G] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad (51)$$

From Eq.s (51) and (12), we get the coefficients of the Eq. (12) as

$$c_1 = c_2 = c_3 = c_4 = 0 \quad (52)$$

The result is very special and implies that the roots of Eq. (12) can be any values. For this special situation to determine the three values we should consider other conditions. From section 2.2 of the References (Huang et al., 2004; and Huang & Fang, 1996) the three roots should all be infinite. That means the three roots,  $h_\alpha$ ,  $h_\beta$  and  $h_\gamma$ , all are  $\infty$ . The three principal screws belong to the sixth special third-order system presented by Hunt (1978). The three mutually perpendicular screws correspond with three independent translational motions. Obviously, along any direction in space there also exists an instant translational motion by linear combination of the three screws.

However, by further analysis we find that only three feasible translational motions can continue along the three coordinate axes, respectively. The feasible translational motions along all other directions in 3-D space are only instantaneous. It is easy to recognize, that when a small finite translation occurs not along the coordinate axis from the initial mechanism configuration, all three UPU chains are not the same as the configuration shown in Fig. 7b, and the three constraint screws will change and not similar that in the first configuration. Not all constrained motions are rotational. Therefore, the finite translation can occur only independently along each one of the three coordinate axes. In other words, three twists with  $\infty$  pitch cannot be linearly combined at this initial position and the mechanism is not the same as the general 3D translational parallel mechanism proposed by Tsai & Stamper, (1996). The mechanism has such a very unusual characteristic.

### 5.3. The second configuration

The parameters of the mechanism are assumed as:  $m = 1.0$  m,  $l = 0.3$  m; and  $a = 0.2$  m is the displacement of the moving pyramid along the X-axis, Fig. 8. In this case we have

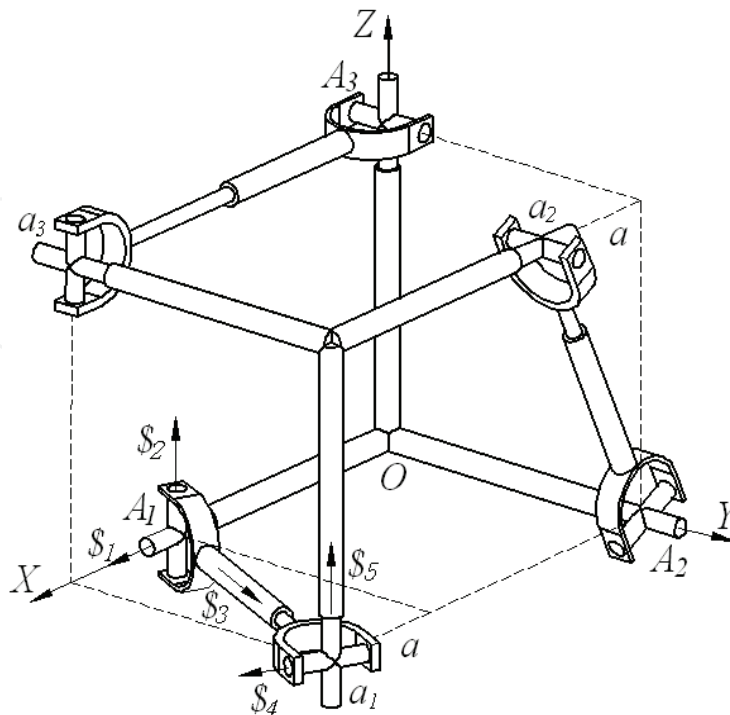


Fig.8. UPU branch after moving along the X-axis

$$\begin{aligned}
 [G'] &= \begin{bmatrix} 0 & 0 & 0 \\ 0.00567188 & 0.170156 & -0.0344828 \\ 0.170156 & 0.0567188 & 1.03448 \end{bmatrix} \\
 [G] &= \begin{bmatrix} -0.0567188 & -0.170156 & 1.03448 \\ 0.850782 & 0.0283594 & -0.172414 \\ 0.0351657 & 1.05497 & -0.213793 \end{bmatrix}
 \end{aligned} \tag{53}$$

Substituting  $[G]$  and  $[G']$  into the Eq. (8) and according to the Eq. (10), we have

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0 \tag{54}$$

Expanding and solving the equation, we have

$$\begin{aligned}
 h_\alpha &= 5.6 \\
 h_\beta &= -5.6 \\
 h_\gamma &= -2.16318 \times 10^{17}
 \end{aligned} \tag{55}$$

where one is infinite, the other two are finite values with opposite signs. Therefore any screw in the screw system is the linear combination of the three principal screws and its pitch is inside the scope,  $-5.6 \leq h \leq 5.6$ . Three principal kinematic screws are

$$\begin{aligned}
 \$\alpha &= (0 \ 1.0 \ 1.0; 88.4053 \ 5.0 \ 6.2) / \sqrt{2} \\
 \$\beta &= (0 \ -1.0 \ 1.0; 71.7085 \ 5.0 \ -6.2) / \sqrt{2} \\
 \$\gamma &= (0 \ 0 \ 0; 1 \ 0 \ 0)
 \end{aligned} \tag{56}$$

and the vector equations of three axes are

$$\begin{aligned}
 \mathbf{r} \times \mathbf{S}_\alpha &= (88.4053 \ 5 \ 6.2)^T / \sqrt{2} \\
 \mathbf{r} \times \mathbf{S}_\beta &= (71.7085 \ 5 \ 6.2)^T / \sqrt{2} \\
 \mathbf{r} \times \mathbf{S}_\gamma &= (0 \ 0 \ 0)^T
 \end{aligned} \tag{57}$$

where  $\mathbf{S}_\alpha$ ,  $\mathbf{S}_\beta$  and  $\mathbf{S}_\gamma$  are three direction vectors of the three principal screws. Comparing with Eq. (15) in Reference Huang et al., (2004), the three screws in that Eq.(15) are just the linear combination of the three principle screws in Eq. (56). That means the result is correct and proved mutually. This system belongs to a third special three-system screw.

When different  $h$  value is substituted into Eq. (10), we may obtain different quadratic equation. Giving one set of input  $(u \ w \ 1)$ , the corresponding pitch of output motion is shown in Fig.9. Figure 9 illustrates the full-scale feasible instantaneous motion at that moment.

We know that each pitch of the screw determines a quadratic equation, Eq. (10). Here all quadratic equations degenerate into a pair of intersecting straight lines, when  $h$  lies within the range  $-5.6 < h < 5.6$ ; It is because that two invariants of all the quadratic equations, Eq. (10), satisfy  $D = 0$  and  $\delta < 0$ . Similarly, when  $h = 5.6$  and  $h = -5.6$  both quadratic equations collapse into two pairs of superposed straight lines, they are respectively

$$u = w$$

$$0.173127u + 0.173127w = 0.0679061 \quad (58)$$

The quadratic equation collapses into a point which is just the intersecting point of all the straight lines, as shown in Fig. 9, when  $h = h_\gamma = -2.16381 \times 10^{17} = -\infty$ .

Fig. 9 illustrates the finite-and-infinite pitch graph of the third special three-system screw including the finite pitches in the scope from -5.6 to 5.6 and an infinite pitch.

Each point in the figure indicates the relation between the input  $(u \ w \ 1)$  and the output pitch,  $h$ . It is necessary to point out that, for a six-DOF mechanism, infinite pitches of its infinite feasible instant motions distribute in an infinite scope  $(-\infty \ \infty)$ , but for this 3-UPU mechanism its infinite possibility is only in a limited scope  $(-5.6, 5.6)$  plus a point with infinite pitch value.

From Fig. 9, we can find that all the straight lines pass through a common point, which is a very special point. The pitch values of all the straight lines are finite, but at the special point, the pitch suddenly becomes infinite

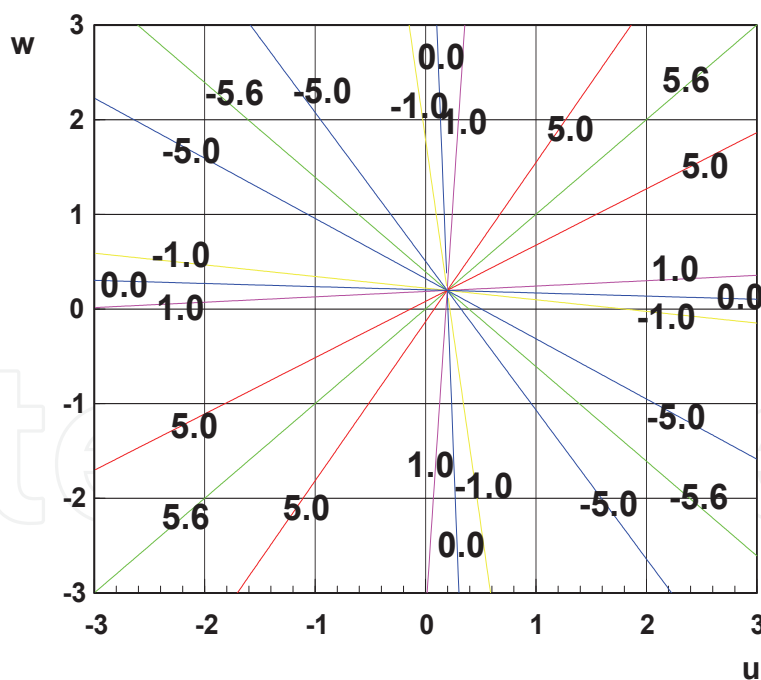


Fig. 9. The Pitch of the Twist at the Second Configuration of 3-TPT Mechanism.

## 6. Future research

Based on this principle many three-degrees of freedom parallel mechanisms need to be further analyzed.

## 7. Conclusions

This chapter presents a study on the full-scale instant twists motions of 3-DOF parallel manipulators. The study is of extremely benefit to understand and correctly apply a mechanism. It is based on principal screws of the screw system. The key problem is to derive three principal screws from a given 3-DOF mechanism. It needs to set the relation between the pitches of the principal screws and the three linear inputs of the mechanism.

In this chapter, the effective method to identify the principal screws of a third-order screw system of 3-DOF mechanisms is presented. For obtaining the principal screws there introduce two methods, the quadratic curve degenerating theory and quadric degenerating theory. Besides, the imaginary-mechanism influence coefficient principle is also used.

In the following sections two mechanisms are discussed using the principle. Analyzing the full-scale screws the planar representations of pitches and the spatial distributions of the axes are illustrated.

It is necessary to conclude that the special 3-UPU mechanism has some exceptional interesting characteristics. At the initial configuration, the moving pyramid can continually translate along the X- or Y- or Z-axis, however, for all other directions the translational freedom is only instantaneous. At a general configuration, all the straight lines with different pitch pass through a common point, a very special point. The pitch values of all the straight lines are finite, at the intersecting point, however, the pitch is infinite.

## 8. Acknowledgement

The research work reported here is supported by NSFC under Grant No. 59575043 and 50275129.

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