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## Mechanics and Simulation of Six-Legged Walking Robots

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## 1. Introduction

Legged locomotion is used by biological systems since millions of years, but wheeled locomotion vehicles are so familiar in our modern life, that people have developed a sort of dependence on this form of locomotion and transportation. However, wheeled vehicles require paved surfaces, which are obtained through a suitable modification of the natural environment. Thus, walking machines are more suitable to move on irregular terrains, than wheeled vehicles, but their development started in long delay because of the difficulties to perform an active leg coordination.
In fact, as reported in (Song and Waldron, 1989), several research groups started to study and develop walking machines since 1950, but compactness and powerful of the existent computers were not yet suitable to run control algorithms for the leg coordination. Thus, E ASV (Adaptive-Suspension-Vehicle) can be considered as the first comprehensive example control, gait analysis and mechanical design in terms of legs, actuation and vehicle structure. Moreover, ASV belongs to the class of "statically stable" walking machines because a static equilibrium is ensured at all times during the operation, while a second class is represented by the "dynamically stable" walking machines, as extensively presented in (Raibert, 1986). Later, several prototypes of six-legged walking robots have been designed and built in the world by using mainly a "technical design" in the development of the mechanical design and control. In fact, a rudimentary locomotion of a six-legged walking robot can be achieved by simply switching the support of the robot between a set of legs that form a tripod. Moreover, in order to ensure a static walking, the coordination of the six legs can be carried out by imposing a suitable stability margin between the ground projection of the center of gravity of the robot and the polygon among the supporting feet.
A different approach in the design of six-legged walking robots can be obtained by referring to biological systems and, thus, developing a biologically inspired design of the robot. In fact, according to the "technical design", the biological inspiration can be only the trivial observation that some insects use six legs, which are useful to obtain a stable support during the walking, while a "biological design" means to emulate, in every detail, the locomotion of a particular specie of insect. In general, insects walk at several speeds of locomotion with a
variety of different gaits, which have the property of static stability, but one of the key characteristics of the locomotion control is the distribution.
Thus, in contrast with the simple switching control of the "technical design", a distributed gait control has to be considered according to a "biological design" of a six-legged walking robot, which tries to emulate the locomotion of a particular insect. In other words, rather than a centralized control system of the robot locomotion, different local leg controllers can be considered to give a distributed gait control.
Several researches have been developed in the world by referring to both "cockroach insect", or Periplaneta Americana, as reported in (Delcomyn and Nelson, 2000; Quinn et al., 2001; Espenschied et al., 1996), and "stick insect", or Carausius Morosus, as extensively reported in (Cruse, 1990; Cruse and Bartling, 1995; Frantsevich and Cruse, 1997; Cruse et al., 1998; Cymbalyuk et al., 1998; Cruse, 2002; Volker et al., 2004; Dean, 1991 and 1992).
In particular, the results of the second biological research have been applied to the development of TUM (Technische-Universität-München) Hexapod Walking Robot in order to emulate the locomotion of the Carausius Morosus, also known as stick insect. In fact, a biological design for actuators, leg mechanism, coordination and control, is much more efficient than technical solutions.
Thus, TUM Hexapod Walking Robot has been designed as based on the stick insect and using a form of the Cruse control for the coordination of the six legs, which consists on distributed leg control so that each leg may be self-regulating with respect to adjacent legs. Nevertheless, this walking robot uses only Mechanism 1 from the Cruse model, i.e. "A leg is hindered from starting its return stroke, while its posterior leg is performing a return stroke", and is applied to the ispilateral and adjacent legs.
TUM Hexapod Walking Robot is one of several prototypes of six-legged walking robots, which have been built and tested in the world by using a distributed control according to the Cruse-based leg control system. The main goal of this research has been to build biologically inspired walking robots, which allow to navigate smooth and uneven terrains, and to autonomously explore and choose a suitable path to reach a pre-defined target position. The emulation of the stick insect locomotion should be performed through a straight walking at different speeds and walking in curves or in different directions.
Therefore, after some quick information on the Cruse-based leg controller, the present chapter of the book is addressed to describe extensively the main results in terms of mechanics and simulation of six-legged walking robots, which have been obtained by the authors in this research field, as reported in (Figliolini et al., 2005, 2006, 2007). In particular, the formulation of the kinematic model of a six-legged walking robot that mimics the locomotion of the stick insect is presented by considering a biological design. The algorithm for the leg coordination is independent by the leg mechanism, but a three-revolute ( $3 R$ ) kinematic chain has been assumed to mimic the biological structure of the stick insect. Thus, the inverse kinematics of the $3 R$ has been formulated by using an algebraic approach in order to reduce the computational time, while a direct kinematics of the robot has been formulated by using a matrix approach in order to simulate the absolute motion of the whole six-legged robot.
Finally, the gait analysis and simulation is presented by analyzing the results of suitable computer simulations in different walking conditions. Wave and tripod gaits can be observed and analyzed at low and high speeds of the robot body, respectively, while a transient behaviour is obtained between these two limit conditions.

## 2. Leg coordination

The gait analysis and optimization has been obtained by analyzing and implementing the algorithm proposed in (Cymbalyuk et al., 1998), which was formulated by observing in depth the walking of the stick insect and it was found that the leg coordination for a sixlegged walking robot can be considered as independent by the leg mechanism.
Referring to Fig. 1, a reference frame $G^{\prime}\left(x_{G}^{\prime} y^{\prime}{ }_{G}^{\prime} z_{G}^{\prime}\right)$ having the origin $G^{\prime}$ coinciding with the projection on the ground of the mass center $G$ of the body of the stick insect and six reference frames $O_{S i}\left(x_{S i} y_{S i} z_{S i}\right)$ for $i=1, \ldots, 6$, have been chosen in order to analyze and optimize the motion of each leg tip with the aim to ensure a suitable static stability during the walking.
Thus, in brief, the motion of each leg tip can be expressed as function of the parameters ${ }^{s i} p_{i x}$ and $s_{i}$, where ${ }^{S i} p_{i x}$ gives the position of the leg tip in $O_{S i}\left(x_{S i} y_{S i} z_{S i}\right)$ along the $x$-axis for the stance phase and $s_{i} \in\{0 ; 1\}$ indicates the state of each leg tip, i.e. one has: $s_{i}=0$ for the swing phase and $s_{i}=1$ for the stance phase, which are both performed within the range [ $P E P_{i}$, $\left.A E P_{i}\right]$, where $P E P_{i}$ is the Posterior-Extreme-Position and $A E P_{i}$ is the Anterior-Extreme-Position of each tip leg. In particular, $L$ is the nominal distance between $P E P_{0}$ and $A E P_{0}$.
The trajectory of each leg tip during the swing phase is assigned by taking into account the starting and ending times of the stance phase.


Fig. 1. Sketch and sizes of the stick insect: $d_{1}=18 \mathrm{~mm}, d_{2}=20 \mathrm{~mm}, d_{3}=15 \mathrm{~mm}, l_{1}=l_{3}=24$ $\mathrm{mm}, L=20 \mathrm{~mm}, d_{0}=5 \mathrm{~mm}, l_{0}=20 \mathrm{~mm}$

## 3. Leg mechanism

A three-revolute (3R) kinematic chain has been chosen for each leg mechanism in order to mimic the leg structure of the stick insect through the coxa, femur and tibia links, as shown in Fig. 2.
A direct kinematic analysis of each leg mechanism is formulated between the moving frame $O_{T i}\left(x_{T i} y_{T i} z_{T i}\right)$ of the tibia link and the frame $O_{0 i}\left(x_{0 i} y_{0 i} z_{0 i}\right)$, which is considered as fixed frame before to be connected to the robot body, in order to formulate the overall kinematic model of the six-legged walking robot, as sketched in Fig. 3.
In particular, the overall transformation matrix $\mathbf{M}_{T i}^{0 i}$ between the moving frame $O_{T i}\left(x_{T i} y_{T i}\right.$ $\left.z_{T i}\right)$ and the fixed frame $O_{0 i}\left(x_{0 i} y_{0 i} z_{0 i}\right)$ is given by

$$
\mathbf{M}_{T i}^{0 i}\left(\vartheta_{1 i}, \vartheta_{2 i}, \vartheta_{3 i}\right)=\left[\begin{array}{cccc}
r_{11} & r_{12} & r_{13} & { }^{0 i} p_{i x}  \tag{1}\\
r_{21} & r_{22} & r_{23} & { }^{0 i} p_{i y} \\
r_{31} & r_{32} & r_{33} & { }^{0 i} p_{i z} \\
0 & 0 & 0 & 1
\end{array}\right] .
$$

This matrix is obtained as product between four transformation matrices, which relate the moving frame of the tibia link with the three typical reference frames on the revolute joints of the leg mechanism.
Thus, each entry $r_{j k}$ of $\mathbf{M}_{T i}^{0 i}$ for $j, k=1,2,3$ and the Cartesian components of the position vector $\mathbf{p}_{i}$ in frame $O_{0 i}\left(x_{0 i} y_{0 i} z_{0 i}\right)$ are given by

$$
\begin{aligned}
r_{11} & =\mathrm{c} \alpha_{0} \mathrm{~s} \vartheta_{1 i} ; \quad r_{21}=-\mathrm{c} \vartheta_{1 i} ; \quad r_{31}=-\mathrm{s} \vartheta_{1 i} \mathrm{~s} \alpha_{0} \\
r_{12} & =\mathrm{s} \vartheta_{3 i}\left(\mathrm{~s} \alpha_{0} \mathrm{~s} \vartheta_{2 i}-\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}\right)-\mathrm{c} \vartheta_{3 i}\left(\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i}+\mathrm{s} \alpha_{0} \mathrm{c} \vartheta_{2 i}\right) \\
r_{22} & =-\mathrm{s} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i} \mathrm{~s} \vartheta_{3 i}-\mathrm{s} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i} \mathrm{c} \vartheta_{3 i} \\
r_{32} & =\mathrm{s} \vartheta_{3 i}\left(\mathrm{c} \alpha_{0} \mathrm{~s} \vartheta_{2 i}+\mathrm{s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \vartheta_{2 i}\right)+\mathrm{c} \vartheta_{3 i}\left(\mathrm{~s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i}+\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{2 i}\right) \\
r_{13} & =\mathrm{c} \vartheta_{3 i}\left(\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}-\mathrm{s} \alpha_{0} \mathrm{~s} \vartheta_{2 i}\right)-\mathrm{s} \vartheta_{3 i}\left(\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i}+\mathrm{s} \alpha_{0} \mathrm{c} \vartheta_{2 i}\right) \\
r_{23} & =\mathrm{s} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i} \mathrm{c} \vartheta_{3 i}-\mathrm{s} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i} \mathrm{~s} \vartheta_{3 i} \\
r_{33} & =-\mathrm{c} \vartheta_{3 i}\left(\mathrm{~s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}+\mathrm{c} \alpha_{0} \mathrm{~s} \vartheta_{2 i}\right)+\mathrm{s} \vartheta_{3 i}\left(\mathrm{~s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i}-\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{2 i}\right) \\
{ }^{0}{ }^{0 i} p_{i x} & =\left[\mathrm{c} \vartheta_{3 i}\left(\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}-\mathrm{s} \alpha_{0} \mathrm{~s} \vartheta_{2 i}\right)-\mathrm{s} \vartheta_{3 i}\left(\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i}+\mathrm{s} \alpha_{0} \mathrm{c} \vartheta_{2 i}\right)\right] a_{3}+ \\
& +\left(\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}-\mathrm{s} \alpha_{0} \mathrm{~s} \vartheta_{2 i}\right) a_{2}+\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{1 i} a 1_{1} \\
{ }^{0}{ }^{0} p_{i v} & =a_{3}\left(\mathrm{~s} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i} \vartheta_{3 i}-\mathrm{s} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i} \mathrm{~s} \vartheta_{3 i}\right)+\left(\mathrm{s} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}\right) a_{2}+\mathrm{s} \vartheta_{1 i} a_{1} \\
{ }^{0 i} p_{i 2} & =\left[-\mathrm{c} \vartheta_{3 i}\left(\mathrm{~s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}+\mathrm{c} \alpha_{0} \mathrm{~s} \vartheta_{2 i}\right)+\mathrm{s} \vartheta_{3 i}\left(\mathrm{~s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{~s} \vartheta_{2 i}-\mathrm{c} \alpha_{0} \mathrm{c} \vartheta_{2 i}\right)\right] a_{3}+ \\
& -\left(\mathrm{s} \alpha_{0} \mathrm{c} \vartheta_{1 i} \mathrm{c} \vartheta_{2 i}-\mathrm{c} \alpha_{0} \mathrm{~s} \vartheta_{2 i}\right) a_{2}-\mathrm{s} \alpha_{0} \mathrm{c} \vartheta_{1 i} a_{1}
\end{aligned}
$$

where $v_{1 i}, \vartheta_{2 i}$ and $v_{3 i}$ are the variable joint angles of each leg mechanism $(i=1, \ldots, 6), \alpha_{0}$ is the angle of the first joint axis with the axis $z_{0 i}$, and $a_{1}, a_{2}$ and $a_{3}$ are the lengths of the coxa, femur and tibia links, respectively.

The inverse kinematic analysis of the leg mechanism is formulated through an algebraic approach. Thus, when the Cartesian components of the position vector $\mathbf{p}_{i}$ are given in the frame $O_{F i}\left(x_{F i} y_{F i} z_{F i}\right)$, the variable joint angles $\vartheta_{1 i}, \vartheta_{2 i}$ and $\vartheta_{3 i}(i=1, \ldots, 6)$ can be expressed as

$$
\begin{equation*}
\vartheta_{1 i}=\operatorname{atan} 2\left({ }^{F i} p_{i y},{ }^{F i} p_{i x}\right) \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
\vartheta_{3 i}=\operatorname{atan} 2\left(\mathrm{~s} \vartheta_{3 i}, \mathrm{c} \vartheta_{3 i}\right), \tag{4}
\end{equation*}
$$

where

$$
\begin{aligned}
& \mathrm{c} \vartheta_{3 i}=\frac{\left({ }^{F i} p_{i x}\right)^{2}+\left({ }^{F i} p_{i y}\right)^{2}+\left({ }^{F i} p_{i z}\right)^{2}+a_{1}^{2}-2 a_{1} \sqrt{\left({ }^{F i} p_{i x}\right)^{2}+\left({ }^{F i} p_{i y}\right)^{2}}-a_{2}{ }^{2}-a_{3}^{2}}{2 a_{2} a_{3}}, \\
& \mathrm{~s} \vartheta_{3 i}= \pm \sqrt{1-\mathrm{c}^{2} \vartheta_{3 i}}
\end{aligned}
$$



Fig. 2. A 3R leg mechanism of the six-legged walking robot
and, in turn, by


Therefore, the Eqs. (1-7) let to formulate the overall kinematic model of the six-legged walking robot, as proposed in the following.

## 4. Kinematic model of the six-legged walking robot

Referring to Figs. 2 and 3, the kinematic model of a six-legged walking robot is formulated through a direct kinematic analysis between the moving frame $O_{T i}\left(x_{T i} y_{T i} z_{T i}\right)$ of the tibia link and the inertia frame $O(X Y Z)$.
In general, a six-legged walking robot has 24 d.o.f.s, where 18 d.o.f.s are given by $\vartheta_{1 i}, \vartheta_{2 i}$ and $\vartheta_{3 i}(i=1, \ldots, 6)$ for the six 3 R leg mechanisms and 6 d.o.f.s are given by the robot body, which are reduced in this case at only 1 d.o.f. that is given by $X_{G}$ in order to consider the pure translation of the robot body along the $X$-axis.
Thus, the equation of motion $X_{G}(t)$ of the robot body is assigned as input of the proposed algorithm, while $\vartheta_{1 i}(t), \vartheta_{2 i}(t)$ and $\vartheta_{3 i}(t)$ for $i=1, \ldots, 6$ are expressed through an inverse kinematic analysis of the six 3R leg mechanisms when the equation of motion of each leg tip is given and the trajectory shape of each leg tip during the swing phase is assigned.
In particular, the transformation matrix $\mathbf{M}_{G}$ of the frame $G\left(x_{G} y_{G} z_{G}\right)$ on the robot body with respect to the inertia frame $O(X Y Z)$ is expressed as

$$
\mathbf{M}_{G}\left(X_{G}\right)=\left[\begin{array}{cccc}
1 & 0 & 0 & p_{G X}  \tag{8}\\
0 & 1 & 0 & p_{G Y} \\
0 & 0 & 1 & p_{G Z} \\
0 & 0 & 0 & 1
\end{array}\right],
$$

where $p_{G X}=X_{G}, p_{G Y}=0$ and $p_{G Z}=h_{G}$.
The transformation matrix $\mathbf{M}_{B i}^{G}$ of the frame $O_{B i}\left(x_{B i} y_{B i} z_{B i}\right)$ on the robot body with respect to the frame $G\left(x_{G} y_{G} z_{G}\right)$ is expressed by

where $l_{1}=l_{4}=-l_{0}, l_{2}=l_{5}=0, l_{3}=l_{6}=l_{0}$.
Therefore, the direct kinematic function of the walking robot is given by

$$
\begin{equation*}
\mathbf{M}_{T i}\left(X_{G}, \vartheta_{1 i}, \vartheta_{2 i}, v_{3 i}\right)=\mathbf{M}_{G}\left(X_{G}\right) \quad \mathbf{M}_{B i}^{G} \quad \mathbf{M}_{0 i}^{B i} \quad \mathbf{M}_{T i}^{0 i}\left(\vartheta_{1 i}, \vartheta_{2 i}, v_{3 i}\right) \tag{10}
\end{equation*}
$$

where $\mathbf{M}_{0 i}^{B i}=\mathbf{I}$, being I the identity matrix.
The joint angles of the leg mechanisms are obtained through an inverse kinematic analysis. In particular, the position vector ${ }^{S i} \mathbf{p}_{i}(t)$ of each leg tip in the frame $O_{S i}\left(x_{S i} y_{S i} z_{S i}\right)$, as shown in Fig.2, is expressed in the next section along with a detailed motion analysis of the leg tip. Moreover, the transformation matrix $\mathbf{M}_{S i}^{B i}$ is given by

$$
\mathbf{M}_{S i}^{B i}=\left\{\begin{array}{lll}
{\left[\begin{array}{rrrr}
0 & -1 & 0 & L_{i} \\
1 & 0 & 0 & d_{i} \\
0 & 0 & 1 & -h_{i} \\
0 & 0 & 0 & 1
\end{array}\right] \quad \text { for } i=1,2,3}  \tag{11}\\
{\left[\begin{array}{rrrr}
0 & 1 & 0 & L_{i-3} \\
-1 & 0 & 0 & -d_{i-3} \\
0 & 0 & 1 & -h_{i} \\
0 & 0 & 0 & 1
\end{array}\right] \quad \text { for } i=4,5,6}
\end{array}\right.
$$

where $L_{1}=l_{1}-l_{0}, L_{2}=0$ and $L_{3}=l_{3}-l_{0}$ with $L_{i}$ shown in Fig.2.
Finally, the position of each leg tip in the frame $O_{F i}\left(x_{F i} y_{F i} z_{F i}\right)$ is given by

$$
\begin{equation*}
{ }^{F i} \mathbf{p}_{i}(t)=\mathbf{M}_{0 i}^{F i} \mathbf{M}_{B i}^{0 i} \mathbf{M}_{S i}^{B i}{ }^{S i} \mathbf{p}_{i}(t) \tag{12}
\end{equation*}
$$

where the matrix $\mathbf{M}_{0 i}^{F i}$ can be easily obtained by knowing the angle $\alpha_{0}$.

Therefore, substituting the Cartesian components of ${ }^{F i} \mathbf{p}_{i}(t)$ in Eqs. (3), (5) and (7), the joint angles $\vartheta_{1 i}, \vartheta_{2 i}$ and $\vartheta_{3 i}(i=1, \ldots, 6)$ can be obtained.


Fig. 3. Kinematic scheme of the six-legged walking robot

## 5. Motion analysis of the leg tip

The gait of the robot is obtained by a suitable coordination of each leg tip, which is fundamental to ensure the static stability of the robot during the walking. Thus, a typical motion of each leg tip has to be imposed through the position vector ${ }^{S i} \mathbf{p}_{i}(t)$, even if a variable gait of the robot can be obtained according to the imposed speed of the robot body. Referring to Figs. 2 to 4 , the position vector ${ }^{S i} \mathbf{p}_{i}(t)$ of each leg tip can be expressed as

$$
{ }^{s i} \mathbf{p}_{i}(t)=\left[\begin{array}{llll}
{ }^{s i} p_{i x} & { }^{s i} p_{i y} & { }^{s i} p_{i z} & 1 \tag{13}
\end{array}\right]^{T}
$$

in the local frame $O_{S i}\left(x_{S i} y_{S i} z_{S i}\right)$ for $i=1, \ldots, 6$, which is considered as attached and moving with the robot body.
Referring to Fig. 4, the $x$-coordinate ${ }^{S i} p_{i x}$ of vector ${ }^{S i} \mathbf{p}_{i}(t)$ is given by the following system of difference equations

$$
{ }^{S i} p_{i x}(t+\Delta t)= \begin{cases}p_{i x}(t)-V_{r} \Delta t & \text { for } s_{i}(t)=1  \tag{14}\\ p_{i x}(t)+V_{p} \Delta t & \text { for } s_{i}(t)=0\end{cases}
$$

where $V_{r}$ is the velocity of the tip of each leg mechanism during the retraction motion of the stance phase, even defined power stroke, since producing the motion of the robot body, and $V_{p}$ is the velocity along the robot body of the tip of each leg mechanism during the protraction motion of the swing phase, even defined return stroke, since producing the forward motion of the leg mechanism.


Fig. 4. Trajectory and velocities of the tip of each leg mechanism during the stance and swing phases

Parameter $s_{i}$ defines the state of the $i$-th leg, which is equal to 1 for the retraction state, or stance phase, and equal to 0 for the protraction state, or swing phase.
Both velocities $V_{p}$ and $V_{r}$ are supposed to be constant and identical for all legs, for which the speed $V_{G}$ of the center of mass of the robot body is equal to $V_{r}$ because of the relative motion. In fact, during the stance phase (power stroke), each tip leg moves back with velocity $V_{r}$ with respect to the robot body and, consequentially, this moves ahead with the same velocity. Thus, the function ${ }^{S i} p_{i x}(t)$ of Eq. (2) for $i=1, \ldots, 6$ is linear periodic function.
Moreover, it is quite clear that the static stability of the six-legged walking robot is obtained only when $V_{r}\left(=V_{G}\right)<V_{p}$, because the robot body cannot move forward faster than its legs move in the same direction during the swing phase. Likewise, ${ }^{S i} p_{i y}$ is equal to zero in order to obtain a vertical planar trajectory, while ${ }^{S i} p_{i z}$ is given by

$$
{ }^{s_{i}} p_{i z}(t)= \begin{cases}0 & \text { for } s_{i}(t)=1  \tag{15}\\ h_{T} \sin \left(\pi \frac{t-t_{0}^{i}}{t_{f}^{i}-t_{0}^{i}}\right) & \text { for } s_{i}(t)=0\end{cases}
$$

where $h_{T}$ is the amplitude of the sinusoid and time $t$ is the general instant, while $t_{0}^{i}$ and $t_{f}^{i}$ are the starting and ending times of the swing phase, respectively.
Times $t_{0}^{i}$ and $t_{f}^{i}$ take into account the mechanism of the leg coordination, which give a suitable variation of $A E P_{i}$ and $P E P_{i}$ in order to ensure the static stability.
Thus, referring to the time diagrams of $V_{G}$ in Fig. 5, the time diagrams of Figs. 6 to 8 have been obtained. In particular, Fig. 5 shows the time diagrams of the robot speeds $V_{G}=0.05$, $0.1,0.5$ and $0.9 \mathrm{~mm} / \mathrm{s}$ for the case of constant acceleration $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$. Thus, the transient periods $t=25,50,250$ and 450 s for the speeds $V_{G}=0.05,0.1,0.5$ and $0.9 \mathrm{~mm} / \mathrm{s}$ of the robot body are obtained respectively before to reach the steady-state condition at
constant speed. The time diagrams of Figs. 6 to 8 show the horizontal $x$-displacement, the $x$ component of the velocity, the vertical $z$-displacement, the $z$-component of the velocity, the magnitude of the velocity and the trajectory of the leg tip 1 (front left leg) of the six-legged walking robot. Thus, before to analyze in depth the time diagrams of Figs. 6 to 8, it may be useful to refer to the motion analysis of the leg tip and to remind that the protraction speed $V_{p}$ along the axis of the robot body has been assigned as constant and equal to $1 \mathrm{~mm} / \mathrm{s}$ for the swing phase of the leg tip. In other words, only the retraction speed $V_{r}$ can be changed since related and equal to the robot speed $V_{G}$, which is assigned as input data. Consequently, the range time during the stance phase between two consecutive steps of each leg can vary in significant way because of the different imposed speeds $V_{r}=V_{G}$, while the time range to perform the swing phase of each leg is almost the same because of the same speed $V_{p}$ and similar overall $x$-displacements.
In particular, Fig. 6 show computer simulations between the time range 200-340 s, which is after the transient periods of 25 and 50 s for $V_{G}=0.05$ and $0.1 \mathrm{~mm} / \mathrm{s}$, respectively.
Thus, both $x$-component of the velocity, protraction speed $V_{p}=1 \mathrm{~mm} / \mathrm{s}$ and retraction speed $V_{r}=V_{G}=0.05$ and $0.1 \mathrm{~mm} / \mathrm{s}$, are constant versus time. Instead, Figs. 7 and 8 show two computer simulations between the time ranges of $0-400 \mathrm{~s}$ and $200-600 \mathrm{~s}$, which are greater than the transient periods of 250 and 450 s for the robot speeds $V_{G}$ of 0.5 and $0.9 \mathrm{~mm} / \mathrm{s}$, respectively. Thus, the transient behavior of the velocities is also shown at the constant acceleration of $0.002 \mathrm{~mm} / \mathrm{s}^{2}$. In fact, during these time ranges of 250 and 450 s , the protraction speed $V_{p}$ is always constant and equal to $1 \mathrm{~mm} / \mathrm{s}$, while the retraction speed $V_{r}$ varies linearly according to the constant acceleration, before to reach the steady-state condition and to equalize the speed $V_{G}$ of the robot body. The same effect is also shown by the time diagrams of Figs. 7 e and 8 e , which show the magnitude of the velocity.
Moreover, single loop trajectories are shown in the simulations of Figs. 6e and 6 m , because one step only is performed by the leg mechanism 1, while multi-loop trajectories are shown in the simulations of Figs. 7 f and 8 f , because 3 (three) and 7 (seven) steps are performed by the leg mechanism 1, respectively. The variation of the step length is also evident in Figs. 7 f and 8 f because of the influence mechanisms for the leg coordination.


Fig. 5. Time diagrams of the robot body speed for a constant acceleration $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ and $V_{G}=0.05,0.1,0.5$ and $0.9 \mathrm{~mm} / \mathrm{s}$


Fig. 6. Computer simulations for the motion analysis of the leg tip of a six-legged walking robot when $V_{G}=0.05$ and $0.1[\mathrm{~mm} / \mathrm{s}]$ : a) and g) horizontal $x$-displacement; b) and h) vertical $z$-displacement; c) and i) $x$-component of the velocity; d) and l) $z$-component of the velocity; e) and m) planar trajectory in the $x z$-plane; f) and $n$ ) magnitude of the velocity


Fig. 7. Computer simulations within the time range 0-400 s for the motion analysis of the leg tip when $V_{G}=0.5[\mathrm{~mm} / \mathrm{s}]$ : a) horizontal $x$-displacement; b) $x$-component of the velocity; c) vertical $z$-displacement; d) $z$-component of the velocity; e) magnitude of the velocity; f planar trajectory in the $x z$-plane


Fig. 8. Computer simulations within the time range 200-600 s for the motion analysis of the leg tip when $V_{G}=0.9[\mathrm{~mm} / \mathrm{s}]$ : a) horizontal $x$-displacement; b) $x$-component of the velocity; c) vertical $z$-displacement; d) $z$-component of the velocity; e) magnitude of the velocity; f) planar trajectory in the $x z$-plane

## 6. Gait analysis

A suitable overall algorithm has been formulated as based on the kinematic model of the six-legged walking robot and on the leg tip motion of each leg mechanism. This algorithm has been implemented in a Matlab program in order to analyze the absolute gait of the sixlegged walking robot, which mimics the behavior of the stick insect, for different speeds of the robot body. Thus, the absolute gait of the robot is analyzed by referring to the results of suitable computer simulations, which have been obtained by running the proposed algorithm. In particular, the results of three computer simulations are reported in the following in the form of time diagrams of the $z$ and $x$-displacements of the tip of each leg mechanism. These three computer simulations have been obtained for three different input parameters in terms of speed and acceleration of the robot body.
The same constant acceleration $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ has been considered along with three different speeds $V_{G}=0.05,0.1$ and $0.9 \mathrm{~mm} / \mathrm{s}$ of the center of mass of the robot body, as shown in the time diagram of Fig. 5. Of course, the transient time before to reach the steadystate condition is different for the three simulations because of the same acceleration which has been imposed. Moreover, the protraction speed $V_{p}$ along the axis of the robot body has been assigned equal to $1 \mathrm{~mm} / \mathrm{s}$ for the swing phase. Thus, only the retraction speed $V_{r}$ of the tip of each leg mechanism is changed since related and equal to the speed $V_{G}$ of the center of mass of the robot body. Consequently, the range time during the stance phase between two consecutive steps of each leg varies in significant way because of the different imposed speeds $V_{r}=V_{G}$, while the time range to perform the swing phase of each leg is almost the same because of the same speed $V_{p}$ and similar overall $x$-displacements.
The time diagrams of the $z$ and $x$-displacements of each leg of the six-legged walking robot are shown in Figs. 9 to 11, as obtained for $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ and $V_{G}=0.05 \mathrm{~mm} / \mathrm{s}$. It is noteworthy that the maximum vertical stroke of the tip of each leg mechanism is always equal to 10 mm , while the maximum horizontal stroke is variable and different for the tip of each leg mechanism according to the leg coordination, which takes into account the static stability of the six-legged walking robot. However, these horizontal strokes of the tip of each leg mechanism are quite centered around 0 mm and similar to the nominal stroke $L=24$ mm , which is considered between the extreme positions $A E P_{0}$ and $P E P_{0}$.
Moreover, the horizontal $x$-displacements are represented through liner periodic functions, where the slope of the line for the swing phase is constant and equal to the speed $V_{p}=1$ $\mathrm{mm} / \mathrm{s}$, while the slope of the line for the stance phase is variable according to the assigned speed $V_{G}$, as shown in Figs. 9,10 and 11 for $V_{G}=0.05,01$ and $0.9 \mathrm{~mm} / \mathrm{s}$, respectively.
In particular, referring to Fig. 11, the slopes of both linear parts of the linear periodic function of the $x$-displacement are almost the same, as expected, because the protraction speed of $1 \mathrm{~mm} / \mathrm{s}$ is almost equal to the retraction speed of $0.9 \mathrm{~mm} / \mathrm{s}$.
Moreover, three different gait typologies of the six-legged walking robot can be observed in the three simulations, which are represented in the diagrams of Figs. 9 to 11.
In particular, the simulation of Fig. 9 show a wave gait of the robot, which is typical at low speeds and that can be understood with the aid of the sketch of Fig. 12a. In fact, referring to the first peak of the diagram of leg 1 of Fig. 9, which takes place after 400 s and, thus, after the transient time before to reach the steady-state condition of $0.05 \mathrm{~mm} / \mathrm{s}$, the second leg to move is leg 5 and, then, leg 3 . Thus, observing in sequence the peaks of the $z$-displacements of the six legs, after leg 3 , it is the time of the leg 4 and, then, leg 2 in order to finish with leg 6, as sketched in Fig. 12a, before to restart the wave gait.


Fig. 9. Displacements of the leg tip for $V_{G}=0.05 \mathrm{~mm} / \mathrm{s}$ and $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}:$ a) vertical $z$ displacement; b) horizontal $x$-displacement


Fig. 10. Displacements of the leg tip for $V_{G}=0.1 \mathrm{~mm} / \mathrm{s}$ and $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ : a) vertical $z$ displacement; b) horizontal $x$-displacement


Fig. 11. Displacements of the leg tip for $V_{G}=0.9 \mathrm{~mm} / \mathrm{s}$ and $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ : a) vertical $z$ displacement; b) horizontal $x$-displacement


Fig. 12. Typical gaits of a six-legged walking robot: a) wave; b) transient gait; c) tripod
The simulation of Fig. 10 show the case of a particular gait of the robot, which is not wave and neither tripod, as it will be explained in the following. The sequence of the steps for this particular gait can be also understood with the aid of the sketch of Fig. 12b. This gait typology of the six-legged walking robot can be considered as a transient gait between the two extreme cases of wave and tripod gaits.
In fact, the tripod gait can be observed by referring to the time diagrams reported in Fig. 11. The tripod gait can be understood by analyzing the sequence of the peaks of the $z$-displacements for the tip of each leg mechanism and with the aid of the sketch of Fig. 12c. The tripod gait is typical at high speeds of the robot body. In fact, the simulation of Fig. 11 has been obtained for $V_{G}=0.9 \mathrm{~mm} / \mathrm{s}$, which is almost the maximum speed ( $V_{p}=1 \mathrm{~mm} / \mathrm{s}$ ) reachable by the robot before to fall down because of the loss of the static stability. In particular, legs 1,5 and 3 move together to perform a step and, then, legs 4, 2 and 6 move together to perform another step of the six-legged walking robot. Both steps are performed with a suitable phase shift according to the input speed.

## 7. Absolute gait simulation

This formulation has been implemented in a Matlab program in order to analyze the performances of a six-legged walking robot during the absolute gait along the $X$-axis of the inertia frame $O(X Y Z)$.
Figures 13 and 14 show two significant simulations for the wave and tripod gaits, which have been obtained by running the proposed algorithm for $V_{G}=0.05 \mathrm{~mm} / \mathrm{s}$ and $V_{G}=0.9$ $\mathrm{mm} / \mathrm{s}$, respectively. In particular, six frames for each simulation are reported along with the inertia frame, which can be observed on the right side of each frame, as indicating the starting position of the robot. Thus, the robot moves toward the left side by performing a transient motion at constant acceleration $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ before to reach the steady-state condition with a constant speed.
In particular, for the wave gait cycle of Fig. 13, all leg tips are on the ground in Fig. 13a and leg tip 4 performs a swing phase in Fig. 13b before to touch the ground in Fig. 13c. Then, leg tip 2 performs a swing phase in Fig. 13d before to touch the ground in Fig. 13e and, finally, leg tip 6 performs a swing phase in Fig. 13f.
Similarly, for the tripod gait cycle of Fig. 14, all leg tips are on the ground in Figs. 14a, 14c and 14 e . Leg tips $4-2-6$ perform a swing phase in Fig. 14b and 14 f between the swing phase performed by the leg tips 1-5-3 in Fig. 14d.


Fig. 13. Animation of a wave gait along the $X$-axis for $V_{G}=0.05 \mathrm{~mm} / \mathrm{s}$ and $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ : a), c) and e), all leg tips are on the ground; b) leg tip 4 performs a swing phase; d) leg tip 2 performs a swing phase; f) leg tip 6 performs a swing phase


Fig. 14. Animation of a tripod gait along the $X$-axis for $V_{G}=0.9 \mathrm{~mm} / \mathrm{s}$ and $a=0.002 \mathrm{~mm} / \mathrm{s}^{2}$ : a), c) and e), all leg tips are on the ground; b) leg tips 4-2-6 perform a swing phase; d) leg tips 1-5-3 perform a swing phase; f) leg tips 4-2-6 perform a swing phase

## 8. Conclusions

The mechanics and locomotion of six-legged walking robots has been analyzed by considering a simple "technical design", in which the biological inspiration is only given by the trivial observation that some insects use six legs to obtain a static walking, and considering a "biological design", in which we try to emulate, in every detail, the locomotion of a particular specie of insect, as the "cockroach" or "stick" insects.
In particular, as example of the mathematical approach to analyze the mechanics and locomotion of six-legged walking robots, the kinematic model of a six-legged walking robot, which mimics the biological structure and locomotion of the stick insect, has been formulated according to the Cruse-based leg control system.
Thus, the direct kinematic analysis between the moving frame of the tibia link and the inertia frame that is fixed to the ground has been formulated for the six 3R leg mechanisms, where the joint angles have been expressed through an inverse kinematic analysis when the trajectory of each leg tip is given. This aspect has been considered in detail by analyzing the motion of each leg tip of the six-legged walking robot in the local frame, which is considered as attached and moving with the robot body.
Several computer simulations have been reported in the form of time diagrams of the horizontal and vertical displacements along with the horizontal and vertical components of the velocities for a chosen leg of the robot. Moreover, single and multi-loop trajectories of a leg tip have been shown for different speeds of the robot body, in order to put in evidence the effects of the Cruse-based leg control system, which ensures the static stability of the robot at different speeds by adjusting the step length of each leg during the walking. Finally, the gait analysis and simulation of the six-legged walking robot, which mimics the locomotion of the stick insect, have been carried out by referring to suitable time diagrams of the $z$ and $x$-displacements of the six legs, which have shown the extreme typologies of the wave and tripod gaits at low and high speeds of the robot body, respectively.

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With the advancement of technology，new exciting approaches enable us to render mobile robotic systems more versatile，robust and cost－efficient．Some researchers combine climbing and walking techniques with a modular approach，a reconfigurable approach，or a swarm approach to realize novel prototypes as flexible mobile robotic platforms featuring all necessary locomotion capabilities．The purpose of this book is to provide an overview of the latest wide－range achievements in climbing and walking robotic technology to researchers， scientists，and engineers throughout the world．Different aspects including control simulation，locomotion realization，methodology，and system integration are presented from the scientific and from the technical point of view．This book consists of two main parts，one dealing with walking robots，the second with climbing robots． The content is also grouped by theoretical research and applicative realization．Every chapter offers a considerable amount of interesting and useful information．

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