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Parallel Manipulators with Lower Mobility

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1. Introduction

Parallel manipulators with lower mobility (LM-PMs) are multi-loop mechanisms with less than six degrees of freedom (dofs). This type of manipulators has attracted the attention both of academic researchers and of industries since the early appearance of the DELTA robot (Clavel 1988).

The DELTA robot showed that, when the manipulation task requires less than six dofs, the use of an LM-PM may bring some advantages (simple architecture of the machine, very fast machine, etc.) that are added to the known appealing features (high stiffness, good positioning precision, etc) of parallel manipulators (PMs).Planar motions, translational motions and spherical motions are important examples of motion tasks that require less than six dofs and are often necessary in industrial applications. Each of these types of motion has generated a class of LM-PMs. So, today, there is a great variety of planar PMs (PPMs), of translational PMs (TPMs) and of spherical PMs (SPMs).

This chapter attempts to provide a unified frame for the study of this type of machines together with a critical analysis of the vast literature about them.

The chapter starts with the classification of the LM-PMs, and, then, analyzes the specific subjects involved in the functional design of these machines. Special attention is paid to the definition of the limb topology, the singularity analysis and the discussion of the characteristics of some machines.

2. Classification of the Parallel Manipulators with Lower Mobility

Addressing the problem of classifying manipulators is neither a useless nor a trivial task. Indeed, an exhaustive classification is able to guide the designer towards the technical answers he is looking for.

Lower-mobility manipulators (LM-M) can be classified according to the type of motion their end effector performs by using the group theory (Hervé 1978, 1999). The set of rigid-body displacements (motions), {D}, is a six-dimensional group which, in addition to the identity subgroup, {E}, that corresponds to absence of motion, contains the following ten motion subgroups with dimension greater than zero and less than six (the generic element of a displacement sub-

group can be represented by the screw identifying the finite or infinitesimal motion belonging to the subgroup; the dimension of the subgroup is the number of independent scalar parameters that, in the analytic expression of the generic-element's screw, must be varied to generate the screws of all the elements of the subgroup):

(a) Subgroups of dimension 1:

(a.1) <u>linear translation subgroup</u>, $\{T(\mathbf{u})\}$, that collects all the translations parallel to the unit vector $\mathbf{u}$. As many $\{T(\mathbf{u})\}$ as unit vectors, $\mathbf{u}$, can be defined. The identity subgroup $\{E\}$ is included in all the $\{T(\mathbf{u})\}$. A prismatic pair (hereafter denoted with P) with sliding direction parallel to $\mathbf{u}$ physically generates the motions of $\{T(\mathbf{u})\}$.

(a.2) <u>revolute subgroup</u>, $\{R(O, \mathbf{u})\}$, that collects all the rotations around an axis (rotation axis) passing through point O and parallel to the unit vector $\mathbf{u}$. As many $\{R(O, \mathbf{u})\}$ as rotation axes, $(O, \mathbf{u})$, can be defined. The identity subgroup $\{E\}$ is included in all the $\{R(O, \mathbf{u})\}$. A revolute pair (hereafter denoted with R) with rotation axis $(O, \mathbf{u})$ physically generates the motions of $\{R(O, \mathbf{u})\}$.

(a.3) <u>helical subgroup</u>, $\{H(O, \mathbf{u}, p)\}$, that collects all the helical motions with axis $(O, \mathbf{u})$ and finite pitch p that is different from zero and constant. As many $\{H(O, \mathbf{u}, p)\}$ as sets of helix parameters, $(O, \mathbf{u}, p)$, can be defined. The identity subgroup $\{E\}$ is included in all the $\{H(O, \mathbf{u}, p)\}$. A helical pair (hereafter denoted with H) with helix parameters $(O, \mathbf{u}, p)$ physically generates the motions of $\{H(O, \mathbf{u}, p)\}$.

(b) Subgroups of dimension 2:

(b.1) <u>planar translation subgroup</u>, $\{T(\mathbf{u}_1, \mathbf{u}_2)\}$, that collects all the translations parallel to a plane perpendicular to $\mathbf{u}_1 \times \mathbf{u}_2$ where $\mathbf{u}_1$ and $\mathbf{u}_2$ are two orthogonal unit vectors. As many $\{T(\mathbf{u}_1, \mathbf{u}_2)\}$ as unit vectors $\mathbf{u}_1 \times \mathbf{u}_2$ can be defined. The identity subgroup $\{E\}$ and all the linear translation subgroups $\{T(\mathbf{v})\}$ with $\mathbf{v}$ equals to $a\mathbf{u}_1 + \mathbf{u}_2\sqrt{1-a^2}$ are included in $\{T(\mathbf{u}_1, \mathbf{u}_2)\}$. Two prismatic pairs in series, whose sliding directions are respectively parallel to two independent vectors that are linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$, physically generate the motions of $\{T(\mathbf{u}_1, \mathbf{u}_2)\}$.

(b.2) <u>cylindrical subgroup</u>, $\{C(O, \mathbf{u})\}$, that collects all the motions obtained by combining a rotation around a rotation axis $(O, \mathbf{u})$ and a translation parallel to the unit vector $\mathbf{u}$. As many $\{C(O, \mathbf{u})\}$ as $(O, \mathbf{u})$ axes can be defined. The subgroups $\{E\}$, $\{T(\mathbf{u})\}$, $\{R(O, \mathbf{u})\}$ and $\{H(O, \mathbf{u}, p)\}$ are all included in $\{C(O, \mathbf{u})\}$. A cylindrical pair (hereafter denoted with C) or, which is the same, a revolute pair with rotation axis $(O, \mathbf{u})$ in series with a prismatic pair with sliding direction parallel to $\mathbf{u}$ physically generate the motions of $\{C(O, \mathbf{u})\}$.

(c) Subgroups of dimension 3:

(c.1) spatial translation subgroup, {T}, that collects all the spatial translations. Only one subgroup {T} can be defined. The identity subgroup {E}, all the {T($\mathbf{u}$)} subgroups and all the {T($\mathbf{u}_1$, $\mathbf{u}_2$)} subgroups are included in {T}. Three prismatic pairs in series whose sliding directions are respectively parallel to three independent unit vectors, $\mathbf{u}_1$, $\mathbf{u}_2$ and $\mathbf{u}_3$, physically generate the motions of {T}.

(c.2) spherical subgroup, {S(O)}, that collects all the spherical motions with center O. As many {S(O)} as O points can be defined. The identity subgroup {E} and all the {R(O, $\mathbf{u}$)} subgroups are included in {S(O)}. A spherical pair (hereafter denoted with S) or, which is the same, three revolute pairs in series with rotation axes that intersect one another in O physically generate the motions of {S(O)}.

(c.3) planar subgroup, {G($\mathbf{u}_1$, $\mathbf{u}_2$)}, that collects all the planar motions with motion plane perpendicular to $\mathbf{u}_1 \times \mathbf{u}_2$ where $\mathbf{u}_1$ and $\mathbf{u}_2$ are two orthogonal unit vectors. As many {G($\mathbf{u}_1$, $\mathbf{u}_2$)} as unit vectors $\mathbf{u}_1 \times \mathbf{u}_2$ can be defined. The subgroups {E}, {T($\mathbf{u}_1$, $\mathbf{u}_2$)}, {R(O, $\mathbf{u}_1 \times \mathbf{u}_2$)} and {T($\mathbf{v}$)} with $\mathbf{v}$ equals to $a\mathbf{u}_1 + \mathbf{u}_2\sqrt{1-a^2}$ are included in {G($\mathbf{u}_1$, $\mathbf{u}_2$)}. A PPR kinematic chain where the sliding directions of the two prismatic pairs are respectively parallel to two independent vectors that are linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$, and the revolute-pair axis is orthogonal both to $\mathbf{u}_1$ and to $\mathbf{u}_2$ physically generates the motions of {G($\mathbf{u}_1$, $\mathbf{u}_2$)}.

(c.4) pseudo-planar subgroup, {Y($\mathbf{u}_1$, $\mathbf{u}_2$, p)}, that collects all the motions obtained by combining a planar translation belonging to {T($\mathbf{u}_1$, $\mathbf{u}_2$)} with a helical motion belonging to {H(O, $\mathbf{u}_1 \times \mathbf{u}_2$, p)}. As many {Y($\mathbf{u}_1$, $\mathbf{u}_2$, p)} as sets of parameters ($\mathbf{u}_1 \times \mathbf{u}_2$, p) can be defined. The subgroups {E}, {T($\mathbf{u}_1$, $\mathbf{u}_2$)}, {H(O, $\mathbf{u}_1 \times \mathbf{u}_2$, p)} and {T($\mathbf{v}$)} with $\mathbf{v}$ equals to $a\mathbf{u}_1 + \mathbf{u}_2\sqrt{1-a^2}$ are included in {Y($\mathbf{u}_1$, $\mathbf{u}_2$, p)}. A RRH kinematic chain where the axes of the two revolute-pairs and the helical-pair's axis are all parallel to one another and orthogonal both to $\mathbf{u}_1$ and to $\mathbf{u}_2$ physically generates the motions of {Y($\mathbf{u}_1$, $\mathbf{u}_2$, p)}.

(d) Subgroups of dimension 4:

(d.4) Schoenflies subgroup, {X($\mathbf{u}_1$, $\mathbf{u}_2$)}, that collects all the motions obtained by combining a planar translation belonging to {T($\mathbf{u}_1$, $\mathbf{u}_2$)} with a cylindrical motion belonging to {C(O, $\mathbf{u}_1 \times \mathbf{u}_2$)}. As many {X($\mathbf{u}_1$, $\mathbf{u}_2$)} as unit vectors $\mathbf{u}_1 \times \mathbf{u}_2$ can be defined. The subgroups {E}, {T}, {G($\mathbf{u}_1$, $\mathbf{u}_2$)}, {Y($\mathbf{u}_1$, $\mathbf{u}_2$, p)}, {T($\mathbf{u}_1$, $\mathbf{u}_2$)}, {C(O, $\mathbf{u}_1 \times \mathbf{u}_2$)}, {H(O, $\mathbf{u}_1 \times \mathbf{u}_2$, p)} and {T($\mathbf{v}$)} with $\mathbf{v}$ equals to $a\mathbf{u}_1 + \mathbf{u}_2\sqrt{1-a^2}$ are included in {X($\mathbf{u}_1$, $\mathbf{u}_2$)}. A RRC kinematic chain where the axes of the two revolute pairs and the cylindrical-pair's

axis are all parallel to one another and orthogonal both to $\mathbf{u}_1$ and to $\mathbf{u}_2$ physically generates the motions of $\{X(\mathbf{u}_1, \mathbf{u}_2)\}$.

According to this (Rico et al. 2006), the set of the LM-Ms can be separated into two subsets: (i) the subset of the pure-motion LM-Ms and (ii) the subset of the mixed-motion LM-Ms. The first subset collects all the LM-Ms whose end effector exhibits motions that belong to only one out of the ten motion subgroups of {D}, whereas the second one collects all the other LM-Ms.

The pure-motion LM-Ms can be further spread into ten pure-motion subsets: one for each pure motion identified by the ten subgroups of {D}. In (Hervé 1978, 1999), a kinematic chain is called mechanical bond when it connects one rigid body (end effector) to another (frame) so that the relative motion between end effector and frame is constrained. A mechanical bond is called mechanical generator when all the allowed relative motions between end effector and frame belong to only one of the ten subgroups of {D}.

The nature of an LM-M can be identified by analysing its workspace, {W} (the workspace is the connected set of poses (positions and orientations) the end effector can assume without disassembling the LM-M). In fact, if any couple of poses belonging to {W} defines an end-effector motion that belongs to the same motion subgroup of {D}, then the LM-M is a pure-motion LM-M, otherwise it is a mixed-motion LM-M. Hereafter, if a set of motions, {M}, only collects the motions identified by all the couples of poses that belong to the same connected set of poses, {P}, then it will be said that "{P} corresponds to {M} and vice versa" (it is worth noting that different set of poses may correspond to the same set of motions).

When serial manipulators with lower mobility (LM-SMs) are considered, the end-effector motion is obtained by composing the motions of all the manipulator's joints (Hervé 1978). Therefore, a pure-motion LM-SM can be obtained only by using joints whose motions belong to the same motion subgroup. Moreover, the sum of the degrees of freedom (dofs) of the joints must be equal to the dimension of that motion subgroup.

When a parallel manipulator with lower mobility (LM-PM) is considered, the identification of the set of motions, {L}, the end effector can perform is a bit more complex. From a structural point of view, a parallel manipulator is a machine where the end effector is connected to the frame through a number, n, of kinematic chains (limbs) acting in parallel. Therefore, in an LM-PM with n limbs, both {L} and {W} are subsets of the common intersection of the n sets, respectively, of motions, $\{L_j\}$, j=1,...,n, and of poses, $\{W_j\}$, j=1,...,n, the j-th limb would allow to the end effector if it were the only kinematic chain joining end effector and frame. If all the $\{W_j\}$ are restricted to the end effector poses that can be reached without disassembling the LM-PM and all the corresponding $\{L_j\}$ are accordingly restricted (hereafter, if it is not differently specified, this restriction will be implicitly assumed), then the following relationships hold:

$$\{W\} = \bigcap_{j=1,n} \{W_j\} \tag{1}$$

$$\{L\} = \bigcap_{j=1,n} \{L_j\} \tag{2}$$

This fact discloses a wide scenario of possible machine architectures even for pure-motion LM-PMs since conditions (1) and (2), where $\{L\}$ is a subset of a motion subgroup, can be satisfied by using, in the limbs, joints of any type (not necessarily belonging to the same motion subgroup), and, as limbs, kinematic chains with number of dof (limb's connectivity) greater than the manipulator's dofs.

Each subset of LM-PMs can be further separated into two classes: the class of the overconstrained manipulators and the class of the non-overconstrained manipulators. Overconstrained manipulators are machines whose dof number is higher than the one computed as if the constraints due to the joints were in-dependent. An overconstrained LM-PM can be obtained by using, as limbs, a number, n, of serial kinematic chains with the same number of dofs as the LM-PM, provided that n be equal to the LM-PM's dofs, and the limbs be arranged so that they warranty a non-empty intersection, $\{W\}$, among the n sets, $\{W_j\}$, j=1,…,n. This principle has guided the design of many overconstrained LM-PMs among which the most known is certainly the 3RRR wrist (Gosselin & Angeles 1989) which uses, as limbs, three serial wrists of type RRR with the same spherical-motion center (see Fig. 1). The advantage of a pure-motion LM-PM obtained by using this principle, with respect to the serial manipulator that have the same topology as the one of the LM-PM's limbs, is that the LM-PM has all the actuators located on the frame, which allows fast manipulators to be manufactured. This advantage is paid with a reduced workspace and with an architecture that need an high manufacturing precision to avoid jamming of the joints and high internal loads in the links.
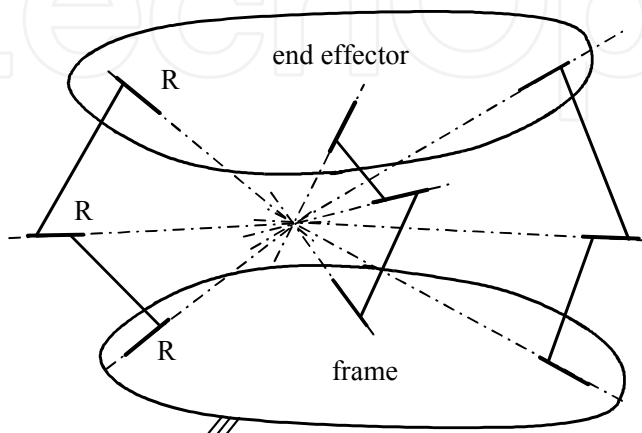

Figure 1. 3RRR wrist (Gosselin & Angeles 1989)

3. Determination of Limbs' Topologies for an LM-PM

Conditions (1) and (2) guide the determination of limbs' topologies suitable for making the end effector of an LM-PM perform a given type of motion. In the literature, the analyses that bring to identify such topologies have been mainly addressed through three different approaches: (i) group theory (Hervé 1978, 1995, 1999, 2004; Hervé & Sparacino 1991; Karouia & Hervé 2000, 2002; Huynh & Hervé 2005; Lee & Hervé 2006; Rico et al. 2006), (ii) screw theory (Tsai 1999; Fang & Tsai 2002; Frisoli et al. 2000; Kong & Gosselin 2002, 2004a, 2004b, 2005; Huang & Li 2002, 2003; Carricato 2005) and (iii) velocity-loop equations (Di Gregorio & Parenti-Castelli 1998; Di Gregorio 2001a, 2001b, 2002, 2003; Carricato & Parenti-Castelli 2002, 2003).

The first approach (group theory) determines the generic $\{L_j\}$ by composing the set of motions generated by each joint of the kinematic chain that is candidate to be a limb, and, then, searches for the geometric conditions the potential limb must satisfy in order to make a subset of an assigned motion subgroup of $\{D\}$ be a subset of $\{L_j\}$. The result of this type of analysis is the determination (topology plus geometric conditions) of all the generators of a given motion subgroup. Each generator of a given motion subgroup of $\{D\}$ can be used as limb in an LM-PM that must bound the end-effector motions to a subset of that motion subgroup.

The second approach (screw theory) determine the screw (twist), $\$_j$, which represents the generic element of $\{L_j\}$, as a linear combination of the twists of all the joints of the kinematic chain that is candidate to be a limb. Then, the screw (wrench), ξ_j, reciprocal to $\$_j$, which represents the system of forces exerted on the end effector by the j-th limb, is computed. Finally, the wrench system obtained as linear combination of the ξ_j is considered, and the geometric conditions that make it coincide with the wrench system reciprocal to all the elements of the motion subgroup, which $\{L\}$ must be a subset of, are deduced.

The third approach (velocity-loop equations) consists in writing n times both the end-effector angular velocity, ω, and the velocity, $\dot{P}$, of an end-effector point by exploiting the kinematic properties of the n limbs of the generic LM-PM topology under analysis. So doing n expressions of the couple of vectors $(\omega, \dot{P})$ are obtained where the j-th expression, j=1,…,n, is a linear combination of the joint rates of the j-th limb. The analysis of these $(\omega, \dot{P})$ expressions is sufficient to determine the geometric conditions that each limb has to satisfy in order to make (I) all the n expressions compatible, and (II) the end-effector's motion characteristics $(\omega, \dot{P})$ respect the conditions which warranty that all the end effector motions belong to an assigned motion subgroup of $\{D\}$. Since this approach deduces geometric conditions by analysing the instantaneous end-effector motion, the characteristics of the finite end-effector motion are stated by demonstrating that those conditions are sufficient to warranty an infinite

sequence of instantaneous motion of the same type provided that no singular configuration is encountered.

The first and the third approaches rely on purely kinematic considerations, whereas the second one takes into account both kinematic and static considerations which is typical of approaches based on screw theory. The three approaches are all able to find the singular configurations of any LM-PM architecture, even though the second and the third ones directly give the singularity conditions as a result of the analysis that identifies the limb topologies, which make them more appropriate for the singularity analysis.

4. Singularity Analysis

Singularities are manipulator configurations where the input-output instantaneous relationship fails (Gosselin & Angeles 1990; Ma & Angeles 1991; Zlatanov et al. 1995). If the input-output instantaneous relationship is considered (Gosselin & Angeles 1990), they are of three types: (I) singularities of the inverse kinematic problem, (II) singularities of the direct kinematic problem, and (III) singularities both of the inverse and of the direct kinematic problems.

Type-(I) singularities occur when at least one out of the input-variable rates (actuated-joint rates) are undetermined even though all the output-variable rates (end-effector's motion characteristics $(\omega, \dot{P})$) are assigned. All the manipuator configurations where the end effector reaches the border of the workspace are type-(I) singularities, and finding type-(I) singularities is one way to determine the workspace border. From a static point of view, in type-(I) singularities, at least one component of output torque (force), applied to the end effector, is equilibrated by the manipulator structure without applying any input torque (force) in the actuated joints.

Type-(II) singularities occur when at least one component of end-effector's motion characteristics, $(\omega, \dot{P})$, is undetermined even though all the actuated-joint rates are assigned. These singularities may be present only in the PMs and fall inside the workspace. From a static point of view, in type-(II) singularities, a (finite or infinitesimal) output torque (force), applied to the end effector, needs at least one infinite input torque (force) in the actuated joints to be equilibrated. Since, long before the input torque (force) becomes infinite, the manipulator breaks down, type-(II) singularities must be found during design and avoided during operation.

This singularity classification has been extended in (Zlatanov et al. 1995) by taking into account the rates of the non-actuated joints.

In the literature (Di Gregorio & Parenti-Castelli 2002; Di Gregorio 2001a, 2001b, 2002a, 2003, 2004a, 2004c; Zatlanov et al. 2001, 2002), the possibility of changing the type of motion, the end effector performs, in correspondence of par-

ticular type-(II) singularities, named constraint singularities, has been high-lighted. Constraint singularities affect only LM-PMs where the limbs' connectivity is greater than the manipulator's dofs.

Conditions (1) and (2) can explain why constraint singularities may occur in LM-PMs where the limbs' connectivity is greater than the manipulator's dofs. If m_j and m with $m < m_j \le 6$ are the connectivity of the j-th limb and the LM-PM's dofs respectively, then the $\{W_j\}$ and the $\{L_j\}$ sets have dimension m_j whereas $\{W\}$ and $\{L\}$ have dimension m. A continuous subset with dimension m of a continuous set with dimension m_j ($>m$) can be generated in $\infty^{m_j - m}$ ways; hence, it can happen that $\{L_j\}$ have m-dimensional subsets, $\{L_{kj}\}$, k=1,…,s_j, of different motion subgroups of $\{D\}$ and of mixed motions among its m-dimensional subsets, and that the corresponding m-dimensional subsets, $\{W_{kj}\}$, k=1,…,s_j, of $\{W_j\}$ have a non-empty intersection $\{C_j\}$ (i.e. they constitute a connected set). When this condition occurs, the j-th limb can move the end effector from a pose of $\{C_j\}$ by making it perform motions that belong to different m-dimensional motion subgroups of $\{D\}$ (that belong either to a m-dimensional motion subgroup of $\{D\}$ or to a mixed-motion subsets of $\{W_j\}$). Since, according to condition (1) the set $\{C\}$, defined as follows

$$\{C\} = \bigcap_{j=1,n} \{C_j\}, \tag{3}$$

must be a subset of $\{W\}$, if $\{C\}$ is a non-empty set and $\{W\}$ contains subsets that belong to different m-dimensional motion subgroups (or to m-dimensional subsets of mixed motions together with subsets of a m-dimensional motion subgroup), then, there is a non-empty subset, $\{S\}$, of $\{C\}$ whose elements are end-effector poses from which the LM-PM can move the end effector by making it perform motions that belong to different m-dimensional motion subgroups of $\{D\}$ (that belong to either a m-dimensional motion subgroup of $\{D\}$ or a mixed-motion subsets of $\{W\}$). The end-effector's possibility of leaving a pose by performing motions that belong to disjoint m-dimensional subsets of $\{D\}$ implies that the end effector locally has at least one additional dof (i.e. $m+h$ dofs with $h \ge 1$) when the end effector assumes that pose. Therefore, when the end effector assumes that pose, the end effector's motion characteristics, ($\boldsymbol{\omega}$, $\dot{\mathbf{P}}$), are not determined even though the m actuated-joint rates are assigned (i.e. the LM-PM's configuration with the end effector located at that pose is a particular type-(II) singularity).

In (Zatlanov et al. 2001), it has been presented a three-dof LM-PM with topology 3URU (i.e. with three limbs of type URU (U stands for universal joint)), named DYMO (Fig. 2), that, by crossing constraint singularities, can become either a TPM, or an SPM, or a PPM, or a three-dof mixed-motion LM-PM.
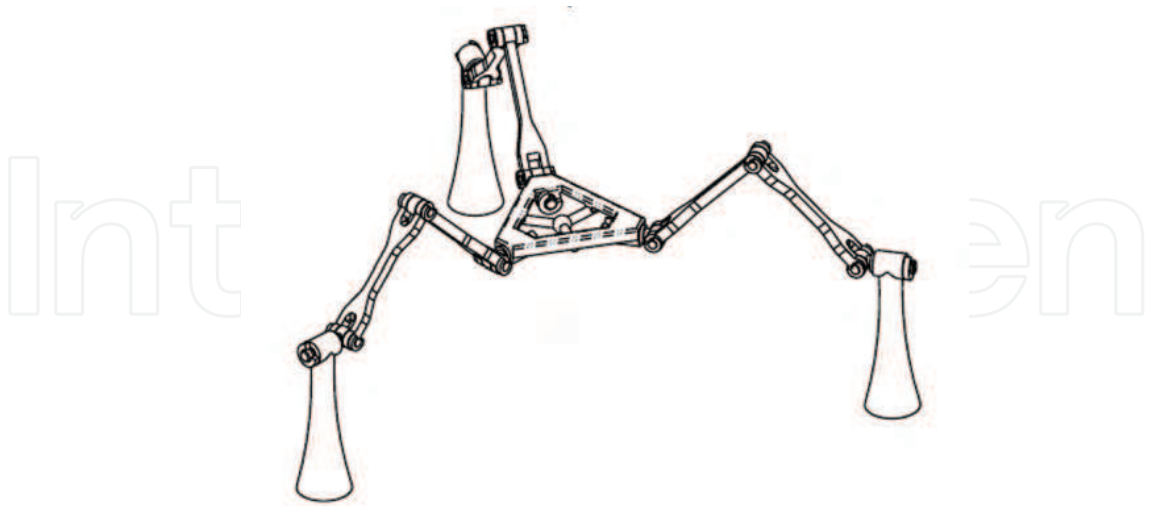
Figure 2. 3URU DYMO (Zatlanov et al. 2001)

A method to avoid the presence of constraint singularities is to choose the limb topologies and to assemble the limbs so that the subset {S} is an empty set. This idea guided the proposal of the 3RRS wrist (Di Gregorio 2004b), that has three limbs of type RRS, where the axes of the six (two per limb) revolute pairs pass through the center of the spherical motion (see Fig. 3). The three RRS limbs are assembled so that {L} contains only motions belonging to the spherical sub-group which make {S} empty even though {C} is not empty.

All type-(II) singularities must be individuated during the LM-PM design and, when possible, eliminated by suitably choosing the manipulator geometry. Moreover, the end effector must be kept as far as possible from these singularities during operation.

From an analytic point of view, the research of the type-(II) singularities can be implemented either through a static analysis (Di Gregorio 2004a) or through a kinematic analysis (Di Gregorio 2003). The static analysis studies the relationship between the system of external loads applied to the end effector and the set of the generalised torques applied in the actuated joints to equilibrate those loads. The kinematic analysis studies the relationship between the end-effector's motion characteristics (ω, $\dot{P}$) and the m-dimensional vector (m (m<6) is the dof number of the LM-PM), $\dot{q}$, that collects all the actuated-joint rates, $\dot{q}_p$, p=1,...,m.
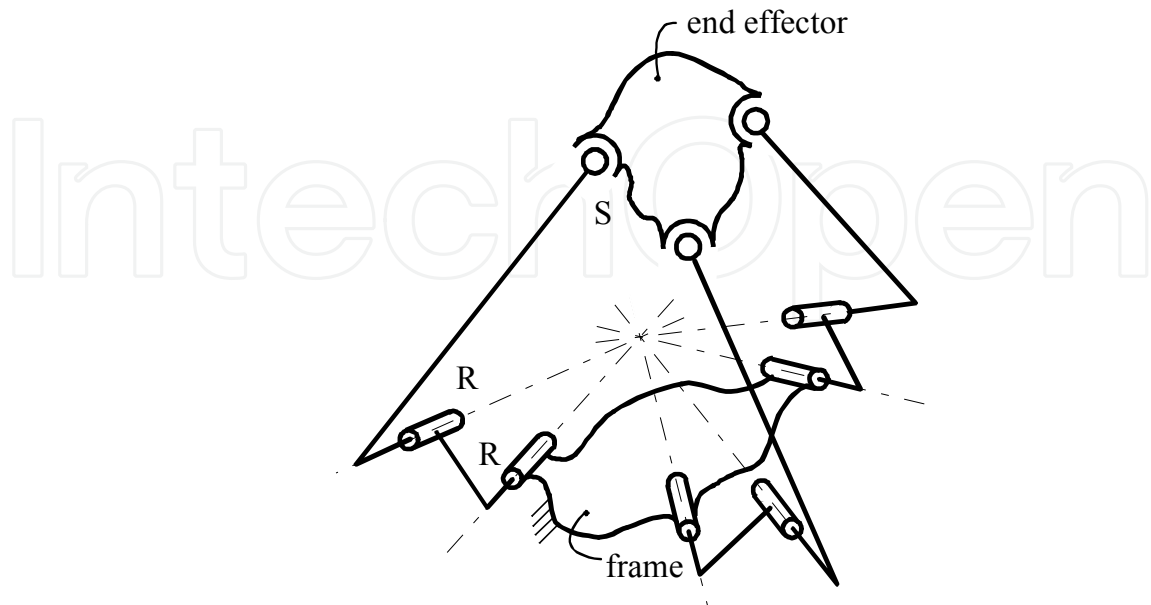
Figure 3. 3RRS wrist (Di Gregorio 2004b)

By following the method based on the kinematic analysis, the relationship to be studied is

$$\mathbf{A} \left\{ \begin{matrix} \dot{\mathbf{P}} \\ \boldsymbol{\omega} \end{matrix} \right\} = \mathbf{B}\dot{\mathbf{q}} \tag{4}$$

where **A** and **B** are a 6×6 matrix and a 6×m matrix respectively, and both, in general, depend on the m-dimensional vector **q** which collects the m actuated-joint variables, q_p, p=1,…,m, (i.e. they depend on the manipulator configuration). Since the LM-PM has m dofs, 6–m equations of system (4) simply state that $\boldsymbol{\omega}$ and $\dot{\mathbf{P}}$ cannot be arbitrarily chosen.

A non-singular configuration is characterised by rank(**A**)=6 and rank(**B**)=m. A type-(I) singularity is characterised by rank(**A**)=6 and rank(**B**)<m. A type-(II) singularity is characterised by rank(**A**)<6 (i.e. det(**A**)=0) and rank(**B**)=m. A type-(III) singularity is characterised by rank(**A**)<6 and rank(**B**)<m.

In order to find the type-(II) singularities the values of **q** that solve the equation

$$\det(\mathbf{A})=0 \tag{5}$$

must be determined. Moreover, the condition number of **A** evaluated for an assigned value of **q** (i.e. an assigned configuration) can be used to judge how far is the configuration individuated by that value of **q** from type-(II) singular-

ity conditions (Gosselin & Angeles 1991): the nearer to one the condition number is, the farther from type-(II) singularity conditions that configuration is (the condition number ranges from 1 to infinity). The configurations where the condition number of $\mathbf{A}$ is equal to one are the farthest from type-(II) singularity conditions. Such configurations are called isotropic configurations. In an isotropic configuration, the matrix $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is proportional to the 6×6 identity matrix, $\mathbf{I}_6$, or, which is the same, the singular values of $\mathbf{A}$ are all equal.

In an LM-PM has a matrix $\mathbf{A}$ that is constant (i.e. does not depend on $\mathbf{q}$) and non singular, then all the manipulator configurations have the same condition number and are not singular. Such a manipulator will be called constant-isotropy LM-PM. In addition, if, in a constant-isotropy LM-PM, the constant value of the condition number of $\mathbf{A}$ is one, then all the manipulator configurations are isotropic and the manipulator is called fully isotropic.

The appealing properties of constant-isotropy or fully isotropic LM-PMs pushed researchers to determine their topologies (Kong & Gosselin 2002a; Carricato & Parenti-Castelli 2002; Carricato 2005; Gogu 2004). Among all the proposed fully-isotropic architecture, the Cartesian 3PRRR (Kong & Gosselin 2002b; Di Gregorio 2002b; Kim & Tsai 2003) is certainly the most known. With reference to Fig. 4, the Cartesian 3PRRR has three limbs of type PRRR where the prismatic pair is actuated. In the j-th limb, j=1, 2, 3, of type PRRR, the three revolute-pair axes and the prismatic-pair sliding direction are all parallel. Finally, the sliding directions of the three prismatic pairs are mutually orthogonal.
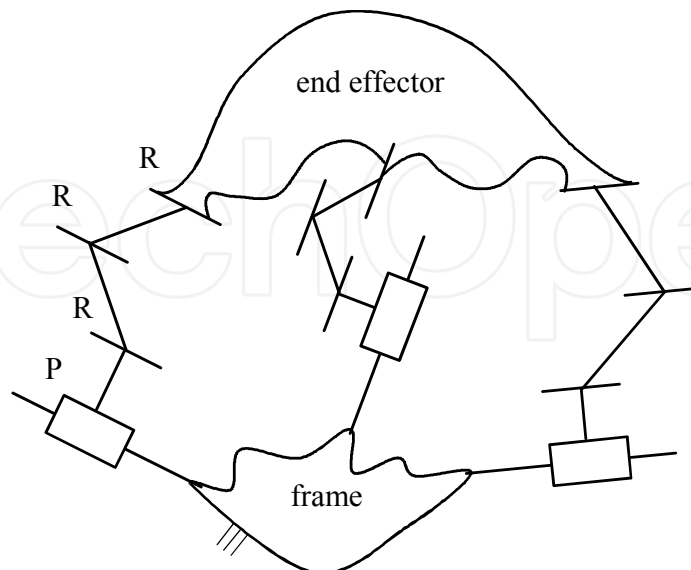

Figure 4. Cartesian 3PRRR (Kong & Gosselin 2002b; Di Gregorio 2002b; Kim & Tsai 2003)

5. Conclusions

The functional (kinetostatic) design of a LM-PM starts from the analysis of the manipulation task to accomplish, continues with the identification of the limb topologies and finishes with the determination of the machine architecture passing through the singularity analysis.

In the literature, the methodologies for the identification of the limb topologies suitable for a given manipulation task has been well described. How to combine the limbs in order to avoid singularities has been diffusely discussed at least for the most popular architectures. Nevertheless, comparison criteria among different architectures that perform the same manipulation task are not well established, yet. So that, even though is quite easy to find long lists of limb's topologies that are suitable for a given manipulation task (the works reported in the references are just a sample of the vast literature on this subject), stating which is the best one still is an open problem.

Some authors (Tsai & Joshi 2001) proposed the use of the "global condition number" (defined as the average value, on the workspace, of the inverse of the condition number) as index for evaluating or optimising a machine, but the same authors had to recognise that the comparison among different LM-PMs must take into account also the inertia properties of the machines. In general, it can be said that machines which exhibit good kinetostatic properties do not necessarily provide good dynamic performances.

6. References

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Industrial Robotics: Theory, Modelling and Control

Edited by Sam Cubero

This book covers a wide range of topics relating to advanced industrial robotics, sensors and automation technologies. Although being highly technical and complex in nature, the papers presented in this book represent some of the latest cutting edge technologies and advancements in industrial robotics technology. This book covers topics such as networking, properties of manipulators, forward and inverse robot arm kinematics, motion path-planning, machine vision and many other practical topics too numerous to list here. The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.

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