34

Stochastic Analysis of a System Containing One Robot and (n-1) Standby Safety Units with an Imperfect Switch

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1. Introduction

Robots are increasingly being used in industry to perform various types of tasks. These tasks include material handling, spot welding, arc welding and routing. The word 'Robot' is derived from the Czechoslovakian language, in which it means 'worker'. In 1959, the first commercial robot was manufactured by the Planet Corporation and today there around on million robots in use worldwide [1-4].

Although robots are used to replace humans performing various types of complex and hazardous tasks, unfortunately over the years a number of accidents involving robots have occurred. In fact, many people have been killed or injured [5-7]. In using robots, particularly in the industrial sector, often safety units are included with robots. A robot has to be safe and reliable. An unreliable robot may become the cause of unsafe conditions, high maintenance costs, inconvenient, etc.

As robots contain parts such as electrical, electronic, mechanical, pneumatic and hydraulic their reliability problem become a challenging task because of many different sources of failures. Thus, this paper presents a mathematical model for performing reliability and availability analyses of a system containing one robot and (n-1) standby safety units with a switch in mechanism that can fail. More specifically, the robot system is composed of one robot, n identical safety units and a switch to replace a failed safety unit.

The block diagram of the robot system is shown in Figure1 and its corresponding state space diagram is presented in Figure2. The numerals and letter n in the boxes of Figure2 denote system state.

At time t =0, robot, one safety unit and the switch to replace a failed safety unit start operating and n-1 safety units are on standby. The overall robot-safety system can fail the following two ways:

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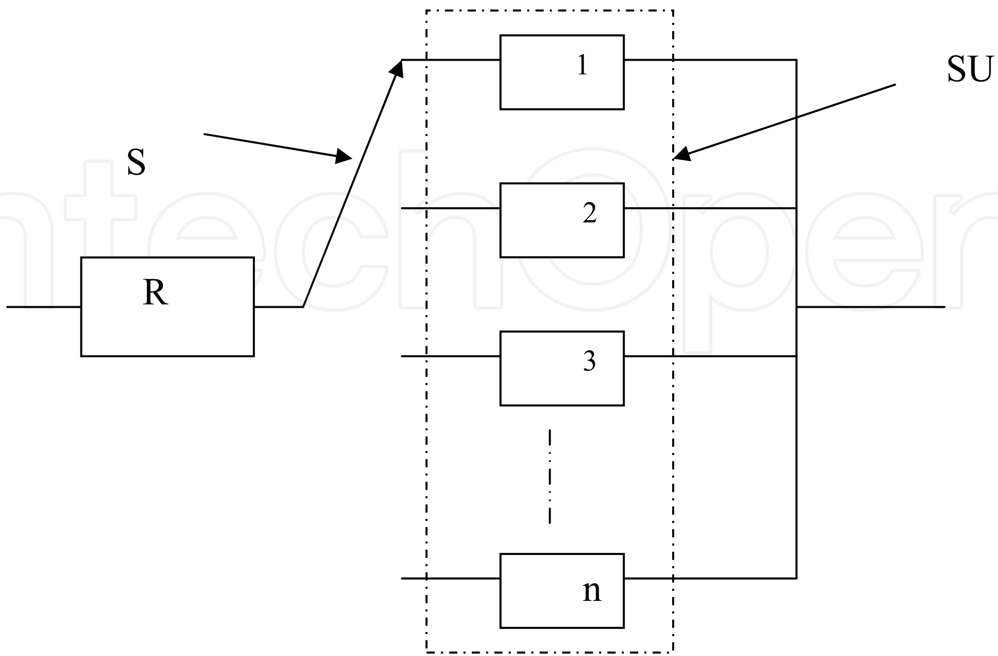


R : Robot
SU : n identical safety units (one operating and n-1 on standby)
S : Switch for replacing a failed safety unit and it can also fail.

Figure 1. The block diagram of the robot-safety system

- The robot fails with a normally working safety unit and the switch. In addition zero or more safety units are on standby.
- The robot fails with one or more safety units failed or considered failed and the switch is either working or failed.
- The following assumptions are associated with this model:
- The robot-safety system is composed of one robot, n identical safety units (only one operates and the rest remain on standby) and a switch.
- Robot, switch and one safety unit start operating simultaneously.
- The completely failed robot-safety system and its individually failed units (i.e. robot, switch and safety unit) can be repaired. Failure and repair rates of robot, switch and safety units are constant.
- The failure robot-safety system repair rates can be constant or non-constant.
- All failures are statistically independent.
- A repaired safety unit, robot, switch or the total robot-safety system is as good as new.

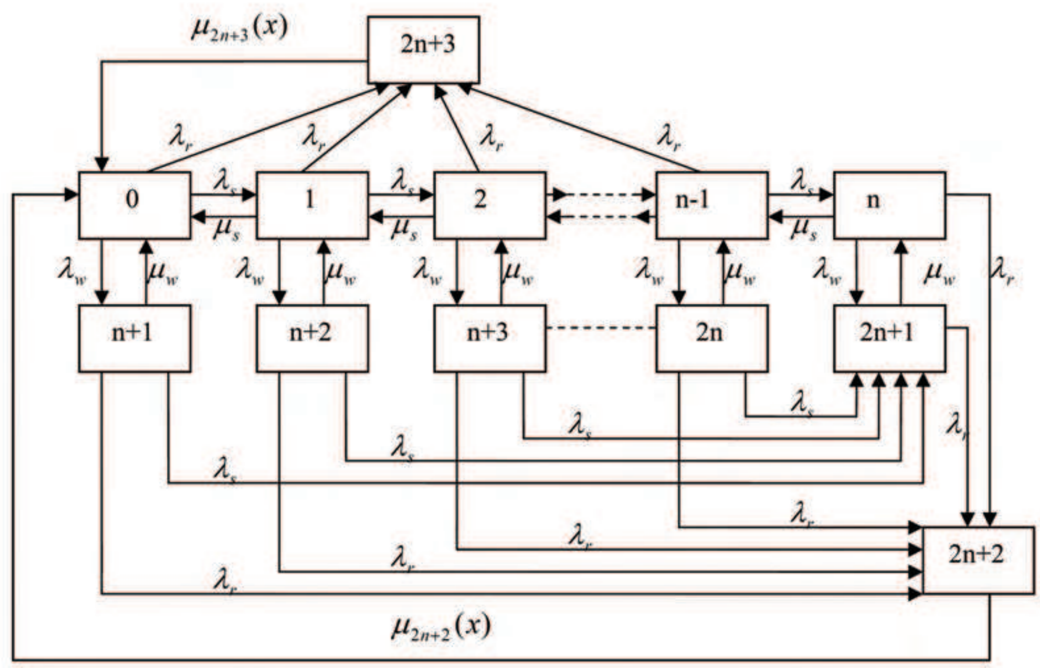


Figure 2. The state space diagram of the robot-safety system

1.1 Notation

The following symbols are associated with the model:

i $\quad$ i^{th} state of the robot-safety system.

for i = 0, means the robot, the switch and one safety unit are working normally;

for i = 1, means the robot, the switch, one safety unit are working normally and one safety unit has failed;

for i = k, means the robot, the switch, one safety unit are working normally and k safety units have failed; (i.e., k = 2,3……n-1);

for i = n, means the robot work, the switch are working normally and all safety units have failed;

for i = h, means the robot, one safety unit still work normally and h-n safety units and the switch have failed; (i.e., h = n+1, n+2,…… 2n)

for i = 2n+1, means the robot work normally and all the safety units and the switch have failed;

j	j^{th} state of the robot–safety system: for j = 2n+2, means the total robot-safety system has failed (i.e. the robot , one or more safety units have failed or considered failed and the switch is either working or failed); for j = 2n+ 3, means the robot-safety system has failed (i.e. the robot has failed while a safety unit and the switch are working normally. In addition, zero or more safety units are on standby);
t	time.
λ_s	Constant failure rate of a safety unit.
λ_γ	Constant failure rate of the robot.
λ_w	Constant failure rate of the switch.
μ_s	Constant repair rate of a safety unit.
μ_w	Constant repair rate of the switch.
Δ_x :	Finite repair time interval.
$\mu_j(x)$	Time dependent repair rate when the failed robot-safety system is in state j:and has an elapsed repair time of x; for j = 2n+2, 2n+3.
$P_j(x.t)\Delta_x$	The probability that at time t, the failed robot-safety system is in state j and the elapsed repair time lies in the interval [x, x+Δx]; for j = 2n+2, 2n+3 .
pdf	Probability density function.
$w^j(x)$	Pdf of repair time when the failed robot-safety system is in state j and has an elapsed time of x; for j = 2n+2, 2n+3 .
$P^j(t)$	Probability that the robot safety system is in state j at time t; for j = 2n+2, 2n+3.
$P^i(t)$	Probability that the robot-safety system is in state i at time t; for i = 0,1,2…2n+1.
P^i	Steady state probability that the robot-safety system is in state i; for i=0,1,..2n+1.
P^j	Steady state probability that robot-safety system is in state j; for j = 2n+2, 2n+3.
s	Laplace transform variable.
$P^i(s)$	Laplace transform of the probability that the robot-safety system is in state i; for i = 0,1,2…2n+1.
$P^j(s)$	Laplace transform of the probability that the robot-safety system is in state j; for j = 2n+2, 2n+3.

AVrs(s)	Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot.
AVr(s)	Laplace transform of the robot-safety system availability with or without a normally safety unit.
AVrs (t)	Robot-safety system time dependent availability with one normally working safety unit, the switch and the robot.
AVr(t)	Robot-safety system time dependent availability with or without a normally working safety unit.
SSAVrs	Robot-safety system steady state availability with one normally working safety unit, the switch and the robot.
SSAVr	Robot-safety system steady state availability with or without a normally working safety unit.
Rrs(s)	Laplace transform of the robot-safety system reliability with one normally working safety unit, the switch and the robot.
Rr(s)	Laplace transform of the robot safety system reliability with or without a normally working safety unit.
MTTFrs	Robot-safety system mean time to failure when the robot working normally with one normally working safety unit.
MTTFr	Robot-safety system mean time to failure with or without a normally working safety unit.

2. Generalized robot-safety system analysis

Using the supplementary method [8,9],the equations of the system associated with Fig.2 can be expressed as follows:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_s P_1(t) + \mu_w P_{n+1}(t) + \sum_{j=2n+2}^{2n+3} P_j(x,t)\mu_j(x)dx \tag{1}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_s P_{i-1}(t) + \mu_s P_{i+1}(t) + \mu_w P_{i+n+1}(t) \tag{2}$$

(for i = 1,2,……..,n-1)

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) + \mu_w P_{2n+1}(t) \tag{3}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_w P_{i-n-1}(t) \quad (\text{for } i = n+1, n+2, \ldots\ldots, 2n) \tag{4}$$

$$\frac{dP_{2n+1}(t)}{dt} + a_{2n+1} P_{2n+1}(t) = \lambda_s \sum_{i=n+1}^{2n} P_i(t) + \lambda_w P_n(t) \tag{5}$$

where

$a_0 = \lambda_s + \lambda_w + \lambda_r$

$a_i = \lambda_s + \lambda_w + \lambda_r + \mu_s$ (for i = 1,2,……..,n-1)

$a_n = \lambda_w + \lambda_r + \mu_s$

$a_i = \lambda_s + \lambda_r + \mu_w$ (for i = n+1,n+2,……..,2n)

$a_{2n+1} = \lambda_r + \mu_w$

$$\frac{\partial P_j(x,t)}{\partial t} + \frac{\partial P_j(x,t)}{\partial x} + \mu_j(x)\, P_j(x,t) = 0 \quad (\text{for } j = 2n+2, 2n+3) \tag{6}$$

The associated boundary conditions are as follows:

$$P_{2n+2}(0,t) = \lambda_r \sum_{i=n}^{2n+1} P_i(t) \tag{7}$$

$$P_{2n+3}(0,t) = \lambda_r \sum_{i=0}^{n-1} P_i(t) \tag{8}$$

At time t = 0, $P_0(0) = 1$, and all other initial state probabilities are equal to zero.

3. Generalized Robot-Safety System Laplace Transforms of State Probabilities

By solving Equations (1)-(8) with the Laplace transform method, we get the following

Laplace transforms of state probabilities:

$$P_0(s) = [\, s(1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} a_j(s) \frac{1 - W_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{9}$$

$$P_i(s) = Y_i(s)\, P_0(s) \quad (\text{for } i = 1,2,\ldots\ldots,n) \tag{10}$$

$$P_i(s) = V_i(s)\, P_0(s) \quad (\text{for } i = n+2, n+2, \ldots\ldots, 2n+1) \tag{11}$$

$$P_{n+1}(s) = \frac{\lambda_w}{s + a_{n+1}}\, P_0(s) \tag{12}$$

$$P_j(s) = a_j(s)\frac{1-W_j(s)}{s} P_0(s) \qquad (\text{for } j = 2n+2, 2n+3) \tag{13}$$

where

$$L_i(s) = (s + a_i) - \frac{\lambda_w \mu_w}{s + a_{i+n+1}} \qquad (\text{for } i = 1,2,\ldots\ldots,n)$$

$$D_1(s) = L_1(s)$$

$$D_i(s) = L_i(s) - \frac{\lambda_s \mu_s}{D_{i-1}(s)} \qquad (\text{for } i = 2,\ldots\ldots,n)$$

$$A_i(s) = \frac{\lambda_s^i}{\prod_{h=1}^{i} D_h(s)} \qquad (\text{for } i = 1,2,\ldots\ldots,n-1)$$

$$B_i(s) = \frac{\mu_s}{D_i(s)} \qquad (\text{for } i = 1,2,\ldots\ldots,n-1)$$

$$Y_i(s) = \sum_{h=i}^{n-1} A_h(s) \prod_{k=i}^{h-1} B_k(s) + \prod_{h=i}^{n-1} B_h(s) Y_n(s)$$

(for i = 1,2,……..,n-1)

$$V_i(s) = \frac{\lambda_w}{s + a_i} Y_{i-n-1}(s) \qquad (\text{for } i = n+2,\ldots\ldots,2n)$$

$$V_{2n+1}(s) = \frac{\lambda_s \lambda_w}{(s + a_{n+1})(s + a_{2n+1})} + \frac{\lambda_s}{s + a_{2n+1}} \sum_{i=1}^{n-1} \frac{\lambda_w}{s + a_{i+n+1}} Y_i(s) + \frac{\lambda_w}{s + a_{2n+1}} Y_n(s)$$

$Y_n(s) =$

$$\frac{\lambda_s A_{n-1}(s) + \dfrac{\lambda_s \lambda_w \mu_w}{(s + a_{n+1})(s + a_{2n+1})} + \dfrac{\lambda_s \mu_w}{s + a_{2n+1}} \sum_{i=1}^{n-1} \dfrac{\lambda_w}{s + a_{i+n+1}} \sum_{h=i}^{n-1} [A_h(s) \prod_{k=i}^{h-1} B_k(s)]}{L_n(s) - \lambda_s B_{n-1}(s) - \dfrac{\lambda_s \mu_w}{s + a_{2n+1}} \sum_{i=1}^{n-1} \dfrac{\lambda_w}{s + a_{i+n+1}} \prod_{h=i}^{n-1} B_h(s)}$$

$$a_{2n+2}(s) = \lambda_r [\, Y_n(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) \,]$$

$$a_{2n+3}(s) = \lambda_r [1+\sum_{i=1}^{n-1} Y_i(s)]$$

$$G(s) = s(1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} a_j(s)\frac{1-W_j(s)}{s}) \tag{14}$$

$$W_j(s) = \int_0^\infty e^{-sx} w_j(x)dx \quad \text{for } j = 2n+2, 2n+3)$$

$$\tag{15}$$

$$w_j(x) = \exp[-\int_0^x \mu_j(\delta)d\delta]\ \mu_j(x)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function
The Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot is given by:

$$AV_{rs}(s)=\sum_{i=0}^{n-1} P_i(s) + \sum_{i=n+1}^{2n} P_i(s) = \frac{1+\sum_{i=1}^{n-1} Y_i(s) + \frac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n} V_i(s)}{G(s)} \tag{16}$$

The Laplace transform of the robot-safety system availability with or without a normally working safety unit:

$$AV_r(s) = \sum_{i=0}^{2n+1} P_i(s) = \frac{1+\frac{\lambda_w}{s+a_{n+1}} + \sum_{i=1}^{n} Y_i(s) + \sum_{i=n+2}^{2n+1} V_i(s)}{G(s)} \tag{17}$$

Taking the inverse Laplace transforms of the above equations, we can obtain the time dependent state probabilities, $P_i(t)$and $P_j(t)$, and robot-safety system availabilities,

AVrs(t) and AVr(t).

3.1 Robot Safety System Time Dependent Analysis For A Special Case

For two safety units (i.e., one working, other one on standby)
Substituting n=2 into Equations (9)-(16), we get

$$P_0(s) = \frac{1}{s[1+\sum_{i=1}^{2} Y_i(s) + \frac{\lambda_w}{s+a_3} + \sum_{i=4}^{5} V_i(s) + \sum_{j=6}^{7} a_j(s)\frac{1-W_j(s)}{s}]} = \frac{1}{G(s)} \quad (18)$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad (\text{for } i = 1,2) \quad (19)$$

$$P_3(s) = \frac{\lambda_w}{s+a_3} P_0(s) \quad (20)$$

$$P_i(s) = V_i(s) P_0(s) \qquad (\text{for } i = 4,5) \quad (21)$$

$$P_j(s) = a_j(s)\frac{1-W_j(s)}{s} P_0(s) \quad (22)$$

where

$$Y_2(s) = \frac{\lambda_s \frac{\lambda_s}{L_1(s)} + \frac{\lambda_s\lambda_w\mu_w}{(s+a_3)(s+a_5)} + \frac{\lambda_s\lambda_w\mu_w}{(s+a_4)(s+a_5)}\frac{\mu_s}{L_1(s)}}{L_2(s) - \lambda_s\frac{\mu_s}{L_1(s)} - \frac{\lambda_s\lambda_w\mu_w}{(s+a_4)(s+a_5)}\frac{\mu_s}{L_1(s)}}$$

$$Y_1(s) = \frac{\lambda_s}{L_1(s)} + \frac{\mu_s}{L_1(s)} Y_2(s)$$

$$V_5(s) = \frac{\lambda_s\lambda_w}{(s+a_3)(s+a_5)} + \frac{\lambda_s}{s+a_5}\frac{\lambda_w}{s+a_4} Y_1(s) + \frac{\lambda_w}{s+a_5} Y_2(s)$$

$$V_4(s) = \frac{\lambda_w}{s+a_4} Y_1(s)$$

$$a_6(s) = \lambda_r [\, Y_2(s) + \frac{\lambda_w}{s+a_3} + \sum_{i=4}^{5} V_i(s)\,]$$

$$a_7(s) = \lambda_r [\, 1 + Y_1(s)]$$

$$L_1(s) = (s+a_1) - \frac{\lambda_w\mu_w}{s+a_4}$$

$$L_2(s) = (s+a_2) - \frac{\lambda_w\mu_w}{s+a_5}$$

$$G(s) = s[1+\sum_{i=1}^{2} Y_i(s) + \frac{\lambda_w}{s+a_3} + \sum_{i=4}^{5} V_i(s) + \sum_{j=6}^{7} a_j(s)\frac{1-W_j(s)}{s}] \quad (23)$$

The Laplace transform of the robot-safety system availability with one normally working safety unit, the switch and the robot is given by:

$$AV_{rs}(s = \sum_{i=0}^{1} P_i(s) + \sum_{i=3}^{4} P_i(s) = \frac{1+Y_1(s)+\frac{\lambda_w}{s+a_3}+V_4(s)}{G(s)} \quad (24)$$

The Laplace transform of the robot-safety system availability with or without a normally working safety unit is given by:

$$AV_r(s) = \sum_{i=0}^{5} P_i(s) = \frac{1+\sum_{i=1}^{2} Y_i(s)+\frac{\lambda_w}{s+a_3}+\sum_{i=4}^{5} V_i(s)}{G(s)} \quad (25)$$

Taking the inverse Laplace transforms of the above equations, we can obtain the time dependent state probabilities, P_i (t)and P_j (t), and robot-safety system availabilities,

AVrs(t) and AVr(t).

Thus, for the failed robot-safety system repair time x is exponentially distributed repair times, the probability function is expressed by

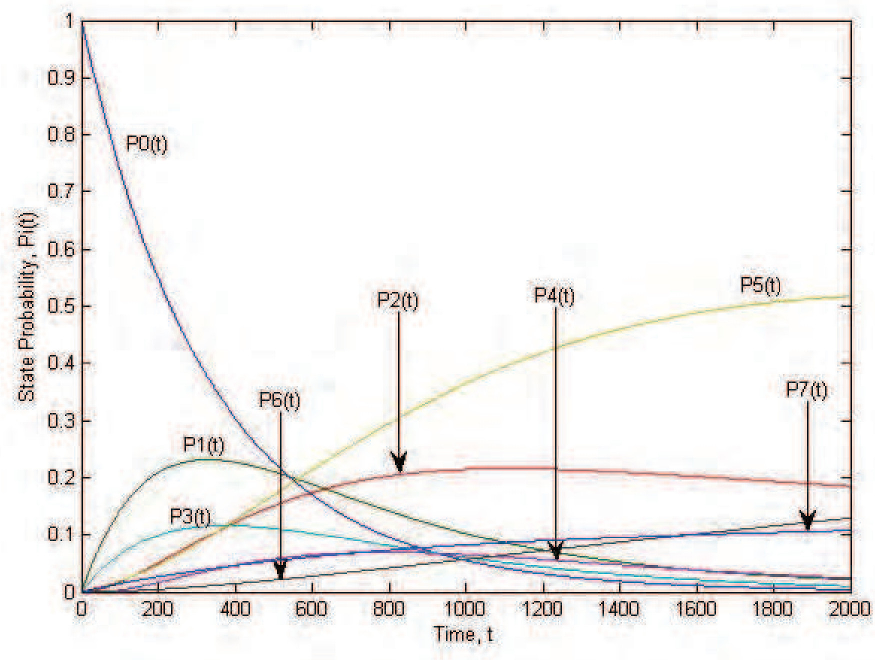
$$w_j(x) = \mu_j e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 6,7) \quad (26)$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j*. Substituting equation (26) into equation (15), we can get

$$W_j(s) = \frac{\mu_j}{s+\mu_j} \qquad (\mu_j > 0, j = 6,7) \quad (27)$$

By inserting Equation (27) into Equations (9)-(13), setting λ_s =0.002, μ_s = 0.00015, λ_w=0.001, μ_w = 0.0003, λ_r = 0.00009, μ_6 = 0.0001, μ_7 = 0.00015; and using Matlab computer program [10], the Figure 3 plots were obtained. These plots show that state probabilities decrease and increase with varying time t.



λ_s =0.002, μ_s = 0.00015, λ_w =0.001, μ_w = 0.0003,
λ_r = 0.00009, μ_6 = 0.0001, μ_7 = 0.00015

Figure 3. Time-dependent probability plots for a robot safety system with exponential distributed failed system repair time.

4. Generalized Robot Safety System Steady State Analysis

As time approaches infinity, all state probabilities reach the steady state. Thus, from

Equations (1)-(8) get:

$$a_0 P_0 = \mu_s P_1 + \mu_w P_n + \sum_{j=2n+2}^{2n+3} P_j(x)\mu_j(x)dx \tag{28}$$

$$a_i P_i = \lambda_s P_{i-1} + \mu_s P_{i+1} + \mu_w P_{i+n+1} \tag{29}$$

(for i = 1,2,……..,n-1)

$$a_n P_n = \lambda_s P_{n-1} + \mu_w P_{2n+1} \tag{30}$$

$$a_i P_i = \lambda_w P_{i-n-1} \tag{31}$$

(for i = n+1,n+2,……..,2n-k-1)

$$a_{2n+1} P_{2n+1} = \lambda_s \sum_{i=n+1}^{2n} P_i + \lambda_w P_n \quad (32)$$

where

$a_0 = \lambda_s + \lambda_w + \lambda_r$

$a_i = \lambda_s + \lambda_w + \lambda_r + \mu_s$ (for i = 1,2,……..,n-1)

$a_n = \lambda_w + \lambda_r + \mu_s$

$a_i = \lambda_s + \lambda_r + \mu_w$ (for i = n+1,n+2,……..,2n)

$a_{2n+1} = \lambda_r + \mu_w$

$$\frac{dP_j(x)}{dx} + \mu_j(x) P_j(x) = 0 \quad (\text{ for } j = 2n+2, 2n+3) \quad (33)$$

The associated boundary conditions are as follows:

$$P_{2n+2}(0) = \lambda_r \sum_{i=n}^{2n+1} P_i \quad (34)$$

$$P_{2n+3}(0) = \lambda_r \sum_{i=0}^{n-1} P_i \quad (35)$$

Solving Equations (28) - (33), and together with

$$\sum_{i=0}^{2n+1} P_i + \sum_{j=2n+2}^{2n+3} P_j = 1 \quad (36)$$

We get:

$$P_0 = \left(1 + \sum_{i=1}^{n} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n} V_i + \sum_{j=2n+2}^{2n+3} a_j E_j[x]\right)^{-1} = \frac{1}{G} \quad (37)$$

$$P_i = Y_i P_0 \quad (\text{ for } i = 1,2,\ldots\ldots,n) \quad (38)$$

$$P_i = V_i P_0 \quad (\text{ for } i = n+2,\ldots\ldots,2n+1) \quad (39)$$

$$P_{n+1} = \frac{\lambda_w}{a_n} P_0 \quad (40)$$

$$P_j = a_j E_j[x] P_0$$

(for j = 2n+2, 2n+3) (41)

where

$$L_i = \lim_{s\to 0} L_i(s) \qquad (\text{ for } i = 1,2,\ldots\ldots,n)$$

$$D_1 = L_1$$

$$D_i = L_i - \frac{\lambda_s \mu_s}{D_{i-1}} \qquad (\text{ for } i = 2,\ldots\ldots,n)$$

$$A_i = \frac{\lambda_s^i}{\prod_{h=1}^{i} D_h} \qquad (\text{ for } i = 1,2,\ldots\ldots,n-1)$$

$$B_i = \frac{\mu_s}{D_i} \qquad (\text{ for } i = 1,2,\ldots\ldots,n-1)$$

$$Y_i = \sum_{h=i}^{n-1} A_h \prod_{k=i}^{h-1} B_k + \prod_{h=i}^{n-1} B_h Y_n$$

(for i = 1,2,……..,n-1)

$$V_i = \frac{\lambda_w}{a_i} Y_{i-n-1} \qquad (\text{ for } i = n+2,\ldots\ldots,2n)$$

$$V_{2n+1} = \frac{\lambda_s \lambda_w}{a_{n+1} a_{2n+1}} + \frac{\lambda_s}{a_{2n+1}} \sum_{i=1}^{n-1} \frac{\lambda_w}{a_{i+n+1}} Y_i + \frac{\lambda_w}{a_{2n+1}} Y_{n-k}$$

$$Y_n = \frac{\lambda_s A_{n-1} + \dfrac{\lambda_s \lambda_w \mu_w}{a_n a_{2n+1}} + \dfrac{\lambda_s \mu_w}{a_{2n+1}} \sum_{i=1}^{n-1} \dfrac{\lambda_w}{a_{i+n+1}} \sum_{h=i}^{n-1} A_h \prod_{k=i}^{h-1} B_k}{L_n - \lambda_s B_{n-1} - \dfrac{\lambda_s \mu_w}{a_{2n+1}} \sum_{i=1}^{n-1} \dfrac{\lambda_w}{a_{i+n+1}} \prod_{h=i}^{n-1} B_h}$$

$$a_{2n+2} = \lambda_r \left(Y_n + \sum_{i=n+2}^{2n+1} V_i + \frac{\lambda_w}{a_{n+1}}\right)$$

$$a_{2n+3} = \lambda_r \left(1 + \sum_{i=1}^{n-1} Y_i\right)$$

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j E_j[x] \quad (42)$$

$$E_j[x] = \int_0^{\infty} \exp[-\int_0^x \mu_j(\delta)d\delta]dx$$
$$= \int_0^{\infty} x w_j(x)dx \qquad (\text{ for } j = 2n+2, 2n+3) \quad (43)$$

where

$w_j(x)$ is the failed robot safety system repair time probability density function
$E_j[x]$ is the mean time to robot safety system repair when the failed robot safety system is in state j and has an elapsed repair time x.
The generalized steady state availability of the robot safety system with one normally working normally safety unit, the switch and the robot is given by

$$SSAVrs = \sum_{i=0}^{n-1} P_i + \sum_{i=n+1}^{2n} P_i = \frac{1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n} V_i}{G} \quad (44)$$

Similarly, the generalized steady state availability of the robot safety system with or without a working safety units is

$$SSAVr = \sum_{i=0}^{2n+1} P_i = \frac{1 + \sum_{i=1}^{n} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i}{G} \quad (45)$$

For different failed robot-safety system repair time distributions, we get different expressions for G as follows:

1) For the failed robot-safety system Gamma distributed repair time x, the probability

density function is expressed by

$$w_j(x) = \frac{\mu_j^{\beta} x^{\beta-1} e^{-\mu_j x}}{\Gamma(\beta)} \quad (\beta > 0, j = 2n+2, 2n+3) \quad (46)$$

where

x is the repair time variable, $\Gamma(\beta)$ is the gamma function, μ_j is the scale parameter and β is the shape parameter.
Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty x w_j(x)dx = \frac{\beta}{\mu_j} \qquad (\beta > 0, j = 2n+2, 2n+3) \tag{47}$$

Substituting equation (47) into equation (42), we get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \frac{\beta}{\mu_j} E_j[x] \tag{48}$$

2) For the failed robot-safety system Weibull distributed repair time x, the probability

density function is expressed by

$$w_j(x) = \mu_j \beta x^{\beta-1} e^{-\mu_j (x)^\beta} \qquad (\beta > 0, j = 2n+2, 2n+3) \tag{49}$$

where

x is the repair time variabl, μ_j is the scale parameter and β is the shape parameter .
Thus, the mean time to robot-safety system repair is given by

$$E_j[x] = \int_0^\infty x W_j(x)dx = \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \qquad (\beta > 0, j = 2n+2, 2n+3) \tag{50}$$

Substituting (50) into equation (42), we can get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \left(\frac{1}{\mu_j}\right)^{1/\beta} \frac{1}{\beta} \Gamma\left(\frac{1}{\beta}\right) \tag{51}$$

3) For the failed robot-safety system Rayleigh distributed repair time x, the probability

density function is expressed by

$$w_j(x) = \mu_j x e^{-\mu_j x^2/2} \qquad (\mu_j > 0, \ j = 2n+2, 2n+3) \tag{52}$$

where

x is the repair time varable, μ_j is the scale parameter.
Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty x W_j(x)dx = \sqrt{\frac{\pi}{2\mu_j}} \qquad (\mu_j > 0, j = 2n+2, 2n+3) \tag{53}$$

Substituting (53) into equation (42), we can get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \sqrt{\frac{\pi}{2\mu_j}} \tag{54}$$

4) For the failed robot system Lognormal distributed repair time x, the probability

density function is expressed by

$$w_j(x) = \frac{1}{\sqrt{2\pi} x \sigma_{y_j}} e^{[\frac{-(Inx-\mu_{y_j})^2}{2\sigma_{y_j}^2}} \qquad (\text{for } j = 2n+2, 2n+3) \tag{55}$$

where

x is the repair time variable, Inx is the natural logarithm of x with a mean μ and
variance σ^2. The conditions μ and σ^2 on parameters are:

$$\sigma_{y_j} = \text{In}\sqrt{1 + (\frac{\sigma_{x_j}}{\mu_{x_j}})^2} \tag{56}$$

$$\mu_{y_j} = \text{In}\sqrt{\frac{\mu_{x_j}^4}{\mu_{x_j}^2 + \sigma_{x_j}^2}} \tag{57}$$

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \qquad (\text{for } j = 2n+2, 2n+3) \tag{58}$$

Substituting (58) into equation (42), we can get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \, e^{(\mu_{y_j} + \frac{\sigma_{y_j}^2}{2})} \quad \text{(for } j = 2n+2, 2n+3\text{)} \tag{59}$$

5) For the failed robot system exponentially distributed repair time x, the probability

density function is expressed by

$$w_j(x) = \mu_j \, e^{-\mu_j x} \qquad (\mu_j > 0,\ j = 2n+2, 2n+3) \tag{60}$$

where

x is the repair time variable and μ_j is the constant repair rate of state *j.*

Thus, the mean time to robot-safety system repair is given by

$$E_j(x) = \int_0^\infty x w_j(x) dx = \frac{1}{\mu_j} \qquad (\beta > 0, j = 2n+2, 2n+3) \tag{61}$$

Substituting equation (61) into equation (42), we can get

$$G = 1 + \sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i + \sum_{j=2n+2}^{2n+3} a_j \frac{1}{\mu_j} \tag{62}$$

4.1 The Robot-Safety System Steady State Analysis For A Special Case

For n = 2, from Equations (37)-(45), we get

$$P_0 = \frac{1}{1 + \sum_{i=1}^{2} Y_i + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i + \sum_{j=6}^{7} a_j E_j[x]} \tag{63}$$

$$P_i = Y_i \, P_0 \qquad \text{(for } i = 1,2\text{)} \tag{64}$$

$$P_3 = \frac{\lambda_w}{a_3} P_0 \tag{65}$$

$$P_i = V_i P_0 \qquad \text{(for } i = 4,5\text{)} \tag{66}$$

$$P_j = a_j E_j[x] P_0 \tag{67}$$

where

$$Y_2(s) = \frac{\lambda_s \frac{\lambda_s}{L_1} + \frac{\lambda_s \lambda_w \mu_w}{a_3 a_5} + \frac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \frac{\mu_s}{L_1}}{L_2 - \lambda_s \frac{\mu_s}{L_1} - \frac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \frac{\mu_s}{L_1}}$$

$$Y_1 = \frac{\lambda_s}{L_1} + \frac{\mu_s}{L_1} Y_2$$

$$V_5 = \frac{\lambda_s \lambda_w}{a_3 a_5} + \frac{\lambda_s \lambda_w}{a_4 a_5} Y_1 + \frac{\lambda_w}{a_5} Y_2$$

$$V_4 = \frac{\lambda_w}{a_4} Y_1$$

$$a_6 = \lambda_r \left[Y_2 + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i \right]$$

$$a_7 = \lambda_r [1 + Y_1]$$

$$L_1 = a_1 - \frac{\lambda_w \mu_w}{a_4}$$

$$L_2 = a_2 - \frac{\lambda_w \mu_w}{a_5}$$

$$G = 1 + \sum_{i=1}^{2} Y_i + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i + a_j E_j[x] \tag{68}$$

$$\text{SSAVrs} = \sum_{i=0}^{1} P_i + \sum_{i=3}^{4} P_i = \frac{1 + Y_1 + \frac{\lambda_w}{a_3} + a_4}{G} \tag{69}$$

$$\text{SSAVr} = \sum_{i=0}^{5} P_i = \frac{1 + \sum_{i=1}^{2} Y_i + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i}{G} \tag{70}$$

For exponentially distributed failed robot-safety system repair Equation (61) into
Equations (69) and (70), setting:

λ_s =0.0002, λ_w =0.001, μ_w = 0.0003, λ_r = 0.00009, μ_6 = 0.0001, μ_7 = 0.00015;

and using matlab computer program [10], the Figure 4 plot were obtained. The plot shows, as expected, that $SSAV_r$ is greater than $SSAV_{rs}$ and both of them increase slightly with the increasing values of the safety unit repair rate.

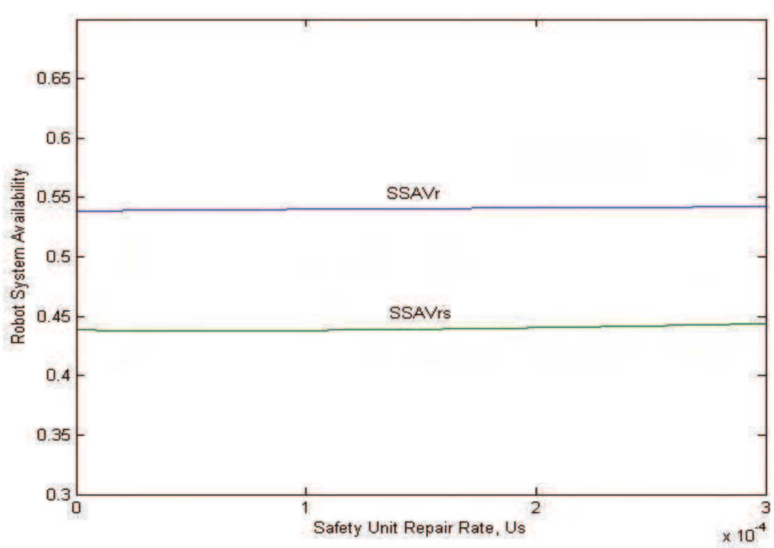
5. Robot-Safety System Reliability and MTTF Analysis

Setting μ_j = 0, (for j = 2n+2, 2n+3), in Figure 2 and using the Markov method[11], we write the following equations for the modified figure:

$$\frac{dP_0(t)}{dt} + a_0 P_0(t) = \mu_s P_1(t) + \mu_w P_{n+1}(t) \qquad (71)$$

$$\frac{dP_i(t)}{dt} + a_i P_i (t = \lambda_s P_{i-1}(t) + \mu_s P_{i+1}(t) + \mu_w P_{i+n+1}(t) \qquad (72)$$

(for i = 1,2,……..,n-1)



λ_s =0.0002, λ_w =0.001, μ_w = 0.0003, λ_r = 0.00009,
μ_6 = 0.0001, μ_7 = 0.00015

Figure 4. Robot system steady state availability versus safety unit repair rate (μ_s) plots with exponentially distributed failed system repair time

$$\frac{dP_n(t)}{dt} + a_n P_n(t) = \lambda_s P_{n-1}(t) + \mu_w P_{2n+1}(t) \tag{73}$$

$$\frac{dP_i(t)}{dt} + a_i P_i(t) = \lambda_w P_{i-n-1}(t) \qquad (\text{ for } i = n+1, n+2, \ldots\ldots, 2n) \tag{74}$$

$$\frac{dP_{2n+1}(t)}{dt} + a_{2n+1} P_{2n+1}(t) = \lambda_s \sum_{i=n+1}^{2n} P_i(t) + \lambda_w P_n(t) \tag{75}$$

$$\frac{dP_{2n+2}(t)}{dt} = \lambda_r \sum_{i=n}^{2n+1} P_i(t) \tag{76}$$

$$\frac{dP_{2n+3}(t)}{dt} = \lambda_r \sum_{i=0}^{n-1} P_i(t) \tag{77}$$

where

$a_0 = \lambda_s + \lambda_w + \lambda_r$

$a_i = \lambda_s + \lambda_w + \lambda_r + \mu_s$ (for i = 1,2,……..,n-1)

$a_n = \lambda_w + \lambda_r + \mu_s$

$a_i = \lambda_s + \lambda_r + \mu_w$ (for i = n+1,n+2,……..,2n)

$a_{2n+1} = \lambda_r + \mu_w$

At time t = 0, $P_0(0)$ =1 and all other initial conditions state probabilities are equal to zero.
By solving Equations (71) – (77) with the aid of Laplace transforms, we get:

$$P_0(s) = P_0(s) = [\, s(1 + \sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s + a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} \frac{a_j(s)}{s})]^{-1} = \frac{1}{G(s)} \tag{78}$$

$$P_i(s) = Y_i(s)\, P_0(s) \qquad (\text{ for } i = 1,2,\ldots\ldots,n) \tag{79}$$

$$P_i(s) = V_i(s)\, P_0(s) \qquad (\text{ for } i = n+2, n+2, \ldots\ldots, 2n+1) \tag{80}$$

$$P_{n+1}(s) = \frac{\lambda_w}{s + a_{n+1}} P_0(s) \tag{81}$$

$$\mathrm{P}_j(\mathrm{s}) = \frac{a_j(s)}{s}\,\mathrm{P}_0(\mathrm{s}) \quad (\text{for } j = 2n+2, 2n+3) \tag{82}$$

$$G(s)=s[1+\sum_{i=1}^{n} Y_i(s) + \frac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n+1} V_i(s) + \sum_{j=2n+2}^{2n+3} \frac{a_j(s)}{s}] \tag{83}$$

The Laplace transform of the robot-safety system reliability with one normally working safety unit, the switch and the robot is given by:

$$\mathrm{R}_{rs}(\mathrm{s})=\sum_{i=0}^{n-1} P_i(s) + \sum_{i=n+1}^{2n} P_i(s) = \frac{1+\sum_{i=1}^{n-1} Y_i(s) + \frac{\lambda_w}{s+a_{n+1}} + \sum_{i=n+2}^{2n} V_i(s)}{G(s)} \tag{84}$$

Similarly, the Laplace transform of the robot safety system reliability with or without a working safety unit is

$$\mathrm{R}_r(\mathrm{s})=\sum_{i=0}^{2n+1} P_i(s) = \frac{1+\frac{\lambda_w}{s+a_{n+1}} + \sum_{i=1}^{n} Y_i(s) + \sum_{i=n+2}^{2n+1} V_i(s)}{G(s)} \tag{85}$$

Using Equation (83) and Reference [11], the robot-safety system mean time to failure with one normally working safety unit, the switch and the robot is given by

$$\mathrm{MTTF}_{rs} = \lim_{s\to 0} \mathrm{R}_{rs}(\mathrm{s}) = \frac{1+\sum_{i=1}^{n-1} Y_i + \frac{\lambda_w}{a_{n+1}} + \sum_{i=n+2}^{2n} V_i}{\sum_{j=2n+2}^{2n+3} a_j} \tag{86}$$

Similarly, using Equation (84) and Reference [11], the robot safety system mean time to failure with or without a working safety unit

$$\text{is}\mathrm{MTTF}_r = \lim_{s\to 0} \mathrm{R}_r(\mathrm{s}) = \frac{1}{\lambda_r} \tag{87}$$

5.1 Robot-Safety System MTTF Analysis for a Special Case

Substituting n = 2 into Equation (86) and (87), we get

$$\text{MTTF}_{rs} = \frac{1 + Y_1 + \frac{\lambda_w}{a_3} + V_4}{\sum_{j=6}^{7} a_j} \tag{88}$$

$$\text{MTTF}_{r} = \frac{1}{\lambda_r} \tag{89}$$

where

$$Y_2 = \frac{\lambda_s \frac{\lambda_s}{L_1} + \frac{\lambda_s \lambda_w \mu_w}{a_3 a_5} + \frac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \frac{\mu_s}{L_1}}{L_2 - \lambda_s \frac{\mu_s}{L_1} - \frac{\lambda_s \lambda_w \mu_w}{a_4 a_5} \frac{\mu_s}{L_1}}$$

$$Y_1 = \frac{\lambda_s}{L_1} + \frac{\mu_s}{L_1} Y_2$$

$$V_5 = \frac{\lambda_s \lambda_w}{a_3 a_5} + \frac{\lambda_s \lambda_w}{a_4 a_5} Y_1 + \frac{\lambda_w}{a_5} Y_2$$

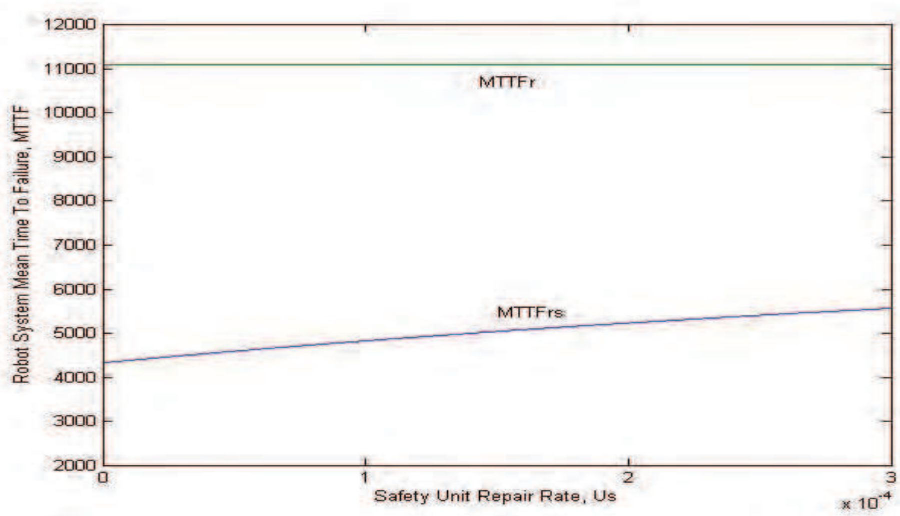
$$V_4 = \frac{\lambda_w}{a_4} Y_1$$

$$a_6 = \lambda_r \left[Y_2 + \frac{\lambda_w}{a_3} + \sum_{i=4}^{5} V_i \right]$$

$$a_7 = \lambda_r [1 + Y_1]$$

$$L_1 = a_1 - \frac{\lambda_w \mu_w}{a_4}$$

$$L_2 = a_2 - \frac{\lambda_w \mu_w}{a_5}$$



For λ_s =0.0002, λ_w =0.001, μ_w = 0.0003, λ_r = 0.00009,and using Equations (88)-(89) and Matlab computer program [10], in Figure 5 MTTF$_{rs}$ and MTTF$_r$ plots were obtained. λ_s =0.0002, λ_w =0.001, μ_w = 0.0003, λ_r = 0.00009

Figure 5. The robot-safety system mean time to failure plots for the increasing value of the safety unit repair rate (μ_s).

These plots demonstrate that MTTF$_r$ is greater than MTTF$_{rs}$, but just MTTF$_{rs}$ increases with the increasing value of μ_s.

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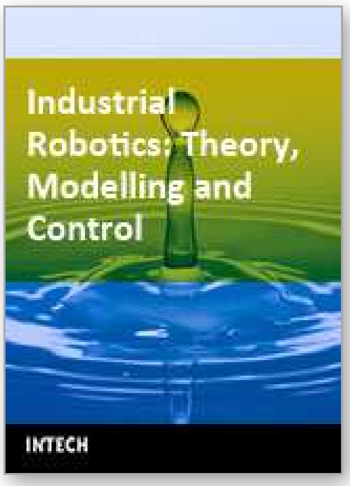

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This book covers a wide range of topics relating to advanced industrial robotics, sensors and automation technologies. Although being highly technical and complex in nature, the papers presented in this book represent some of the latest cutting edge technologies and advancements in industrial robotics technology. This book covers topics such as networking, properties of manipulators, forward and inverse robot arm kinematics, motion path-planning, machine vision and many other practical topics too numerous to list here. The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.