The Constructability of Cheops Pyramid

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Abstract

The aim of this paper is to forecast the methodology by which Cheops Pyramid as a historic building was built, based on notions of modular patterns discovered within the following planar illustrations, starting at the apex and descending to its grand base.

Some of Cheops' heritage in terms of form and function has existed for millennia, as its mark was recapped with a front line that was constructed thousands of years before. Such a voyage through the missing captures of the building techniques wouldn't have occurred without the incidental outcome that resulted from overlaying the various pixelated plans of block arrangements on the floor maps of Cheops. This building is believed to exhibit norms similar to those governing the balance between matter and void in the universe.

By assessing this historical masonry structure of Giza, we have come to find some regulatory approaches concerning how to asses the construction of such a building by predicting their interiors, spatial definitions, customs, and layouts. As there are no major records of their plans, the best practice anticipated in this paper is to modularize the figurative appearance of such buildings, by imitating themes of the repetitive patterns of past regional Egyptian architecture seen as pixels. This led to the fundamental solution to such a dilemma in building construction.

Keywords: Cheops, Pyramid, Balance, Historical Buildings

1.0 Historical context of Cheops Pyramid

"The Great Pyramid has lent its name as a sort of by-word for paradoxes; and, like moths to a candle, so are theorizers attracted to it. The very fact that the subject was so generally familiar, and yet so little was accurately known about it, made it the more enticing; there were plenty of descriptions from which to choose, and yet most of them were so hazy that their support could be claimed for many varying theories" (Petrie, 1887).
"Egyptologists believe the Pyramid was built as a tomb for the Fourth Dynasty Egyptian pharaoh Khufu (often Hellenized as "Cheops") and was constructed over a 20-year period. Khufu's vizier, Hemiunu (also called Hemon), is believed by some to be the architect of the Great Pyramid" (Shaw, 2003).

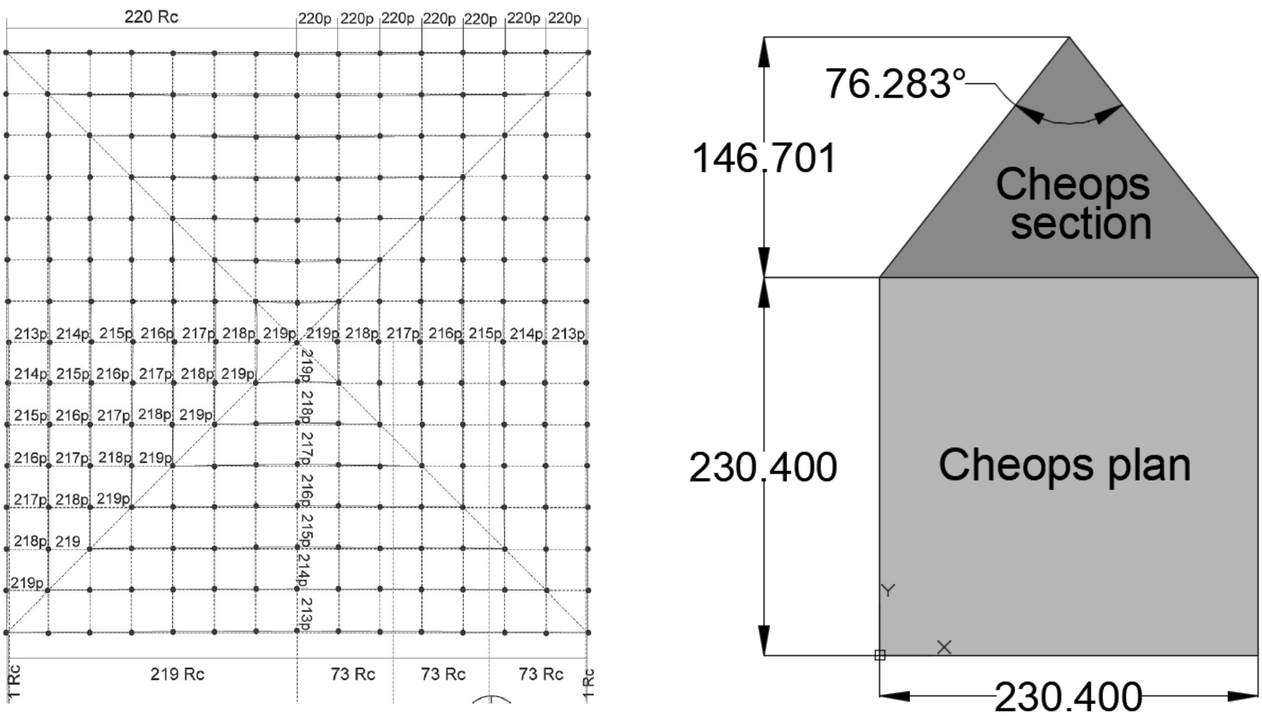
The pyramids still signify, among their many aspects, the accountability of the measuring apparatus used by the ancient Egyptians to plan the complexities of their structures; many alternative and often contradictory theories have been proposed regarding the Pyramid's construction techniques (BBC, 2006).

Nevertheless, the pyramids of Giza were built thousands of years ago, yet they are still in their upright positions and largely preserved in their original condition. More errors in workmanship are considered to exist compared with the Great or Second Pyramid of Giza but rather less than those of the Third Pyramid (Petrie, 1887).

The secrets that lie within the framework of Cheops Pyramid have contributed directly to the regional structural vocabulary/context by which elemental figures within the architectural edifice such as lintels, corridors, empty chambers, shafts, symmetricity, modules, and so on later spread widely across the region and were used by the contemporary architects who followed.

Egyptian pyramids were once regarded as piles of stones, necessitating that each stone be placed on top of others (Bryn, 2010). However, the assumption of any kind of piling has been proved erroneous. The geometric principle at work in the Cheops Pyramid is the arrangement of blocks of stone on a horizontal plane according to a particular geometric organization.
This is a remarkable representation of the gradual derivative of balanced states in the depiction of the Cheops Pyramid. It sheds further light on the realms of mystery that have encompassed this remarkable landmark and is of great concern to studies related to the prehistoric intelligence of a great civilization that once inhabited the ancient world (Ghoussayni et al., Cheops, 2018).

Wherever you look at Cheops, you find configurations of blocks accumulated around one main principle, the foundation of an immobile tombstone. This is due to the arrangements of its building blocks, which are explained extensively in the following sections. A universal pattern exists in the way this pyramid is built, as, to a large extent, throughout the construction phases and time taken to add each layer of blocks, the Pyramid maintained a good condition of equilibrium. This type of block arrangement eliminates all risks that may emerge during the pyramid construction sequence. The stone organization enhances the timeframe for builders, while the modular composition eliminates excessive design and proposes pattern-type templates that cover projects in the sequence of the builder’s plans. The grand depiction of a pyramid’s base in square geometry is highly efficient to the best extent possible in maintaining symmetrical alignment in four directions (horizontal, vertical, and two crossed diagonals). This symmetricity is well preserved, along with the variant layers of building blocks.
How the pyramids were built is forecast directly from methods that governed how they were planned. However, the planning phase would not have been discovered without the involvement of advanced mathematics. This phase is deduced from the inventory of a building grid that best represents the separation of the building’s measuring system and its constituent parts.



(a) (b)

Fig. 1.0 (a) The “diamond matrix” in the plan (Ole J. Bryn, 2010), (b) Cheops dimensions in meters (Petrie, 1883), (Cole, 1925) (Camacho and Sánchez, 2018)

The clearly appealing question of our study is to discover to what extent the Cheops Pyramid is related to the buildings in the vicinity of Giza. As previously explained, the only compression structures that exist such as vaults are, to a large extent, derived from the geometric normalities embedded in the pyramids. The load distribution is equally disseminated in both cases, and they are both made of stone-based material.

2.0 Safety evaluation of Cheops masonry structure

Historical landmarks like the ancient world's Cheops Pyramid are in urgent need of re-evaluation, and only the intense efforts of researchers can save them from being misunderstood. Much of Cheops Pyramid's architectural heritage consists of unreinforced masonry buildings, which must be conserved for future generations. Many of its structures have complex geometries, and their static equilibrium is still poorly understood. To understand the structural behavior patterns of masonry assemblies, the simple void from the king's chamber is a good example, as it figures repeatedly among the many appealing structures of the Giza plateau and ruins. A simple equilibrium state, characterized by the addition of pixelated square structures known as stone blocks, led to the analysis of this masonry building, is a well-determined problem, and allows for the drawing of conclusions.

From the previous dialogue, few key resolutions can be extracted. Indeed, there is a mathematical challenge associated with the pyramid's image transformation throughout various states. The Giza plateau that surrounds the historical Cheops has become a pole of attraction, unlimited by the presence of its geometrical force. It is an attractive image from the outside. This provision is, of course, the result of old and more recent historical, spatial, and social processes that are likely to grow stronger given the importance of touristic recomposition.

A situation that involves building with blocks calls for work on the integration of spaces and masses and a certain geometrical mix, to achieve a figure of cohesion and exchange. In this way, the composition of modular patterns and their prolongation over the Pyramid's grid as norms are found intact with the historical context and are being projected further over the void found with the absence of historical recognition due to the lack of recent technologies and communal neglect.

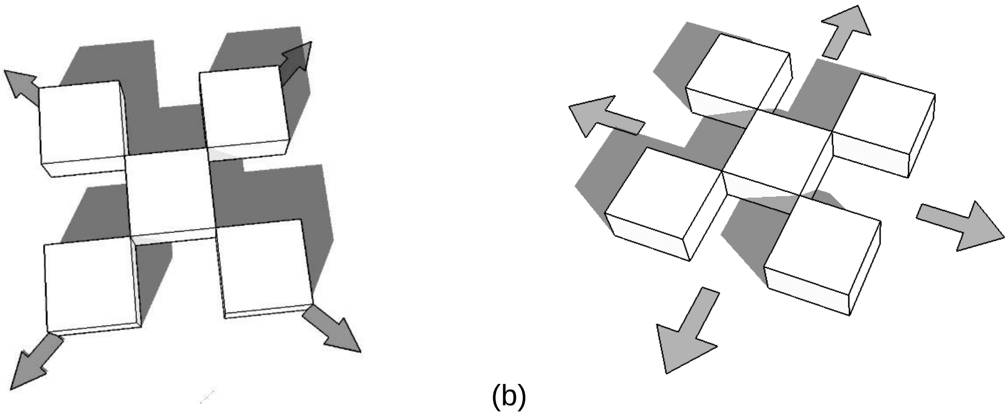
3.0 Equilibrium analysis of Cheops patterns

The condition of equilibrium is necessary for all structure types, since stability is important for traditional pattern-type structures. With a better understanding of foundation blocks, unnecessary interventions and strain in the design methods of such buildings can be avoided.

The Egyptians had clearly developed tools for understanding and explaining the basic behavior of masonry structures, like the patterns frequently found at Giza.

With time, the fundamental structures have maintained their integrity, despite the surrounding civic conditions which have been relentless, as the conscience of a place rooted within these dilapidated structures is to be reclaimed alongside the memory of this landmark.

Providing structural indemnity against the factors of time and neglect, which is strongly attributed to the continuous occurrence of repetitive patterns, is considered a strong shield against deteriorating factors, since the nature of such patterns is to oversimplify any building calculations and calls for strong coverage of the different entities in any building type. These patterns are equational solutions to different complex problems and establish a clear grid that can be easily understood. Patterns that have been discovered in Cheops are non-differential and accredited to the repetitive occurrence of a homogeneous spatial dimension. In our case, the span of multiple royal cubits has been detected in many Giza buildings as Isaac Newton established an average 0.525 m for the length of the Egyptian royal cubit (Newton, 1737); the architectural palette of the region is greatly noted among the design throughout the podium of this city.



(a) (b)

Fig 2.0 (a) Cross-vault pixelation showing the direction of lateral forces. (b) Pyramid pixelation showing the result of outward expansion (Source: Authors)

4.0 Pattern language

Traditionally and by definition, a void is an empty space located in a floor: it develops in height and is fully open and is normally oriented to true north, as is the case of the Cheops Pyramid. Everything differentiates these spaces from those of modern buildings, where networks are chained to the culture, and there is a daily transition between the narrow and dark corridors and open-air spaces that separate the blank interior—the private—from the outside— the public. Void and mass are correlated through various superimpositions that intervene in a pattern-like composition.

"What we call now the 'overall pattern language approach' in architecture, urban design, and planning has grown from its original principle of 'pattern and pattern language' into a large and solid body of theory and professional work" (Neis, Joachim, Baumgartner, 2015).

"The idea of a pattern in this specific understanding is first based on the observation of repetition of particular building elements" such as typology, function, color, material, or structural methodology that "form substantial knowledge of a building culture or culture in general. Hundreds of these patterns may define the building knowledge of a given culture, and thousands might describe close to the overall cultural knowledge of a given culture" (Neis, Joachim, Baumgartner, 2015).

Cheops Pyramid, already densely occupied by building blocks, was thought to be divided into layers in patterns attributed to the different methodologies of building techniques. The first scale map of the Pyramid is another addition to the cluster of units enjoyed during a period of low technological advancement, reflected by the creation of low-resolution maps. Research under modern scientific empires did overcome its historical limits and undergo great changes. The "importance of using a Pattern Language is that it combines all of the different visions and needs of the people involved in a common language that everyone can understand. … The simplest form of applying a pattern language to a given design project—for example, a residence—is to select a set of patterns and use these solution-patterns as archetypal starting points for a design and building process" (Neis, Joachim, Baumgartner, 2015).

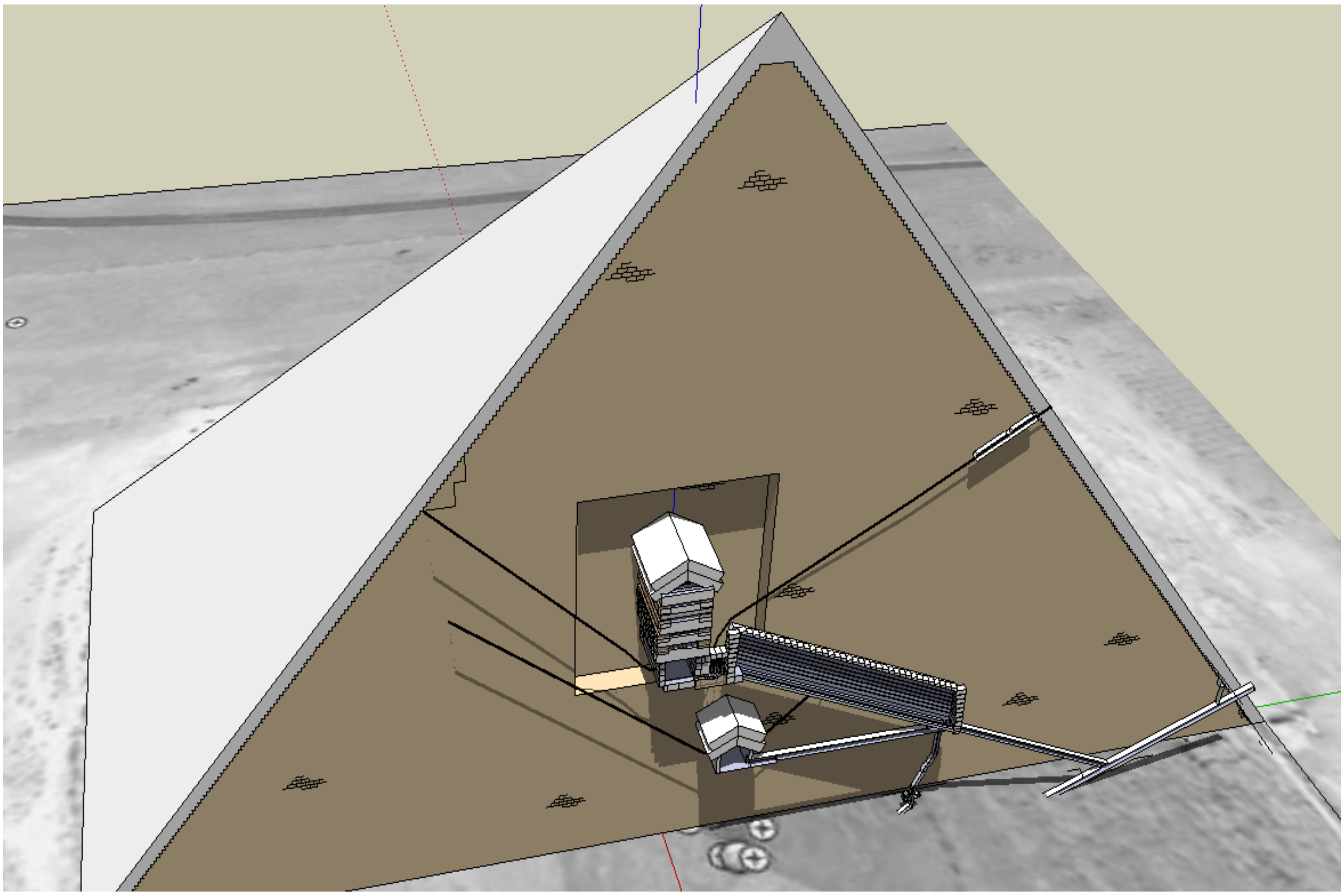
Regarding a common typology of openings, it is very difficult to draw up an exhaustive "inventory" of those encountered in this Pyramid. As a result of the changes that have affected the scientific fabric, new forms of studies emerged and brought the record up to date. We will go through the consecutive sections, as patterns and modules are simple in construction, which led architects to use them in almost all building elements, from ornaments to spaces, as they "focused more extensively on other aspects of architecture, such as form" (Abdullahi, 2013).

The building's geometry is intrinsically integrated within the grand plateau depiction of Giza, as the routes are, to a large extent, perpendicular to one another. By knowing the visual screen to set over the map of the Cheops Pyramid obtained from some of the references, we were able to understand the homogeneous settings of the historical norms of building design.

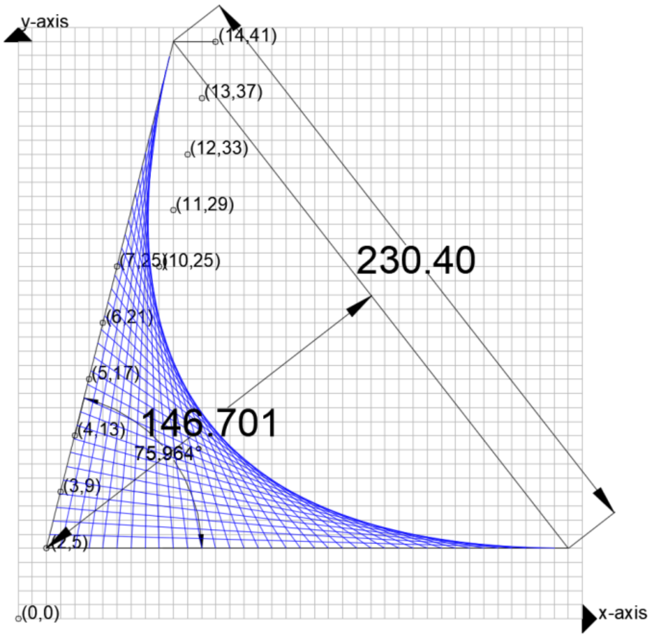
5.0 Cheops void patterns

The story of this space is historically attributed to fundamental structures that could not have been constructed without the consequent knowledge of mass and its hidden void by the earliest known civilizations that inhabited the ancient world.

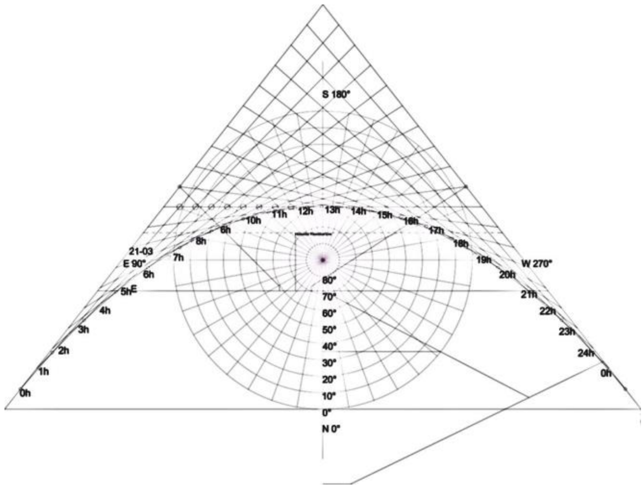
Despite the progression of current technological norms in computer-aided design, one of the historic components of space can be accredited to the pyramids of Giza and the revelation that their mysteries relate to the fundamental constituents of mathematics, arithmetic, and geometry. In other words, the geometric configuration of the space and void incubated in Cheops Pyramid provides a road map to understanding the most prominent tools described in previous sections and lays the foundation for essential methodology when setting the choice to be taken on the expanding textile which is in our case the void that is within the intent of this pyramid.



(a)



(b)



(c)

Fig. 3.0 (a) 3D visualization: (GoogleSketchup-3D Warehouse, 2016).(b) Relationship of the dimensions, side = 230.4 m and height = 146,701 m, of the pyramid, concerning the curves that result on Cheops triangle. (c) Construction of a quadratic Bézier curve using a graph of Cheops Pyramid overlayed on the solar representation of equinox at the site of the Giza pyramids (Author).

The sequence of discoveries can be correlated with the static condition of a pyramid, wherein the different states of static and dynamic figures have been well underlined; as such, the lost space is emphasized and identified as an empty figure (void) denoted as the basic parameter in an orthogonal-based system of space definition.

The pre-existence of the void is conditioned by the existence of matter, since a void without matter cannot be detected and its variable notion in accordance with time and space. Hence, the approach to such a conclusion of the static or dynamic state of the void can only be described and calculated based on the presence of matter formerly defined by a filled square, where zero solid states are associated with infinite empty states. In this condition, there exists a whole state of dynamism because what falls between the two ends of balance vibrates and interchanges positions until it acquires one of the two states.

The void pixelation of the pyramid's interior is well determined in the above figure, where the corridors and shafts are structured in accordance with the highly unwavering methodology of the erection. In this way, some of the pyramid building methods can be highlighted.

The approach used to obtain the findings is based on geometric and mathematical representations illuminating the principle that void and matter are in constant competition for balance, from the first formation of an apex of a given pyramid to the endless ramifications of its representation.

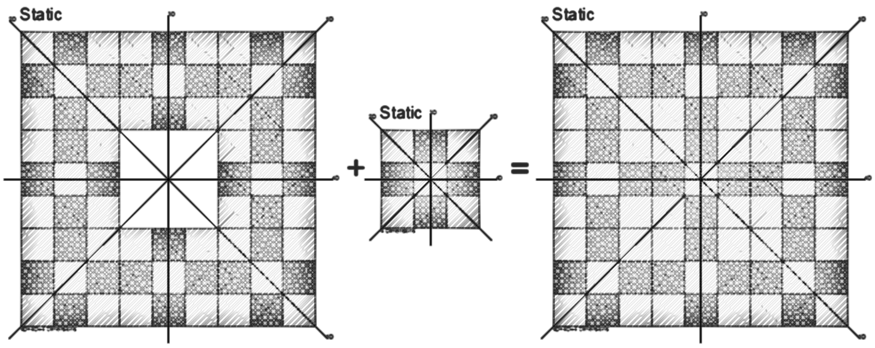


Fig. 4.0 Void pixelation in the static state (Source: Authors)

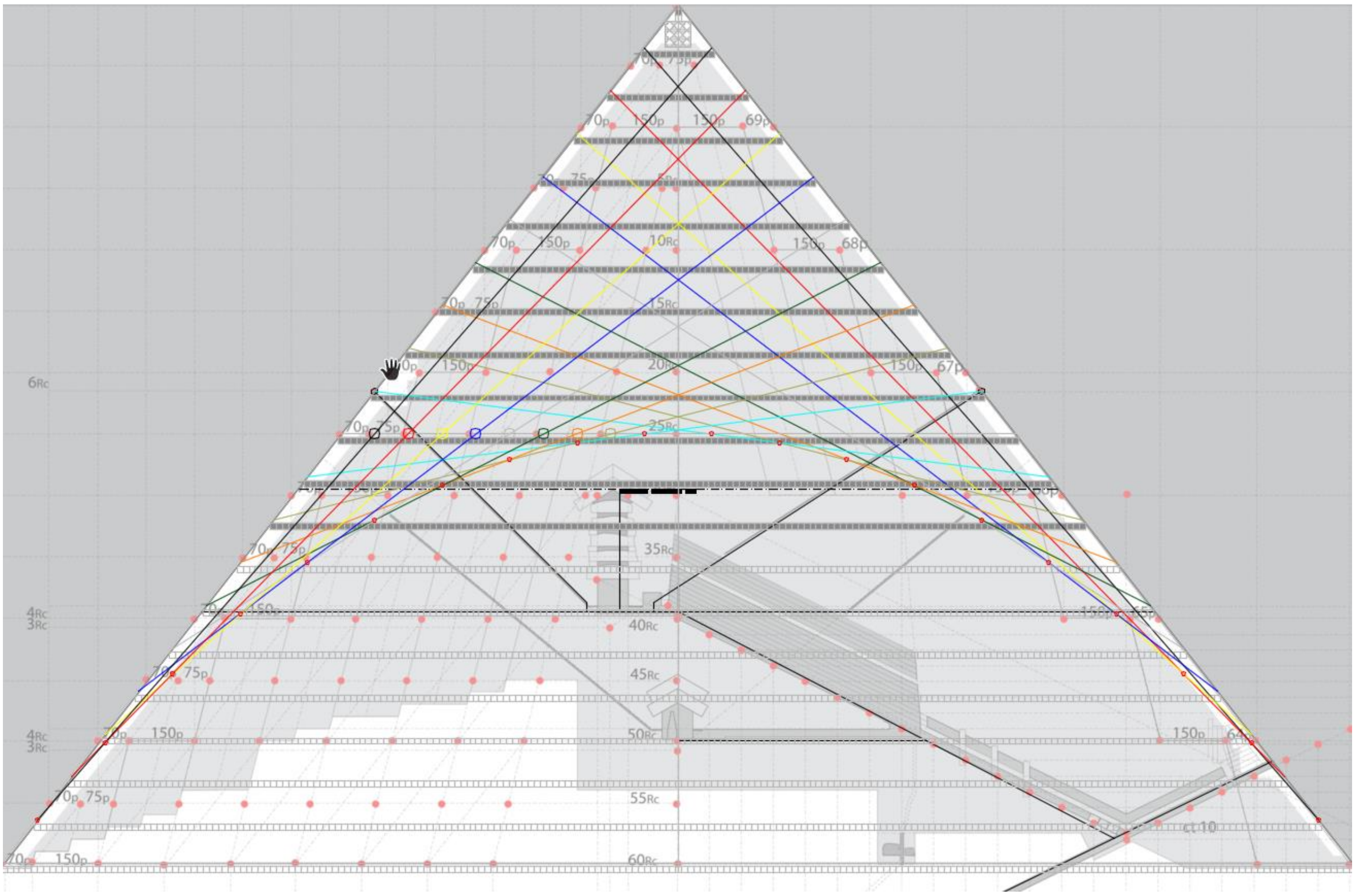


Fig. 5.0 Superimposition of Bézier curve over the solar representation of equinox in Giza (Bryn 2010, Bézier Visualization by Authors)

As shown in the figure above, “a Bézier curve is a parametric curve frequently used in computer graphics and related fields. Generalizations of Bézier curves to higher dimensions are called Bézier surfaces, of which the Bézier triangle is a special case. In vector graphics, Bézier curves are used to model smooth curves that can be scaled indefinitely. ‘Paths,’ as they are commonly referred to in image manipulation programmes, are combinations of linked Bézier curves. Paths are not bound by the limits of rasterized images and are intuitive to modify” (Wikipedia, 2017). To correlate the above with the search for the void within historical norms, it is pertinent to conclude that the void pixels (King’s Chamber, Queen’s Chamber, paths, etc.) fall beneath the Bézier curve formation.
From another perspective, the void formed in the Pyramid under the Bézier arc can be clearly correlated to the vaults in Figure 6.0, as the question immediately arises concerning how these vaults stand and how stable they are, given the huge void beneath. They have complex three-dimensional geometry, unknowable material properties, and numerous cracks and voids due to movement of their supports over the centuries. The safety of these historic structures is difficult to determine and current computational tools are unsatisfactory (Block, 2009).

(a) (b) (c)

Fig. 6.0 Complex three-dimensional masonry vaults: (a) Hieronymites Monastery Church (Jero’nimos), Lisbon, Portugal, 1499–1528; (b) King Henry VII’s Lady Chapel, Westminster Abbey, London, England, 1503–1519; and (c) Pfarrkirche, Ko¨nigswiesen, Austria, 1520 (P. Block, 2009)

Thrust line analysis is a “method for understanding and exploring the range of lower-bound equilibrium solutions” of masonry vaults to better understand the pyramid model, by “showing the paths of the resultant compressive forces throughout the structure.” Thrust line analysis is “primarily a two-dimensional technique and is, therefore, most appropriate for the analysis of arches, flying buttresses or any structure which can be reduced to a sectional analysis” (Nikolinakou et al., 2005). The main advantage of using graphic statics is the clear visual representation of the possible compressive forces in the system through the use of funicular polygons and thrust lines.

Any three-dimensional vault can be analyzed using thrust line analysis when combined with the slicing technique, which was first discussed in Ungewitter [1890]. It slices the vault into two-dimensional imaginary strips, reducing the vault’s structural behavior to a combination of two-dimensional problems that can be more easily analyzed. Figure 7.0 shows one of Wittmann’s applications of combining thrust line analysis with the slicing technique to analyze spatial vaults (Wittmann, 1879). According to Huerta [2008], this is the first known application of this approach” (Block, 2009).

The slicing technique shown in the below figure correlates with the founding masonry equilibrium of vaults, as previously explained, and provides a better understanding of the structural implications.

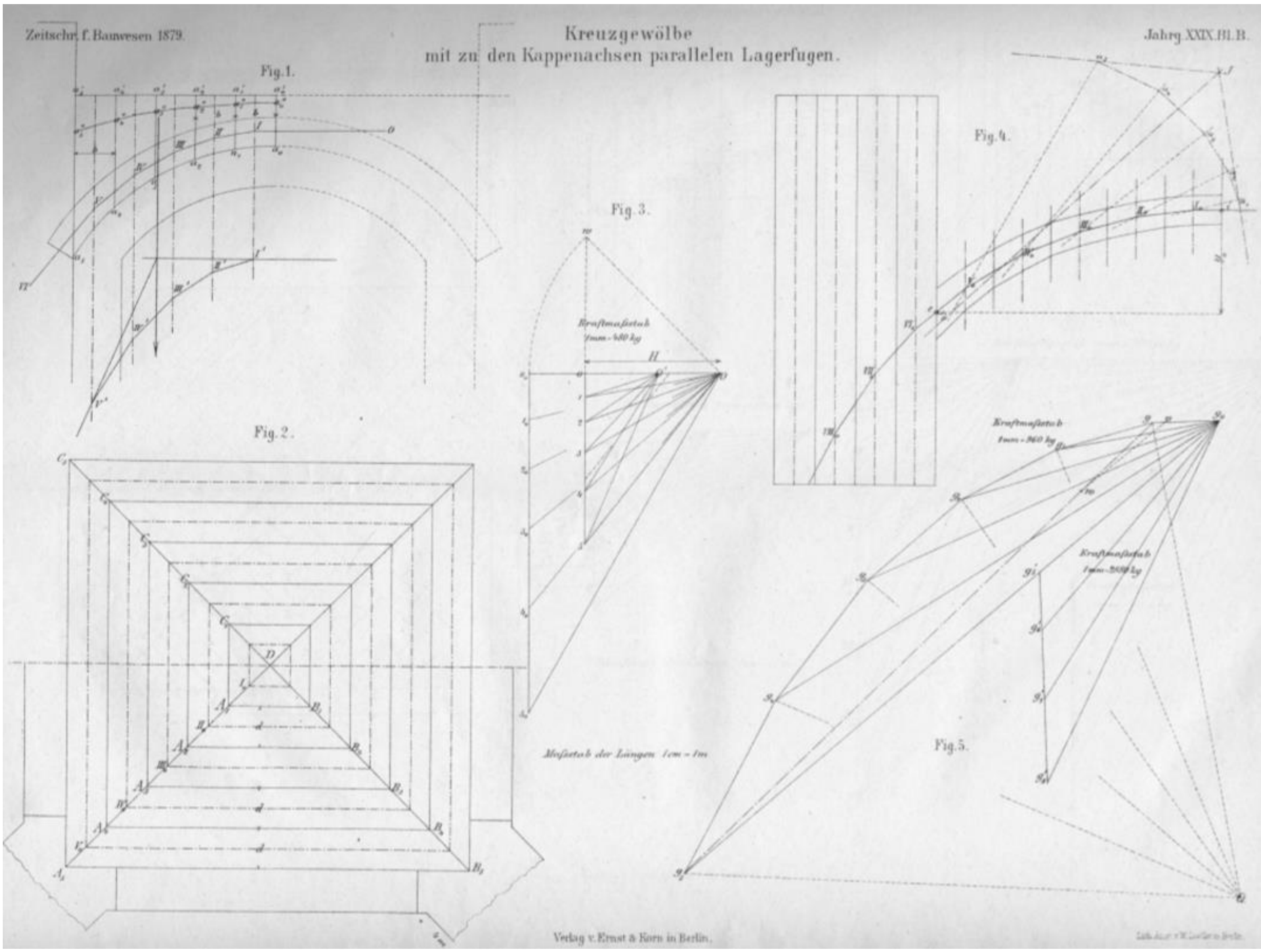
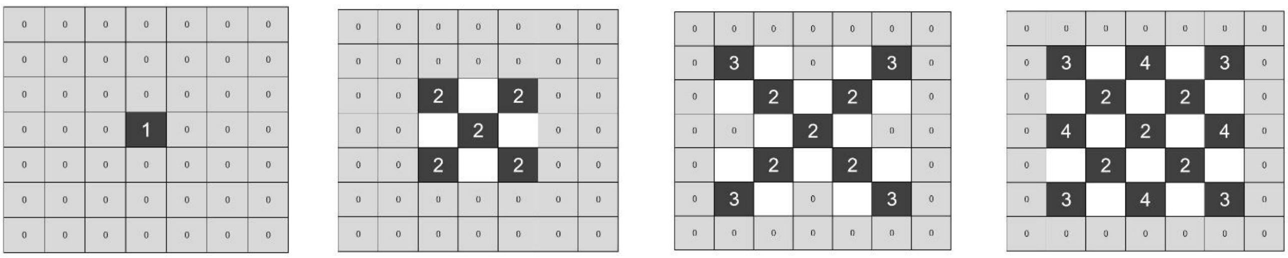


Fig. 7.0 The first known application of thrust-line analysis combined with the slicing technique by Wittmann (1879)

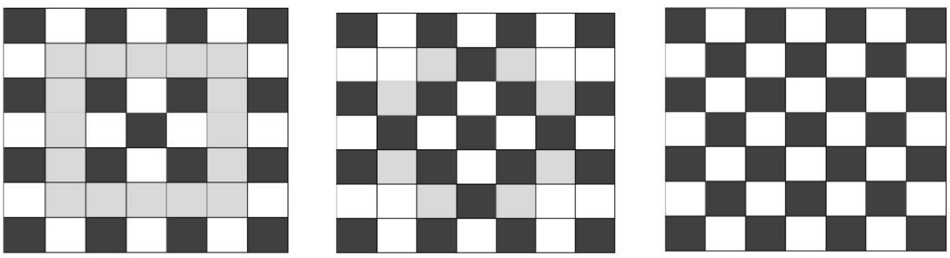
6.0 Decryption of Cheops construction

Given that the simplest constitution of a pyramid is a block/cube with orthogonal geometry, a two-dimensional composition is the simplest familiar figure in the field of graphical understanding, whereas a void is visualized as an empty pixel. Symmetrical representation in any square should be in at least four directions, namely the vertical, horizontal, and diagonal directions. Meanwhile, the orthogonal system necessitates four-directional symmetry to attain static representation, and by this, we mean the balance of the building blocks accumulating one on top of the other (Ghoussayni et al., 2018).
Any state that does not constitute four-axial symmetries will definitely be considered a non-static state. The present study expands the figures and their states to calculate the different schemes, with respect to the consideration of single symmetry or sole axis as movement and of four-way symmetry or four-dimensional axis as static.

Starting with the graphical presentation of the results obtained from the configuration of equal and stable alignments of the different black placements and their underlying progression, the study illustrates the findings below.



(a) (b) (c) (d)



(e) (f) (g)

Fig. 8.0 (a) (First step)=x=1 (b) Second Step=x=2, y=(No. of blocks)=5 (c) x=3, y=9 (d) x=4, y=13 (e) x=5, y=17 (f) (6,21) (g) (7,25) (Source: Authors)

With the superimposition of the stepwise configuration on Cheops Pyramid, when expanding the figures obtained in the above figure, and as proof of the importance of the above findings, seen as (X, Y), we observed that many superimpositions occur when overlaying the Cheops Pyramid section and graphical representation resulting from the coordinates in Figure 8.0 (Ghoussayni et al., 2018), as shown in Figure 9.0.

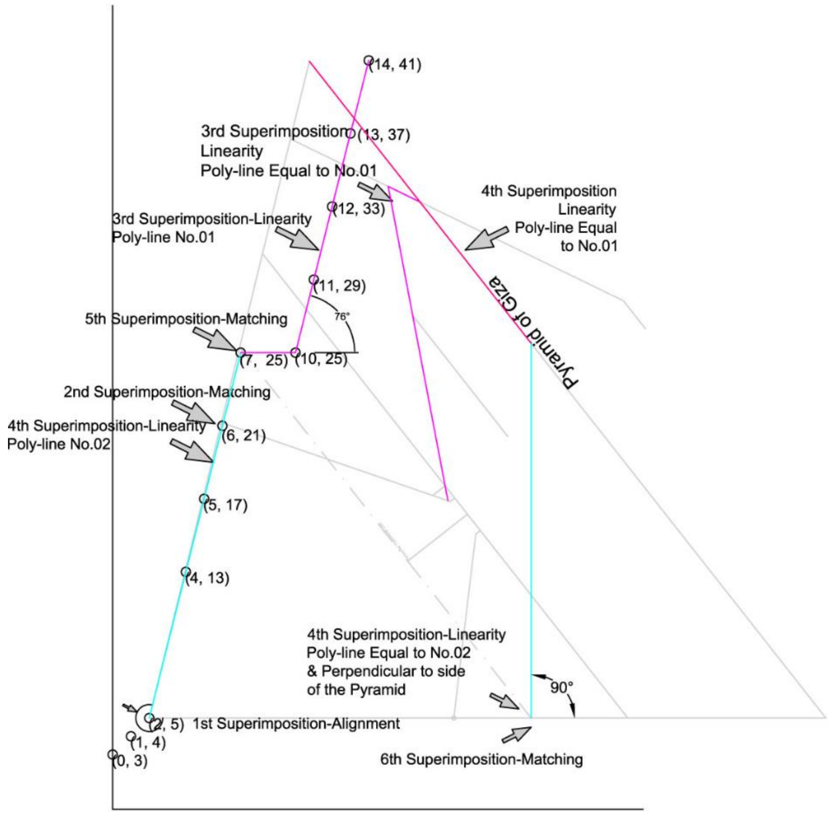


Fig. 9.0 Demonstration of the superimpositions (Ghoussayni et al., 2018)

In the search for the relation between the base of each triangle, by drawing the same apex triangle with different base widths, it was found that the width of the base from the midpoints of the original sides downwards has the same value as the number of positive blocks at the small base's level (new finding).

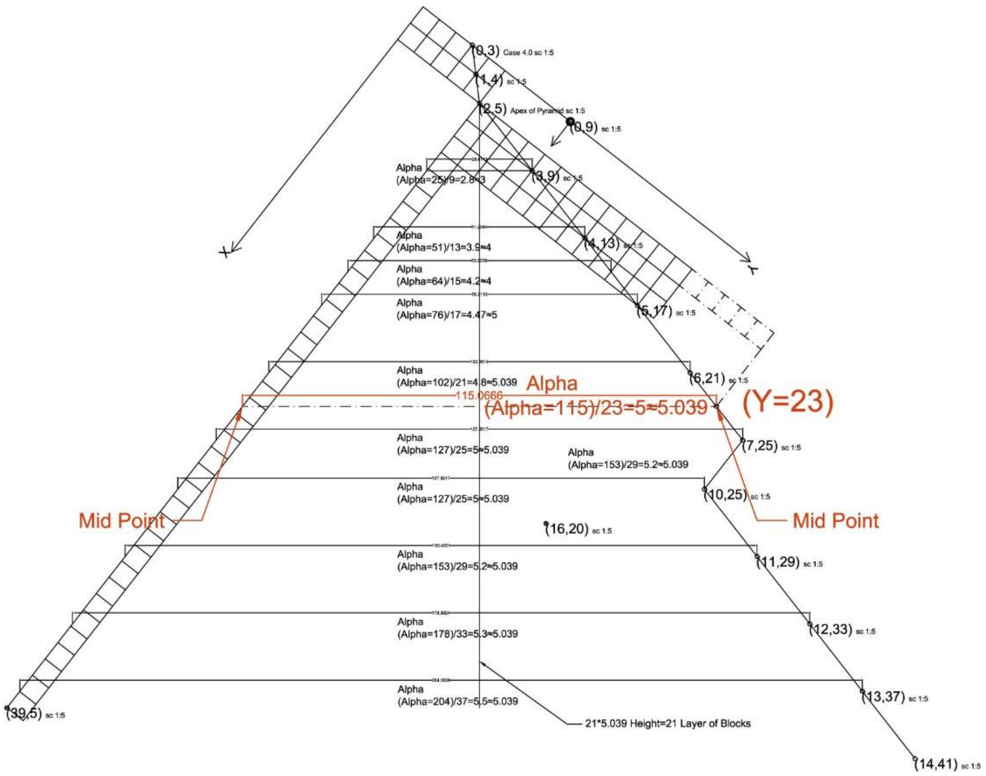


Fig. 10.0 Relation between the width of the base and the number of positive blocks at that base (Source: Authors)

Total no. of pixel blocks = ∑(Y^2) (as Y = 25 till 200) = n (n+1) (2n+1) / 6 (by convention), where n = Y (proven in the case of Cheops Pyramid)
To solve Y^2, this study considered a pyramid whose base is equal to Y (proven hereinafter), with an apex of (10, 25); hence, the sum of Y^2 = sum of n^2 = 2,686,700 pixel blocks (where each block is 1 x 1 x 1 pixel). As such, the question of how many quadratic pixel blocks are supposed to be in the Cheops Pyramid has been answered. Namely, to find the volume, the number of blocks should be determined and then multiplied by the volume of each pixel block, which may vary from 1 x 1 x 1 royal cubit to 2.42 x 2.42 x 1.355 royal cubit. (Petrie, 1883)

On the other hand, the pharaohs had clearly developed tools for providing structural indemnity against the factors of time, which is strongly attributed to the continuous occurrence of repetitive patterns. This was considered a strong shield against deteriorating factors, since mathematical patterns by their nature oversimplify building calculations and calls for strong coverage of the different entities of any building type.
The patterns found in the Great Pyramid of Giza (Cheops) and many other buildings that will be revealed in the progression of this paper are similar in nature to those exhibited in the course of Egyptian monument building; it is clear that such recurrences in building design established the fundamental idea that a state of equilibrium is always a condition of symmetrical forms of grid-like modules that turn out to be mathematical patterns from the past.
In Figures 11.0 and 12.0, the expanding modular base and built-up area from the top of the Cheops Pyramid section down to its large base exhibit a set of calculations that can be initialized to account for the total number of building blocks in the Cheops Pyramid.
This measures the exact arrangement of pixelated spaces and mathematical pattern-like structures. It also predicts the building form that best exhibits an equilibrium state, and by which the transfer of the loads can be easily maintained along the vertical alignments of structural elements.
The Cheops Pyramid is considered a good model of stability and perseverance and provides the best response to fundamental questions about how to maintain an equilibrium state for any given structure.

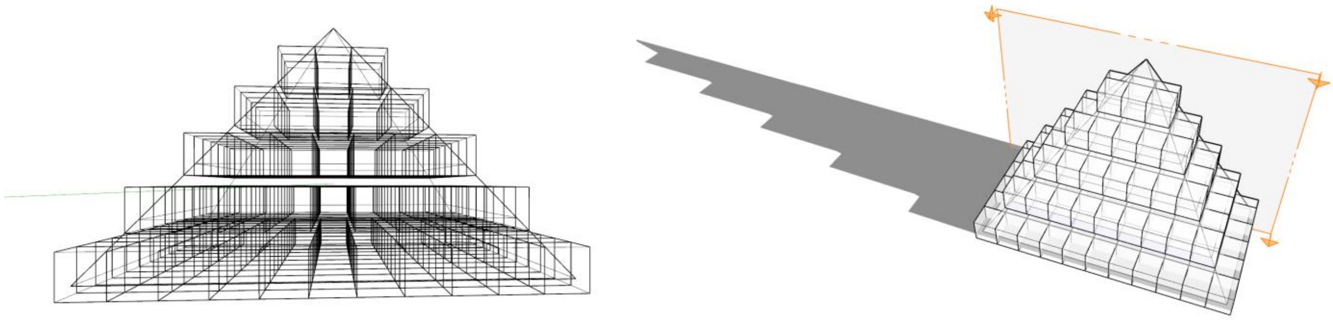


Fig. 11.0 (a) Demonstration of superimpositions in the form of a pyramid (Source: Authors)

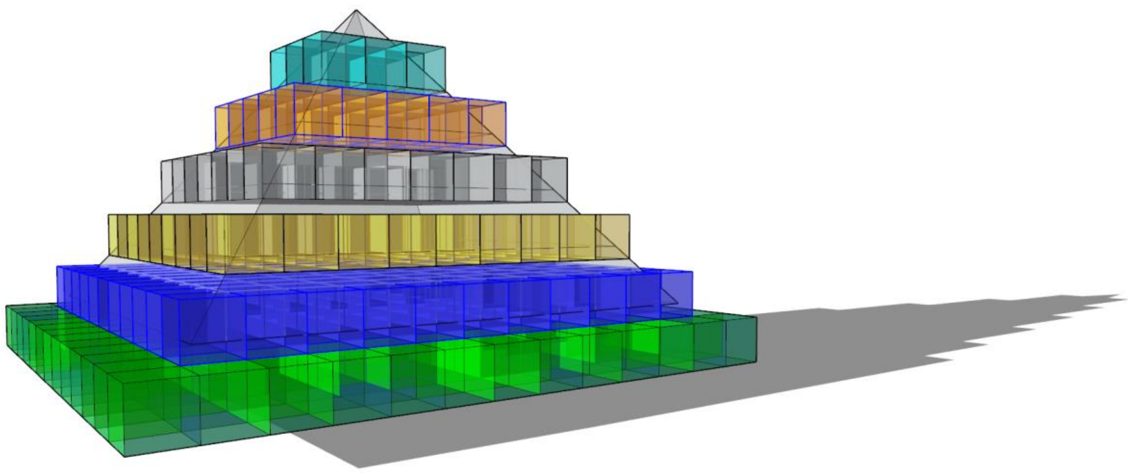


Fig. 12.0 Demonstration of superimpositions of the different layers of a pyramid (Source: Authors)

Hence, the figures/coordinates ((2,5), (3,9), (4,13),…etc.) that show the elemental modules/pixels arrangement as segregated into different levels, where each level weights its percentage out of the total accumulation of pixels, are depicted in Figure 13.0 below.
This produces an example of a six-story pyramid, whose total volume is the sum of Y ordinates (Y total=5+9+13+17+21+25), where the percentage of floor/layer No. 6 of its volume constitutes 7% of the total volume of the building and continues with floor/layer No.5, which constitutes 11%, and so on.

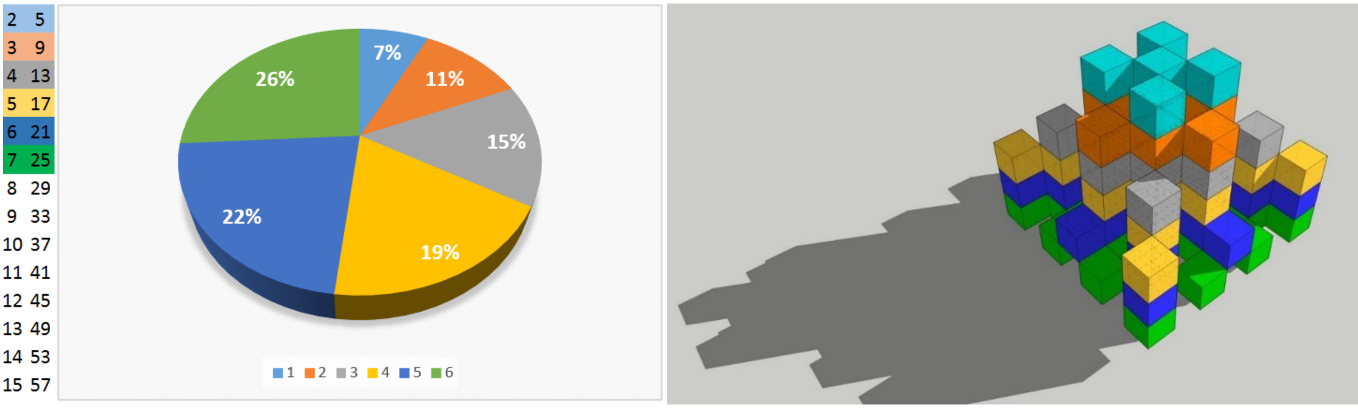


Fig. 13.0 Built-up area statistics per each level of Cheops Pyramid (Source: Authors)

On the other hand, to highlight the structural emphasis in the method used by the ancient Egyptians who built the Cheops Pyramid and achieved high accuracy within the projected timeframe, Figure 14 below is a representation of colors depicting the phases of block layering in accordance with the following steps: green, then red, then yellow, then blue and black. Then, the layering starts over again, with the condition of keeping a state of balance when layering the blocks that will result in patterns within the graph.

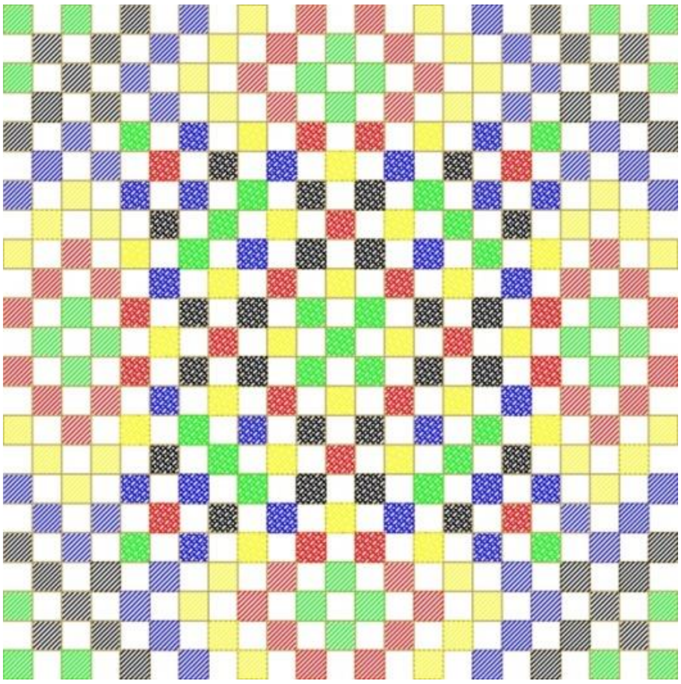


Fig. 14.0 Diminishing much space by interpolating the random coordinates and saving space and time (Source: Authors)

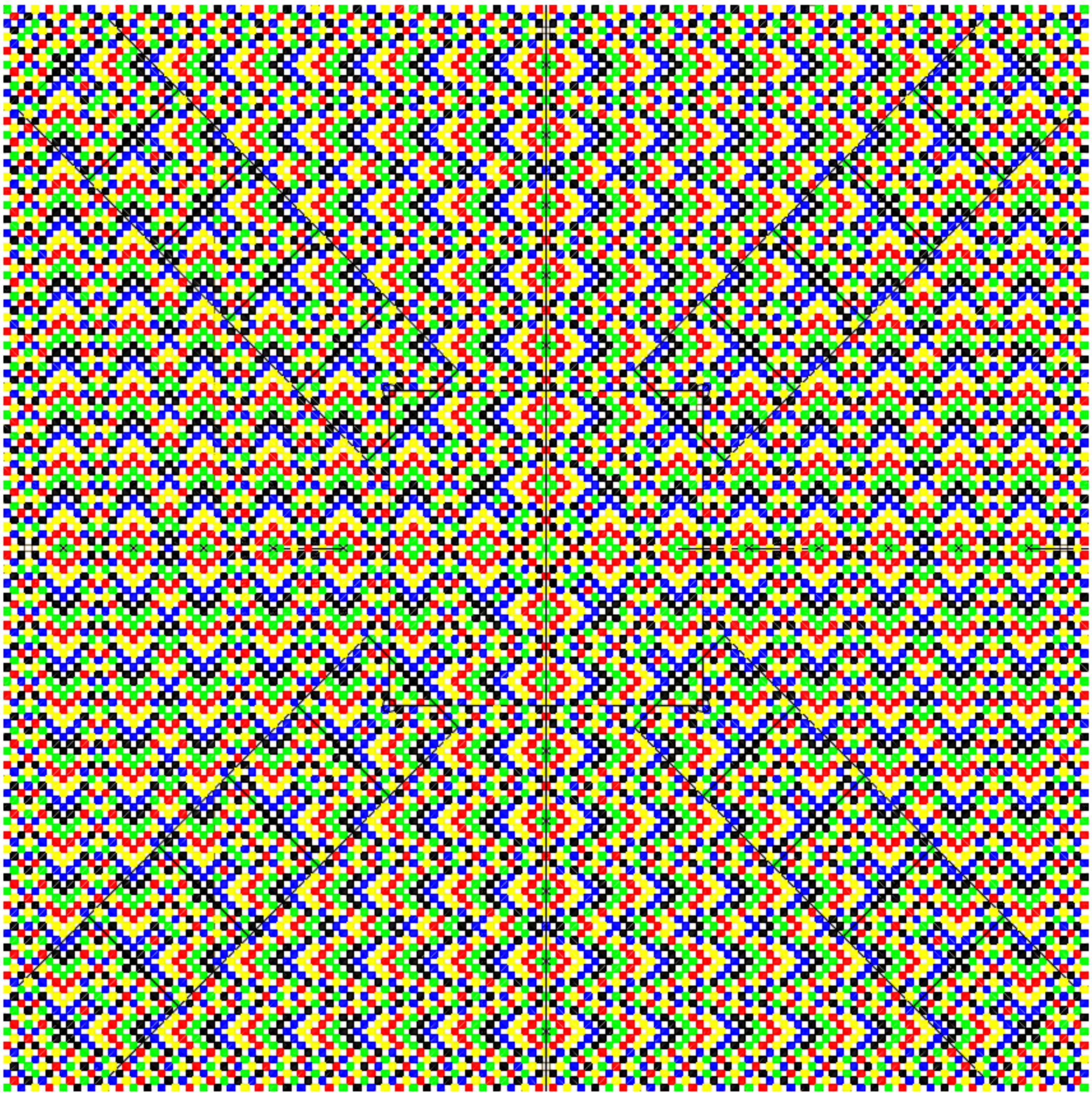


Fig.15.0 Diminishing much space by interpolating/approximating the random coordinates and saving space and time (Source: Authors)

Figure 16 below is a representation of the different shades of grey generated when changing states (GRYBB). The result is that certain blocks, highlighted in dark grey, are considered to be the last layer of blocks to be put in place; thus, when running this method on the overall pixel plan in Figure 15.0, various appealing highlights emerge among certain patterns in the void.

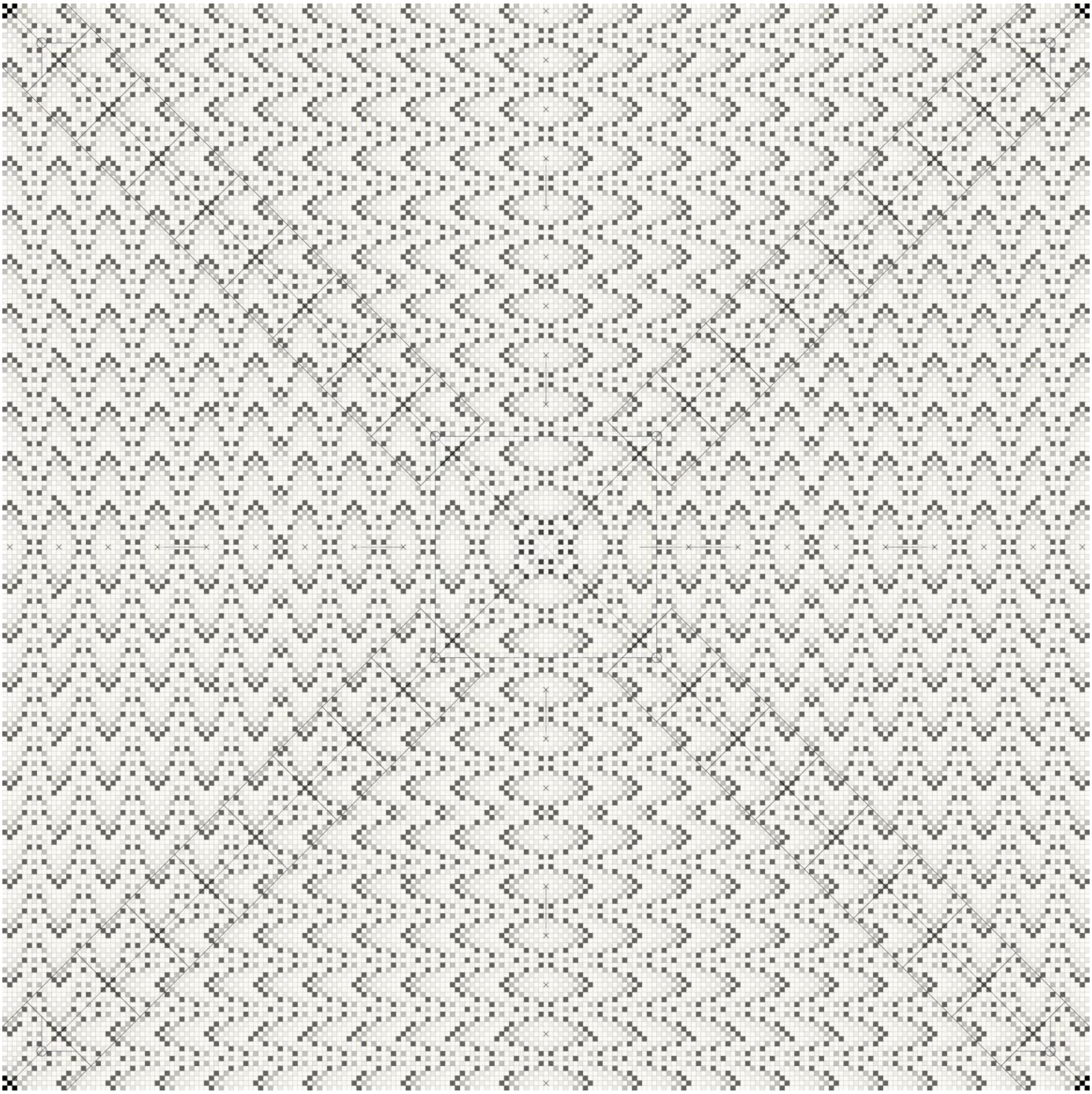


Fig. 16.0 Depiction of layering the blocks from lightest to darkest shade of black (Source: Authors)

The void depicted in the darkest grey is constituted upon the interpolation of building block placement that sets the framework for the buildings of this study's subject and concludes with a recommendation of best practices to reconcile the emptiness, or, in other words, the void.

7.0 Conclusions

This study presents derivative values for sensible states in the Cheops Pyramid. New contributions to the subject confirm the remarkable engineering details that shed new light on the true nature of the Pyramid. Approaches can only be valid if they are made with scientific integrity and mathematical precision, and, therefore, this amazing structure and beacon of architectural accuracy must be studied with appropriate methodologies. In the future, experts researching the exceptional construction of the pyramids should employ straightforward techniques and practices but be open to the possibility of unimagined results.

Space is the nurturing factor of human observations on distance, time, and velocity. In an experiment, a working platform is needed— one with a clear, fixed, and reliable context. To advance from one level to another, a small void between them is required. Occupational property is an important key to explaining existential acuteness and the extent to which it inhabits the transition. Regarding the Cheops Pyramid, transitional occupation from the apex down shows matter in its conditional state of balance. This finding highlights the importance of implementing the patterns of void and matter depicted in the Cheops Pyramid as a factor within any equation.

If humans are looking for a definitive conclusion on the blank space in the universe, we may state that darkness is built from matter as much as light is but that the difference between darkness and light is that darkness is not within human perceived vision.

The representation of the gradual derivative of balanced states in the depiction of the Cheops Pyramid, which sheds further light on the realms of mystery that encompass this remarkable landmark is very significant and of great concern to studies related to the historic intelligence of this great civilization that once inhabited part of the ancient world.

This innovative model parades by the techniques used in consequent sections, as it represents a novel understanding of repetitive elements and detectable patterns. This is considered to be a new finding, as it could predetermine the space framework, to a large extent, from tombs to religious edifices, in addition to the basis of pixelating the established commodities of any of ancient Egypt’s built environment. It could simplify the material at hand, to a large degree, and predetermine the architectural patterns to be overcast on the understanding. Moreover, it will help to dissipate the vagueness that has veiled the overall aspects of the pyramids, which were always intended to be rediscovered. While we had no idea of their structure, interior space, or outlines at the beginning of our quest, eventually, with the help of the abovementioned innovative methodologies, we were able to determine, to a large extent, the main framework and map for our journey.

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