

## **Augmented Feedforward and Feedback Control Scheme for Input Tracking and Vibration Control**

*M. S. Alam<sup>1</sup>, F. M. Aldebrez<sup>2</sup> and M. O. Tokhi<sup>3</sup>*

*<sup>1</sup>Dept. of Applied Physics, Electronics and Communication Engineering, The University of Dhaka, Bangladesh, <sup>2,3</sup>Dept. of Automatic Control & Systems Engineering, University of Sheffield, UK*

### **Abstract**

*This paper presents an investigation into the development of an augmented control scheme for input tracking and vibration suppression in the vertical movement of a twin rotor multi-input multi-output system (TRMS). A parametric model of the TRMS in hovering mode employed for design and implementation of an augmented feedforward and feedback control law for vibration suppression and setpoint tracking. A PID controller is developed for control of rigid body motion. This is then extended to incorporate feedforward control for vibration suppression of the TRMS. A 4-impulse input shaper is used as a feedforward control method to pre-process the command signal to the system, based on the identified modes of vibration. Simulation results of the response of the TRMS with the controllers are presented in time and frequency domains. The performance of the proposed control scheme is assessed in terms of input tracking and level of vibration reduction. This is accomplished by comparing the system response to that without the feedforward components. The approach has shown to result in satisfactory vibration reduction.*

### **Keywords**

Command shaping, feedback control, feedforward control, genetic algorithms, vibration suppression.

### **I. INTRODUCTION**

The vibration in a flexible system is normally induced due to fast motion. The occurrence of vibration leads to an additional settling time before the new maneuver can be initiated.

Therefore, in order to achieve a fast system response to command input signals, it is imperative that this vibration is reduced. Various approaches have been proposed to reduce vibration in flexible systems.<sup>1,2</sup> They can be broadly categorized as feedforward, feedback or a combination of both methods. This paper addresses the later approach in which an augmented feedforward and feedback control strategy is proposed for vibration suppression and command tracking performance. The former is used for vibration suppression and the latter for input tracking.

Initially, a Proportional-Integral-Derivative (PID)<sup>3</sup> controller is developed for control of rigid body motion. This is then extended to incorporate feedforward control for vibration suppression of the flexible system. The feedforward control method based on command shaping techniques is used for motion-induced vibration suppression. The command shaping technique is widely employed in control of flexible manipulators and aircraft.<sup>4,5,6,7</sup> In this investigation, an input shaper with four-impulse sequences is considered. The input shaper is designed based on the natural frequency and damping ratio of the system and used for pre-processing the input.

Singer et al.<sup>4</sup>, have proposed an input-shaping strategy, which is currently receiving considerable attention in vibration control.<sup>5,6,8</sup> The method involves convolving a desired command with a sequence of impulses known as an input shaper. The shaped command that results from the convolution is then used to drive the system. Design objectives are to determine the amplitude and time locations of the impulses, so that the shaped command reduces the detrimental effects of system flexibility. These parameters are obtained from the natural frequencies and damping ratios of the system. Using this method, a response without vibration can be achieved, but with a slight time delay approximately equal to the length of the impulse sequence. The method has been shown to be effective in reducing motion-induced vibration. With more impulses, the system becomes more robust to flexible mode parameter changes, but this will result in a longer delay in the system response. In this work, a 4-impulse input shaper and a PID compensator are combined to form an augmented feedforward and feedback control strategy for vibration and rigid body motion control of a twin rotor multi-input multi-output system (TRMS).<sup>9</sup>

## II. EXPERIMENTAL SET-UP

The TRMS,<sup>9</sup> shown in Figure 1, is a laboratory set-up designed for control experiments.<sup>1,6,10</sup> In certain aspects it behaves like a helicopter. The TRMS rig consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical directions producing yaw and pitch movements, respectively. At both ends of the beam there are two rotors driven by two d.c. motors. The main rotor produces a lifting force allowing the beam to rise vertically (*Pitch angle/movement*), while, the tail rotor (smaller than the main rotor) is used to make the beam turn left or right (*Yaw angle/movement*).

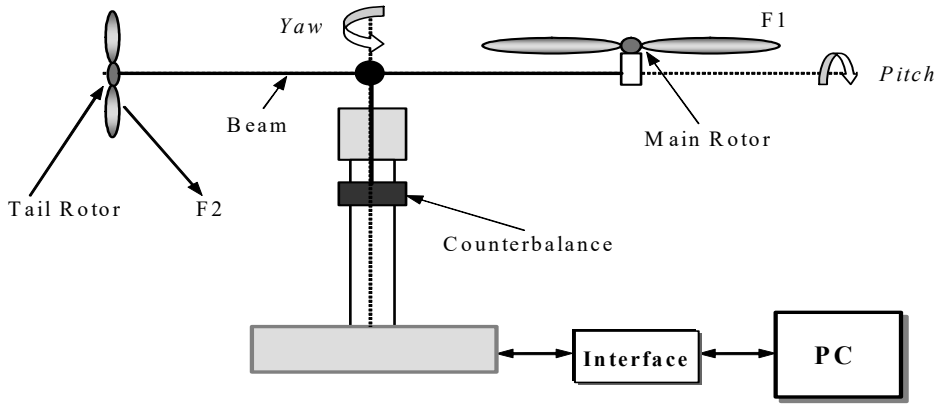


Figure 1: Schematic diagram of the TRMS

Although the TRMS permits multi-input multi-output experiments, this paper addresses the problem of modelling and control of the system in a single-input single-output mode in the longitudinal axis (i.e. vertical movement). The horizontal movement caused by the tail rotor is physically locked and as a result there is no cross-coupling effect between the two channels of the TRMS. The problem of MIMO modelling and control is interesting, and will be looked at in future studies. A 4<sup>th</sup> order transfer function characterizing the vertical movement of the TRMS utilised in this work which is given as<sup>11</sup>:

$$\frac{y(s)}{u(s)} = \frac{-0.08927s^3 + 2.249s^2 - 45.57s + 595.1}{s^4 + 3.469s^3 + 519.6s^2 + 35.95s + 2189} \quad (1)$$

where  $u(s)$  represents the main rotor input (volt) and  $y(s)$  represents pitch angle (radians). The main resonance frequency of the system occurs at 0.3516Hz.<sup>11</sup>

### III. FEEDFORWARD VIBRATION CONTROL

Feedforward methods have been considered in vibration control for flexible systems where the control signal is developed by considering the physical and vibrational properties of the flexible system.<sup>12</sup> In this work, input-shaper with a sequence of four impulses is introduced as a feedforward technique for vibration control in the vertical movement of the TRMS.

The input shaping method involves convolving a desired command with a sequence of impulses.<sup>4,8</sup> The design objectives are to determine the amplitude and time location of the impulses. A vibratory system can be modeled as a superposition of second order systems each with a transfer function:<sup>4,8</sup>

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (2)$$

where,  $\omega_n$  is the natural frequency and  $\zeta$  is the damping ratio of the system. Thus, the impulse response of the system at time  $t$  is:

$$y(t) = \frac{A\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n(t-t_0)} \sin\left[\omega_n\sqrt{1-\zeta^2}(t-t_0)\right] \quad (3)$$

where,  $A$  and  $t_0$  are the amplitude and time-location of the impulse, respectively. Furthermore, the response to a sequence of impulses can be obtained using the superposition principle. Thus, for  $N$  impulses, with  $\omega_d = \omega_n\sqrt{1-\zeta^2}$ , the impulse response can be expressed as:

$$y(t) = M \sin(\omega_d t + \alpha) \quad (4)$$

where,  $M = \sqrt{\left(\sum_{i=1}^N B_i \cos \phi_i\right)^2 + \left(\sum_{i=1}^N B_i \sin \phi_i\right)^2}$ ;  $B_i = \frac{A_i \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n (t_N - t_i)}$ ;

$\phi_i = \omega_d t_i$ ;  $\alpha = \tan^{-1}\left(\frac{\sum_{i=1}^N B_i \cos \phi_i}{\sum_{i=1}^N B_i \sin \phi_i}\right)$ ; and  $A_i$  and  $t_i$  are the magnitudes and time-location

of the impulses, respectively. The residual single-mode vibration amplitude of the impulse response is obtained at the time of the last impulse,  $t_N$ , as:

$$V = \sqrt{V_1^2 + V_2^2} \quad (5)$$

where,  $V_1 = \sum_{i=1}^N \frac{A_i \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n (t_N - t_i)} \cos(\omega_d t_i)$ ;  $V_2 = \sum_{i=1}^N \frac{A_i \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n (t_N - t_i)} \sin(\omega_d t_i)$

To achieve zero vibration after the last impulse, it is required that both  $V_1$  and  $V_2$  in equation (5) are independently zero. Furthermore, to ensure that the shaped command input produces the same rigid-body motion as the unshaped command, it is required that the sum of amplitudes of the impulses is unity. To avoid response delay, the first impulse is selected at time  $t_1=0$ . Hence, setting  $V_1$  and  $V_2$  in equation (5) to zero,  $\sum_{i=1}^N A_i = 1$  and solving for the second derivative of the vibration in equation (5), will produce a four-impulse sequence with a set parameters given by:<sup>8</sup>

$$\begin{aligned} t_1 = 0; \quad t_2 = \frac{\pi}{\omega_d}; \quad t_3 = \frac{2\pi}{\omega_d}; \quad t_4 = \frac{3\pi}{\omega_d}; \quad A_1 = \frac{1}{1+3K+3K^2+K^3}; \\ A_2 = \frac{3K}{1+3K+3K^2+K^3}; \quad A_3 = \frac{3K^2}{1+3K+3K^2+K^3}; \quad A_4 = \frac{K^3}{1+3K+3K^2+K^3} \end{aligned} \quad (6)$$

where,  $K = e^{-\zeta \pi / \sqrt{1-\zeta^2}}$

To handle higher vibration modes, an impulse sequence for each vibration mode can be designed independently. Then the impulse sequences can be convoluted together to form a sequence of impulses that attenuate vibration at higher modes. In this manner, for a vibratory system, the vibration reduction can be accomplished by convolving a desired

system input with the impulse sequence. This yields a shaped input that drives the system to a desired location without vibration.

The 4-impulse input shaper was designed on the basis of vibration frequency and damping ratio of the main rotor system. A damping ratio of 0.04146 was analytically obtained from the extracted transfer function of the system, shown in equation (1). This corresponds to the main resonance frequency, which is 0.3516Hz.<sup>11</sup> The designed input shaper was used for pre-processing the input signal, depicted in Figure 3, applied to the system in an open-loop configuration. A single-switch bang-bang input signal, referred to as unshaped input, used in this work is shown in Figure 2. This signal, which has amplitude  $\pm 0.2$  volt, is used as an input to the system. The magnitudes and time locations of the impulses were obtained by solving equation (6). Table 1 shows the amplitudes of the four impulses and their corresponding time locations. For discrete implementation of the input-shaper, locations of the impulses were selected at the nearest sample time-step.

Table 1. Amplitudes and time locations of the input shaper

Amplitudes (volt)		Time location (sec)	
a <sub>1</sub>	0.1510	t <sub>1</sub>	0
a <sub>2</sub>	0.3977	t <sub>2</sub>	0.7
a <sub>3</sub>	0.3491	t <sub>3</sub>	1.4
a <sub>4</sub>	0.1022	t <sub>4</sub>	2.1

The time-domain unshaped and shaped input signals are depicted in Figure 2 and the corresponding system response is shown in Figure 3. With the four-impulse sequence, the oscillations in the system response (pitch angle) were found to be significantly reduced. These can be observed by comparing the system response to the unshaped input. It can also be noticed from the power spectral density (PSD) plot shown in Figure 4 that the magnitude of vibration of the system was attenuated by 7.16dB.

#### IV. AUGMENTED FEEDBACK AND FEEDFORWARD CONTROL SCHEME

An augmented control structure for rigid body motion control and vibration suppression in the vertical movement is proposed. A block diagram of the proposed control structure

is shown in Figure 5. The feedforward control law is developed utilizing command shaping technique.

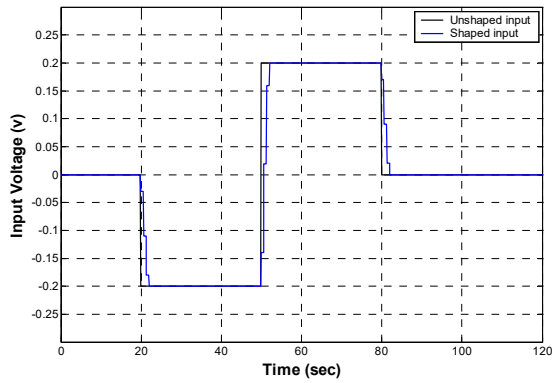


Figure 2: Shaped and Unshaped bang-bang signals

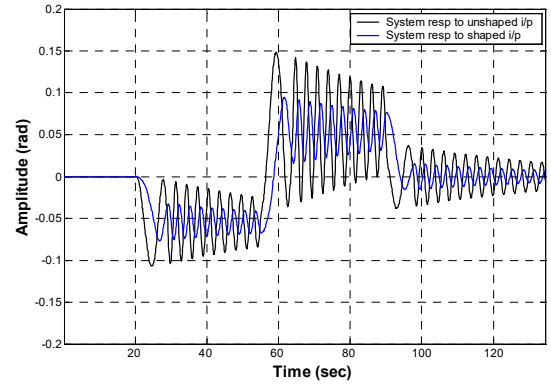


Figure 3: TRMS response to unshaped and shaped input with 4-impulse input-shaper

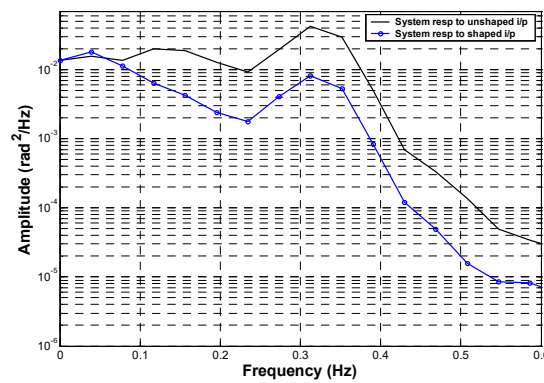


Figure 4: Power spectral density (at the dominant mode) of the TRMS response to unshaped and shaped input with 4-impulse input-shaper

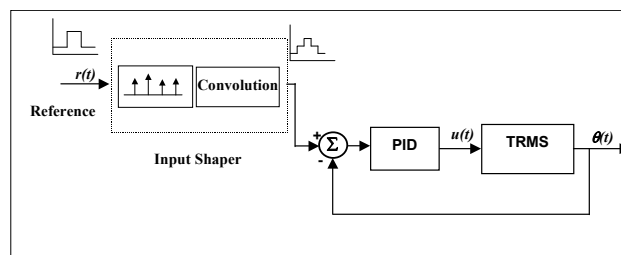


Figure 5: Augmented feedforward and feedback control

A four-impulse sequence input shaper was designed and formulated based on the natural frequency and damping ratio of the system. The natural frequency of the TRMS was

0.3516 Hz and the damping ratio at the main mode of the TRMS was deduced as 0.0414. The feedback control law is designed using PID compensator.

The parameters of the PID controller<sup>3</sup> were tuned using Ziegler-Nichols method.<sup>13</sup> The step response of the system with several sets of PID parameters is shown in Figure 6. To measure the potential effect of the feedforward components in reducing the oscillation from the system response, the worst case PID parameters, giving a high oscillatory response, was chosen. The values of  $K_p$ ,  $K_i$  and  $K_d$  were deduced as 10, 6 and 2 respectively.

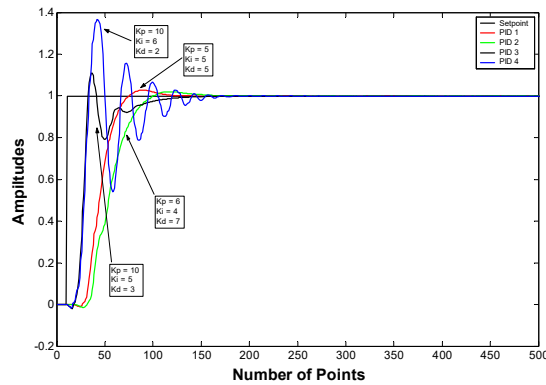


Figure 6: System step response with PID control

## V. IMPLEMENTATION AND RESULTS

The proposed control scheme was implemented and tested within the simulation environment of the TRMS and the corresponding results are presented in Figure 7. The performance of the augmented controller is assessed in terms of input tracking and vibration suppression in comparison to a PID controller. The TRMS is required to follow a trajectory path represented by a square wave input signal.

As noted, the performance of the system with the PID compensator is characterized by an underdamped response with a considerable overshoot (36.8%), rise time (1.55 sec), and a settling time (13 sec). When the input shaper was added, a significant improvement to the system response was achieved. It can be seen that the attenuation in the level of vibration was significant, where the system response has no overshoot with the augmented control structure. It has also performed better than the PID in terms of settling time (12 sec).



However, this is at the cost of a greater rise time (8.4 sec). This is attributed to the length of the impulse sequence (4 impulses in this case).

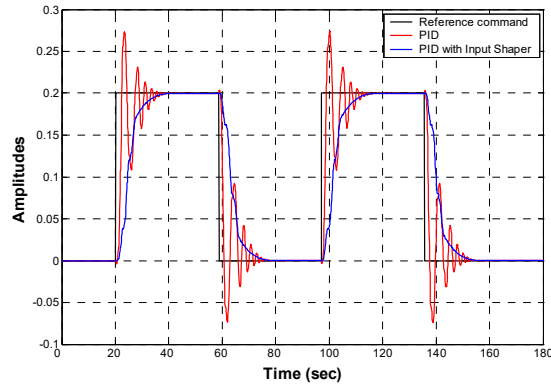


Figure 7: The system performance

## VI. CONCLUSION

This paper has investigated the development of an augmented feedforward and feedback control strategy for vibration suppression and rigid body motion control. The performance of the feedback control has been improved by adding the feedforward control component, which was utilized for preprocessing the reference command signal so that the system vibration (oscillations) are reduced. This is important for fast maneuvering platforms, where the command signals change rapidly. The advantage of the augmented feedforward and feedback control structure is that it does not change the feedback control law in order to attenuate the system vibration. Thus, an appropriately designed feedforward and feedback control scheme is a practical approach to satisfy the design specifications.

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