The Dhaka University Journal of Science, (2008), 57 (2), pp. 151-156

System identification of a twin rotor multi-input multi-output system using adaptive filters with pseudo random binary input

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Abstract

This paper presents an investigation into the development of a parametric model of pitch movement of a twin rotor multi-input multi-output system (TRMS) using adaptive finite impulse response (FIR) models. The TRMS is a laboratory platform designed for control experiments. In certain aspects, its behaviour resembles that of a helicopter. It typifies a high-order nonlinear system with significant cross coupling between its two channels. The system is initially excited with PRBS signals of different bandwidths to ensure that all resonance modes are captured. The PRBS magnitude is selected so that it does not drive the system out of its linear operating range. Then, an adaptive FIR filter structure with LMS, NLMS, and genetic algorithm (GA) with LMS algorithms is investigated to identify the system and extract its parametric model. Effects of filter taps, step-size and system convergence are also studied. Performances of the employed techniques are assessed and presented in time and frequency domains.

1. INTRODUCTION

Adaptive filters are digital filters capable of self-adjustment. They can change in accordance with their input signals. They have been used in a number of applications, including noise cancellation, system identification, and adaptive control [1, 5].

In system identification using adaptive filtering techniques, the unknown system is modelled by an adaptive filter with adjustable coefficients. Both the unknown system and adaptive filter model are excited by an input sequence $x(n)$, as shown in Figure 1. At each time interval, an input signal sample $x(n)$ is processed by the time-varying filter generating a predicted output $y(n)$. The output is compared with the desired output $d(n)$ to produce an error signal $e(n)$. The error signal is then used as input to an adaptive control algorithm, which modifies tap weights of the filter. This process is repeated through several iterations until the error signal *e(n)* becomes sufficiently small. The objective is to minimize the cost function, mean-square-error $\xi=E[e^2(n)]$, where *e(n)* is defined as: $e(n)=d(n)-y(n)$.

Genetic algorithm (GA) is one of the global stochastic search algorithms based on natural biological evolution [6, 8]. Since their introduction by Holland [8], there has been growing interest among scientists and engineers in the use of GAs in identification and control applications [9, 10]. Unlike steepest descent and recursive estimation approaches to nonlinear parameter identification, GA requires no calculation of the gradient and is not susceptible to local minimum problems that arise with multimodal error surfaces.
In this study, a finite impulse response (FIR) transversal filter using least mean square (LMS), normalized least mean square (NLMS) and a new algorithm, GA with LMS (GA+LMS) is investigated for a twin rotor multi-input multi-output system (TRMS) in hovering mode.

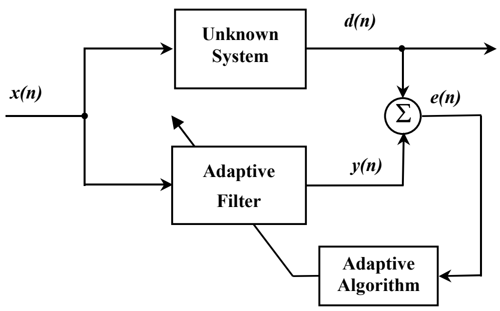


Figure 1: System identification with adaptive filter.

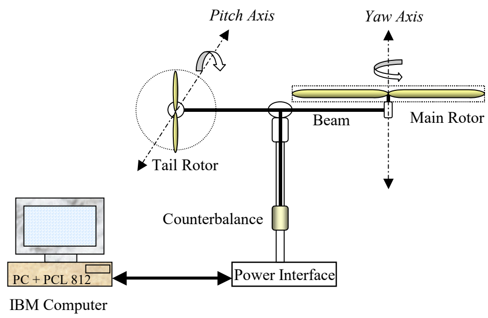


Figure 2: Schematic diagram of the TRMS.

2. EXPERIMENTAL SET-UP

The TRMS, shown in Figure 2, is a laboratory set-up designed for control experiments [3]. In certain aspects it behaves like a helicopter. The TRMS rig consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical directions producing yaw and pitch movements, respectively. At both ends of the beam there are two rotors driven by two d.c. motors. The main rotor produces a lifting force allowing the beam to rise vertically making a rotation around the pitch axis (pitch angle). While, the tail rotor is used to make the beam turn left or right around the yaw axis (yaw angle).

The TRMS is constructed so that the angle of attack of the blades is fixed and the aerodynamic force is controlled by varying the speed of the motors. Therefore, the control inputs are supply voltages of the d.c. motors. A change in the voltage value results in a change in the rotational speed of the propeller, which results in a change in the corresponding position of the beam [3].

The hovering property of the TRMS is the main area of interest in this work. Station keeping or hovering is vital for a variety of flight missions such as load delivery, air-sea rescue etc. Although the TRMS rig reference point is fixed, it still resembles a helicopter, by being highly nonlinear with strongly coupled modes. Such a plant is thus a good benchmark problem to test and explore modern identification and control methodologies.

3. TRANSVERSAL FIR ADAPTIVE FILTERS

In a transversal FIR filter of length *M*, the output *y(n)* is computed by a weighted sum of the current and delayed input samples [7]:

$$y(n) = \sum_{m=0}^{M-1} b_m(n)x(n-m) \tag{1}$$

where, $b_m(n)$ are the adaptive parameters, and *y(n)* and *x(n)* are the predicted output and actual input, respectively. Equation (1) can be rewritten in a vector form as:

$$y(n) = w^T(n)u(n) \tag{2}$$

where, the coefficient vector *w* and the signal vector *u* each have length of *M* and are defined as; $w(n) = [b_0(n),...,b_{M-1}(n)]^T$ and $u(n) = [x(n),...,x(n-M+1)]^T$.

4. ADAPTIVE LEARNING ALGORITHMS

The task of an adaptive algorithm is to find the optimal parameters of the model that minimize the cost function. The performance of the algorithm can be measured by a number of factors such as: accuracy of the obtained solution with respect to the theoretical value, convergence speed, tracking ability, computational complexity and robustness.

4.1 Least mean square algorithm

The LMS algorithm is an iterative gradient algorithm that can be used to adapt the coefficients of an adaptive FIR filter (Figure 1) such that the error *e(n)* is minimized in the mean square sense. The LMS update equation is given as [4, 7]:

$$y(n) = w^T(n-1)u(n) \tag{3}$$

$$e(n) = d(n) - y(n) \tag{4}$$

$$w(n+1) = w(n) + 2\mu\, e^*(n)\, u(n) \tag{5}$$

where, $e^*(n)$ is the complex conjugate of error signal and μ indicates the step size for the gradient descent method.

4.2 Normalised least mean square algorithm

In the LMS algorithm, the correction $[\mu\ u(n)\ e^*(n)]$ applied to the tap-weight vector $w(n)$ at time $n+1$ is normalized with respect to the squared Euclidean norm of the tap-input vector $u(n)$ at time n. The update expression of the NLMS algorithm can be defined as [5, 7]:

$$w(n+1) = w(n) + \frac{\mu}{a + \|u(n)\|^2} u(n) e^*(n) \tag{6}$$

For convergence it is required that: $0 < \mu < 2$, and a is a small positive number.

4.3 Genetic algorithm with LMS (GA+LMS)

Genetic algorithms constitute global and data independent search techniques. They operate on a population of potential solutions by applying the natural evolutionary process (i.e. principles of survival of the fittest) to produce better and better approximation to a solution and as such it is flexible and parallel in nature [2]. The algorithm begins with a collection of parameter estimates, called a chromosome. Each chromosome is evaluated for its fitness in the problem domain. At each generation (algorithm time-step) the most-fit chromosomes are allowed to mate and bear offspring. The new parameter estimates (offspring), then, form the basis for the next generation. GA operators such as selection, crossover and recombination are then re-employed to process the next generation [6]. This process is repeated several times to satisfy some criteria. The mutation feature is often introduced to guard against the local minimum.

The problem of local minima in the gradient-based algorithm like LMS, is very common which leads to biased estimation. In order to overcome the effects of local minima and to improve global searching capability, GA is combined with the LMS estimation. The working principle of GA+LMS algorithm is illustrated in Figure 3. The process starts with data segmentation; dividing the experimental input-output data into n segments of equal length and overlapping each other as shown in Figure 3. Here the number of data segments n is chosen equal to the number of individuals to be used in the

GA optimisation process. The length of the each data segment $D_1, D_2, ...D_n$ is set to N. It is noted that, for consecutive data segments the latter one i.e., D_2 lags the previous one i.e., D_1 by d number of samples. The number of weight for the FIR transversal structure is set at m and conventional LMS algorithm is applied on each set of data set separately and corresponding filter weights are stored in a matrix W at the end of iterations.

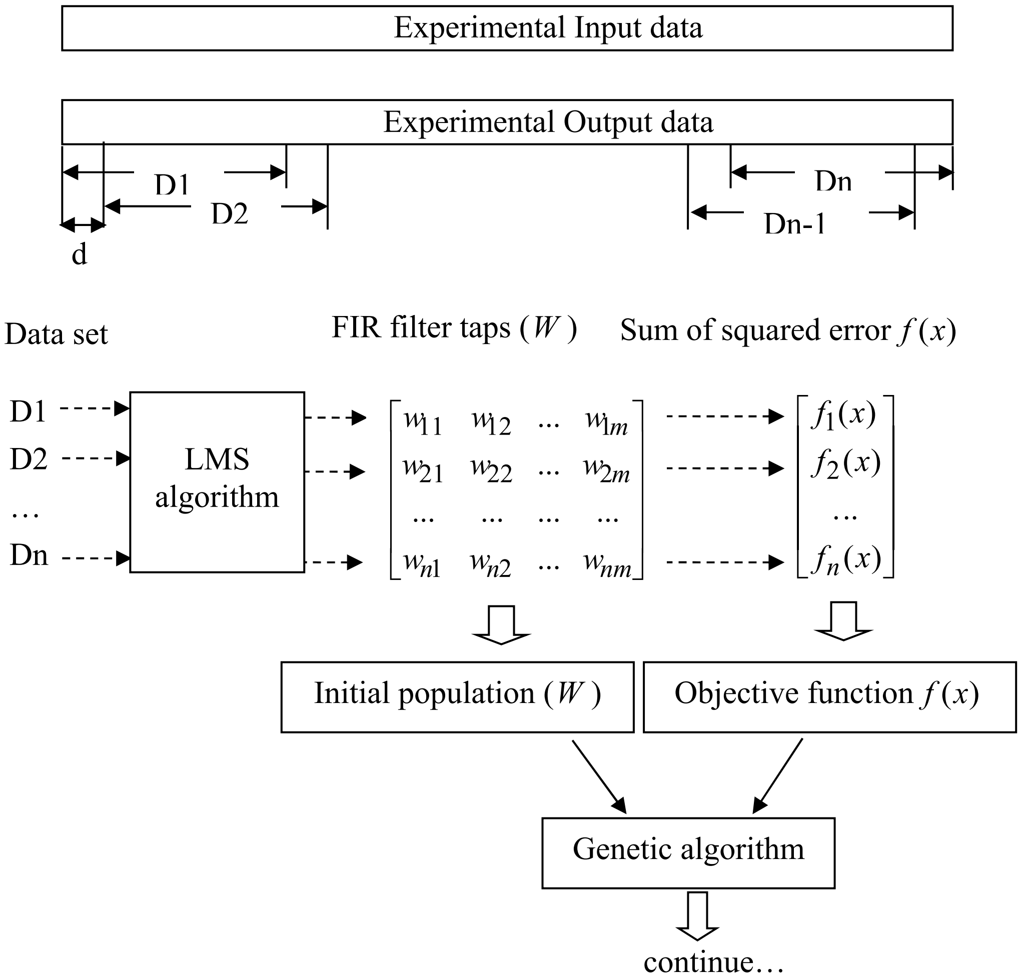


Figure 3: Working principle of GA+LMS algorithm

Similarly, sum of squared error for each data set is stored in a column vector $f(x)$, as shown in Figure 3. Taking matrix W as the initial population and vector $f(x)$ as their corresponding objective functions, GA is applied with any data segment for a predefined number of generations to obtain suitable values of weight values that minimise the objective function further.

5. RESULTS AND DISCUSSION

In this study, three FIR adaptive filters were employed in modelling the TRMS in hovering mode. In each case a different adaptive algorithm based on LMS, NLMS and GA+LMS techniques was used to estimate the parameters of the filter.

The system was excited with a pseudo random binary sequence (PRBS) of bandwidth (0-10 Hz) in order to ensure that all system resonance modes are captured. The PRBS signal level of ±0.2 volts, was selected so that it does not drive the TRMS out of its linear operating range. The input PRBS signal and its corresponding output response of vertical channel of the TRMS is shown in Figure 4. The system is modeled from the input volt to vertical angle/movement with 4000 data samples. The performances of the three algorithms are evaluated in terms of output tracking, resonance mode detection, and minimization of cost function.

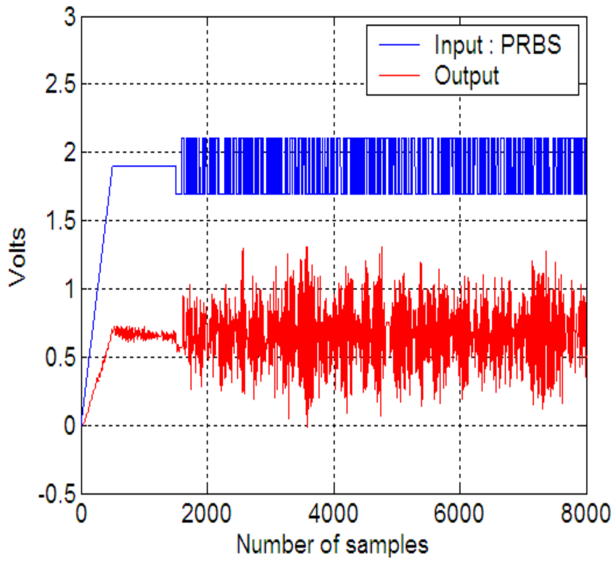


Figure 4: Experimental input-output data of vertical channel of TRMS

5.1 System modelling with LMS algorithm

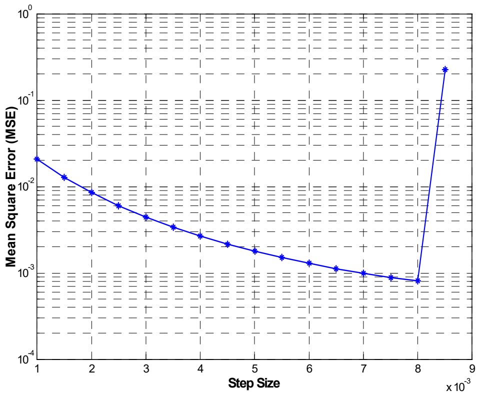
The principal factors that influence the LMS algorithm are: the step-size parameter, μ, the number of taps and the eigenvalues of the correlation matrix of tap-input vector.

5.1.1 Selecting the filter tap

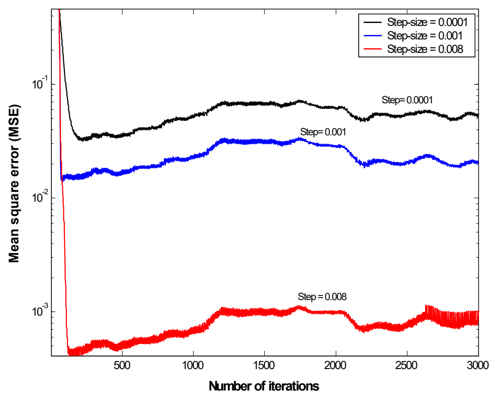
In this study, efforts were made to find the optimum filter order for modelling the system on trial and error basis. Since the transversal structure has only zeros it needs fairly high number of taps to model a practical system, which has complex nonlinear characteristics. It was observed that with a 60th order FIR filter, the system could be modelled with a satisfactory convergence and stability levels, and with further higher model order, the performance degraded. This is because the larger filter order, the larger the eignvalue spread of the correlation matrix of input tap-vector, which in turn decreases the rate of convergence of the LMS to optimal solution. Moreover, the higher number of filter taps adds huge computation, which may not meet real time requirements of the application.

5.1.2 Choice of step-size

The step size parameter μ controls the convergence of the algorithm. If μ is small the adaptation is slow. On the other hand, when μ is large, the adaptation is relatively fast, but at the expense of an increase in the average excess mean squared error after adaptation. For stable adaptation behaviour, the step-size has to be: $0 < \mu < 2/\lambda_{max}$, where, λ_{max} is the maximum eigenvalue of the tap-input correlation matrix. For the 60th order FIR filter, the maximum value of μ according to the above relation is 0.009 for PRBS input. To select the optimum step-size, the experimentation was repeated many times with different step-sizes by monitoring the mean square error (MSE) and the spectrum. The optimum step-size for 60 taps was recorded at 0.008. Figures 5a and 5b demonstrate the effect of step-size on MSE and convergence of LMS algorithm, respectively. It is noted that as the step-size increases, the MSE decreases. This is applicable until a certain value (0.008). After that value, if the step-size is increased, the MSE increases to a very high value.



a) Effect of step size on mean squared error



b) Effect of step size on LMS convergence

Figure 5: Effect of step size on mean-square error and convergence

Genetic algorithm was also used to find the optimum step size of LMS algorithm. The objective function for optimisation was chosen as sum of squared error; $f(x) = sum|d(n) - y(n)|^2$. Multiple crossover was selected with a probability of 0.9 to update the step sizes in the subsequent generations. A high degree of precision (48 bits) was chosen for the step size. An optimum step size was recorded as 0.0081711, for filter taps 60, only after 20 generations. This is almost the same with the value obtained through trial and error.

5.2 System modelling with NLMS algorithm

The normalized LMS algorithm is convergent in the mean-square sense if the adaptation constant μ satisfies the condition: $0 < \mu < 2$. For the 60th order FIR transversal filter using NLMS algorithm, the optimum step-size was found to be 1.75 at which the MSE is minimum.

5.3 System modelling using GA + LMS algorithm

The system was modelled from the input volt to vertical angle/movement. The GA+LMS was designed (see Figure 3) with $N = 4000, n = 30, m = 60, d = 10$, i.e., number of samples in each data segment = 4000, number of individual in initial population = 30, number of FIR filter weights = 60 and lag between consecutive data segments = 10. Satisfactory results were achieved with the following set of parameters: generation gap: 0.8; crossover rate: 0.9; precision: 20; mutation rate: 0.0001 and the maximum number of generations: 100. The convergence curves of GA+LMS and conventional GA in the same problem are shown in Figure 6. It is important to note that the proposed GA+LMS converge to a much lower value of objective function compared to conventional GA in the same number of generation. Moreover it seems that objective function is gradually decreasing in case of GA+LMS while for GA, the objective function remains almost unchanged from generation 30 to onwards. It is clearly evident that the performance of GA+LMS algorithm is much better than conventional GA in this problem.

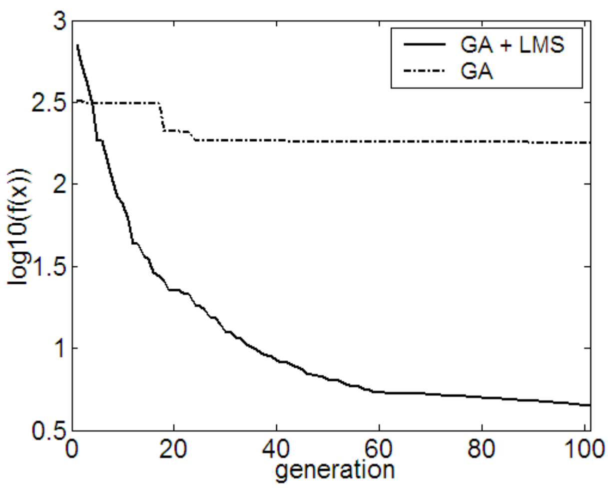
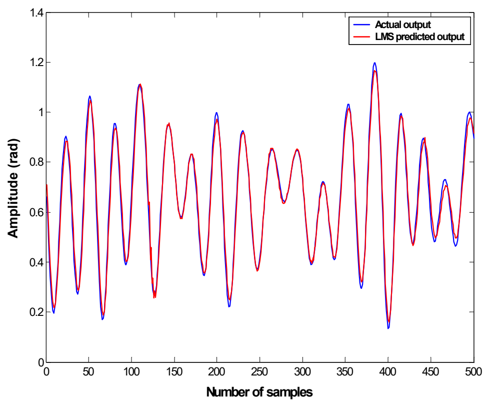


Figure 6: Convergence curves of GA and GA+LMS in system modelling

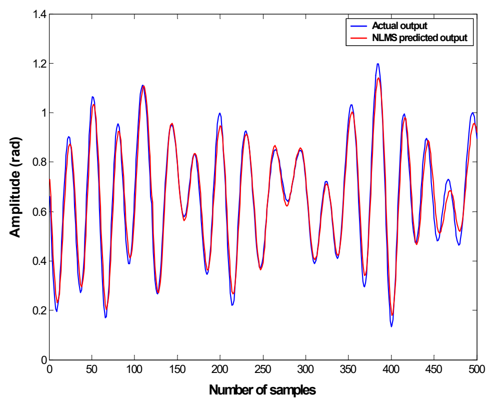
5.4 Comparative assessment

In terms of output tracking, the three algorithms have demonstrated a satisfactory level of performance, as shown in Figures 7. Among the three algorithms, the LMS algorithm has outperformed the other two algorithms, followed by NLMS, and then combined GA+LMS. Figure 4d shows the corresponding power density plot of the three algorithms. The three models have also demonstrated a satisfactory performance in detecting the main system's mode. The system's main mode was clearly detected with the three approaches as 0.3516 Hz. Since, the system has a low frequency modes, the higher frequency region is less significant in terms of system dynamics.

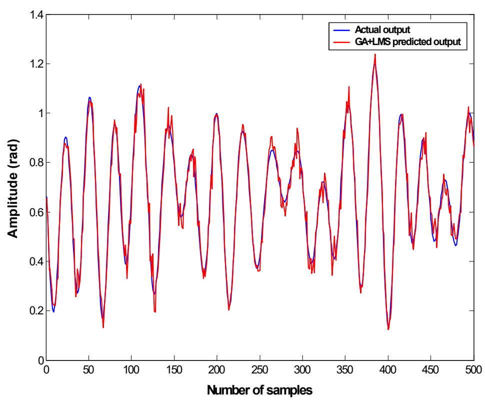
The cost function is minimized by the three algorithms. For the LMS and NLMS based models, the cost function was the MSE. With 60th model order, lowest values of the MSE of 0.0008062 and 0.0008497 were recorded for LMS and NLMS based models, respectively. For the GA+LMS, the objective function was the sum of the squared error (SSE). After 100 generations the SSE was found to be 4.477 for this case.



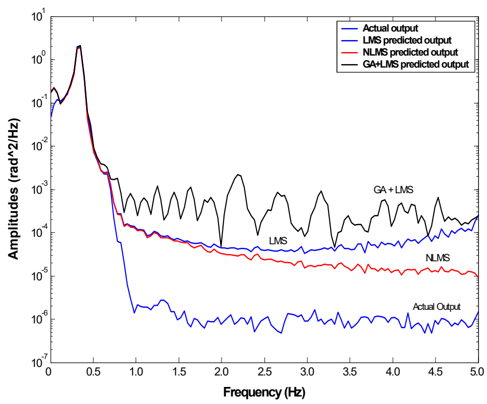
a) LMS based response (Last 500 samples)



b) NLMS based response (Last 500 samples)



c) GA+LMS based response (Last 500 samples)



d) Power spectral density plot

Figure 7: Output tracking and PSD plots for the three algorithms

6. CONCLUSION

This investigation has witnessed the development of dynamic modelling of a twin rotor multi-input multi-output system in a hovering mode. A one degree-of-freedom TRMS model, whose dynamics resemble that of a helicopter has been successfully identified using finite impulse response adaptive filtering formulation. Three adaptive algorithms based on; LMS, NLMS and GA+LMS, were utilized to update the filter's coefficients. The three employed algorithms have demonstrated satisfactory performance and were quite comparable in terms to output tracking and resonance mode detection. The extracted models will be used in subsequent investigations for the development of simulation of rigid-body motion, vibration suppression and control strategies for the twin rotor system.

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