VIBRATION CONTROL OF PITCH MOVEMENT USING COMMAND SHAPING TECHNIQUES: EXPERIMENTAL INVESTIGATION

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Abstract

This paper investigates the development of feedforward control strategies for vibration control of pitch movement (1 DOF) of a twin rotor multi-input multi-output system (TRMS) using command shaping techniques. Command shaping is a feedforward method used to reduce residual vibrations during motion in flexible systems. The TRMS is a laboratory platform designed for control experiments. In certain aspects, its behaviour resembles that of a helicopter. Feedforward controllers are designed for resonance suppression produced by the main rotor, which produces pitch movement around the longitudinal axis, while the lateral axis (yaw movement) is physically constrained. Three feedforward controllers: input-shaper, low-pass filter and band-stop filter are designed based on the natural frequencies and damping ratios of the system. The three controllers are assessed in terms of level of vibration reduction at the system's natural frequencies. Their performances are compared with an unshaped input (single-switch bang-bang signal) that is used to determine the dynamic response of the system.

1   Introduction

The residual motion (vibration) in a flexible system is normally fast motion induced. The occurrence of any vibration after the commanded position has been reached will require additional settling time before the new maneuver can be initiated [1]. Therefore, in order to achieve a fast system response to command input signals, it is imperative that this vibration is reduced. This feature is desirable in fast maneuvering systems, such as fighter aircraft.

Various approaches have been proposed to reduce vibration in flexible systems. They can be broadly categorized as feedforward, feedback or a combination of both methods. For very rapid motions of a flexible system, a satisfactory feedback controller may prove difficult to design. Therefore, augmenting the feedback control system with a feedforward method (Figure 1) can facilitate the controller design, or enable the attainment of better performance [11]. Command shaping is one of the several methods used for feedforward motion-induced vibration control. The input command shaping technique is widely employed in flexible aircraft [8] and helicopter control [7]. In this methodology, the desired command signal is modified/shaped so that it does not contain spectral components at the system's resonance frequencies. A significant amount of work on shaped command input based on filtering techniques has been reported. These include low-pass filters, band-stop filters and notch filters [2, 9, 13-15].
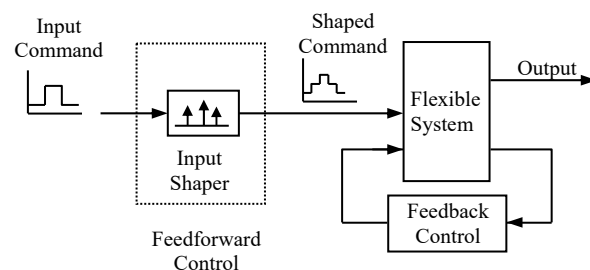

Figure 1. Combined feedforward/feedback control of a flexible system

Singer et al. [12], have proposed an input-shaping strategy, which is currently receiving considerable attention in vibration control [11-13]. The popularity of input shaping is also partly due to its simplicity and its ability to be used with arbitrary actuator commands in real time. Further, since input shaper, shown in Figure 1, resides outside the feedback loop, it is compatible with closed loop vibration reduction scheme. The method involves convolving a desired command with a sequence of impulses known as an input shaper. The shaped command that results from the convolution is then used to drive the system. Design objectives are to determine the amplitude and time locations of the impulses, so that the shaped command reduces the detrimental effects of system flexibility. These parameters are obtained from the natural frequencies and damping ratios of the system. Using this method, a response without vibration can be achieved, but

with a slight time delay approximately equal to the length of the impulse sequence. The method has been shown to be the most effective in reducing motion-induced vibrations [10]. With more impulses, the system becomes more robust to flexible mode parameter changes, but this will result in a longer delay in the system response.

In this work, input shaping with four-impulse sequences and third-order low-pass and band-stop filters is considered. Experimental study is performed on the laboratory-scale twin rotor multi-input multi-output system (TRMS). Initially, to obtain the characteristic parameters of the system, the TRMS is excited with a single-switch bang-bang input signal. Then, an input shaper and low-pass and band-stop digital filters are designed based on the properties of the TRMS and used for pre-processing the desired input so that no energy is fed into the system at the natural frequencies. Performances of the developed controllers are assessed in terms of level of vibration reduction at the natural frequencies. This is accomplished by comparing the system response to that with the unshaped bang-bang input.

2   Experimental Set-Up

The TRMS, shown in Figure 2, is a laboratory set-up designed for control experiments [5]. In certain aspects it behaves like a helicopter. The TRMS rig consists of a beam pivoted on its base in such a way that it can rotate freely both in the horizontal and vertical directions producing yaw and pitch movements, respectively. At both ends of the beam there are two rotors driven by two d.c. motors. The main rotor produces a lifting force allowing the beam to rise vertically making a rotation around the pitch axis (*pitch angle/movement*). While, the tail rotor (smaller than the main rotor) is used to make the beam turn left or right around the yaw axis (y*aw angle/movement*).
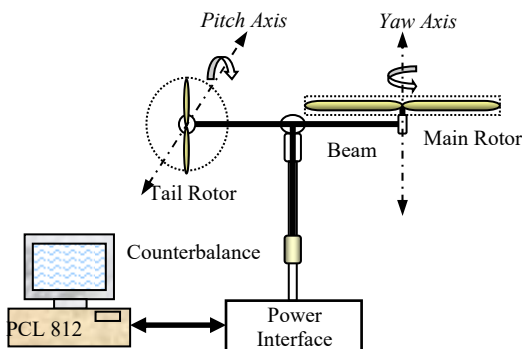

Figure 2. The schematic diagram of the TRMS

In a typical helicopter, the aerodynamic force is controlled by changing the angle of attack of the blades. The laboratory set-up is constructed so that the angle of attack of the blades is fixed. The aerodynamic force is controlled by varying the speed of the motors. Therefore, the control inputs are supply voltages of the d.c. motors. A change in the voltage value results in a change in the rotational speed of the propeller, which results in a change in the corresponding position of the beam [5].

The non-uniform mass distribution due to the rotors and the rotor torque are the main causes of beam deflection in the TRMS at normal operating conditions. The TRMS has an infinite number of normal modes with associated frequencies.

3   Feedforward Vibration Control

Control of flexible structures requires the design of a stable feedback control system. In addition to stability, certain performance measures are integrated into the design. For rapid motion of a flexible system, a satisfactory controller may prove difficult to design. Augmenting the feedback control system with a feedforward method, therefore, can facilitate the controller design or enable the attainment of better performance. Feedforward control methods have been considered in vibration control where the control input is developed by considering the physical and vibrational properties of the flexible system [3]. The main objective of this work is to develop feedforward control method using command shaping techniques to reduce motion and uneven mass induced vibrations in the TRMS during its operation. It is assumed that the motion and the rotor load are the main sources of system vibration. Thus, input profiles, which do not contain energy at system natural frequencies do not excite structural vibration and hence require no additional settling time [2].

In the following sub-sections input-shaping and low-pass and band-stop filtering techniques are introduced as feedforward algorithms used in this work for open-loop vibration control of pitch movement of the TRMS.

3.1   Input Shaping

The input shaping method involves convolving a desired command with a sequence of impulses [9,12]. The design objectives are to determine the amplitude and time location of the impulses. A vibratory system can be modeled as a superposition of second order systems each with a transfer function:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad (1)$$

where, ω_n is the natural frequency and ζ is the damping ratio of the system. Thus, the impulse response of the system at time t is:

$$y(t) = \frac{A\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n(t-t_0)} \sin\left[\omega_n\sqrt{1-\zeta^2}(t-t_0)\right] \qquad (2)$$

where, A and t_0 are the amplitude and time of the impulse, respectively. Furthermore, the response to a sequence of impulses can be obtained using the superposition principle. Thus, for N impulses, with $\omega_d = \omega_n\sqrt{1-\zeta^2}$, the impulse response can be expressed as:

$$y(t) = M\sin(w_d t + \alpha) \qquad (3)$$

where,

$$M = \sqrt{\left(\sum_{i=1}^{N} B_i \cos\phi_i\right)^2 + \left(\sum_{i=1}^{N} B_i \sin\theta_i\right)^2}$$

$$B_i = \frac{A\,\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n(t_N-t_i)}$$

$$\phi_i = \omega_d t_i$$

$$\alpha = \tan^{-1}\left(\sum_{i=1}^{N} \frac{B_i \cos\phi_i}{B_i \sin\phi_i}\right)$$

and A_i and t_i are the magnitudes and times at which the impulses occur.

The residual single-mode vibration amplitude of the impulse response is obtained at the time of the last impulse, t_N, as:

$$V = \sqrt{V_1^2 + V_2^2} \qquad (4)$$

where,

$$V_1 = \sum_{i=1}^{N} \frac{A_i \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n(t_N-t_i)} \cos(\omega_d t_i)$$

$$V_2 = \sum_{i=1}^{N} \frac{A_i \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n(t_N-t_i)} \sin(\omega_d t_i)$$

To achieve zero vibration after the last impulse, it is required that both V_1 and V_2 are independently zero. Furthermore, to ensure that the shaped command input produces the same rigid-body motion as the unshaped command, it is required that the sum of amplitudes of the impulses is unity. To avoid response delay, the first impulse is selected at time $t_1=0$. Hence, by setting V_1 and V_2 in equation (4) to zero, $\sum_{i=1}^{N} A_i = 1$ and taking the second derivatives will produce a four-impulse sequence with parameters as:

$$t_1 = 0 \qquad t_2 = \frac{\pi}{\omega_d} \qquad t_3 = \frac{2\pi}{\omega_d} \qquad t_4 = \frac{3\pi}{\omega_d}$$

$$A_1 = \frac{1}{1+3K+3K^2+K^3} \qquad A_2 = \frac{3K}{1+3K+3K^2+K^3}$$

$$A_3 = \frac{3K^2}{1+3K+3K^2+K^3} \qquad A_4 = \frac{K^3}{1+3K+3K^2+K^3} \qquad (5)$$

where,

$$K = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

To handle higher vibration modes, an impulse sequence for each vibration mode can be designed independently. Then the impulse sequences can be convoluted together to form a sequence of impulses that attenuate vibration at higher modes. In this manner, for a vibratory system, the vibration reduction can be accomplished by convolving a desired system input with the impulse sequence. This yields a shaped input that drives the system to a desired location without vibration.

3.2 Filtering Techniques

Command shaping based on input filtering is developed on the basis of extracting the energy around natural frequencies of the system using filtering techniques [9]. The filters are thus used for pre-processing the input signal so that no energy is fed into the system at the natural frequencies. In this manner, the flexural modes of the system are not excited, leading to a vibration-free motion. This can be realized by employing either low-pass or band-stop filters. In the former, the filter is designed with a cut-off frequency lower than the first natural frequency of the system. In the latter case, band-stop filters with centre frequencies at the natural frequencies

of the system are designed. This will require one filter for each mode of the system. The band-stop filters thus designed are implemented in cascade to pre-process the input signal.

Infinite impulse response (IIR) Butterworth low-pass and band-stop filters are used in this investigation. The magnitude of the frequency response of a low-pass Butterworth filter is given as [6]:

$$|H(j\omega)|^2 = \frac{1}{1+[\omega/\omega_c]^{2m}} = \frac{1}{1+\varepsilon[\omega/\omega_p]^{2m}} \qquad (6)$$

where, m is a positive integer signifying the order of the filter, ω_c is the filter cut-off frequency, ω_p is the pass-band edge frequency and $(1+\varepsilon^2)^{-1}$ is the band edge value of $|H(j\omega)|^2$. Note that $|H(j\omega)|^2$ is monotonic in both the pass-band and stop-band. The order of the filter required to yield attenuation δ_2 at a specified frequency ω_s (stop-band edge frequency) is easily determined from equation (6) as:

$$m = \frac{\log(1/\delta_2^2 - 1)}{2\log(\omega_s/\omega_p)} = \frac{\log(\delta_1/1)}{2\log(\omega_s/\omega_p)} \qquad (7)$$

where, by definition, $\delta_2 = (1+\delta_1^2)^{-0.5}$. Thus, the Butterworth filter is completely characterized by the parameters m, δ_2, ε and the ratio ω_s/ω_p. Equation (7) can be employed with arbitrary $\delta_1, \delta_2, \omega_c,$ and ω_s to yield the required filter order m from which the filter design is readily obtained. The Butterworth approximation results from the requirement that the magnitude response be maximally flat in both the pass-band and the stop-band; i.e. the first ($2m$ - 1) derivatives of $|H(j\omega)|^2$ are specified to be equal to zero at $\omega = 0$ and at $\omega = \infty$. The design relations for the low-pass filters given above can be utilized in normalized form to design the corresponding band-stop filters. This involves a transformation from the low-pass to band-stop filter [4].

4 Experimentation and Results

The feedforward control techniques were designed on the basis of vibration frequencies and damping ratios of the main rotor system. A damping ratio of 0.0414 corresponding to the main resonance frequency mode at 0.34Hz was obtained analytically. The designed input shaper and filters were used for pre-processing the bang-bang command signal applied to the system in an open-loop configuration to reduce the system vibration. To verify the performance of the control techniques, the results were examined in comparison to the unshaped bang-bang command for a similar input level in each case.

4.1 Unshaped Bang-Bang Input

A single-switch bang-bang input command signal, referred to as unshaped input, used in this work is shown in Figure 3. This signal, which has amplitude of ±0.1 volt, is used as a desired input to the system to extract its response and to design and evaluate the performance of the three feedforward controllers.
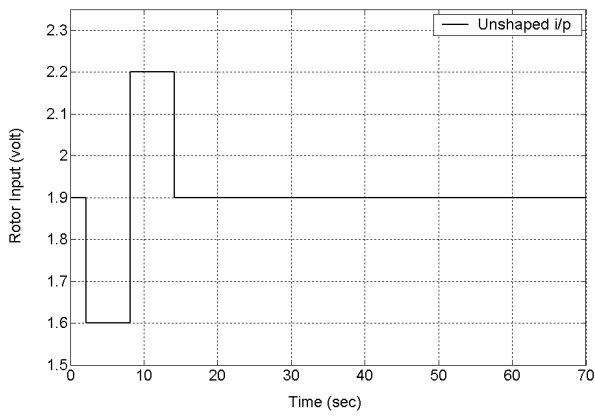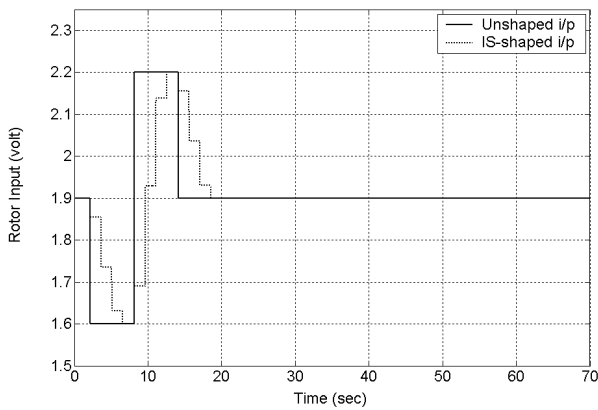
Figure 3. Bang-bang input signal

4.2 Input Shaping

Using the parameters of the system, an input-shaper with four-impulse sequence was designed and examined. The magnitude and time location of the impulses were obtained by solving equation (5). For digital implementation of the input-shaper, locations of the impulses were selected at the nearest sample time-step.
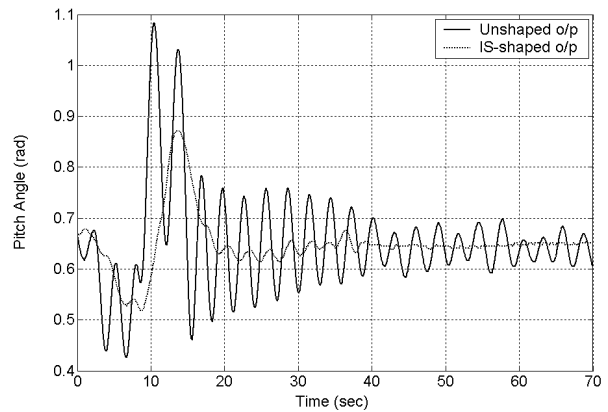

(a) Time domain
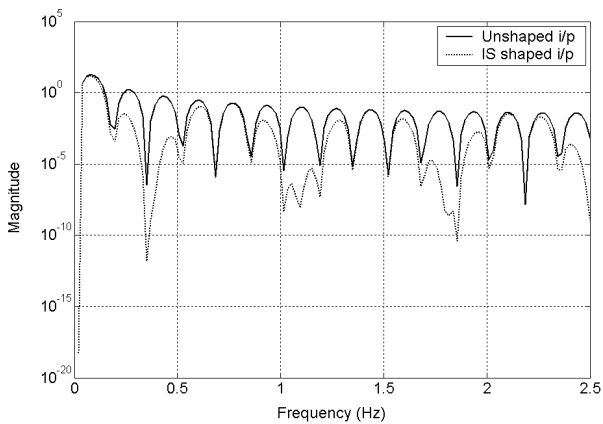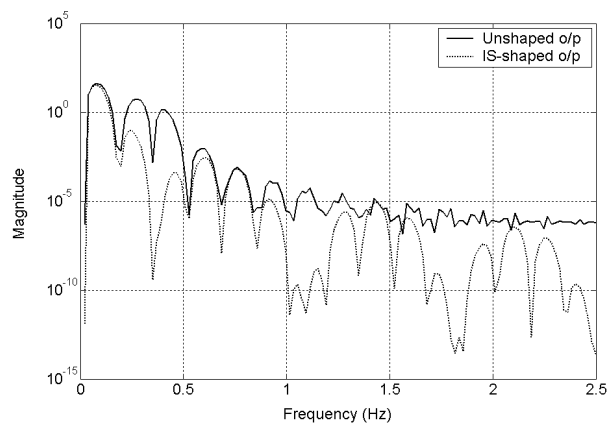

(b) Power spectral density

Figure 4. Shaped input using input-shaper

The shaped input signal and the corresponding power spectral density (PSD) are depicted in Figure 4. Figure 5 shows the response of the main rotor to the shaped input using four-impulse sequences. With the four-impulse sequence, the oscillations in the pitch angle response were found to have almost reduced to zero. These can be observed

by comparing the system response to the unshaped input. It can also be noticed from the PSD, in Figure 5(b), that the magnitude of vibration of the system has been significantly attenuated at the system's resonance modes.
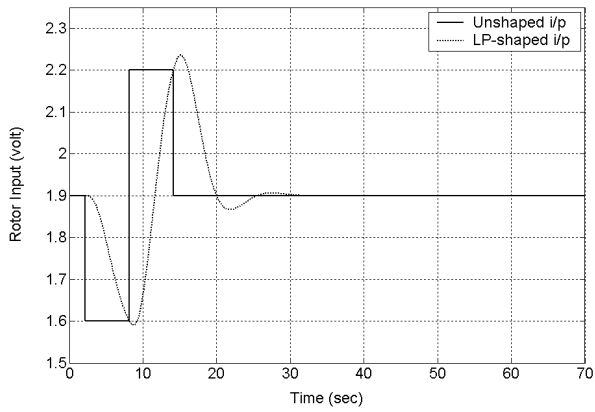

(a) Time domain
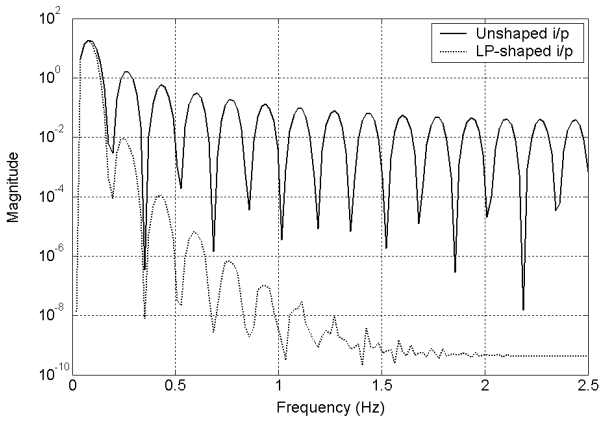

(b) Power spectral density

Figure 5. Pitch response to shaped input using input-shaper

4.3 Filtered Inputs

In this study, a third order Butterworth low-pass filter with cut-off frequency at 0.1Hz was designed to filter the bang-bang input signal. Figure 6 shows the filtered input signal using this filter and Figure 7 shows the response of the main rotor to the filtered input. It can be seen that the system vibrations at the natural frequencies have been considerably reduced in comparison to the unshaped input. Therefore, with such filter, the input energy at all frequencies above the cut-off frequency can be attenuated. Similarly, a third order digital Butterworth band-stop filter with a centre frequency at 0.3Hz and a band-stop frequency ranging between 0.2 and 0.4Hz was designed and implemented. The filtered input thus obtained is shown in Figure 8. The response of the system to the band-stop filtered input is illustrated in Figure 9. As evidenced by the magnitude of the time responses, a relatively small reduction in the system vibration was achieved in comparison to the unshaped input.
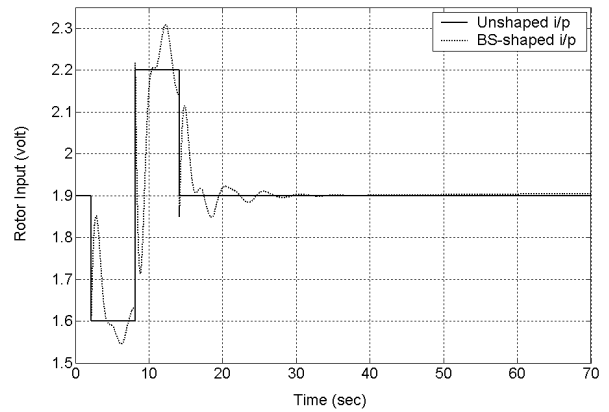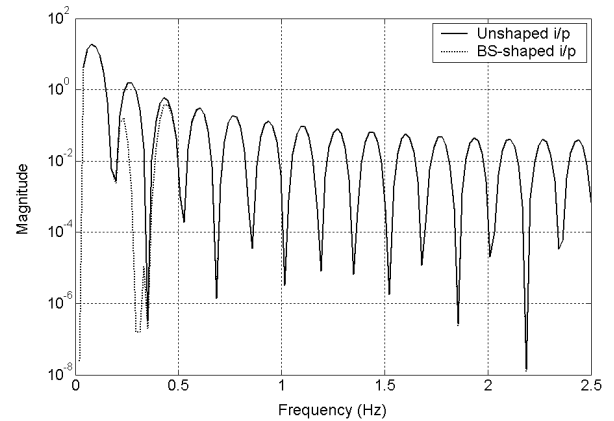
(a) Time domain

(b) Power spectral density

Figure 6. Filtered input using low-pass filter

(a) Time domain

(b) Power spectral density

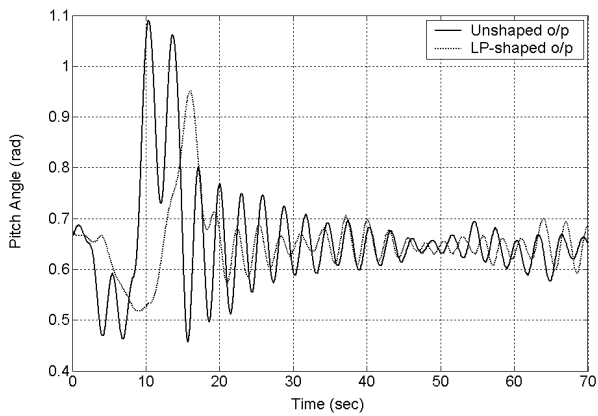Figure 7. Pitch response to filtered input using low-pass filter
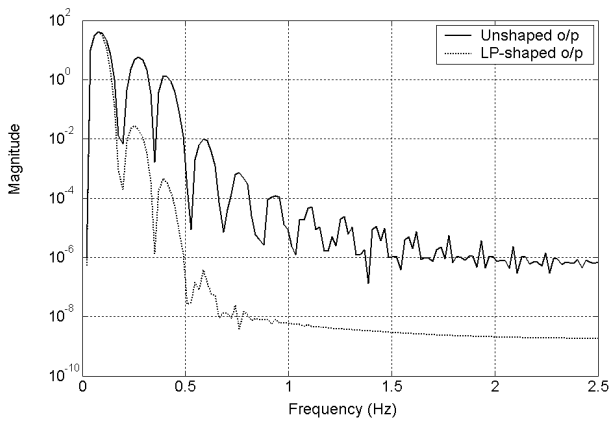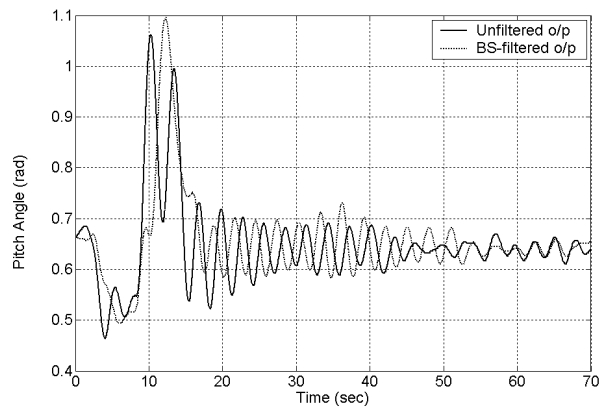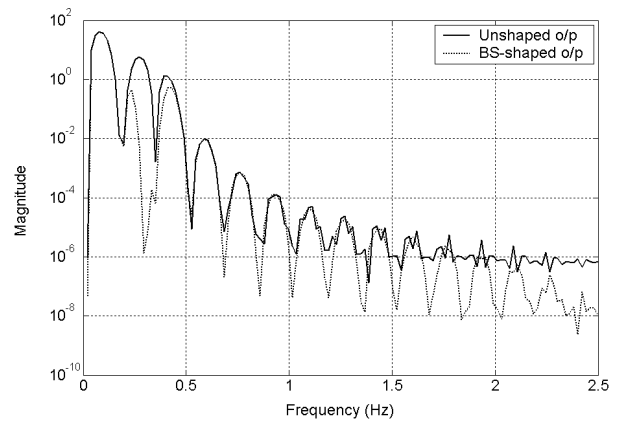

(a) Time domain

(b) Power spectral density

Figure 8. Filtered input voltage using band-stop filter

(a) Time domain

(b) Power spectral density

Figure 9. Pitch response to filtered input with band-stop filter

4.4 Comparative Performance Assessment

Among the three techniques employed for vibration reduction at the resonance frequency of the system, the input-shaper has resulted in better performance than the low-pass and band-stop filters (Table 1). The highest amount of vibration reduction at the resonance mode (at 0.34Hz) was recorded with the input-shaper, which was 65dB, followed by the low-pass filter of 30dB and finally the band-stop filter of 13.3dB. The system response with the input-shaper has the shortest settling time. For a total period of 70sec, it settled at 40sec, followed by the system response with low-pass filter at 48sec and then by the band-stop filtered response at 55sec. Furthermore, the response of the system with the input-shaper and the band-stop filter were faster than with the low-pass filter.

Table 1   Amount of vibration reduction and settling time using the three command shaping techniques

Command shaping technique	Amount of vibration reduction (dB)	Settling time (sec)
Input-shaper	65	40
Low-pass filter	30	48
Band-stop filter	13.3	55

5 Conclusion

The development of feed-forward control strategies for vibration control of pitch movement of a twin rotor MIMO system using input shaping and low-pass and band-stop filtered input techniques has been presented. The system response to the unshaped bang-bang input has been used to determine the parameters of the system for design and evaluation of the control strategies. Performances of the techniques have been evaluated in terms of level of vibration reduction and time response. A significant amount of reduction in the system vibration has been achieved with these control strategies. Among the three techniques, it was noted that the best performance in vibration reduction and time response was achieved with the input shaping technique. The low-pass filtered input resulted in better performance than the band-stop filtered input in terms of vibration reduction. However, with the band-stop filtered input, the system response was faster than with the low-pass filtered input. This study will be extended to include closed-loop control of the system.

References

[1] Ahmad, S. M., Modeling and control of a twin rotor multi-input multi-output system, *PhD thesis*, Department of Automatic Control and Systems Engineering, University of Sheffield, UK, (2001).

[2] Ahmad, S. M., Chipperfield, A. J. and Tokhi, M. O., Dynamic modeling and open-loop control of a twin rotor multi-input multi-output system, *Proc. Inst. Mech. Eng., Part I, J Systems and Control Engineering*, **216**, 477-496, (2002).

[3] Azad, A. K. M., Analysis and design of control mechanism for flexible manipulator systems, *PhD thesis*, Department of Automatic Control and Systems Engineering, University of Sheffield, UK, (1995).

[4] Banks, S., *Signal Processing, Image Processing and Pattern Recognition*. Prentice-Hall International, London, (1990).

[5] Feedback Instruments Ltd, *Twin Rotor MIMO System Manual 33-007-0*. Sussex, UK, (1996).

[6] Jackson, L. B., *Digital Filters and Signal Processing*, Kluwer Academic Publishers, London, (1989).

[7] Landis, H. K., Davis, J. M., Dabundo, C. and Keller, J. F., Advanced flight control research and development at Boeing Helicopter, *In Advances in Aircraft Flight Control*, (Tischler M.B. (Ed.)), 103-141. Taylor and Francis, London. (1996).

[8] Livet, T., Fath, D. and Kubica, F., Robust autopilot design for a highly flexible aircraft, *Proceedings of IFAC World Congress*, San Francisco, California, 279-284, (1996).

[9] Mohamed, Z. and Tokhi, M. O., Vibration control of a single-link flexible manipulator using command shaping techniques, *Proc. Inst. Mech. Eng., Part I, J Systems and Control Engineering*. **216**, 191-210, (2002).

[10] Murphy, B. R. and Watanabe, I., Digital shaping filters for reducing machine vibration, *IEEE Trans. Robotics and Automation*, **8**(2), 285-289, (1992).

[11] Pao, L. Y., Strategies for shaping commands in the control state of flexible structures, *In Proceedings of Japan–USA–Vietnam Workshop on Research and Education in Systems, Computation and Control Engineering*, Vietnam, 309-318, (2000).

[12] Singer, N. C. and Seering, W. P., Preshaping command inputs to reduce system vibration, *Trans. ASME, J. Dynamic Systems, Measurement and Control*, **112**(1), 76-82, (1990).

[13] Singhose, W. E., Singer, N. C. and Seering, W. P., Comparison of command shaping methods for reducing residual vibration, *In Proceedings of European Control Conference*, Rome, 1126-1131, (1995).

[14] Tokhi, M. O. and Azad, A. K. M., Active vibration suppression of flexible manipulator systems: open-loop control methods, *Int. J. Active Control*. **1**(1), 15-43, (1995).

[15] Tokhi, M. O. and Poerwanto, H., Control of vibration of flexible manipulators using filtered command inputs, *In Proceedings of International Congress on Sound and Vibration*, St Petersburg, 1019-1026, (1996).