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GENETIC ALGORITHMS BASED ADAPTIVE ACTIVE VIBRATION CONTROL OF A FLEXIBLE STRUCTURE

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Abstract

This paper investigates the development of an active vibration control (AVC) mechanism for a flexible plate structure using a genetic modelling strategy where the utilisation of genetic algorithms (GAs) for dynamic modelling of the system is considered. The global search technique of GAs is used to obtain a dynamic model of a flexible plate structure based on one-step-ahead (OSA) prediction and verified within the AVC system. The GA based AVC algorithm thus developed is implemented within a flexible plate simulation environment and its performance in the reduction of deflection at the centre of the plate is assessed. The validation of the algorithm is presented in both the time and frequency domains. An assessment of the results thus obtained is given in comparison to AVC system using conventional recursive least squares (RLS) method. Investigations reveal that the developed GA based AVC system performs better in the suppression of vibration of a flexible plate structure compared to an RLS based AVC system.

Keywords: Active vibration control, genetic algorithm, genetic-modelling, flexible plate structure.

1 Introduction

Flexible structure systems are known to demonstrate an intrinsic property of vibration when subjected to disturbance forces, leading to components and/or structural damage [1]. An understanding of the principles involved in the analysis of such systems is crucial so that suitable control laws for effective suppression of undesirable vibrations can be developed.

Many attempts have been made in the past at devising methods of tackling the problems arising due to unwanted structural vibration or disturbance.

Recently, much effort has been expended on active vibration control (AVC) measures for use in this area. These employ generating cancelling source(s) to destructively interfere with the unwanted source and thus result in a reduction in the level of vibration at desired location(s). This is realised by detecting and processing the vibration by a suitable controller so that when superimposed on the disturbances cancellation occurs [2].

Owing to the broadband nature of disturbances, it is vital that the control mechanism in an AVC system realizes suitable frequency-dependent characteristics so that cancellation over a broad range of frequencies is achieved. The spectral contents of the disturbances as well as the characteristics of system components are, in general, subject to variation, giving rise to time-varying phenomena. This implies that the control mechanism is further required to be intelligent enough to track these variations so that the desired level of performance is achieved and maintained [3].

Extensive work on genetic algorithms (GAs) has been reported covering various applications in the last two decades. GAs constitute a global, natural and data independent search procedure that imitates the principle of natural evolution. GAs have received significant interest among researchers and have been applied to various applications such as signal processing, robotics, active noise cancellation, system identification and modelling, adaptive control, engineering design, planning and scheduling, and pattern recognition [4], [5], [6], [7], [8], [9]. Although GAs have gained popularity as parallel, global search techniques, their use in the area of adaptive active control is very limited.

This paper presents an investigation into the use of GAs to estimate the adaptive controller characteristics, where the controller is designed based on the plant model. This is realised by minimising the prediction error of the actual plant output and the model output. A Matlab GA Toolbox is utilised to identify the controller parameters. A comparative performance of the conventional RLS scheme and GA is presented. The adaptive AVC algorithm based on GA is implemented and its performance in vibration suppression of a flexible plate structure is assessed.

2 Simulation of the flexible plate structure

Plates are elements of practical importance in many engineering applications. Study of the natural modes, frequencies and the dynamic behaviour of flexible plates is a subject that has received considerable attention due to its technical importance. In addition to being a problem of academic interest, many applications of thin flat plates are found in industry. Examples include bridge decks, solar panels, and electronic circuit board design. The control of a vibrating plate is, however, a complex problem. This is due to the highly non-linear dynamics of the system, which involve complex processes. Accordingly, there is a growing need for developing suitable modelling and control strategies for such systems. It is crucial to obtain an accurate model of the plate structure in order to control its vibration efficiently. An accurate model will lead to the realisation of satisfactory control.

Dynamic modelling and simulation of a flexible plate structure using the finite difference (FD) method have been developed by Mat Darus et al. [10], where a flat, square plate with all edges clamped has been considered. The thin plate as shown in Figures 1 and 2 is assumed to undergo a small deflection, w. Considering all the forces including the effect of shear forces Q_x and Q_y, in terms of the moments M_x, M_y and M_{xy} on bending, the classical dynamic equations of motion of a thin rectangular plate is obtained as

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{\rho}{D}\frac{\partial^2 w}{\partial t^2} = \frac{q}{D} \tag{1}$$

where w is the lateral deflection in the z direction, ρ is the mass density per unit area, $q = q\ (x, y)$ is the transverse external force at point (x, y) and has dimensions of force per unit area, $\frac{\partial^2 w}{\partial t^2}$ is the acceleration in the z direction, $D = \frac{Eh^3}{12(1-\upsilon)}$ is the flextural rigidity, with υ representing the Poisson ratio, h the thickness of the plate and E the Young's modulus. A simulation algorithm characterising the dynamic behaviour of the plate is developed through discretisation of the partial differential equation (PDE) in equation (1) in time and space, where the plate is divided into several sections and a linear relation for the deflection of each section is then obtained using FD approximations.

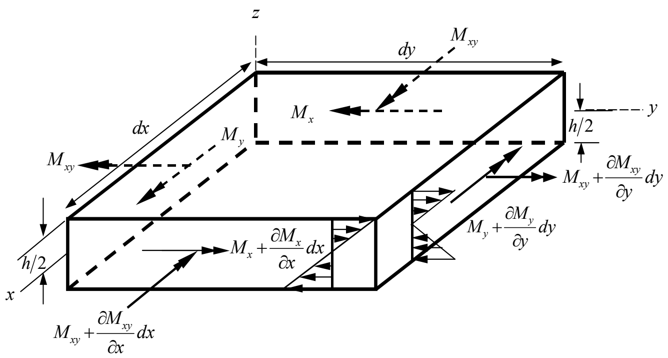


Figure 1: A flexible plate structure with moments

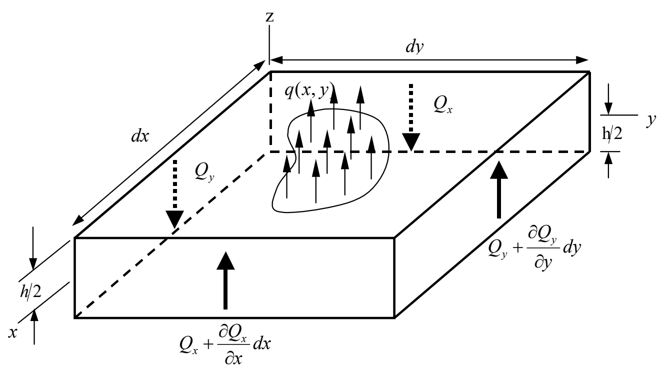


Figure 2: A flexible plate structure with shear forces

The x-axis is represented with the reference index i and the y-axis with the reference index j, where $x = i\Delta x$ and $y = j\Delta y$. In the case of a 2D plate structure, a three dimensional coordinate system is considered. The additional dimension is time t, which is represented with a reference index k, where $t = k\Delta t$. For each nodal point in the interior of the grid (x_i, y_j, t_k), (i = 0,1, …n; j = 0, …m; and k = 0, 1…p), a Taylor series expansion is used to generate the central difference formulae for the partial derivative terms of the response (deflection), $w\ (x, y, t) = w_{i, j, k}$ of the plate at point $x = i\Delta x$, $y = j\Delta y$ and $t = k\Delta t$. Thus, using first-order approximations at the mesh points inside the plate, and second-order approximation at the boundaries, a general solution of the PDE in equation (1) can be obtained in discrete form as [10]:

$$\begin{aligned} w_{i,j,k+1} = -\frac{D\,\Delta t^2}{\rho \Delta x^2 \Delta y^2} \Big(& 20 w_{i,j,k} \\ & - 8\ (w_{i+1,j,k} + w_{i,j+1,k} + w_{i-1,j,k} + w_{i,j-1,k}) \\ & + 2\ (w_{i+1,j+1,k} + w_{i-1,j+1,k} + w_{i-1,j-1,k} + w_{i+1,j-1,k}) \\ & + w_{i+2,j,k} + w_{i,j+2,k} + w_{i-2,j,k} + w_{i,j-2,k} \Big) \\ & + 2\, w_{i,j,k} - w_{i,j,k-1} + \frac{\Delta t^2 q_{i,j}}{\rho} \end{aligned} \tag{2}$$

where $w_{i,\ j,\ k+1}$ is the deflection of nodal point (x_i, y_j) of the plate at time step $k+1$. The boundary condition along a clamped edge, say $y=a$, is

$$w\big|_{y=a} = \frac{\partial w}{\partial y}\bigg|_{y=a} = 0. \tag{3}$$

Using equation (2), a difference equation corresponding to each nodal point is obtained. The algorithm is then developed using an iterative scheme within the Matlab environment, and it allows application and sensing of a disturbance signal at any mesh point on the plate. Such a provision is desirable for the design and development of active vibration control techniques for the system. The simulation algorithm thus developed and validated will be used in this paper as test and verification platform of AVC strategies.

3 Active vibration control system

A schematic diagram of the geometric arrangement of a single-input-single-output (SISO) feedforward AVC structure considered in this investigation is shown in Figure 3.

An unwanted (primary) point source introduces structural vibration into the plate system. This is detected by a detector, processed by a controller of suitable transfer characteristics and fed to a cancelling (secondary) point source. The secondary signal thus generated is superimposed on the primary signal so as to achieve vibration reduction at and in the vicinity of an observation point on the plate.

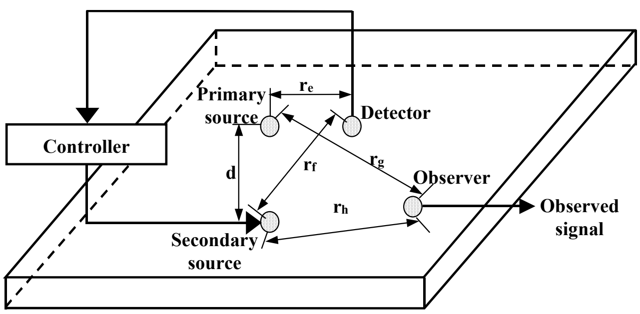


Figure 3. Schematic diagram of the AVC structure.

The objective in Figure 3 is to achieve optimum vibration suppression at the observation point. This is equivalent to the minimum variance design criterion in a stochastic environment. This requires the primary and secondary signals at the observation point to be equal in amplitudes and have a phase difference of 180° relative to one another. Synthesising the controller on the basis of this objective yields [1]:

$$C = \left[1 - \frac{Q_1}{Q_0} \right]^{-1} \tag{4}$$

where Q_0 and Q_1 represent the equivalent transfer characteristics of the system, with input at the detector and output at the observer, when the secondary source is off and on, respectively.

Equation (4) is the required controller design rule which can easily be implemented on-line on a digital processor. This leads to a self-tuning AVC algorithm comprising the processes of identification and control. The process of identification involves obtaining Q_0 and Q_1 using a suitable identification technique whereas the process of control involves designing the controller according to equation (4) and implementing this in real-time.

4 Genetic Algorithm

Kirkpatrick *et al.* (1983) have described Simulated Annealing as "an example of an evolutionary process modelled accurately by purely stochastic means"[11], but this is more literally true of another class of new optimization routines known collectively as GAs. The phenomenon of natural evolution was first observed by Darwin (1959) and later elaborated by Dawkins (1986) [12], [13]. In natural evolution each species searches for beneficial adaptations in an ever-changing environment. As species evolve these new attributes are encoded in the chromosomes of individual members. This information does change by random mutation, but the real driving force behind evolutionary development is the combination and exchange of chromosomal material during breeding. Although sporadic attempts to incorporate these principles in optimization routines have been made since the early 1960s [14], GA was first established on a sound theoretical basis by Holland [15]. The two key axioms underlying this innovative work were that complicated nonbiological structures could be described by simple bit strings and that these structures could be improved by the application of simple transformations to these strings.

Since their introduction as evolutionary algorithms that mimic the natural evolutionary process for problem solving [15], there has been growing interest among scientists and engineers in the use of GAs in various applications such as signal processing, robotics, active noise cancellation, system identification and modelling, adaptive control, engineering design, planning and scheduling, and pattern recognition [16], [17], [18], [19], [20], [21]. A GA is a global, natural and data independent search technique [22].

GAs form one of the prominent members of the broader class of evolutionary algorithms. They are inspired by the mechanism of natural biological evolution, i.e., the principles of survival of the fittest [15]. From an operational perspective, a GA comprises two basic elements; a set of individuals, i.e., potential solutions (the population) and a set of biologically inspired operators active over the population. A new set of approximations/ solutions is created at each generation, by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using the operators. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation [22]. In fact, individuals or current approximations are encoded as strings (typically represented in binary), chromosomes, and then the most promising strings are manipulated using the GA operators for better and better approximation to a solution. The operating mechanism of a GA can be described through the following stages:

1. Creation of initial set of potential solutions (population) as strings.
2. Evaluation of each solution and selection of the best ones.
3. Genetic manipulation to create new population.
4. Go back to step 2.

These stages are shown in Figure 4. At the first stage, an initial population of potential solutions is created. Each element of the population is mapped onto a set of strings (the chromosome) to be manipulated by the genetic operators. In the second stage, the performance of each member of the population is assessed through an objective function imposed by the problem. This establishes the basis for selection of pairs of individuals that will be mated together during reproduction. For reproduction, each individual is assigned a fitness value derived from its raw performance measure, given by the objective function. This value is used in the selection to bias towards more fit individuals. Highly fit individuals, relative to the whole population, have a high probability of being selected for mating, whereas less fit individuals have a correspondingly low probability of being selected [22].

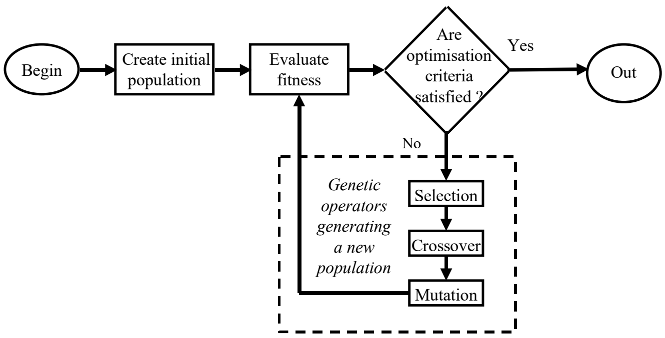


Figure 4. Working principles of GAs.

In the manipulation phase, genetic operators such as crossover and mutation are used to produce a new population of individuals (offspring) by manipulating the "genetic information" usually called genes, possessed by the members (parents) of the current population. The crossover operator is used to exchange genetic information between pairs, on larger groups, of individuals. Mutation is generally considered to be a background operator, which ensures that the search process is not trapped at local minimum, by introducing new genetic structures in the population.

After manipulation by the crossover and mutation operators, the individual strings are then, if necessary, decoded, the objective function evaluated, a fitness value assigned to each individual and individuals selected for mating according to their fitness, and so the process continues through subsequent generations. In this way, the average performance of individuals in a population is expected to increase, as good individuals are preserved and breed with one another and the less fit individuals die out. The GA is terminated when some criteria are satisfied, e.g., a certain number of generations completed or when a particular point in the search space is reached.

5 Identification

The conventional on-line system identification methods, such as least squares, instrumental variable, maximum likelihood etc. are in essence local search techniques. These techniques often fail in the search of global optimum if the search space is not differentiable or linear in the parameters. Also, these techniques do not iterate more than once on each datum received. In many cases when it is difficult to obtain a model structure for a system with traditional system identification techniques, intelligent techniques are desired that can describe the system in the best possible way [11].

GAs constitute one of a few intelligent techniques commonly used for system identification and modelling of dynamical systems. The major advantage of utilising GA for system identification is that it simultaneously evaluates many points in the parameter space and converges towards the global solution [12]. In contrast to conventional methods, GA does not require the search space to be differentiable or continuous and can also iterate several times on each datum received.

The identification process within the control mechanism presented in this paper consists of the processes of estimating the system models Q_0 and Q_1 using GAs in the discrete time domain in parametric form. The global search technique of GAs is used to identify the parameters of a flexible plate structure based on one-step-ahead (OSA) prediction. This is based on the method of minimisation of the prediction error using the fitness function

$$f(e)=\sum_{n=0}^{S}\left|y(n)-\hat{y}(n)\right| \qquad (5)$$

where, S represents the number of input/output samples, $y(n)$ is the desired (plant) output and $\hat{y}(n)$ is the estimated model output.

6 Implementation and results

To assess and verify the AVC algorithm a simulation environment, characterising the feedforward AVC structure described in Figure 3, was utilised. To investigate variations in the detected vibration modes, modelling of the plate system was carried out with the flexible plate simulated responses to a uniformly distributed white noise input. This type of input is chosen to ensure that the dynamic range of interest of the simulated plate system is captured. The performance of the AVC algorithm is assessed based on two different parametric modelling and system identification techniques, the conventional RLS estimation method and the GA optimisation with OSA prediction. The input vector of format:

$$X(t)=\left[y(t-1),\ldots,y(t-n);u(t-1),\ldots,u(t-m)\right]^{T} \qquad (6)$$

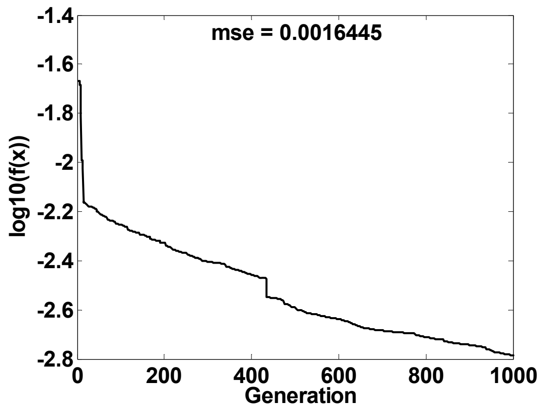
was used with $m=n=10$ and $m=n=12$ for GA and RLS estimation techniques, respectively.

6.1 GA and RLS modelling.

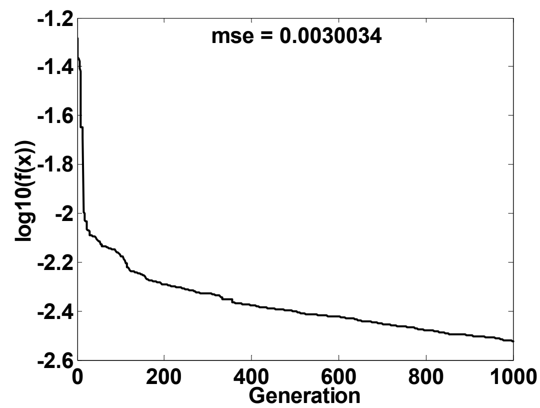
Investigations were carried out using the GA optimisation based on OSA prediction with different initial values and operator rates. From the work carried out it was found that satisfactory results were achieved with the following set of parameters:

- Generation gap: 0.9
- Crossover rate: 0.06
- Mutation rate: 0.01

The deflection model was observed with different orders. The best result was achieved with an order 10. The GA was designed with 100 individuals in each generation. The maximum number of generations was set to 1000. The algorithm achieved the best mean-squared error levels of 0.0016 and 0.0030 for Q_0 and Q_1 respectively in the 1000th generation. Figure 5 shows the algorithm convergence for Q_0 and Q_1 and Figure 6 shows the simulated output with the best parameter set, in the 1000th generation.

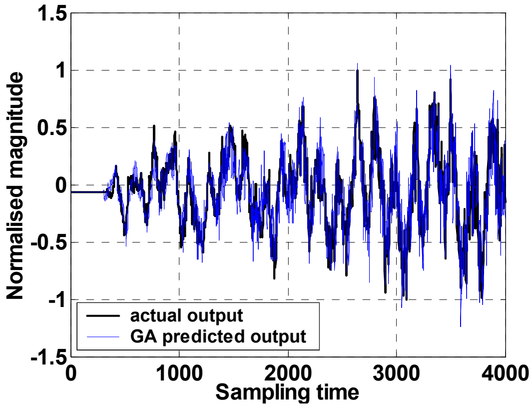


(a) Convergence of Q_0 model

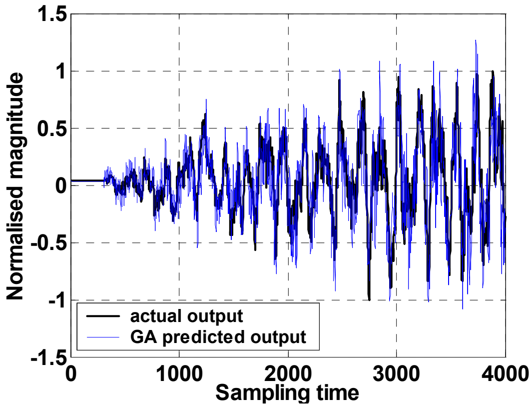


(b) Convergence of Q_1 model

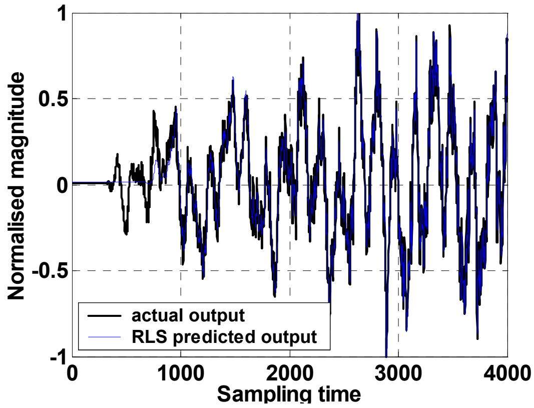
Figure 5. Mean-squared error vs. number of generation.



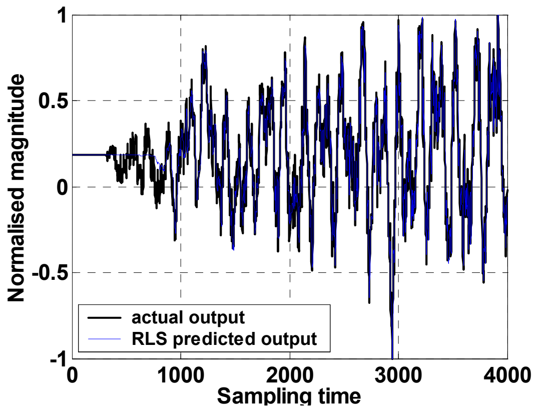
(a) Actual and predicted output for Q_0.



(b) Actual and predicted output for Q_1.
Figure 6. Performance of the GA based models.



(a) Actual and predicted output for Q_0.



(b) Actual and predicted output for Q_1.
Figure 7. Performance of the RLS based models.

For purposes of comparison, the flexible plate was also modelled with RLS algorithm. The deflection model was observed with different orders and the best result was achieved with an order 12. The algorithm achieved the best mean-squared error levels of 0.0068 and 0.0058 for Q_0 and Q_1, respectively. Figure 7 shows the simulated output of Q_0 and Q_1 thus modelled. Comparing these with the corresponding results of GA modelling in Figures 5 and 6 reveals that the identification using GA has performed better than RLS.

6.2 GA and RLS based AVC

The performance of the GA based SISO AVC system in suppressing the vibration of the system at the observation point when the primary source is uniformly distributed white noise is shown in Figure 8. It is noted that, with the uniformly distributed white noise input, the spectral attenuation achieved at the resonance modes with GA based SISO AVC system were 11.858 dB, 21.712 dB and 9.9845 dB at the first, second and third modes respectively. An average attenuation of 9.2832 dB was achieved over the first 100 rad/s frequency range of the disturbance.

Figure 9 shows the performance of the SISO AVC system using conventional RLS in suppressing the vibration of the system at the observation point when the primary source is uniformly distributed white noise.

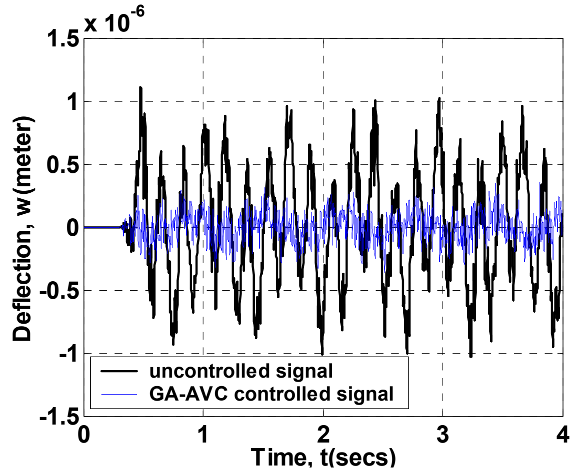
Evidently, the spectral attenuation achieved at the resonance modes with the RLS based SISO AVC system were 7.899 dB, 17.617 dB and 2.92 dB for the first, second and third modes respectively. An average attenuation of 7.434 dB was achieved over the first 100 rad/s frequency range of the disturbance. This reveals that the developed GA based AVC controller has performed better in the suppression of vibration of the flexible plate structure as compared with the RLS based AVC system.

7 Conclusion

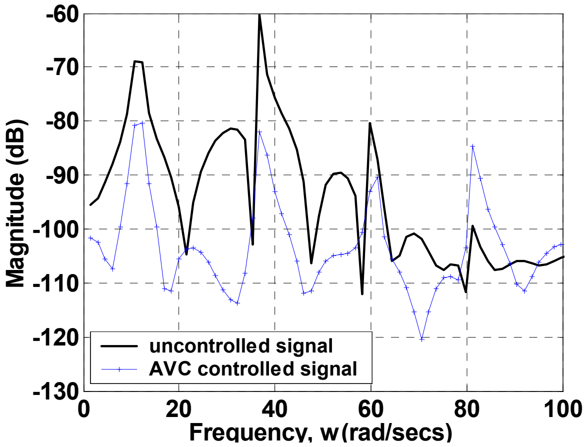
The design and implementation of a GA based adaptive AVC algorithm for flexible plate structures has been presented, discussed and verified through numerical simulation. The performance of the algorithm has been verified in the suppression of broadband vibration in a flexible plate system within SISO AVC structure. The investigation demonstrated that better estimation of the system model is achieved using GAs as compared with a conventional RLS scheme.

The study also revealed that the developed GA based AVC system has performed better in the suppression of vibration of a flexible plate structure as compared with an RLS based AVC system.

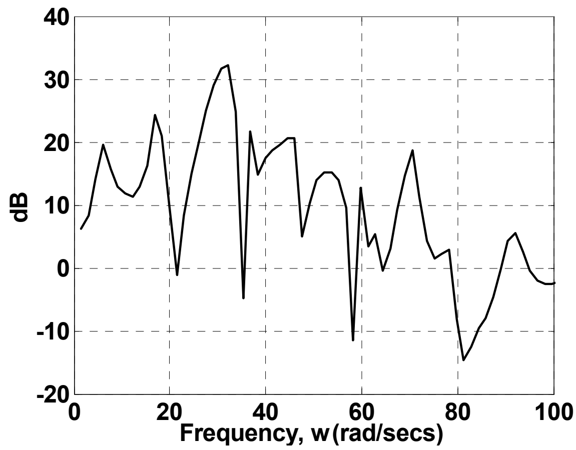
It is noted that the execution time of the GA based algorithm is more than that of the conventional scheme with the same computing platform. However, high performance computing techniques could provide suitable solutions for the real-time implementation of the GA based algorithm.



(a) Time domain.

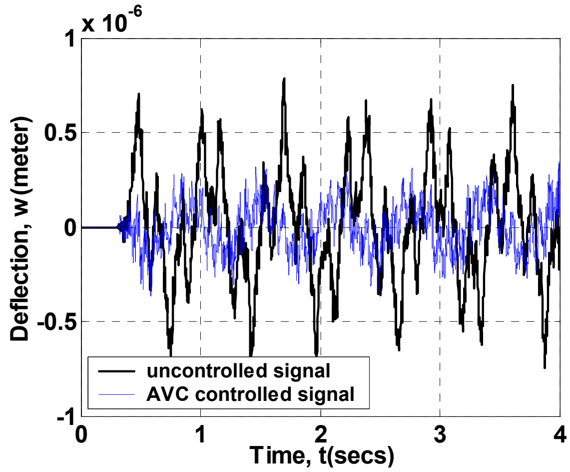


(b) Spectral density.

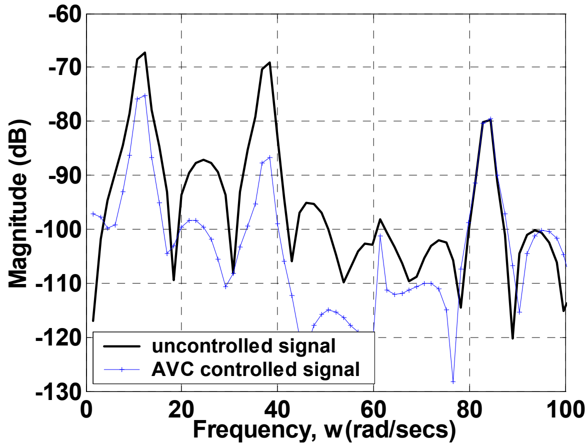


(c) Spectral Attenuation

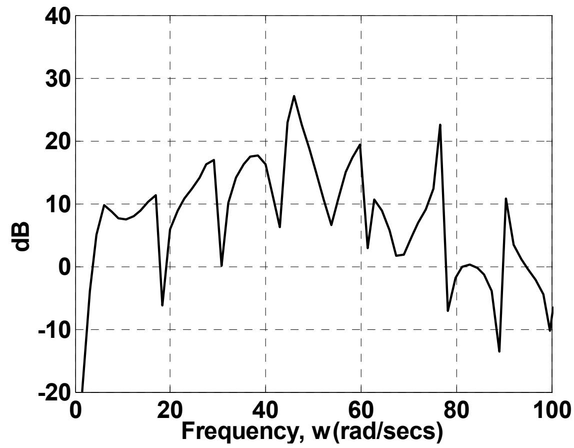
Figure 8. Performance of the GA based SISO AVC system with uniformly distributed white noise input.



(a) Time domain.



(b) Spectral density.



(c) Spectral Attenuation.

Figure 9. Performance of the RLS based SISO AVC system with uniformly distributed white noise input.

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