Original Optimization Procedures of Halbach Permanent Magnet Segmented Array

Sarah Touhami, *IEEE Student Member*
LAPLACE Laboratory, University of Toulouse, CNRS, INPT,UPS
Toulouse, France

Yvan Lefèvre
LAPLACE Laboratory, University of Toulouse, CNRS, INPT,UPS
Toulouse, France

Jean-François Llibre
LAPLACE Laboratory, University of Toulouse, CNRS, INPT,UPS
Toulouse, France

***Abstract*—This paper presents a direct design and optimization procedures of an arbitrary PM Halbach Segmented Array. The direct design is based on the ideal PM Halbach definition. The arc width, magnitude and orientation of polarization of each segment are taken as optimization variables after calculation of PM thickness. A new optimization procedure based on integration of Least Square Method inside the well-known Sequential Quadratic Programming method is proposed to improve the optimization results and to reduce time computation.**

***Keywords*—Halbach Array, Thickness Permanent Magnet Halbach, Optimization).**

I. Introduction

Halbach Permanent Magnets (PMs) are increasingly used in high specific power electric motors due to their many benefits [1]. In literature, the segmented Halbach PM is optimized to reach magnetic flux close to the sinewave or to minimize the rotor weight [2-4]. In these papers, Halbach PMs are optimized by taking as optimization variables the direction of polarization and/or the arc widths and for a given value of the PM thickness. In addition, the relative permeability of PMs is always assumed equal to one. Most often a small number of segments having the same magnetization magnitude are considered in these papers. With a small number of variables, the sequential quadratic programming method proved to be efficient and fast. For a high number of variables, the genetic algorithm is used [3]. Although, the genetic algorithm is more efficient than the sequential quadratic programming method, the computing time of this method may be very huge.

This paper presents the design and optimization of PM Halbach segmented array to reach sinewave airgap flux density of a high specific power electric machine dedicated to an aircraft application. The main sizes of this electric machine are determined using loadability concepts without specify the rotor and stator configurations [5]. The PM thickness is determined using the definition of an ideal Halbach PM and for desired airgap flux density unlike that done in [2]-[5]. The relative permeability of PMs is taking into account. In addition, from ideal Halbach PM definition a direct design of Halbach PM segmented array is proposed to reach a sinewave airgap flux density. The optimization of the Halbach PM segmented array is carried out to improve the airgap flux density. The optimization variables may be the arc widths, the magnetization directions and magnitudes. It can deal with any given number of segments. The optimization procedure used in this paper is based on the integration of Least Square Method (LSM) in Sequential Quadratic Programming Method (SQPM).

II. Direct design segmented Halbach PM array from the ideal Halbach PM

A. Calculation of Permanent Magnet thickness

For a desired maximum airgap sinewave flux density B_m and for a given value of the airgap thickness, the thickness of an ideal Halbach PM can be calculated from equations of 2D analytical model of an ideal Halbach PM. Thereby, the PM inner radius R_1 is governed by:

$$S_{11}R_1^{2p} + cR_1^{1+p} - S_{21} = 0 \quad (1)$$

With:

$$\begin{cases} S_{11} = \frac{(\mu_r+1)R_3^{1-p}}{4p}B_m - \frac{(\mu_r-1)R_3^{1+p}}{4pR_2^{2p}}B_m \\ S_{21} = -\frac{R_2^{1+p}}{(p+1)}J_m + \frac{(\mu_r+1)R_3^{1+p}}{4p}B_m - \frac{(\mu_r-1)R_3^{1-p}R_2^{2p}}{4p}B_m \\ c = -\frac{J_m}{1+p} \end{cases} \quad (2)$$

Where R_3 is the airgap radius, R_2 is the PM external radius, μ_r is the relative permeability of permanent magnet, J_m is the maximum amplitude of polarization, p is the number of pole pair.

By using the Newton method in equation (1), the PM thickness can be directly deduced.

B. Design of Halbach PM array

Ideal Halbach PM is very difficult to realize. For this purpose, PM Halbach can be discretized into several blocs with the same angular width $\delta\theta$ and with different polarization. The polarization of each bloc can be approached by the following polarization:

$$\overrightarrow{J(\theta_k)} = \begin{cases} J_r(\theta_k) = J_m\cos(p\theta_k) \\ J_\theta(\theta_k) = -J_m\sin(p\theta_k) \end{cases} \quad (3)$$

Or by their average value:

$$\overrightarrow{J_{avg}}(\theta_k) = \begin{cases} J_{r_{avg}}(\theta_k) = \frac{1}{\delta\theta_i}\int_{\theta_k-\frac{\delta\theta}{2}}^{\theta_k+\frac{\delta\theta}{2}} J_r(\theta)d\theta = J_m\frac{\sin(0.5p\delta\theta)}{0.5p\delta\theta}\cos(p\theta_k) \\ J_{\theta_{avg}}(\theta_k) = \frac{1}{\delta\theta_i}\int_{\theta_k-\frac{\delta\theta}{2}}^{\theta_k+\frac{\delta\theta}{2}} J_\theta(\theta)\,d\theta = -J_m\frac{\sin(0.5p\delta\theta)}{0.5p\delta\theta}\sin(p\theta_k) \end{cases} \quad (4)$$

Where $k = 1\ldots n_b$, θ_k is the angle at the center of PM bloc k, n_b is the number of segments, $\delta\theta$ is the angular width, $J_{r_{avg}}(\theta_k)$ and $J_{\theta_{avg}}(\theta_k)$ are the magnitude of radial and tangential average polarization respectively.

III. Optimization Procedures from general 2D Model

A. Procedure and methods

By demonstrating that the airgap flux density can be written as a linear superposition of elementary airgap flux density $b_{rk}^{II}(r = R_3, \theta)$ and $b_{\theta k}^{II}(r = R_3, \theta)$ generated respectively by unit radial $J_{rk}(\theta)$ and tangential $J_{\theta k}(\theta)$ polarizations in each segment k, the LSM can then be used in order to find the required polarization magnitude and direction:

$$B_r^{II}(r = R_3, \theta) = \Sigma_k(J_r^k b_{rk}^{II}(r = R_3, \theta) + J_\theta^k b_{\theta k}^{II}(r = R_3, \theta)) \quad (5)$$

As shown on Fig. 1 and contrary to what is done in [5], the LSM is integrated inside the SQPM optimization procedure. In [5], LSM and SQPM are used sequentially.

This integration allows reducing the number of SQPM variables from $3 \times n_b$ to n_b and then reducing time computing.

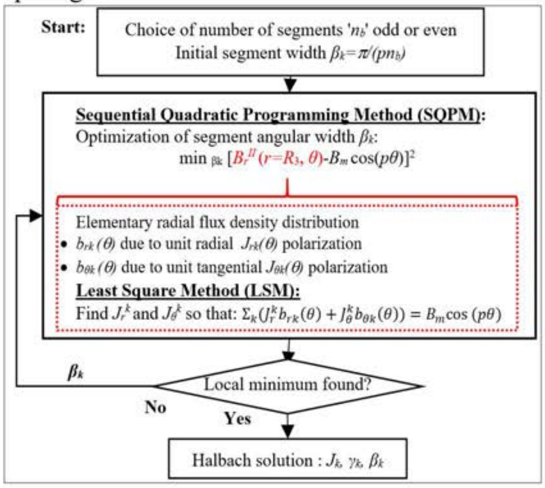

Fig. 1. Optimization procedure: LSM inside the SQPM

B. Application of the optimization procedure

A high specific power electric motor (Table I) has been sized using the tool presented in [6]. Before performed the optimization procedure to reach sinewave airgap flux density, the PM thickness is calculated using equation (1).

TABLE I: SM-PMSM PARAMETERS

Electromagnetic power	P_{em}	1.425 MW
Mechanical Speed	N	15970 rpm
RMS current density	j_{rms}	8.10 A.mm^{-2}
Fill factor	k_{fill}	0.5
Max airgap flux density	B_m	0.9 T
Magnetic shear stress	σ	50000 Pa
Number of poles	$2p$	4
Flux density in stator yoke	B_{sy}	1.21T
Flux density in stator teeth	B_t	1.29 T
Stator outer radius	R_{sout}	163 mm
Stator inner radius	R_{sin}	92 mm
Thickness of airgap	e_g	2.7 mm
Active length	L_m	317.3mm

Table II summarizes the results obtained for 7 parallel segment PM Halbach. Optimization of PM Halbach with 7 segmented parallel polarization with unequal polarization, length and orientation gives the best result compared to the other optimization results and compared to the direct design of equal length ideal PM Halbach. Figure 2 shows the resulting airgap flux densities. The incorporation of LSM method inside SQP has divided by 3 the computation time.

TABLE II: OPTIMIZATION RESULTS FOR 7 SEGMENTS

k	1	2	3	4	5	6	7
Parallel polarization:J^k **=1.15T,**$\mu_{pm} = 1.038\mu_o$**,**$e_{pm} = 8.85mm$							
β^k [°]	15	11.86	12.11	12.04	12.11	11.86	15
γ^k [°]	-9.8	-35.7	-63.4	-90	-116	-144	-170
B_m=0.9T, $THD(\%) = 5.78$ and quadratic error: 4.1%							
Parallel polarization:$\mu_{pm} = 1.038\mu_o$, $e_{pm} = 15.38mm$							
J^k[T]	1.05	0.96	0.83	0.79	0.83	0.96	1.05
β^k[°]	15	13.7	10.7	11	10.7	13.7	15
γ^k[°]	-12.3	-36.8	-61.3	-90	-118	-143	-167
B_m=0.9T and $THD(\%) = 2.98$ and quadratic error: 3.07%							
Radial-Tangential polarization: $\mu_{pm} = 1.038\mu_o$, $e_{pm} = 15.38mm$							
J_r^k [T]	1.05	0.85	0.46	0.00	-0.46	-0.85	-1.05
J_θ^k[T]	-0.04	-0.08	-0.05	-0.08	-0.05	-0.08	-0.04
β^k[°]	15	14.4	10.3	10.4	10.3	14.4	15
B_m=0.9T, $THD(\%) = 4.39$ and quadratic error: 3.25%							
Direct design:$\mu_{pm} = 1.038\mu_o$, $e_{pm} = 15.38mm$, $\beta^k = 12.85°$, $J^k = 1T$							
γ^k[°]	-12.8	-38.5	-64.2	-90	-115	-141	-167
B_m=0.9T and $THD(\%) = 3.5$							

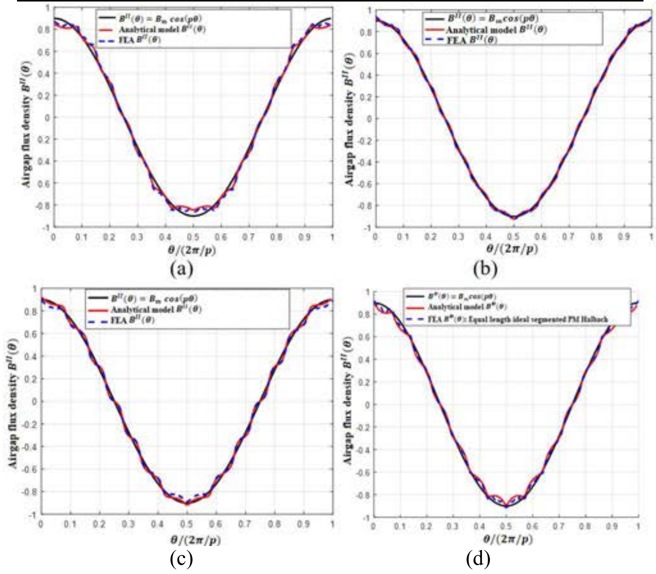

Fig. 2 Airgap flux density (a) Parallel polarization with the same magnetization (b) Parallel polarization with different magnitude (c) Radial-Tangential magnetization, (d) equal length ideal PM Halbach

IV. CONCLUSION

The thickness of PM Halbach is calculated from the definition of ideal PM Halbach. This calculation is used in the direct design of equal length ideal PM Halbach and in the optimization design of an arbitrary Halbach PM. The direct design presented in this paper is efficient in the case of design without optimization. Optimization procedure performed in this paper gives improved airgap flux density. In the optimization procedure, the incorporation of LSM method into SQP method allows to reduce the computation time. 7 segments Halbach PM with unequal polarization, length and orientation gives the best optimization result.

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REFERENCES

[1] Y. Chen, R.Sanchez, A.Yoon, K. Sivasubramniam, "Mechanical design considerations of an Ironless high specific power electric machine," *IEEE Trans. on Transp. Elect*., vol.3, no.4, Dec 2017.

[2] P. H. Mellor, R. Wrobel, "Optimization of a multipolar permanent-magnet rotor comprimising two arc segments per pole," *IEEE Trans. Ind. Appl*., vol.43, no.4, Jul/Aug 2007.

[3] H. Huang, R. Qu, D.Li, "Analysis and application of discrete Halbach magnet array with unequal arc lengths and unequally changed magnetization directions," *IEEE Trans. Appl. Supercond,* vol. 28, no.3, Apr 2018.

[4] M. Markovic, Y. Perriard, "Optimization deisgn of a segmented halbach permanent-magnet motor using an analytical model," *IEEE Trans.On. Magn*., vol.45, no.7, July 2009.

[5] S. Touhami, Y. Lefevre, J.F. Llibre, "Joint Design of Halbach Segmented Array and Distributed Stator Winding", *IEEE International Conference on Electrical Machines (ICEM)*. Sept.3-6, 2018.

[6] Y. Lefevre, S. El Aabid, J.F. Llibre, C. Henaux, S.Touhami, "Performance Assessment Tool Based on Loadabilty Concepts," *International Journal of Applied Electromagetics and Mechanics*, vol. 59, no. 2, pp. 687-694, 2018.