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Gravitational waves emission by millisecond accreting neutron stars

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Summary. — Strong toroidal magnetic fields can be generated by differential rotation induced by r-modes in the core of the rapidly rotating neutron stars (NSs). The intensities of these fields can reach strengths of order 10^{14} G deforming significantly the star. The magnetically deformed NS may radiate gravitational waves (GWs) if the symmetry axis of the generated magnetic field is not aligned with the spin axis. This mechanism may explain the upper limit of the spin frequencies of accreting NSs in Low Mass X-ray Binaries (LMXBs).

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1. – Introduction

The r-mode instability develops in the fastest NSs rotating at millisecond periods. It induces the GW emission which carries away spin angular momentum causing the star to spin down. It also gives rise to differential rotation [1-5] that in turn produces very strong toroidal magnetic fields inside the star. These fields damp the instability converting the energy of the mode into magnetic energy. This mechanism has been investigated in the case of rapidly rotating, isolated, newly born neutron stars in refs. [1-3] and in the case of accreting millisecond neutron and quark stars in refs. [6, 7].

Magnetic fields deform the star and if the magnetic axis is not aligned with the rotation axis the NS undergoes free body precession. The symmetry axis of the precessing NS drifts on a timescale determined by its internal viscosity, eventually becoming an orthogonal rotator [8]. This is an optimal configuration for efficient GW emission, thus enhancing angular momentum losses from the NS.

We show that the GW emission due to the magnetic deformation can effectively limit the spin frequency of accreting NSs in LMXBs. In particular this mechanism could explain the observed cut-off above 730 Hz [9, 10]. We estimate also the strain amplitude of the GW signal emitted from LMXBs in the local group of galaxies to establish their detectability with the next generation of interferometers (*e.g.*, Einstein Telescope).

2. – Magnetic deformation and GW emission

The total angular momentum J of a star can be decomposed into an equilibrium angular momentum J_* and a canonical angular momentum J_c proportional to the r-mode perturbation [11]:

$$J = J_*(M,\Omega) + (1 - K_j)J_c, \qquad J_c = -K_c\alpha^2 J_*, \tag{1}$$

where $K_{(j,c)}$ are dimensionless constants and $J_* \cong I_*\Omega$.

The canonical angular momentum obeys the following equation [6, 12]:

$$dJ_c/dt = 2J_c\left\{F_g^r(M,\Omega) - [F_v(M,\Omega,T_v) + F_{m_i}(M,\Omega,B)]\right\}, \tag{2}$$

where F_g^r is the gravitational radiation growth rate of the r-mode, $F_v = F_s + F_b$ is the sum of the shear and bulk viscous damping rate and F_{m_i} is the damping rate associated with the generation of an internal magnetic field. Finally, $T_v(t)$ is a spatially averaged temperature. Once the condition $F_g > F_v + F_{m_i}$ is satisfied, r-modes are excited and they induce azimuthal drift motions of fluid parcels in the NS core. Magnetic field lines are twisted and stretched by the shearing motions and a new field component B^ϕ, in the azimuthal direction, is generated. The toroidal field B^ϕ deforms the NS into a prolate ellipsoid with ellipticity $\epsilon_B \equiv (I_{zz} - I_{xx})/I_{zz}$ where $I_{jk} = \int_V \rho(r)(r^2\delta_{jk} - x_j x_k)\mathrm{d}V$ is the inertia tensor. For a neutron star with a normal core [8, 13, 14]

$$\epsilon_B \approx -10^{-12}\, R_{10}^4\, M_{1.4}^{-2}\, \bar{B}_{\phi,12}^2. \tag{3}$$

The magnetic deformation dominates the deformation induced by the NS rotation when $|\epsilon_\mathrm{B}| \gtrsim 2\times 10^{-10}\,(\nu_\mathrm{s}/150\,\mathrm{Hz})^2 \equiv \epsilon_{B,\mathrm{min}}$ or equivalently [15]

$$\bar{B}_\phi \gtrsim 1.5\times 10^{13}\,\mathrm{G}\left(\frac{\nu_\mathrm{s}}{150\,\mathrm{Hz}}\right) M_{1.4}\, R_{10}^{-2}, \tag{4}$$

where ν_s is the spin frequency of the NS. Beyond this point, the total NS ellipticity becomes dominated by the magnetically induced, prolate deformation. The freely precessing NS now becomes secularly unstable. In the presence of a finite viscosity of its interior, the wobble angle between the angular momentum J_* and the symmetry (magnetic) axis grows until the two become orthogonal. This occurs on a dissipation (viscous) timescale, τ_v, which can be expressed as [8]

$$\tau_\mathrm{v} = nP_\mathrm{prec} \simeq 10^{11}\times\left(\frac{n}{10^5}\right)\left(\frac{100\,\mathrm{Hz}}{\nu_s}\right)\left(\frac{10^{-8}}{|\epsilon_B|}\right)\,\mathrm{s}. \tag{5}$$

An orthogonal rotator emits gravitational waves at a rate $\dot{E}_\mathrm{gw}^B = -32GI_*^2\epsilon_B^2\Omega^6/5c^5$ [16] which produces spin down at the rate $F_\mathrm{g}^\mathrm{B} \equiv -(\dot{E}_\mathrm{gw}^\mathrm{B}/2E)$. Therefore, if the orthogonality

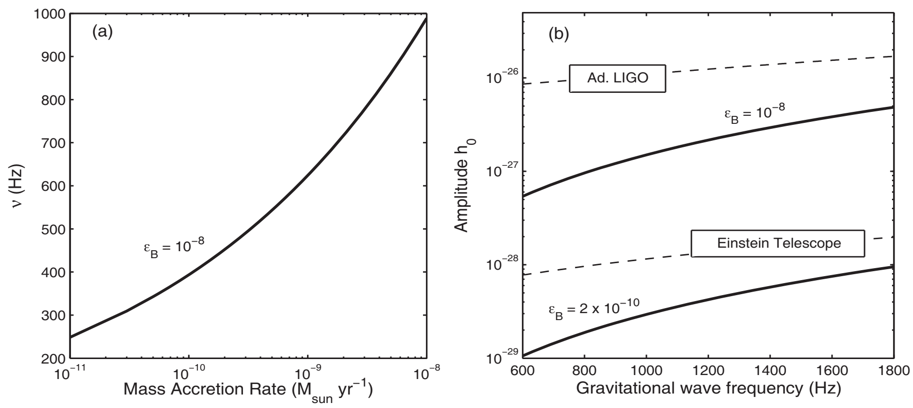

Fig. 1. – (a) Limits on the maximum spin frequencies of accreting neutron stars with an ellipticity $\epsilon_B \sim 10^{-8}$ due to the magnetic deformation. Only stars accreting at stable rates $\dot{M} \gtrsim 10^{-9} \mathrm{M}_\odot\,\mathrm{y}^{-1}$ can be accelerate at frequencies $\nu \gtrsim 650\,\mathrm{Hz}$. (b) Predicted amplitudes of the GW signal for sources at $d = 10\,\mathrm{kpc}$ with ellipticity in the range $|\epsilon_B| = [2 \times 10^{-10}\text{–}10^{-8}]$ (solid lines). We show also the sensitivity curves of Advanced LIGO [17] and of the Einstein Telescope [18] assuming an integration time $T_{obs} = 1\,\mathrm{y}$ (dashed lines).

condition is satisfied, the NS looses significant spin angular momentum to GWs also through the induced magnetic deformation and the total angular momentum of the star satisfies the equation [15]:

$$\mathrm{d}J/\mathrm{d}t = 2J_c F_g^r + \dot{J}_a(t) - I_* \Omega (F_g^B + F_{m_e}), \tag{6}$$

where $\dot{J}_a = \dot{M}(GMR)^{1/2}$ [16] is the rate of variation of angular momentum due to mass accretion and F_{m_e} is the magnetic braking rate associated to the external poloidal magnetic field. Combining eqs. (2) and (6) we obtain the evolution equations of the r-mode amplitude α and of the angular velocity of the star Ω:

$$\begin{aligned}\frac{\mathrm{d}\alpha}{\mathrm{d}t} &= \alpha(F_g^r - F_v - F_{m_i}) + \alpha[K_j F_g^r + (1-K_j)(F_v + F_{m_i})]K_c\alpha^2 \\ &\quad - \frac{\alpha\dot{M}}{2\tilde{I}\Omega}\left(\frac{G}{MR^3}\right)^{\frac{1}{2}} + \frac{\alpha(F_g^B + F_{m_e})}{2},\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\mathrm{d}\Omega}{\mathrm{d}t} &= -2K_c\Omega\alpha^2[K_j F_g^r + (1-K_j)(F_v + F_{m_i})] - \frac{\dot{M}\Omega}{M} + \frac{\dot{M}}{\tilde{I}}\left(\frac{G}{MR^3}\right)^{\frac{1}{2}} \\ &\quad - \Omega(F_g^B + F_{m_e}).\end{aligned} \tag{8}$$

The evolution equations (7), (8) will hold only *after* the symmetry axis of the azimuthal field has become orthogonal to the spin axis.

In the scenario we are describing, an initially slowly rotating NS is secularly spun up by mass accretion. When its spin frequency reaches a few hundred Hz, the NS enters the classical r-modes instability window [6, 15]. As the instability develops, the evolution of the r-mode amplitude becomes coupled to the growth of an internal, toroidal magnetic field B_ϕ. The star becomes secularly unstable when $B_\phi \gtrsim 10^{13}\,\mathrm{G}$ (see eq. (4)). After tilting, the original azimuthal field B_ϕ will have acquired an $r-\theta$ (poloidal) component, whose strength will be comparable to that of the new ϕ-component. The new magnetic field stabilizes the star against r-modes up to frequencies $\nu_{\rm cr} \approx 500\,\text{Hz}$ [15]. Mass accretion will continue to spin up the star, which may eventually enter again the r-mode instability window thus starting the generation of new azimuthal field. The star may be subject to a new instability when the magnetic field exceeds a value of the order of 10^{14} G with an $|\epsilon_B| \approx 10^{-8}$ at frequencies $\nu_{\rm s} \approx 650\,\text{Hz}$ [15]. This *new* internal configuration limits the spin frequencies at values $\nu_{\max} \sim 630\dot{M}_{-9}^{1/5}(\epsilon_B/10^{-8})^{-2/5}\,\text{Hz}$ preventing the star from further increasing their magnetic field by re-entering the r-mode instability region [15]. In fig. 1 (panel a) we show the limits on the maximum spin frequencies when the internal magnetic field reaches strengths of order 10^{14} G. The fastest spin frequencies ($\nu \gtrsim 650\,\text{Hz}$) can be reached only for mass accretion rates $\dot{M} \gtrsim 10^{-9}\,\text{M}_\odot\,\text{y}^{-1}$.

A magnetically deformed NS emits GWs characterized by an instantaneous signal strain $h \sim 1.5 \times 10^{-29}\, k_\epsilon\, d_{10}^{-1}\, B_{\phi,13}^2\, \nu_{500}^2$ [19] where $B_{\phi,13} = B_\phi/(10^{13}\,\text{G})$, $d_{10} = d/(10\,\text{kpc})$ and $\nu_{500} = \nu/(500\,\text{Hz})$. The minimal detectable signal amplitude is $h_0 \approx 11.4(S_n/T_{obs})^{1/2}$ [20], where S_n is the power spectral density of the detector noise and T_{obs} is the observation time. In fig. 1 (panel b) we show the predicted amplitudes of the GWs as a function of the frequency $f = 2\nu$ of the signal. We assume a distance $d = 10$ kpc for the source and $T_{obs} = 1\,\text{y}$.

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