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Kinematics of deep inelastic scattering in the covariant approach

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Summary. — We study the kinematics of deep inelastic scattering corresponding to the rotationally symmetric distribution of quark momenta in the nucleon rest frame. It is shown that the rotational symmetry together with Lorentz invariance can impose constraints on the quark intrinsic momenta. Obtained constraints are discussed and compared with the available experimental data.

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1. – Introduction

The motion of quarks inside the nucleons plays an important role in some effects which are at present intensively investigated both experimentally and theoretically. The actual goal of this effort is to obtain a more consistent 3-D picture of the quark-gluon structure of nucleons. For example the quark transverse momentum creates the asymmetries in particle production in polarized or in unpolarized (Cahn effect) deep inelastic scattering (DIS) experiments. Relevant tool for the study of these effects is the set of the transverse momentum dependent distributions (TMDs). A better understanding of the quark intrinsic motion is also a necessary condition to clarify the role of quark orbital angular momenta in generating nucleon spin.

We have paid attention to these topics in our recent studies, see [1-5] and citations therein. In particular we have shown that the requirements of Lorentz invariance (LI) and the nucleon rotational symmetry in its rest frame (RS), if applied in the framework of the 3-D covariant quark-parton model (QPM), generate a set of relations between parton distribution functions. Recently we obtained within this approach relations between the usual parton distribution functions and the TMDs. The Wanzura-Wilczek approximate relation (WW) and some other known relations between the g_1 and g_2 structure functions were similarly obtained in the same model before [6]. Let us remark that the WW relation has been obtained independently also in other approaches [7,8] in which the LI represents a basic input.

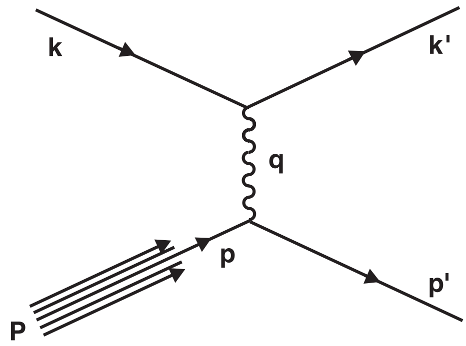

Fig. 1. – Diagram describing DIS as a one photon exchange between the charged lepton and quark. Lepton and quark momenta are denoted by k, p (k', p') in initial (final) state, P is initial nucleon momentum.

The aim of the present report is to consistently apply the assumption LI&RS to the kinematics of DIS and to obtain the constraints on related kinematical variables. This task is complementary to the study of the above mentioned relations between distribution functions, which depend on these variables. So, the report can be considered as an addendum to our former papers related to the covariant QPM [1-5, 9, 6, 10, 11]. However, the results obtained in this report are more general and are independent of any specific model.

2. – Kinematic variables

2·1. *The Bjorken variable and light-cone coordinates.* – First, let us shortly remind the properties of the Bjorken variable

$$x_B = \frac{Q^2}{2Pq}\,, \tag{1}$$

which plays a crucial role in the phenomenology of lepton-nucleon scattering. Regardless of the mechanism of the process, this invariant parameter satisfies

$$0 \leq x_B \leq 1, \tag{2}$$

for complete proof of this general relation see appendix A. Now let us consider QPM, where the process of lepton-nucleon scattering is initiated by the lepton interaction with a quark (see fig. 1), which obeys

$$p' = p + q, \qquad p'^2 = p^2 + 2pq - Q^2; \qquad Q^2 = -q^2. \tag{3}$$

The second equality implies

$$Q^2 = 2pq - \delta m^2; \qquad \delta m^2 = p'^2 - p^2, \tag{4}$$

which, with the use of relation (1), gives

$$\frac{pq}{Pq} = x_B \left(1 + \frac{\delta m^2}{Q^2}\right). \tag{5}$$

The basic input for the construction of QPM is the assumption

$$Q^2 \gg \delta m^2, \tag{6}$$

which allows us to identify

$$x_B = \frac{Q^2}{2Pq} = \frac{pq}{Pq} \tag{7}$$

and to directly relate the quark momentum to the parameters of scattered lepton. Moreover, if one assumes

$$Q^2 \gg 4M^2 x_B^2, \tag{8}$$

where M is the nucleon mass, then one can identify

$$x_B = x \equiv \frac{p_0 - p_1}{P_0 - P_1} \tag{9}$$

in any reference frame in which the direction of the first axis is defined by the vector $\mathbf{q}$ (see appendix A). The last relation expressed in the nucleon rest frame reads

$$x = \frac{p_0 - p_1}{M}, \tag{10}$$

which after inserting into (2) gives

$$0 \le \frac{p_0 - p_1}{M} \le 1. \tag{11}$$

However the most important reason why we require large Q^2 is in physics. If we accept a scenario in which a probing photon interacts with a quark, we need a sufficiently large momentum transfer Q^2 so that the quarks can be considered as effectively free due to asymptotic freedom. At small Q^2 the picture of quarks (with their momenta and other quantum numbers) inside the nucleon disappears.

2˙2. *Rotational symmetry.* – The RS means that the probability distribution of the quark momenta $\mathbf{p} = (p_1, p_2, p_3)$ in the nucleon rest frame depends, apart from Q^2, on $|\mathbf{p}|$. It follows that also $-\mathbf{p}$ is allowed, so together with the inequality (11) we have

$$0 \le \frac{p_0 + p_1}{M} \le 1. \tag{12}$$

The combinations of (11), (12) imply

$$0 \le |p_1| \le p_0 \le M, \qquad |p_1| \le \frac{M}{2}\,. \tag{13}$$

And if we again refer to RS, then further inequalities are obtained:

$$0 \le |p| \le p_0 \le M, \qquad |p| \le \frac{M}{2}\,, \qquad 0 \le p_T \le p_0 \le M \tag{14}$$

and

$$p_T \le \frac{M}{2}\,, \tag{15}$$

where

$$|p| = \sqrt{p_1^2 + p_2^2 + p_3^2}, \qquad p_T = \sqrt{p_2^2 + p_3^2}.$$

Obviously inequality (15) is satisfied also in any reference frame boosted in the directions $\pm\mathbf{q}$. Further, the above inequalities are apparently valid also for average values $\langle p_0 \rangle, \langle p_1 \rangle, \langle |p| \rangle$ and $\langle p_T \rangle$. In addition, if one assumes that the p_T distribution is a decreasing function, then necessarily

$$\langle p_T \rangle \le \frac{M}{4}\,. \tag{16}$$

The above relations are valid for sufficiently high Q^2 suggested by eqs. (6) and (8). Let us note that the on-mass-shell assumption has not been applied for obtaining these relations.

These inequalities can be compared with relations obtained in [11], where the additional on-mass-shell condition $m^2 = p^2 = p_0^2 - \mathbf{p}^2$ had been applied. Corresponding relations are more strict:

$$\frac{m^2}{M^2} \le x \le 1, \qquad p_0 \le \frac{M^2 + m^2}{2M}\,, \qquad |\mathbf{p}| \le \frac{M^2 - m^2}{2M} \tag{17}$$

and

$$p_T^2 \le M^2 \left(x - \frac{m^2}{M^2} \right) (1 - x)\,. \tag{18}$$

However, it is clear that in general the on-mass-shell assumption is not realistic. In the following we will discuss only the off-mass-shell approach.

3. – Discussion

First let us summarize more accurately what we have done in the previous section. We assumed:

a) *Lorentz invariance*

It means that the theoretical description in terms of the standard kinematical variables (see fig. 1)

$$q, x_B, x, p = (p_0, p_1, p_2, p_3), \qquad P = (P_0, P_1, P_2, P_3)$$

can be boosted also to the nucleon rest frame.

b) *Rotational symmetry*

The kinematical region $\mathcal{R}^3$ of the quark intrinsic momenta $\mathbf{p} = (p_1, p_2, p_3)$ in the nucleon rest frame has rotational symmetry (*i.e.* $\mathbf{p} \in \mathcal{R}^3 \Rightarrow \mathbf{p}' = \mathbf{Rp} \in \mathcal{R}^3$, where $\mathbf{R}$ is any rotation in $\mathcal{R}^3$).

c) *Equality* $x_B = x$

It means that all the conditions necessary for eq. (9) are satisfied.

We proved that these assumptions imply bounds (11)–(15) independently of any specific model.

Further we want to make a few comments on the obtained results:

i) The ratio x of light-cone variables (9) has a simple interpretation in a frame, where the proton momentum is large: x is the fraction of this momentum carried by the quark. However an interpretation of the same variable in the nucleon rest frame is more complicated. In this frame the quark transverse momentum cannot be neglected and x depends on both longitudinal and transverse quark momenta components. In the limit of massless quarks the connection between the variable x in (10) and the quark momenta components is given by the relations

$$x = \frac{p_0 - p_1}{M}; \qquad p_0 = \sqrt{p_1^2 + p_T^2}, \tag{19}$$

$$p_1 = -\frac{Mx}{2}\left(1 - \frac{p_T^2}{M^2x^2}\right), \quad p_0 = \frac{Mx}{2}\left(1 + \frac{p_T^2}{M^2x^2}\right).$$

These variables were used in our recent papers on TMDs [2,1]. The value of the invariant variable x does not depend on the reference frame, but its interpretation *e.g.* in the rest frame differs from that in the infinite momentum frame.

ii) The relations (14), (15), which follow from RS, can be compared with the experimental data on $\langle p_T \rangle$ or $\langle \mathbf{p} \rangle$. We have discussed the available data in [2,1] and apparently relation (15) is compatible with the set of lower values $\langle p_T \rangle$ corresponding to the "leptonic data". On the other hand the second set giving substantially greater $\langle p_T \rangle$ and denoted as the "hadronic data", seems to contradict this relation. Actually a conflict with relation (15) would mean a conflict with some of the assumptions a)–c). Let us remark that the failure of the assumption c) means that the Bjorken variable cannot be replaced by the light-cone ratio. And then correspondingly the light-cone formalism itself would be questioned, since the experimentally measured structure functions (x_B dependence) could not be compared with the light-cone calculations (x dependence). In this way, the large intrinsic quark momenta ($p_T > M/2$) are incompatible with the light-cone formalism combined with the RS. No such problem arises provided the data satisfy relations (14), (15).

In [2] we explained why the RS, if applied at the level of QPM, follows from the covariant description. In fact it means that the assumptions a)–c) are common for our QPM and for the approaches like [8,7] where only Lorentz invariance is explicitly required. The predictions of all these models are compatible with the bound (15). This fact is not the result of any specific, simplifying model assumptions, but it is just the consequence of the general conditions a)–c).

iii) The relation (18) is obtained for the quarks on-mass-shell. In a more general case, where only the inequalities (14), (15) hold, this relation is replaced by

$$p_T^2 \leq M^2\left(x - \frac{\mu^2}{M^2}\right)(1-x); \qquad \mu^2 \equiv p_0^2 - \mathbf{p}^2, \tag{20}$$

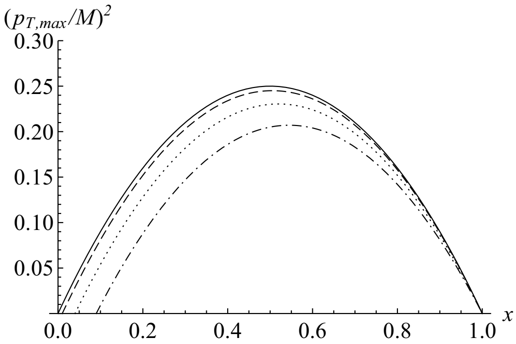

Fig. 2. – Upper limit of the quark transverse momentum as a function of x for $\mu = 0$ (solid line), 0.1 (dashed line), 0.2 (dotted line) and 0.3 (dash-dotted line).

where the term μ^2 is not a parameter corresponding to the fixed mass, but only a number varying within the limits defined by (14). The last relation implies for all μ^2:

$$p_T^2 \leq M^2 x\,(1-x)\,, \tag{21}$$

which is equivalent to the on-mass-shell relation (18) for $m = 0$. This general upper limit for p_T^2 depending on x is displayed in fig. 2. Let us remark that the results on $\langle p_T^2(x)\rangle$ obtained in [7, 8] are compatible also with the bound (21). An equivalent form of this inequality was probably for the first time presented in [12].

To conclude, in the present report we studied the kinematic constraints due to the rotational symmetry of the quark momenta distribution inside the nucleon. In particular, we have shown that the light-cone formalism (which requires the equality $x_B = x$) combined with the assumption on the rotational symmetry in the nucleon rest frame imply $p_T \leq M/2$. Only part of existing experimental data on $\langle p_T\rangle$ satisfies this bound, but another part does not. In general, the reconstruction of $\langle p_T\rangle$ from the DIS data is a model-dependent procedure. These are the reasons why more study is needed to clarify this issue.

Appendix A.

The relation (2) follows from the conditions:

$$k'^2 = k^2, \qquad k' = k - q \tag{A.1}$$

i.e. the mass of the lepton is not changed by the scattering displayed in fig. 1, and

$$P'^2 \geq P^2 = M^2; \qquad P' = P + q, \tag{A.2}$$

which means that the effective mass of the secondary particles coming from the nucleon is greater than the mass M of the primary nucleon. In more detail

$$P'^2 = \left(\sum_{j=1}^{n} P_{0j}\right)^2 - \left(\sum_{j=1}^{n} \mathbf{P}_j\right)^2 = \sum_{j=1}^{n} \left(P_{0j}^2 - \mathbf{P}_j^2\right) + \sum_{j\neq k}^{n} \left(P_{0j}P_{0k} - \mathbf{P}_j\mathbf{P}_k\right) \geq \sum_{j=1}^{n} m_j^2 \geq M^2, \tag{A.3}$$

where P_j, m_j are the momenta and masses of the secondaries. The last inequality is due to the baryon number conservation.

One can check that eqs. (A.1), (A.2) imply

$$q^2 = (k - k')^2 \leq 0, \qquad 2Pq \geq -q^2, \tag{A.4}$$

so we have

$$2Pq \geq Q^2 \geq 0; \qquad Q^2 = -q^2, \tag{A.5}$$

which is equivalent to eq. (2).

The relation (9) can be proved as follows. Let us consider eq. (7) in a frame in which the direction of the first axis is defined by the vector $\mathbf{q}$:

$$x_B = \frac{p_0 q^0 - p_1|\mathbf{q}|}{P_0 q^0 - P_1|\mathbf{q}|}. \tag{A.6}$$

In the nucleon rest frame we have

$$|\mathbf{q}|^2 = \nu^2 + Q^2, \qquad \frac{|\mathbf{q}|^2}{\nu^2} = 1 + \frac{4M^2 x_B^2}{Q^2}. \tag{A.7}$$

It means that for $Q^2 \gg 4M^2 x_B^2$ the relation

$$x_B = \frac{p_0 - p_1}{P_0 - P_1}\left(1 + O\left(\frac{4M^2 x_B^2}{Q^2}\right)\right) \tag{A.8}$$

holds in any reference frame connected with the rest frame by the Lorentz boost in direction $\mathbf{q}$. In this way we have proven that replacement of Bjorken variable by the invariant light-cone ratio in eq. (9) is valid provided the inequality (8) is satisfied.

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